



(Future) weak lensing constraints on chameleon modified gravity

Stephen Appleby
Institute for the early Universe
EWha University
Korea

Workshop on progress in theoretical and
observational cosmology

S.Appleby, J.Weller JCAP 1012:006
S.Thomas, S.Appleby, J.Weller JCAP 1103:036

Institute of High Energy Physics
Chinese Academy of Sciences
Beijing, P.R. China



Overview

- Introduction: Modified gravity
- Theoretical and Observational signatures;
 - Expansion history
 - Growth history
- Observational constraints;
 - Solar system
 - Weak lensing
 - Nonlinear regime: Perturbation theory
 - Nonlinear regime: Simulations
- Conclusions



Introduction

$$S_{EH} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - L_m \right]$$

Cosmic acceleration

Dark energy

$$S_{EH} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - L_m - L_{de} \right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G (T_{\mu\nu} + T_{\mu\nu}^{de})$$

$$3H^2 + \frac{k}{a^2} = 8\pi G (\rho_m + \rho_r + \rho_{de})$$

Examples:

Cosmological constant

Quintessence

Chaplygin gas



Introduction

$$S_{EH} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - L_m \right]$$

Cosmic acceleration

Modified gravity

Dark energy

$$S_{MG} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} + F(R, \phi, C^2, \dots) - L_m \right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + M_{\mu\nu}[g, \partial g, \partial\partial g, \phi, \dots] = 8\pi G T_{\mu\nu}$$

$$3H^2 + \frac{k}{a^2} + B(H, \dot{\phi}) = 8\pi G (\rho_m + \rho_r)$$

Examples:

Higher dimensional models

Weyl gravity

Scalar-tensor gravity

$$S_{EH} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - L_m - L_{de} \right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G (T_{\mu\nu} + T_{\mu\nu}^{de})$$

$$3H^2 + \frac{k}{a^2} = 8\pi G (\rho_m + \rho_r + \rho_{de})$$

Examples:

Cosmological constant

Quintessence

Chaplygin gas



Modified gravity

- Could the late time acceleration be attributed to additional gravitational degrees of freedom?
- Rather than the Einstein Hilbert action

$$S_{EH} = \int_M \sqrt{-g} d^4x \left[\frac{R}{16\pi G} - L_m \right]$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

- One could consider a generalized action of the form

$$S = \int \sqrt{-g} d^4x \left[\frac{R + f(R)}{16\pi G} - L_m \right]$$

- Purely phenomenological; can different (toy) gravitational models fit the data?



f(R) models

- f(R) gravity

$$S = \int \sqrt{-g} d^4x \left[\frac{R + f(R)}{16\pi G} - L_m \right]$$

- Field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + R_{\mu\nu} f_R - \frac{1}{2} g_{\mu\nu} f + [g_{\mu\nu} \square - \nabla_\mu \nabla_\nu] f_R = 8\pi G T_{\mu\nu}$$

