Model-independent tests of dark energy and modified gravity

How good can they get?

Levon Pogosian

Simon Fraser University

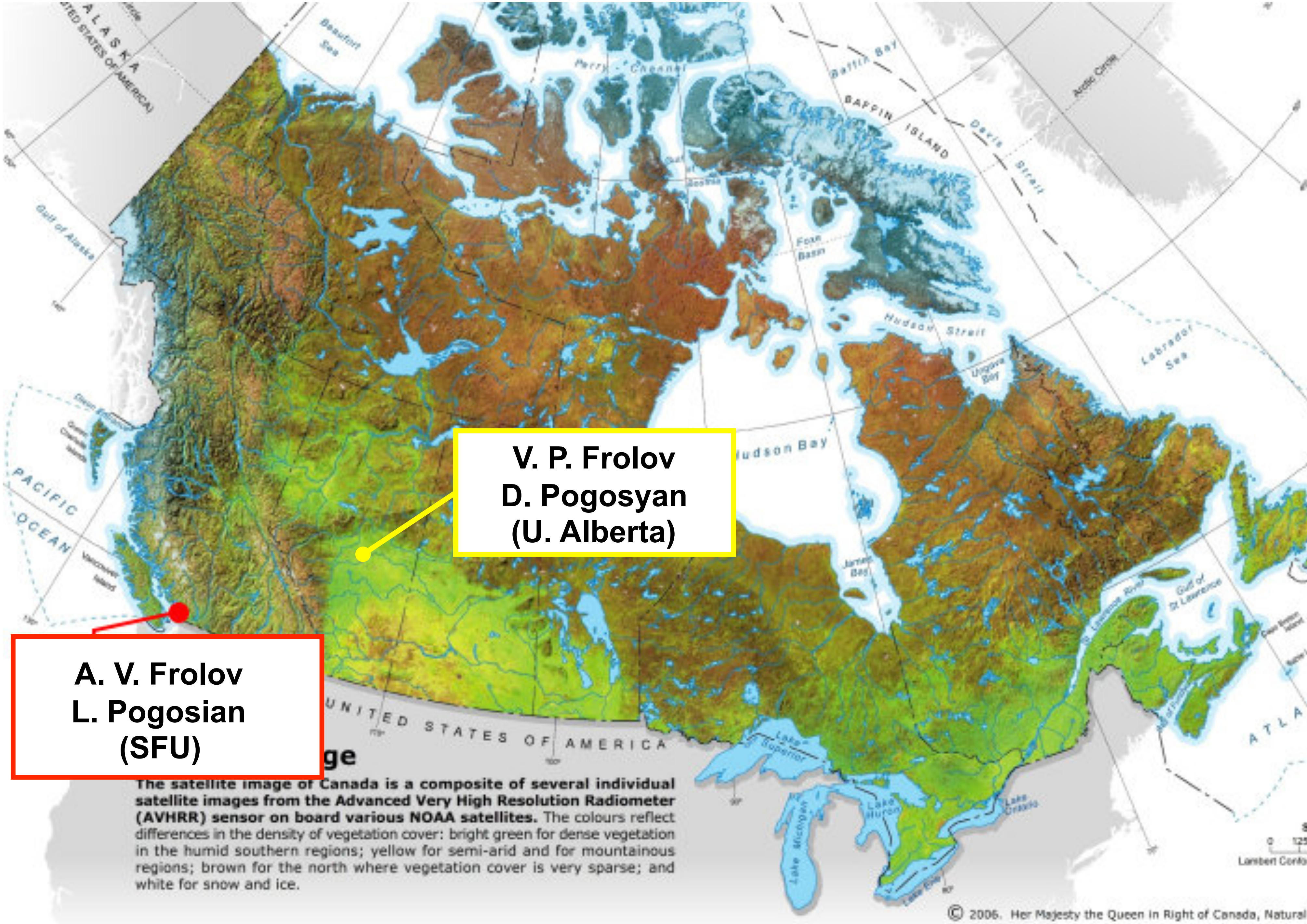
Burnaby, British Columbia, Canada

Robert Crittenden, Gong-Bo Zhao, LP, Lado Samushia, Xinmin Zhang, in preparation

Robert Crittenden, LP, Gong-Bo Zhao, astro-ph/0510293, JCAP 12 (2009) 025

Alireza Hojjati, Gong-Bo Zhao, LP, Alessandra Silvestri, Robert Crittenden, Kazuya Koyama, in preparation

Gong-Bo Zhao, LP, Alessandra Silvestri, Joel Zylberberg, arXiv:0905.1326, Phys. Rev. Lett., 103, 241301 (2009)


V. P. Frolov
D. Pogosyan
(U. Alberta)
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ALASKA
UNITED STATES OF AMERICA
Beaufort Sea
Parry Channel
Baffin Bay
BAFFIN ISLAND
Davis Strait
Arctic Circle
Gulf of Alaska
Foxe Basin
Hudson Strait
Ungava Bay
Labrador Sea
Hudson Bay
James Bay
PACIFIC OCEAN
Vancouver Island
St. Lawrence River
Gulf of St Lawrence
UNITED STATES OF AMERICA
Lake Superior
Lake Michigan
Lake Huron
Lake Ontario
Lake Erie
ATLA
ge
The satellite image of Canada is a composite of several individual satellite images from the Advanced Very High Resolution Radiometer (AVHRR) sensor on board various NOAA satellites. The colours reflect differences in the density of vegetation cover: bright green for dense vegetation in the humid southern regions; yellow for semi-arid and for mountainous regions; brown for the north where vegetation cover is very sparse; and white for snow and ice.
© 2006. Her Majesty the Queen in Right of Canada, Natural

Cosmic Acceleration

- Beyond reasonable doubt (Nobel Prize)

- No compelling alternatives to Lambda

- Lambda already well measured

What else to measure?

- Test specific models – typically a few parameters

- Technology will allow to measure much more

- Test for deviations from Lambda. How model-independent can we be?

The quest for w

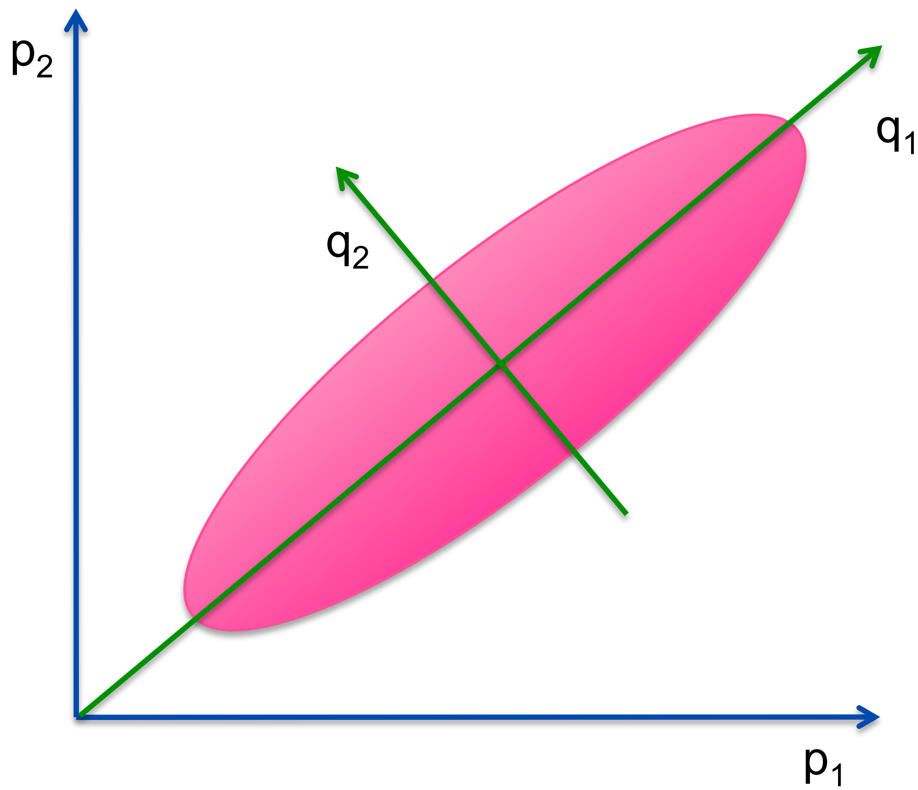
$$w(a) = \frac{p_{\rm DE}(a)}{\rho_{\rm DE}(a)}$$

$$\frac{H^2(a)}{H_0^2} = \Omega_{\rm r} a^{-4} + \Omega_{\rm M} a^{-3} + \Omega_{\rm k} a^{-2} + \Omega_{\rm DE} \exp\left[\int_a^1 3(1+w(a'))\frac{da'}{a'}\right]$$

- General w(a) - infinite # of DoF

- Principal Component Analysis can help

"Principal Components"



Principal Component Analysis of w(z)

Huterer and Starkman, astro-ph/0207517, PRL'03

Crittenden, LP, Zhao, astro-ph/0510293, JCAP'09

Discretize w(z) on a grid in z:

$$1 + w(z) = \sum_{i=1}^{N} w_i s_i(z)$$

$$z_1 < z_2 ... < z_{i-1} < z_i < ... < z_N$$

$$s_i = 1 \text{ if } z_i < z < z_{i+1}; \;\; s_i = 0 \text{ otherwise}$$

Find the covariance matrix:

$$C_{ij} \equiv \langle (w_i - \bar{w}_i)(w_j - \bar{w}_j) \rangle \neq 0 \quad \text{for } i \neq j$$

Decorrelate; work with "rotated" parameters q_i

$$C = W^T \Lambda W \;\; ; \quad \Lambda_{ij} = \lambda_i \delta_{ij}$$

$$q_i = \sum_{j=1}^{N} W_{ij} w_j$$

$$\langle (q_i - \bar{q}_i)(q_j - \bar{q}_j) \rangle = \lambda_i \delta_{ij}$$

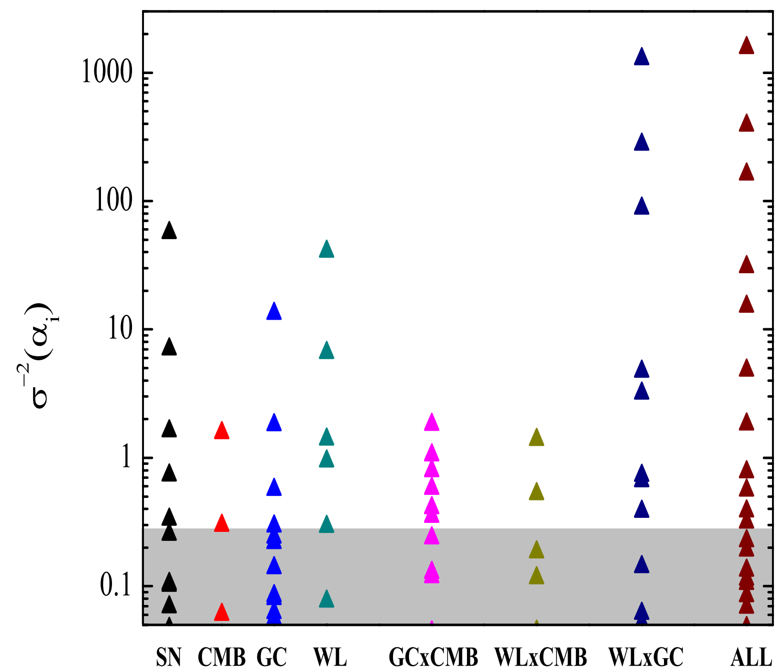
$$\lambda_i = \sigma^2(q_i)$$

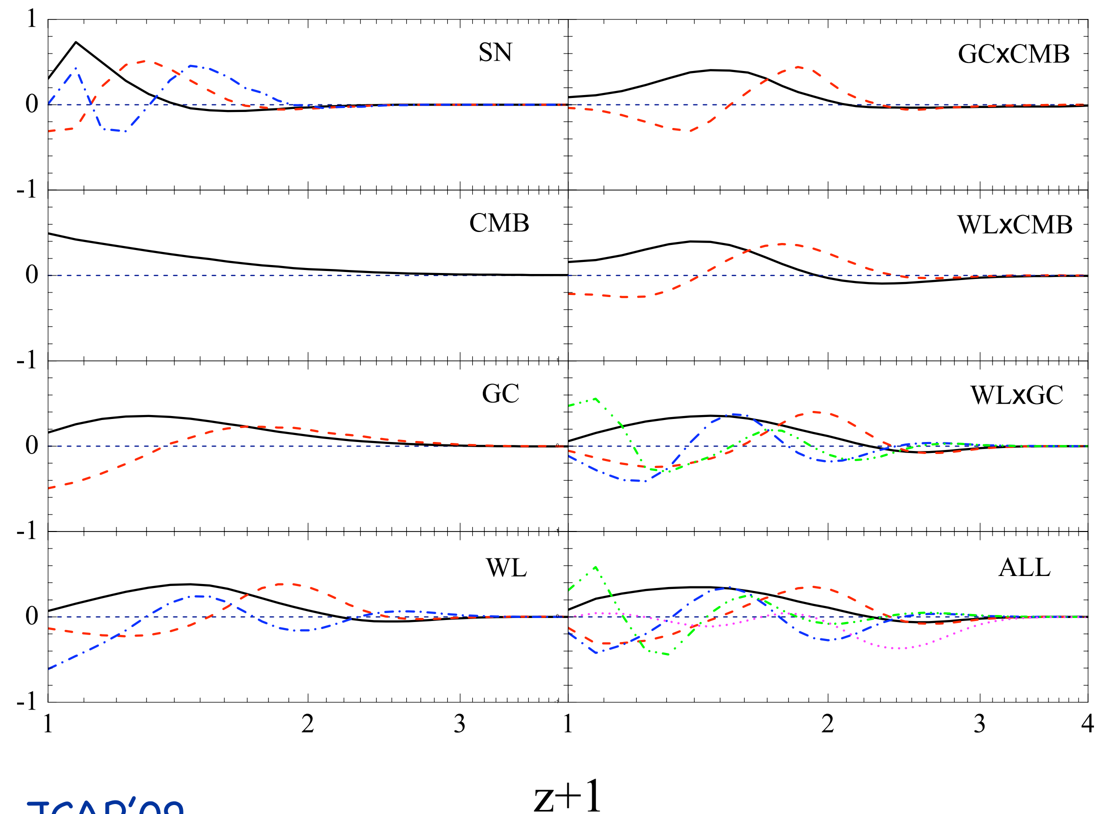
Define eigenmodes $e(z_i)$:

$$w_i \equiv 1 + w(z_i) = \sum_{j=1}^{N} W_{ij}^T q_j \equiv \sum_{j=1}^{N} e_j(z_i) q_j$$

$$N\to\infty \rightarrow 1 + w(z) = \sum_{j=1}^{N} e_j(z) q_j$$

Consider best constrained eigenmodes





Crittenden, LP, Zhao, astro-ph/0510293, JCAP'09

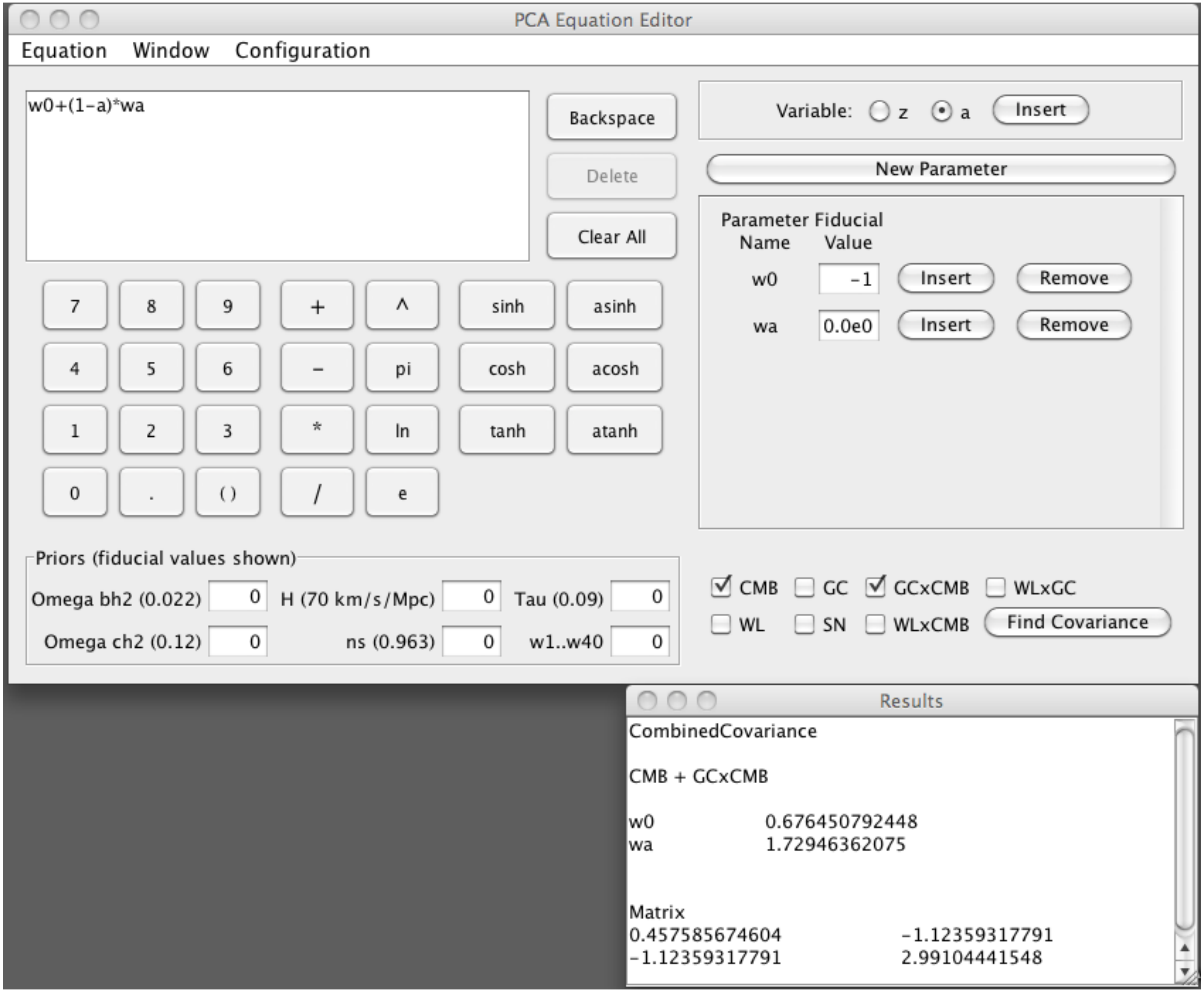
What can PCA do for you?

- Sweet spots – tells what experiments measure best
- A way to compare experiments
- Storage of information – can project on parameters of any w(z)

$$\frac{\partial w(z)}{\partial p^a} = \sum_i \alpha_i^a e_i(z)$$

$$F_{ab} = \alpha_i^a F_{ij} \alpha_j^b = \sum_i \alpha_i^a \alpha_i^b \lambda_i$$

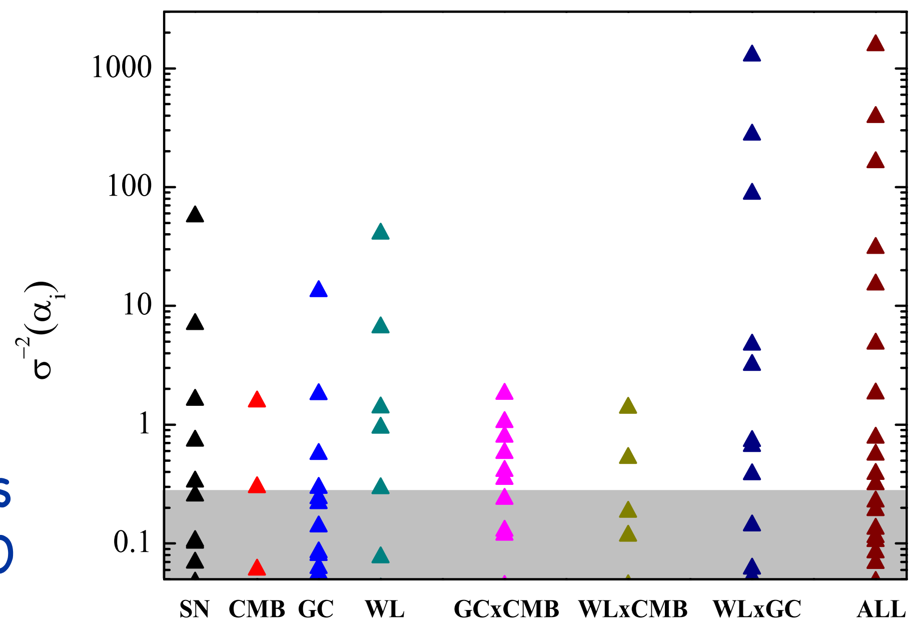
Crittenden, LP, Zhao, astro-ph/0510293, JCAP'09



written by Thomas Wintschel, SFU undergraduate

Issues with PCA

- Which modes are informative?
- Fitting all bins/eigenmodes to data is not an option
- Throwing away poor modes means we assume their amplitudes are 0
- A prior is inevitable