General Relativity

$$f_R = \frac{df}{dR}$$

- The equations now contain fourth order derivatives of the metric

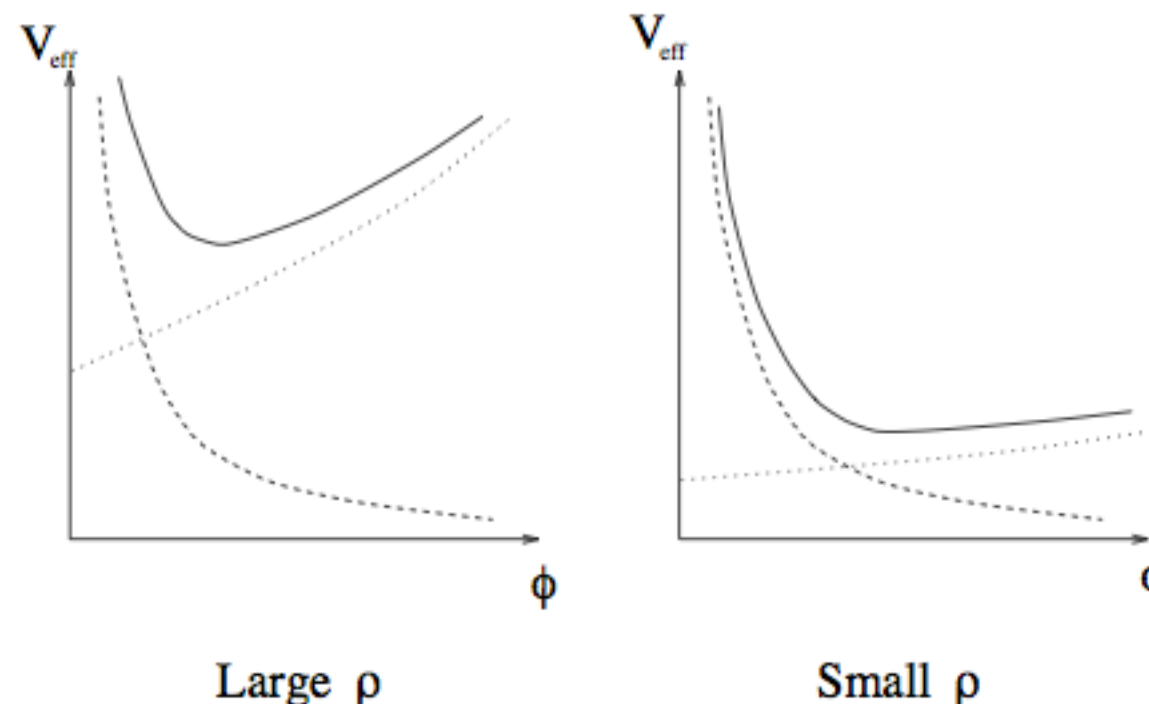
- Can the additional terms drive late time acceleration?

- There is a new, scalar degree of freedom that propagates

Chameleon mechanism

- Chameleon mechanism; the scalar field has a mass that depends on the ‘background’ energy density:

$$\square\phi = \frac{dV}{d\phi} + \beta e^{\beta\phi} \rho_m \quad V_{\text{eff}}(\phi) = V(\phi) + e^{\beta\phi} \rho_m$$
- In regions of high density, the ‘effective’ scalar field potential is very large and the field does not propagate.
- In regions of low density, the potential relaxes.
- The (only important) question; can the scalar field roll on cosmological scales?



- Khoury (2004)



Modified gravity: Cosmology

- By taking an FRW metric ansatz, we can write down the modified Friedmann and acceleration equations

$$ds^2 = -dt^2 + a^2(t) \left[dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

$$3H^2 = 8\pi G(\rho_c + \rho_r) + 3(H^2 + \dot{H})f_R - \frac{f}{2} - 3H\dot{f}_R$$
$$-2\dot{H} - 3H^2 = 8\pi G P_r + \ddot{f}_R + 2H\dot{f}_R + \frac{f}{2} - (\dot{H} + 3H^2)f_R$$

$$\dot{\rho}_c + 3H\rho_c = 0$$

$$\dot{\rho}_r + 4H\rho_r = 0$$

General Relativity

- We have the freedom to specify the $f(R)$ function; we can reproduce any expansion history...
- Introduce an effective density

$$8\pi G\rho_f = 3(\dot{H} + H^2)\frac{\dot{f}}{\dot{R}} - \frac{f}{2} - 3aH^2\left(\frac{\dot{f}}{\dot{R}}\right)$$



Modified gravity: Constraints

- However, $f(R)$ models are subject to stringent theoretical and observational constraints, which restricts the allowed expansion histories
- Theoretical constraints

- No ghost $1 + f_R > 0$

- No early time instability $f_{RR} > 0$

- Dolgov Kawasaki (2004)