Choosing a prior on binned w

$$\mathcal{P}(\mathbf{w}|\mathrm{data}) = \mathcal{P}(\mathrm{data}|\mathbf{w}) \times \mathcal{P}_{\mathrm{prior}}(\mathbf{w})$$

$$\chi^2 = \chi^2_{\mathrm{data}} + \chi^2_{\mathrm{prior}}$$

$$\chi^2_{\mathrm{prior}} = -2\ln \mathcal{P}_{\mathrm{prior}} = (\mathbf{w} - \mathbf{w}^{\mathrm{fid}})^T \mathbf{C}^{-1} (\mathbf{w} - \mathbf{w}^{\mathrm{fid}})$$

What about an independent prior on each bin? $C^{-1}_{ij} = \sigma_i^{-2}\delta_{ij}$

Albrecht et al (JDEM FoM SWG), arXiv:0901.0721

- implies w uncorrelated at neighboring z's, no matter how close
- implies that ALL modes are EQUALLY likely
- the meaning/strength is tied to the bin size
- MCMC takes long time to converge

- Choice of the bin size is a prior – assumes that w is smooth inside

- Binning => a sharp transition from perfect correlation inside the bin to no correlation outside the bin

Smoothness prior

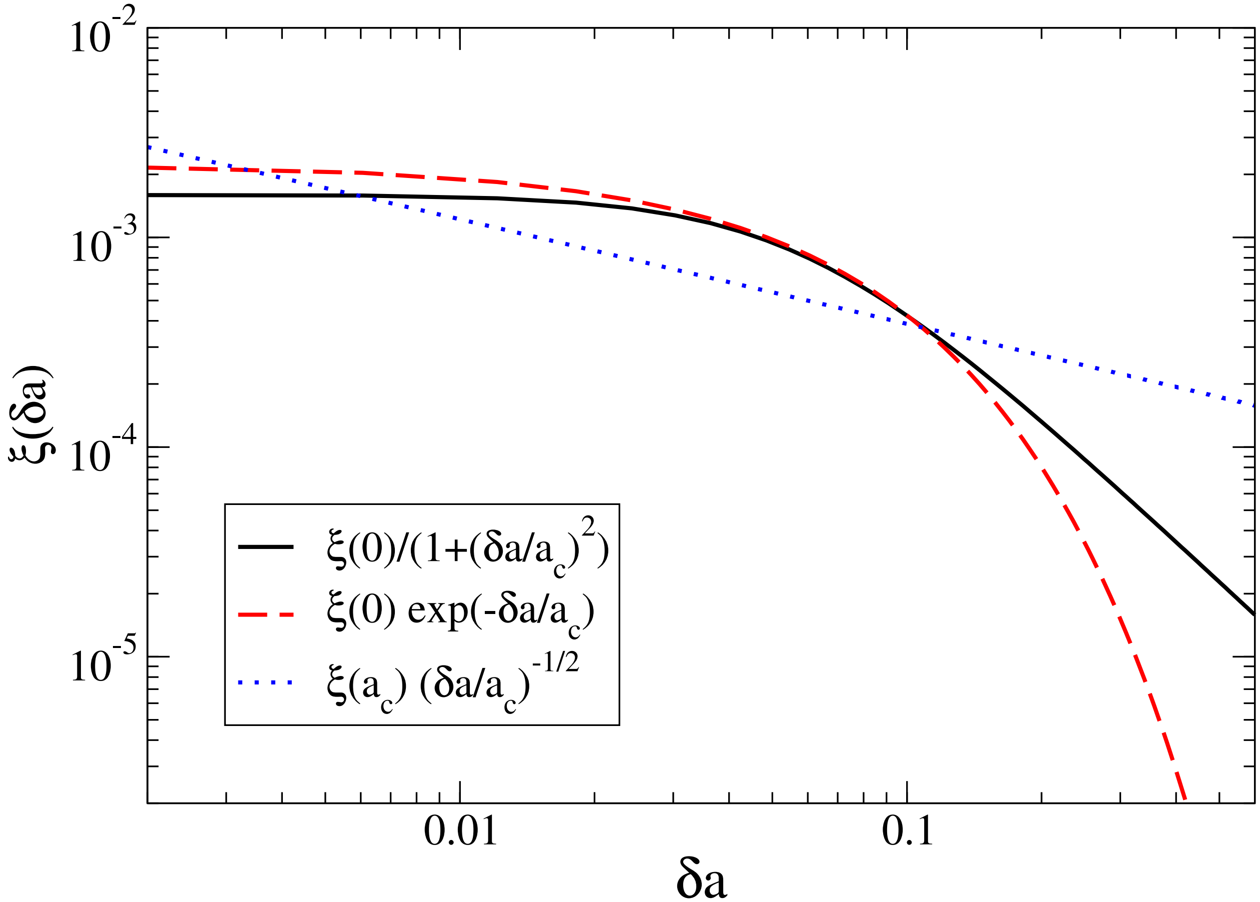
- Let's make the smoothness prior explicit and independent of the binning. Start with a correlation function:

$$\xi_w(|a-a'|) \equiv \left\langle [w(a) - w^{\rm fid}(a)][w(a') - w^{\rm fid}(a')] \right\rangle$$

- Make a reasonable (not unique) choice of a functional form:

$$\xi_w(\delta a) = \frac{\xi_w(0)}{1+(\delta a/a_c)^2} \qquad \delta a \equiv |a-a'|$$

Correlation functions



From correlation function to correlated prior

$$C_{ij} \equiv \langle \delta w_i \delta w_j \rangle = \frac{1}{\Delta^2} \int_{a_i}^{a_i+\Delta} da \int_{a_j}^{a_j+\Delta} da' \xi_w(|a-a'|).$$

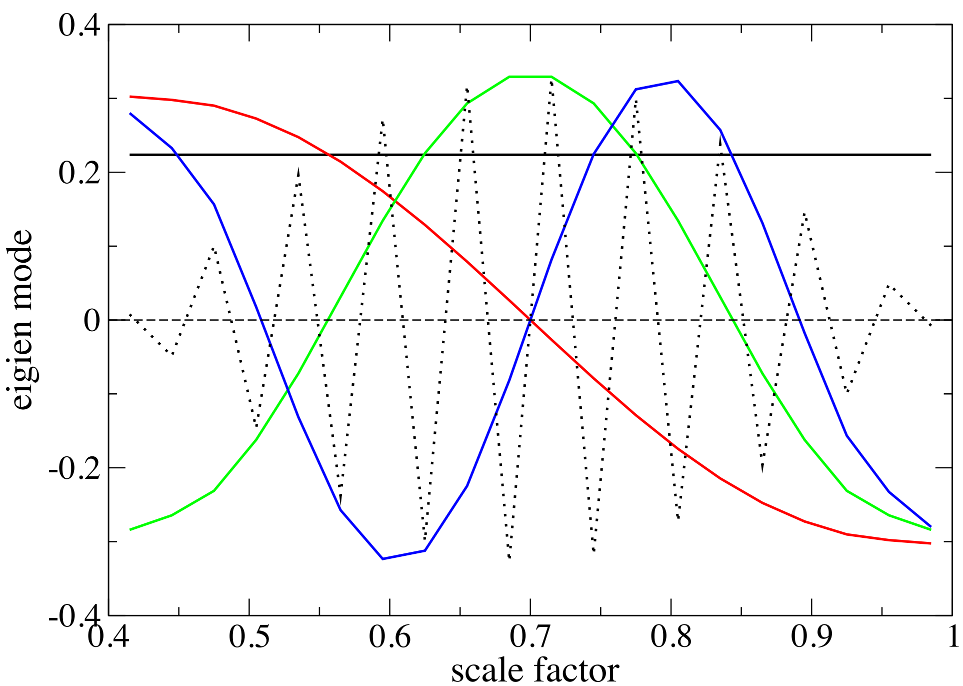
$$\delta w_i = w_i^{\text{true}} - w_i^{\text{fid}}$$

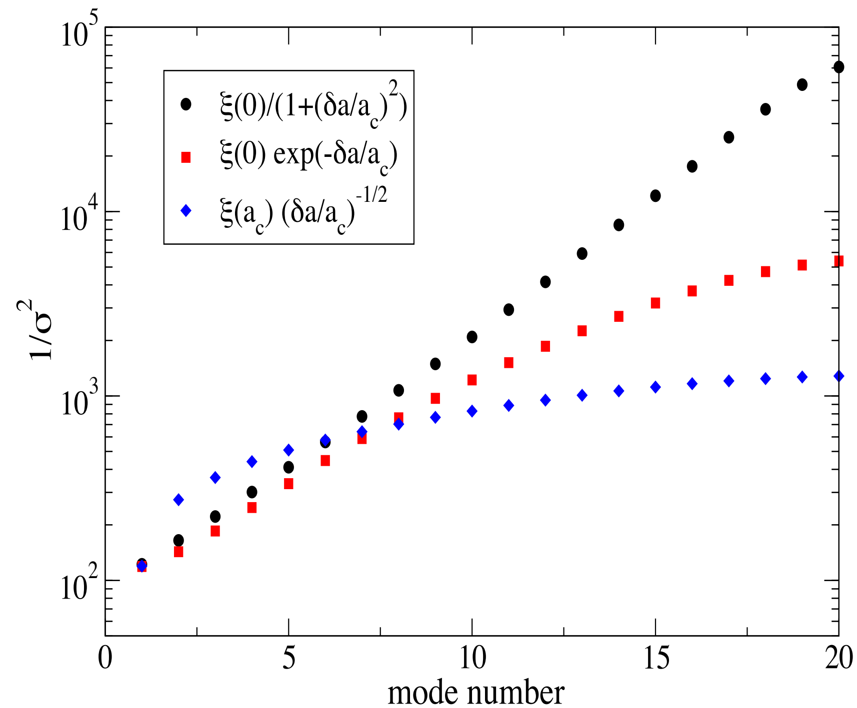
$$\mathcal{P}_{\text{prior}}(\mathbf{w}) \propto e^{-(\mathbf{w}-\mathbf{w}^{\text{fid}})^T \mathbf{C}^{-1} (\mathbf{w}-\mathbf{w}^{\text{fid}})/2}$$

- Two control parameters: "correlation scale" a_c, and the variance in the mean w

$$\sigma_{\bar{w}}^2 \equiv \int_{a_{\min}}^{1} \int_{a_{\min}}^{1} \frac{da\ da'\ \xi_w(a-a')}{(1-a_{\min})^2} \simeq \frac{\pi \xi(0) a_c}{1-a_{\min}}$$

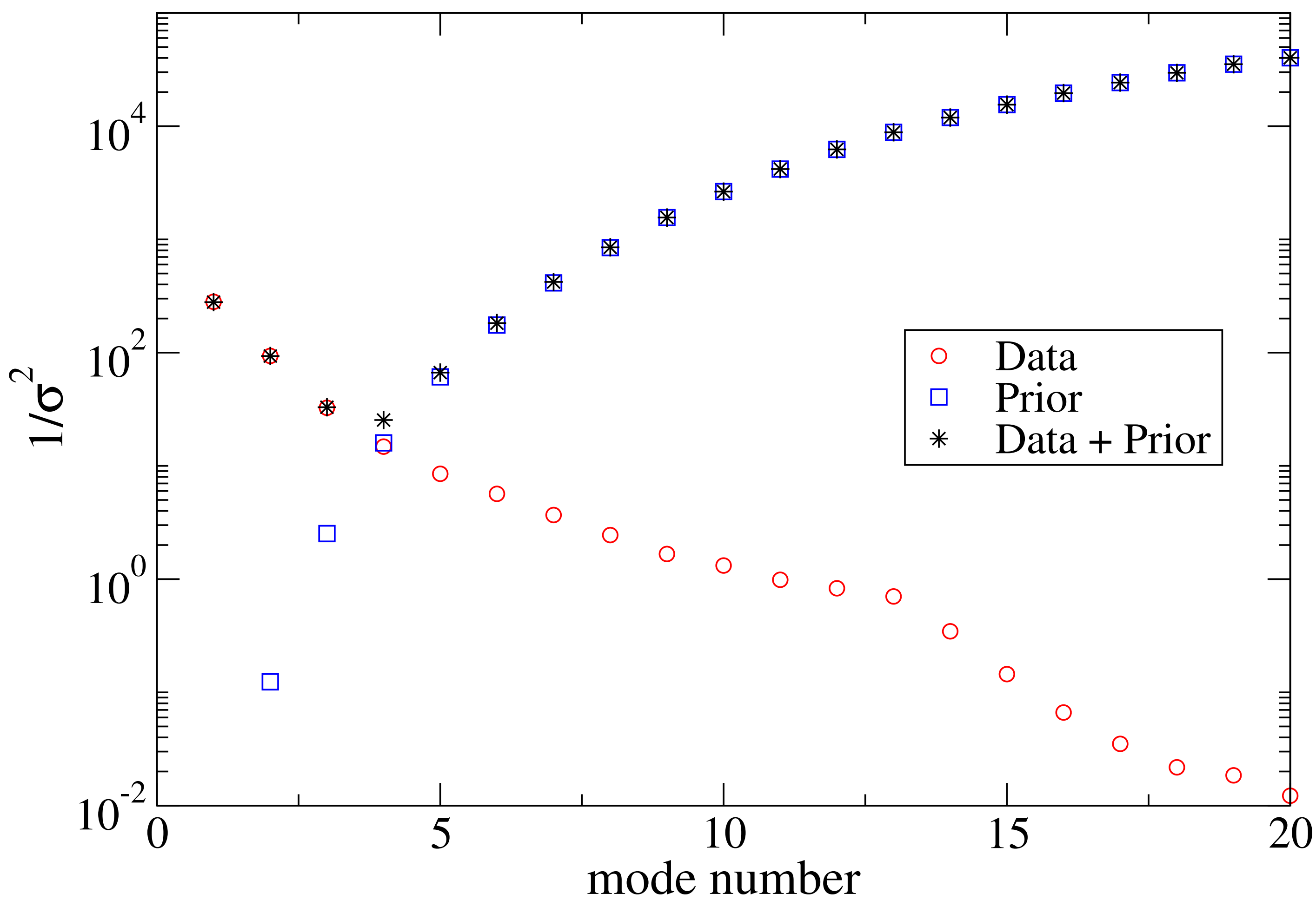
Eigenmodes and eigenvalues of the correlated prior