- No cosmological constant $f(R = 0) = 0$

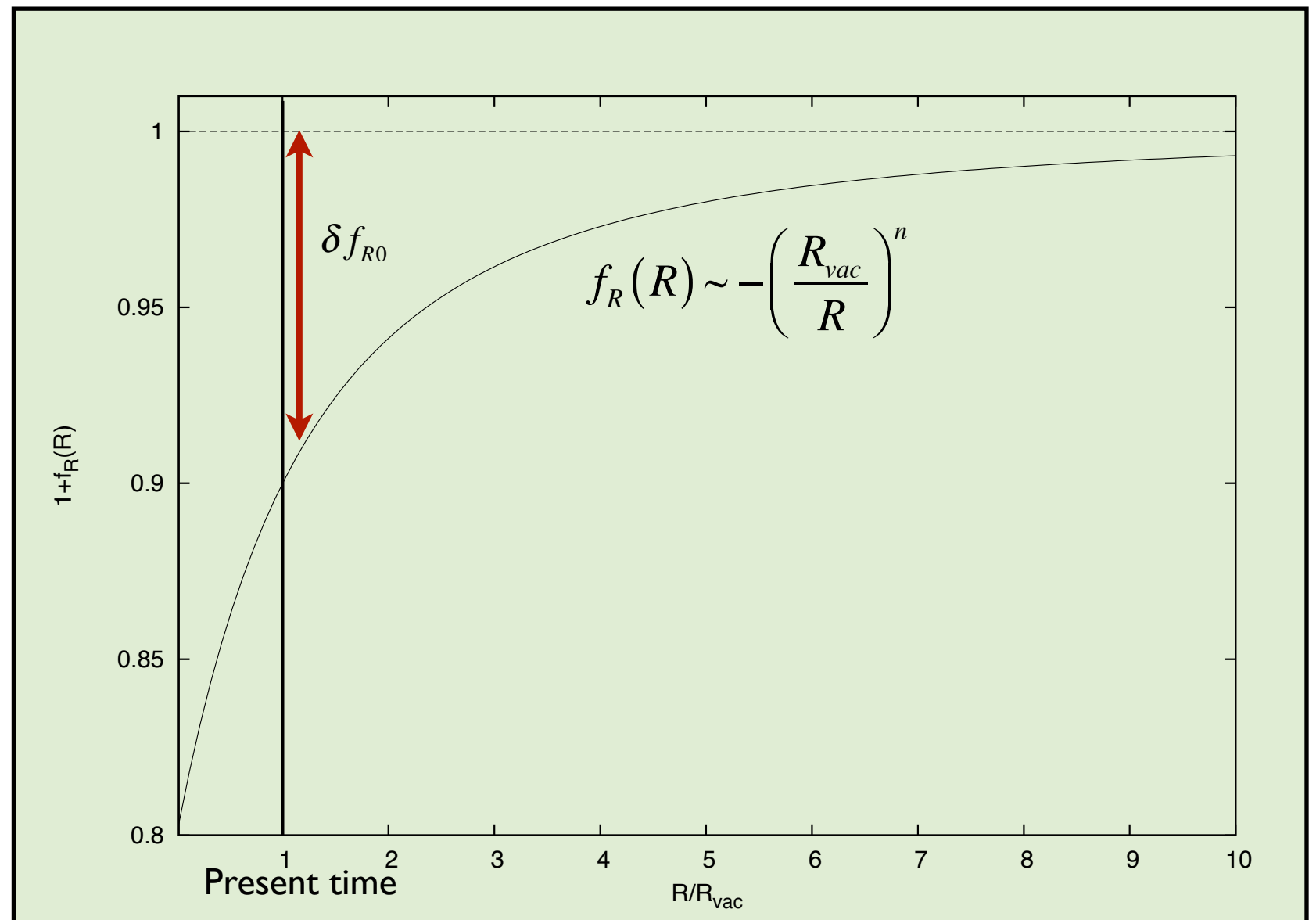
- Return to GR at early times $Rf_{RR} \rightarrow 0$
 $R \rightarrow \infty$



Modified gravity: Theoretical constraints

$$1 + f_R$$

- The only viable functions are monotonic and asymptote to GR to the past.



- We can describe such a function with two ‘modified gravity’ parameters.



$f(R)$ models

- Simple parameterizations of the $f(R)$ curve are considered

$$f(R) = -\frac{R_{vac}}{2} \frac{\lambda (R / R_{vac})^n}{1 + \lambda (R / R_{vac})^n} \quad \lambda > 1 \quad n > 0$$

- Hu et al (2007)
- Starobinsky (2007)

- At large curvatures, $f(R) \simeq -\frac{R_{vac}}{2} \left(1 - \lambda^{-1} \left(\frac{R_{vac}}{R} \right)^n \right)$

- At early times, viable models behave as the standard Λ CDM model
- However, there is no true cosmological constant! Minkowski space is a vacuum solution to the field equations.