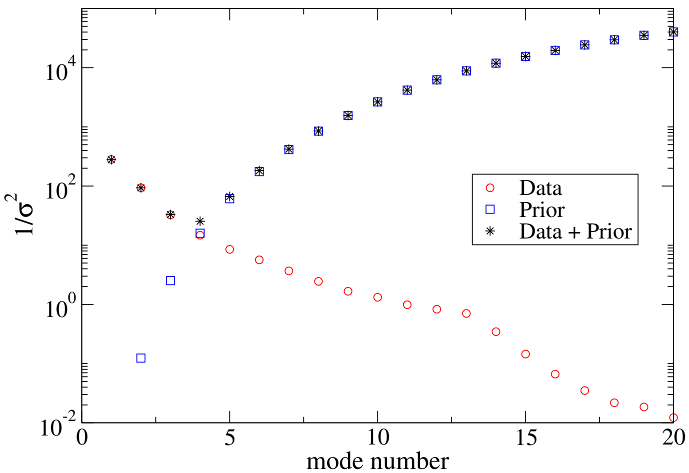


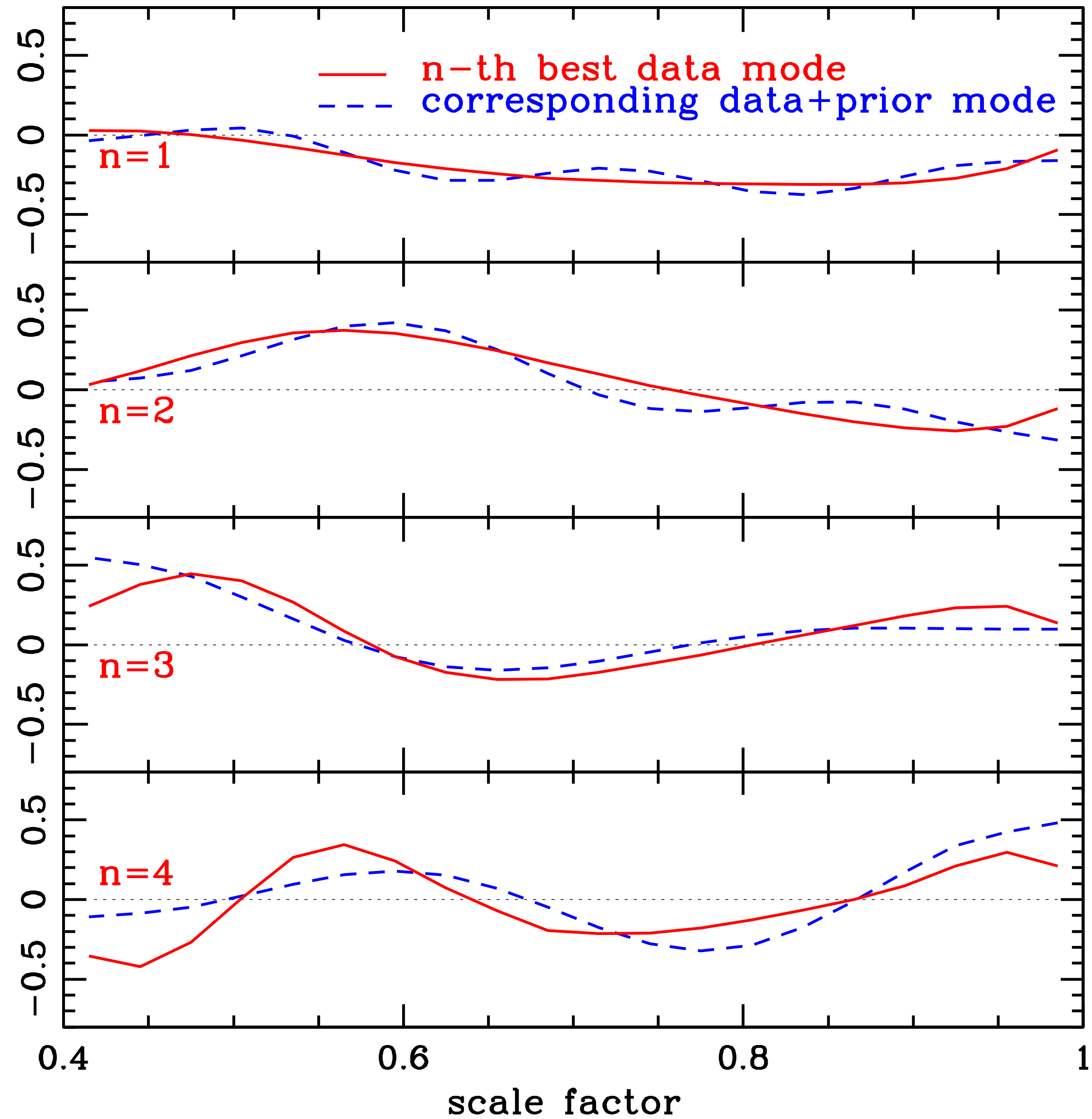
Data meets Prior

EUCLID-like SN, H(z); Planck CMB prior

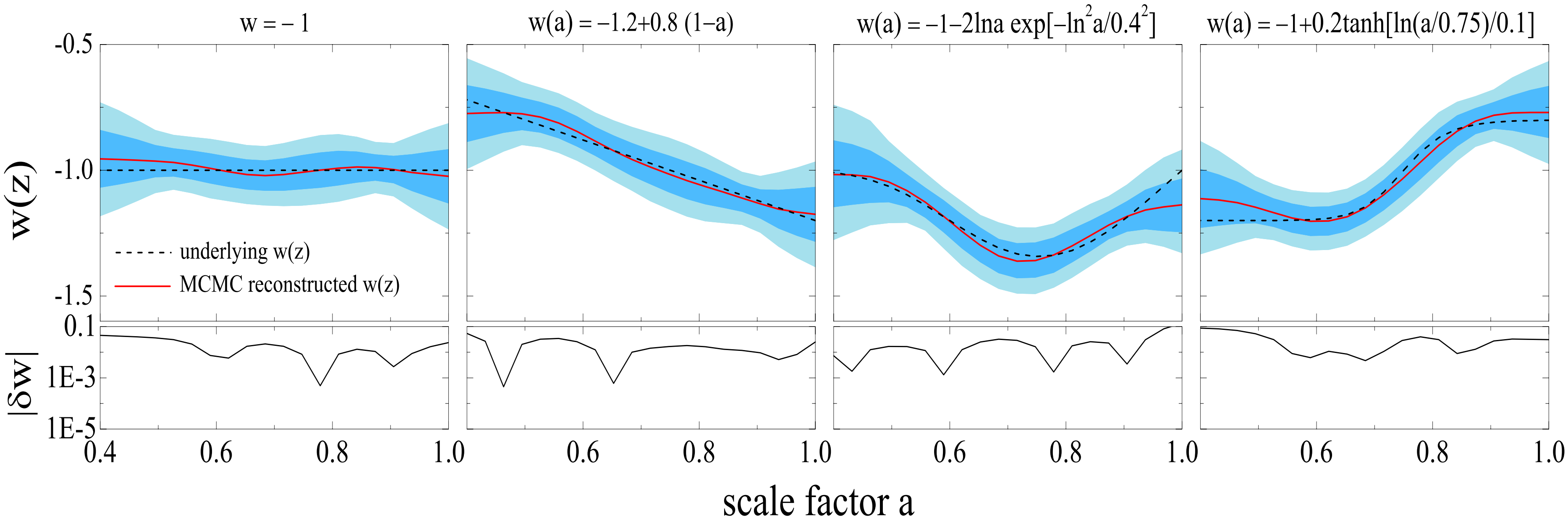


Surviving data eigenmodes





Reconstruction examples

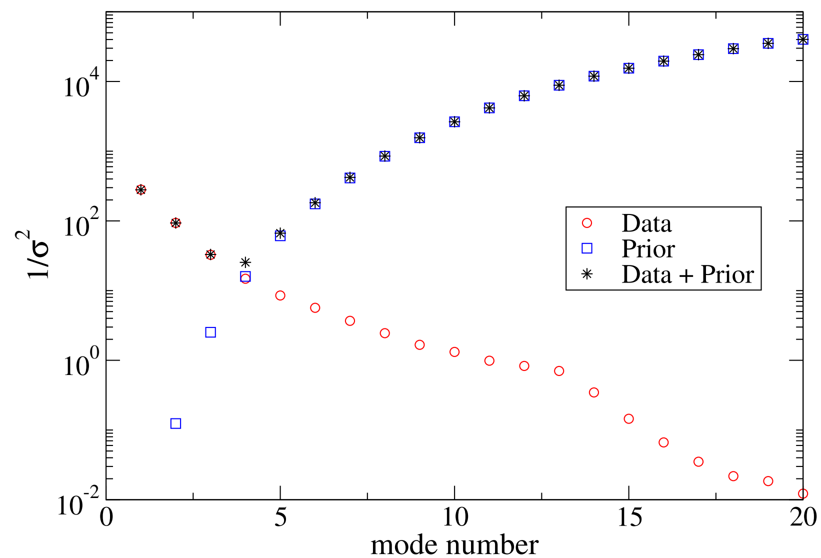


Robert Crittenden, Gong-Bo Zhao, LP, Lado Samushia, Xinmin Zhang, in preparation

Advantages of the correlated prior approach

- Controlled reconstruction bias – high frequency features determined by the prior, low frequency by the data

- This is NOT a low-pass filter !!



- Can perform a PCA forecast to tune the prior

- Independent of the binning scheme

- Fast convergence of MCMC chains with any number of bins

Beyond w

Testing Gravity on Cosmological Scales

because we can

Linear perturbations in FRW universe:

$$\boxed{ds^2 = -a^2(\eta)\left[(1+2\Psi(\vec{x},\eta))\,d\eta^2 - (1-2\Phi(\vec{x},\eta))\,d\vec{x}^2\right]}$$

$$\boxed{D_\mu T^{\mu\nu} = 0} \longrightarrow \boxed{\begin{aligned} \delta' + \frac{k}{aH}V - 3\Phi' &= 0 \\ V' + V - \frac{k}{aH}\Psi &= 0 \end{aligned}}$$

General Relativity

$$\begin{aligned} k^2\Phi &= \\ k^2(\Phi - \Psi) &= \end{aligned}$$

$$\boxed{\begin{aligned} k^2\Phi &= -4\pi G a^2 \rho\Delta \\ \Phi &= \Psi \end{aligned}}$$

$$-4\pi G a^2 \rho\Delta$$

More generally

$$\begin{aligned} k^2\Psi &= -\mu(a,k)4\pi G a^2 \rho\Delta \\ \frac{\Phi}{\Psi} &= \eta(a,k) \quad [= \gamma(a,k)] \end{aligned}$$

GR+ΛCDM: $\mu = \eta = 1$

- E.g. chameleon scalar-tensor and f(R) models:

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_P^2}{2}\tilde{R} - \frac{1}{2}g^{\tilde{\mu}\nu}(\tilde{\nabla}_\mu\phi)\tilde{\nabla}_\nu\phi - V(\phi)\right] + S_i\left(\chi_i, e^{-\kappa\alpha_i(\phi)}\tilde{g}_{\mu\nu}\right)$$

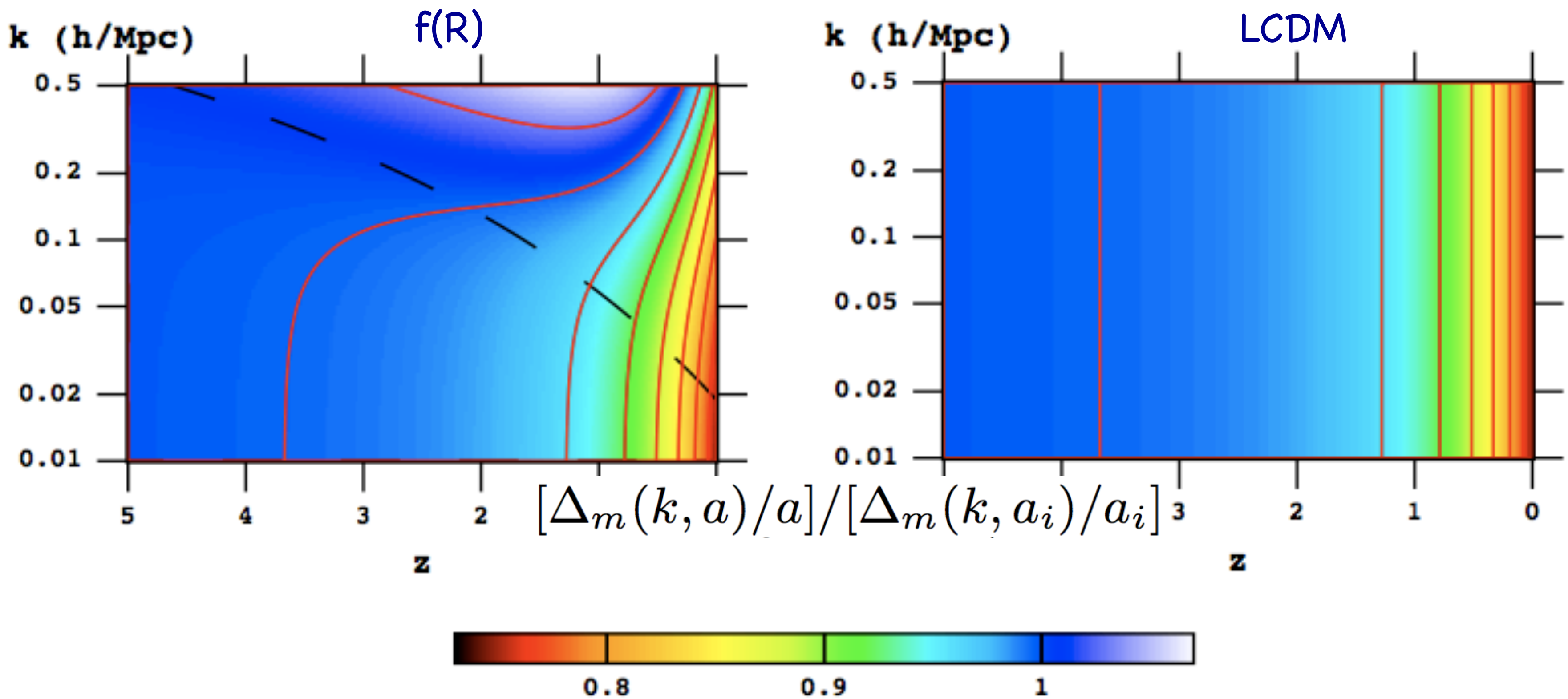
$$\mu(a,k) \approx e^{-\kappa\alpha(\phi)}\frac{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}}$$

$$\eta(a,k) \approx \frac{1+\left(1-\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}$$

in f(R)

$$\alpha' = \frac{d\alpha}{d\phi} = \sqrt{\frac{2}{3}}$$

The linear growth factor in f(R)



LP and A. Silvestri, **arXiv:0709.0296**, PRD'08

An alternative choice

$$\begin{aligned} k^2(\Psi+\Phi) &= -\Sigma(a,k)8\pi G a^2 \rho\Delta \\ \frac{\Phi}{\Psi} &= \eta(a,k) \end{aligned}$$

$$\mu = \frac{2\Sigma}{\eta+1}$$

- Chameleon, f[R]

$$\Sigma(a,k) = e^{-\kappa\alpha(\phi)}$$

$$\eta(a,k) = \frac{1+\left(1-\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}$$

- Degravitation, DGP (from Afshordi, Geshnizjani, Khoury, JCAP'09)

$$\begin{aligned} \Sigma(a,k) &= \frac{1}{1+(kr_c/a)^{2(\alpha-1)}} \\ \eta(a,k) &= \frac{1+(D-2)(Hr_c)^{2(1-\alpha)}\left(1+\frac{\dot{H}}{3H^2}\right)}{(D-3)+(D-2)(Hr_c)^{2(1-\alpha)}\left(1+\frac{\dot{H}}{3H^2}\right)} \end{aligned}$$

Some remarks

- $\mu(a,k)$ and $\eta\,(a,k)$ describe a general departure from GR on linear scales (perhaps too general)

- They can represent SOLUTIONS of theories – depend on the theory AND the initial conditions

- Implemented in MGCAMB (A. Hojjati, G-B. Zhao, LP, 1106.4543)