$f(R)$ models

- Constraints on $f(R)$ models;
 - Local, solar system tests of gravity
 - Cosmological signatures
 - Luminosity distance-redshift relation
 - CMB
 - Matter power spectrum



Modified gravity: Observational constraints

- Gravity is well described by General Relativity in the solar system

$$g_{00} = -1 + 2U$$

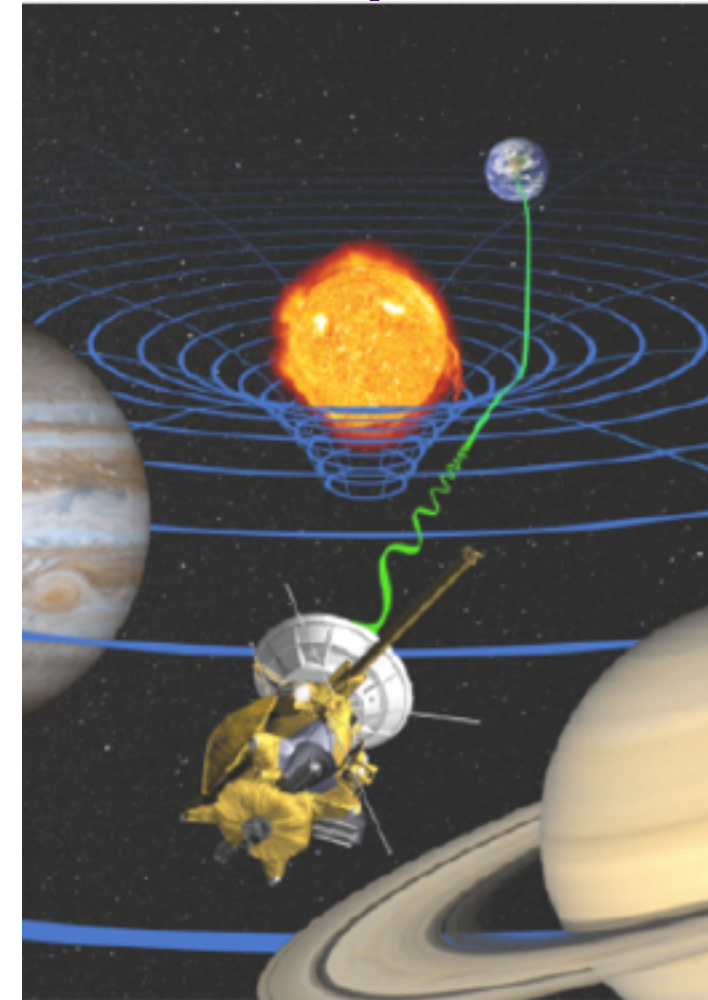
$$g_{ij} = (1 + 2U + (\gamma - 1)U)\delta_{ij}$$

- Shapiro time delay measured by the Cassini probe

$$|\gamma - 1| < 2.3 \times 10^{-5} \quad \text{B. Bertotti, L. Iess, P. Tortora (2003)}$$

Effect due to modified gravity • (Hu et al, 2007)

$$ds^2 = -[1 - 2A(r) + 2B(r)]dt^2 + [1 + 2A(r)](dr^2 + r^2 d\Omega)$$





Modified gravity: Observational constraints

$$\nabla^2 A \simeq -4\pi G \rho_m + \frac{1}{6}(8\pi G \rho_m - R)$$

$$\nabla^2 B \simeq \frac{1}{3}(8\pi G \rho_m - R)$$

$$\nabla^2 f_R \simeq -\frac{1}{3}(8\pi G \rho_m - R)$$

- Solving these equations corresponds to the following constraint: (Hu et al, 2007)