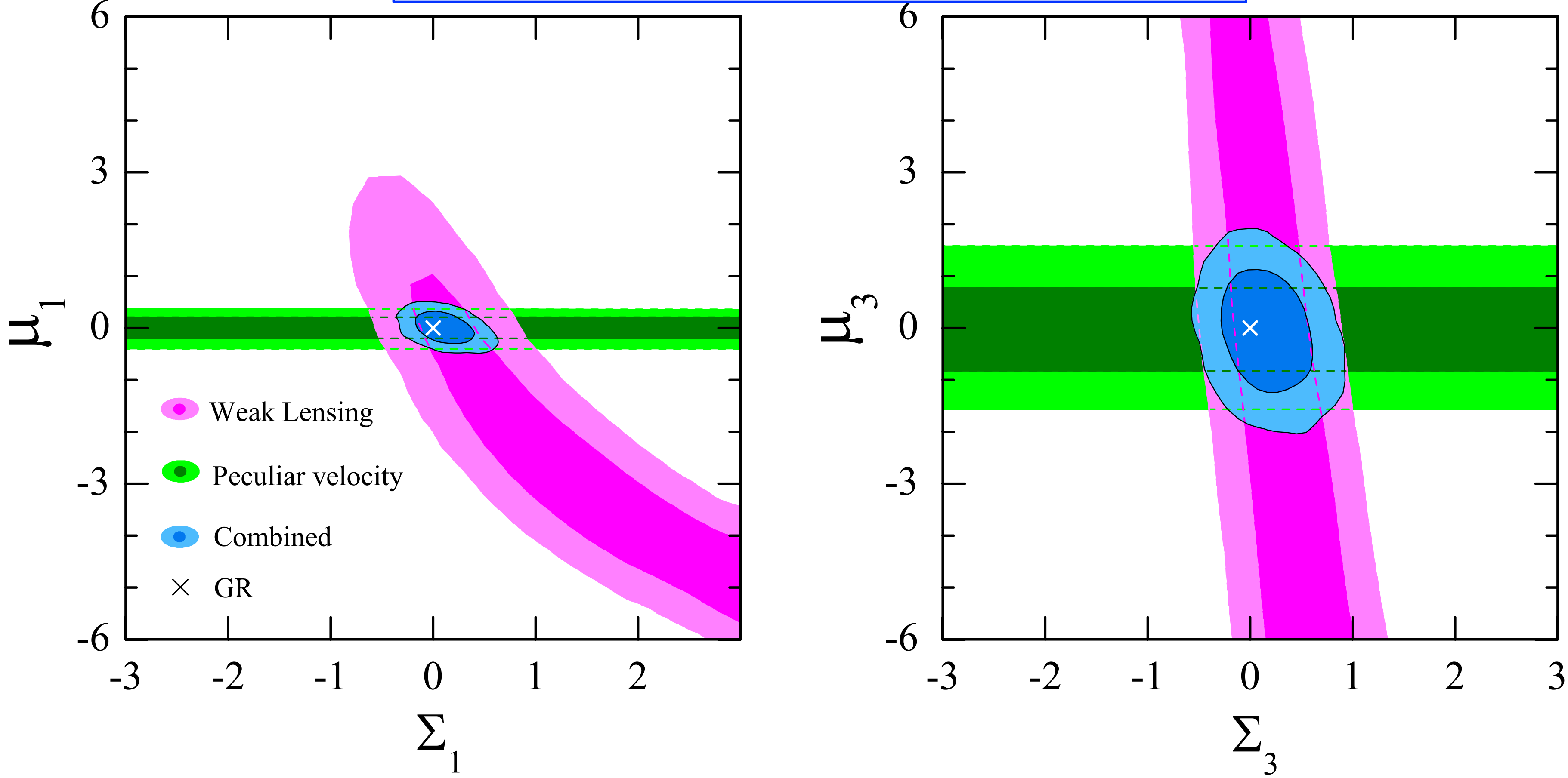
  http://www.sfu.ca/~aha25/MGCAMB.html

Observations

- Supernovae, BAO: expansion history
- CMB: expansion history,
  ISW $(\Phi+\Psi)'$, weak lensing $(\Phi+\Psi)$
- Weak Lensing of galaxies: $(\Phi+\Psi)$
- Galaxy Number Counts: Δ (upto bias)
- Peculiar velocities via z-space distortions: Ψ

Complementarity

$$\Sigma = 1 + \Sigma_s a^s, \quad \mu = 1 + \mu_s a^s$$



WMAP; SN; ISW/Galaxy X-corr; CFHTLS-Wide T003 (Fu et al, 2008); SDSS DR7 peculiar velocity dispersion

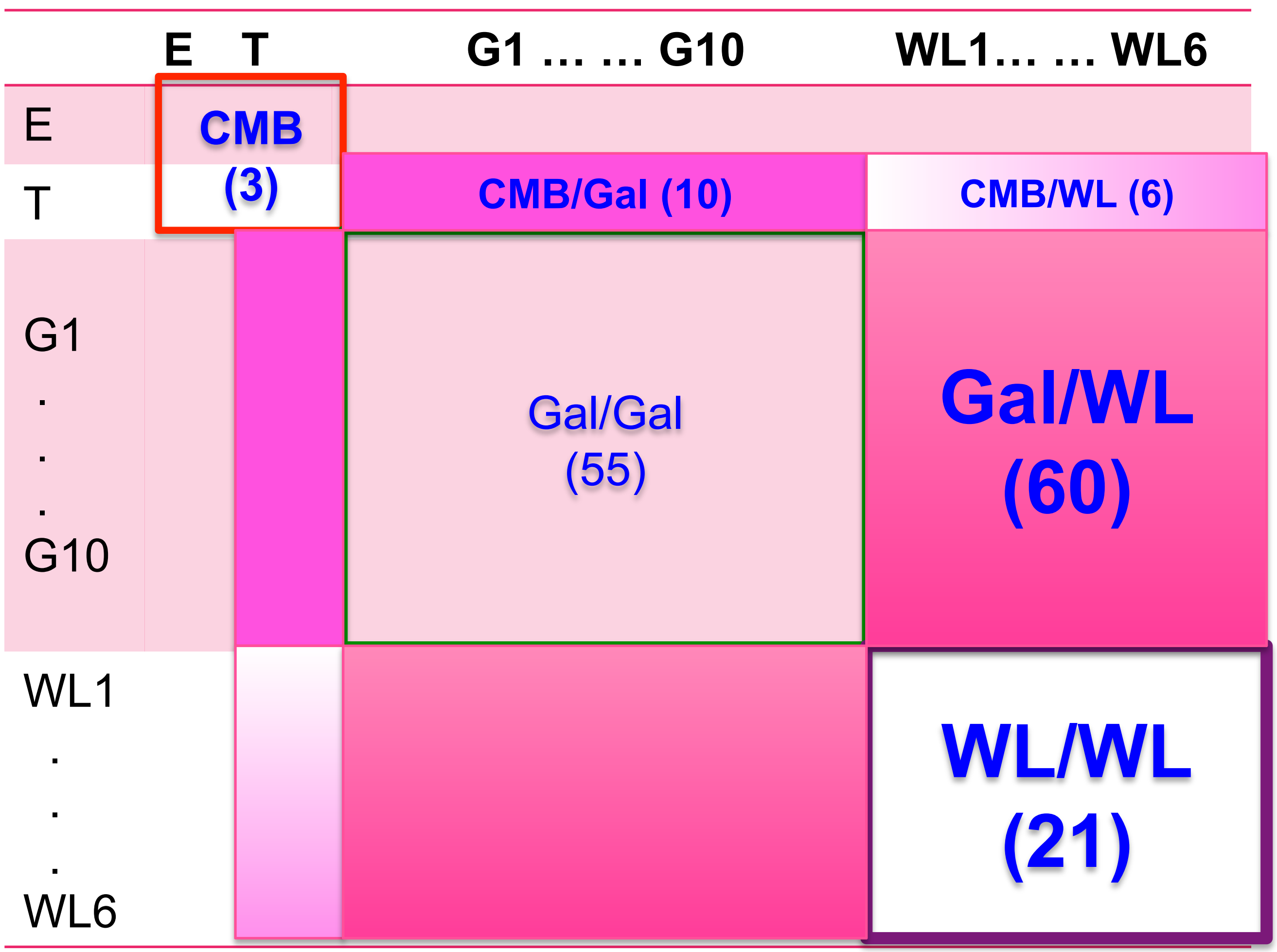
Y.-S. Song, G.-B. Zhao et al, arXiv:1011.2106

- Today's data constrains at best one or two numbers, depending on the assumptions

- μ and η are unknown functions of time and scale

- What can we learn about them in a model-independent way, e.g. with DES and LSST?

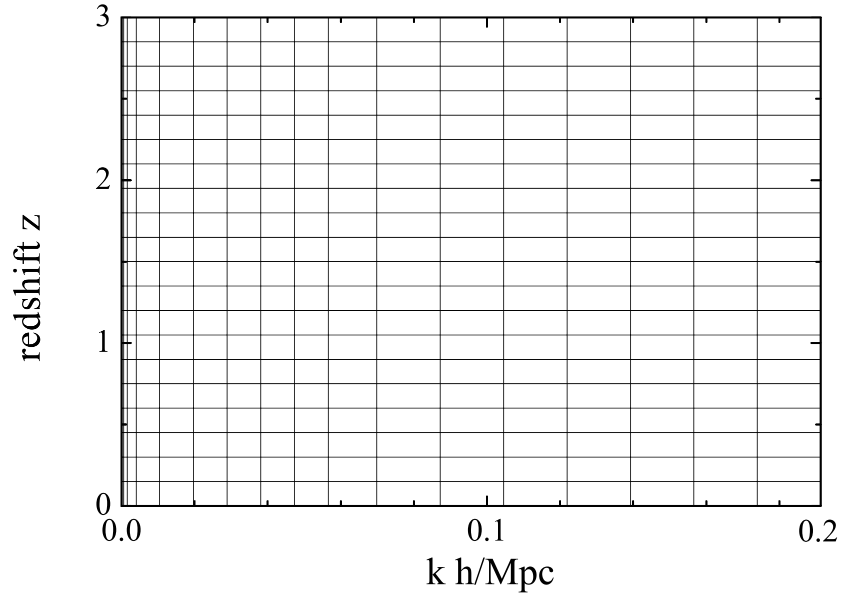
Planck+LSST


E T
G1 … … G10
WL1… … WL6
E
T
G1
.
.
.
G10
WL1
.
.
.
WL6
CMB (3)
CMB/Gal (10)
CMB/WL (6)
Gal/Gal (55)
Gal/WL (60)
WL/WL (21)

PCA forecast for $\mu(z,k)$ and $\eta(z,k)$

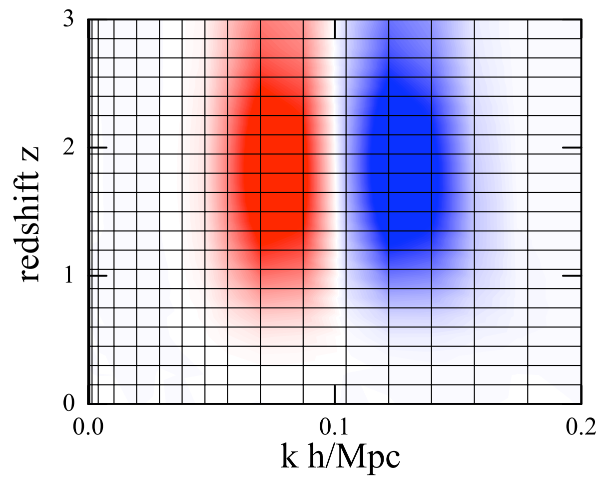
Zhao, LP, Silvestri, Zilberberg, **arXiv:0905.1326,** PRL'09
Hojjati, Zhao, LP, Silvestri, Crittenden, Koyama, in preparation

- Discretize μ and η on a (z,k) grid
- Treat each pixel, μ_{ij} and η_{ij}, as a free parameter
- Discretize w(z) on the same z-grid and treat w_i as free parameters



- Also vary the 6 "vanilla" parameters: Ω_c, Ω_b, h, n_s, A_s, τ and linear bias parameters: one for each redshift bin
- Calculate the Fisher Matrix to forecast the error matrix of ~ 840 parameters

Principal Component Analysis (of μ)



- Diagonalize the μ block of the covariance matrix to find uncorrelated combinations of pixels μ_{ij} – the eigenmodes

- Some eigenmodes are well-constrained, most are not

- Equivalent to expanding $\mu(z,k)$ into an orthonormal basis:

$$\boxed{\mu(z,k) - 1 = \sum_m \alpha_m e_m(z,k)}$$

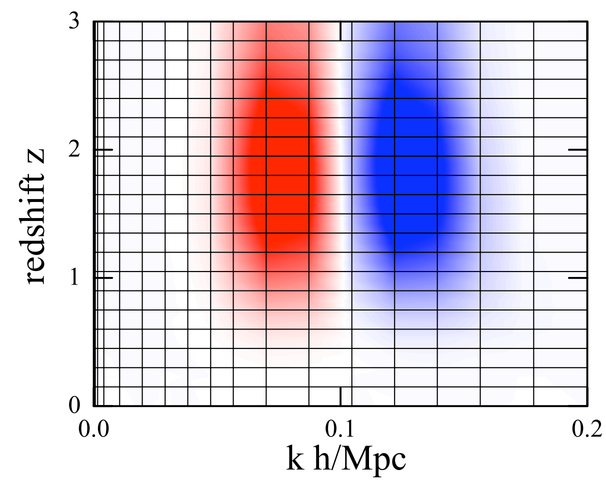
- PCA provides variances of uncorrelated parameters α_m

What do we gain with PCA?

- # of well-constrained modes ~ # of new parameters

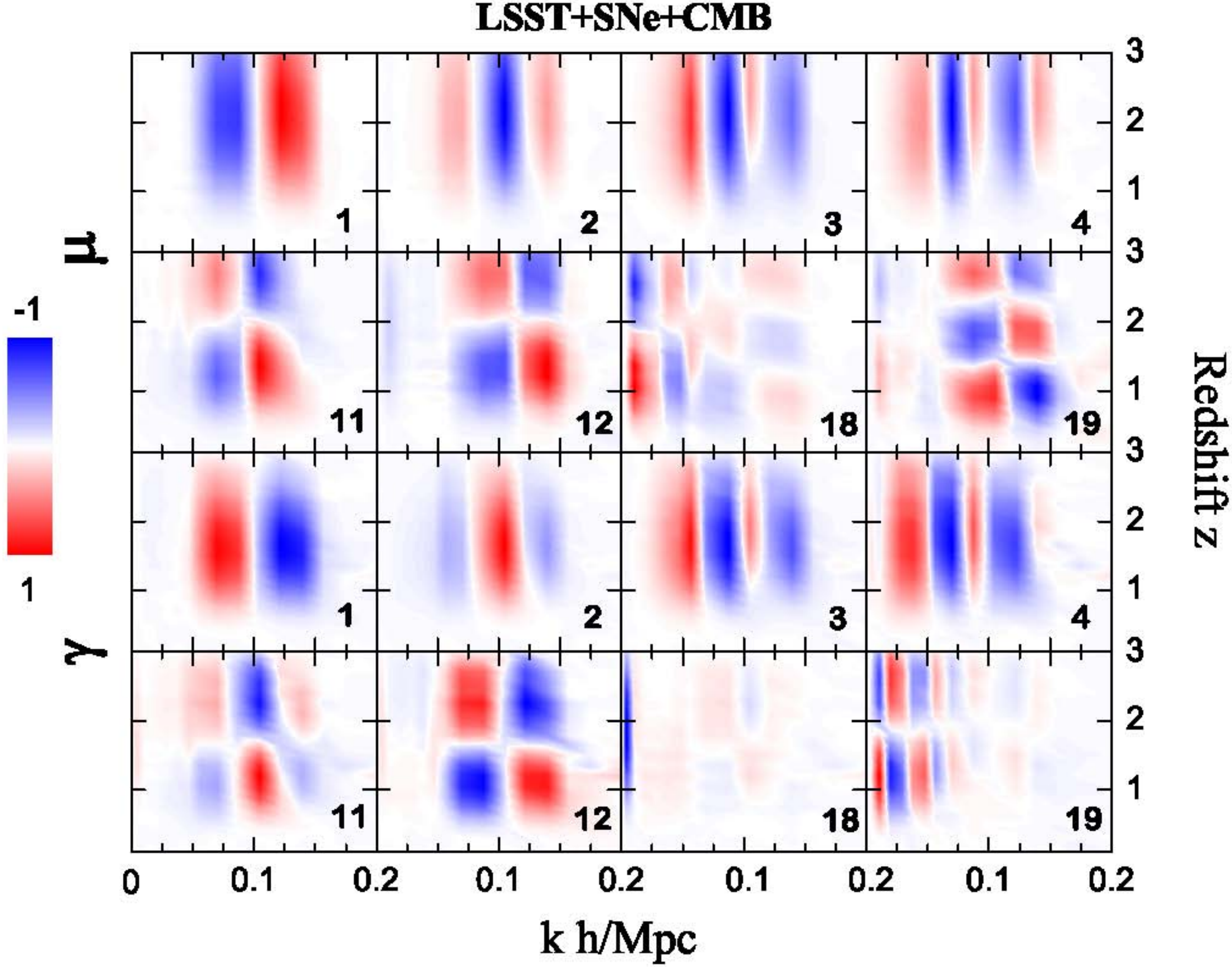
- "Sweet spots" in (k,z) space

- Information storage – can ``project" on parameters of other models

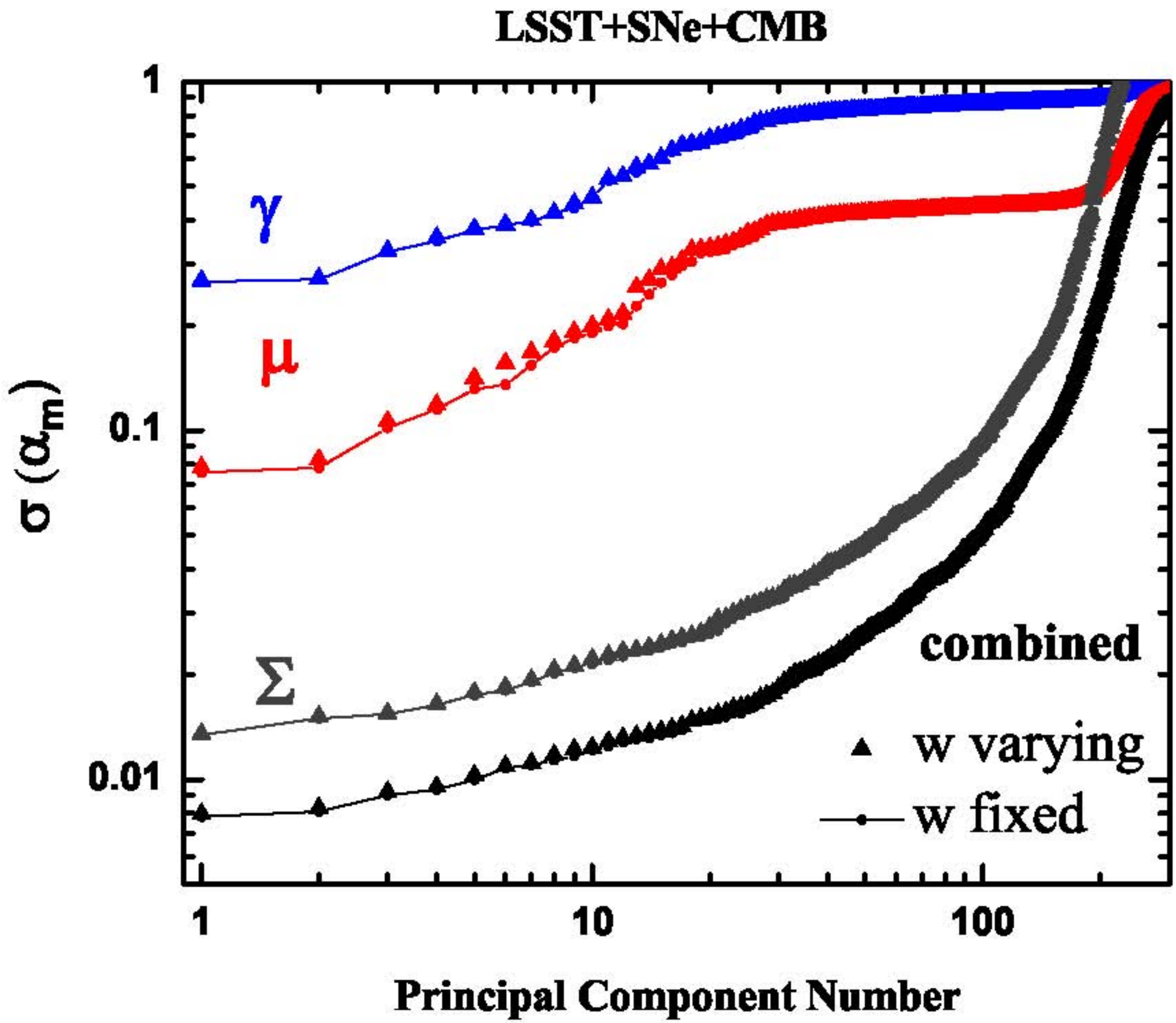


$$\frac{\partial \mu(a,k)}{\partial p^a} = \sum_i \alpha_i^a e_i(a,k).$$

$$F_{ab} = \alpha_i^a F_{ij} \alpha_j^b = \sum_i \alpha_i^a \alpha_i^b \lambda_i.$$

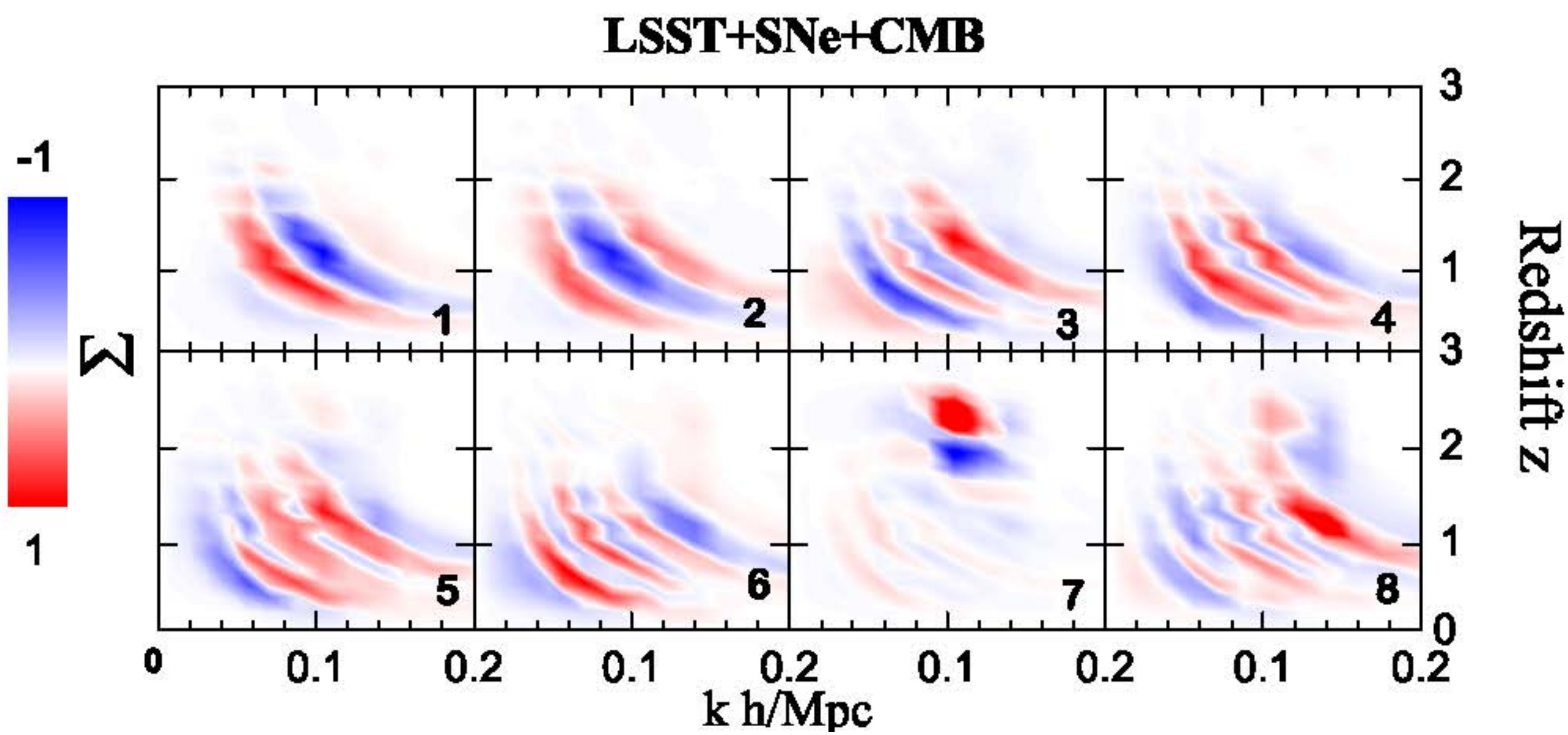


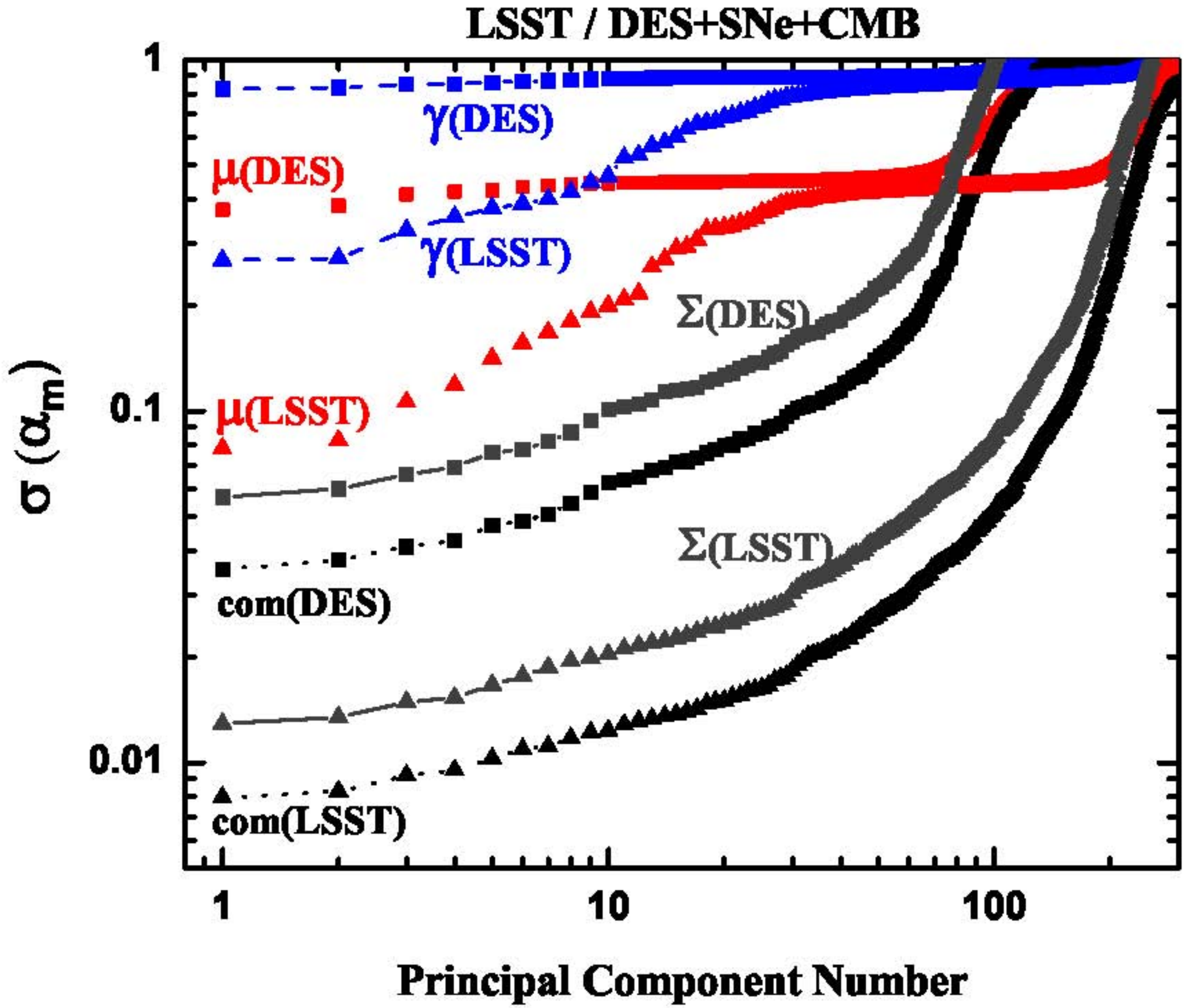
LSST+SNe+CMB
μ
γ
-1
1
Redshift z
k h/Mpc
0
0.1
0.2
1
2
3
4
11
12
18
19



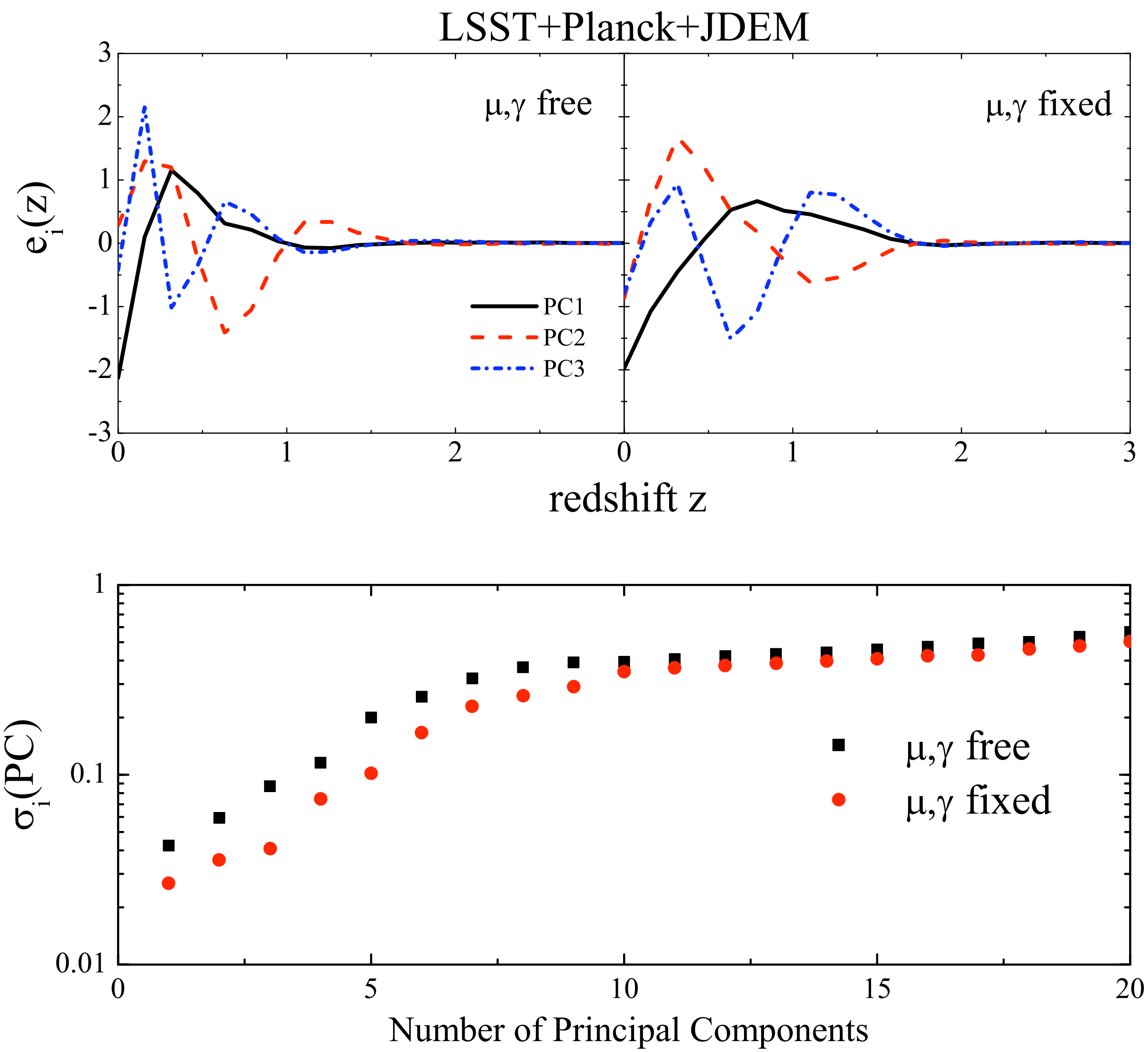
LSST+SNe+CMB
1
0.1
0.01
σ (α_m)
γ
μ
Σ
combined
w varying
w fixed
1
10
100
Principal Component Number

Eigenmodes of Σ




LSST / DES+SNe+CMB
γ(DES)
μ(DES)
γ(LSST)
Σ(DES)
μ(LSST)
com(DES)
Σ(LSST)
com(LSST)
1
0.1
0.01
σ (α_m)
1
10
100
Principal Component Number

Can we still measure w(z)?



What we have done

- Studied the parameter degeneracies
- Studied impact of some of the WL systematic effects
- Projected constraints on f(R) model
- Compared DES and LSST

Hojjati, Zhao, LP, Silvestri, Crittenden, Koyama, in preparation

Summary

- Future surveys can test GR in a model-independent way
- Combining different probes brakes degeneracies between modified gravity parameters, makes it possible to differentiate among theoretical proposals
- Particularly sensitive to <u>scale-dependent</u> modifications (which most modified gravity models predict)
- Using simple parameters can miss information in data
- Need to develop a set of “reasonable” theoretical priors.