$$|\gamma - 1| < 2.3 \times 10^{-5}$$

$$f_R(R_{sol}) \sim (\gamma - 1) \frac{GM_s}{r_s}$$

$$\frac{GM_s}{r_s} = 2.12 \times 10^{-6}$$

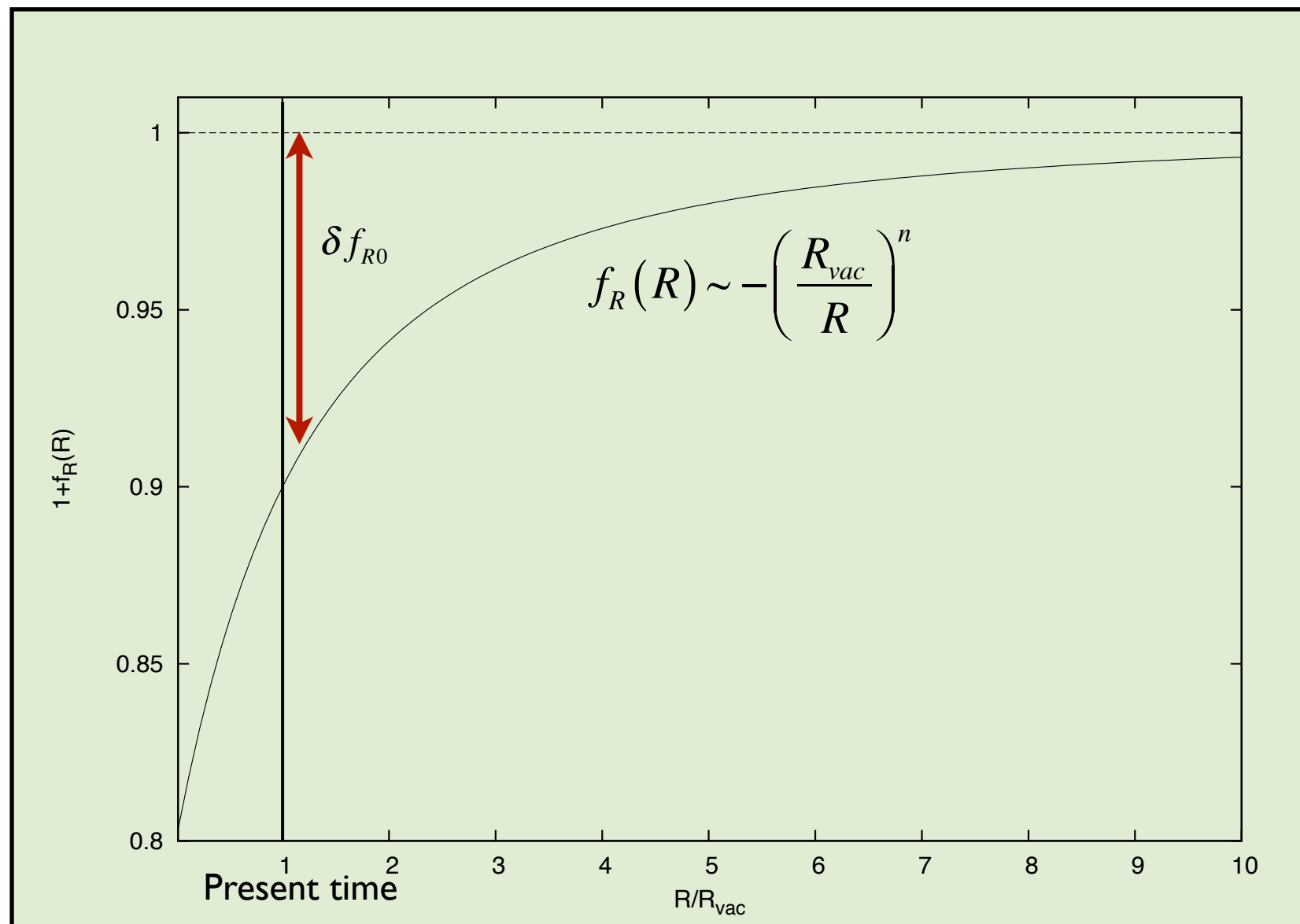
$$R_{sol} \sim 8\pi G \rho_{sol}$$

$$\rho_{sol} \sim 10^{-24} \text{ g cm}^{-3} \quad \rho_{crit} \sim 10^{-29} \text{ g cm}^{-3}$$

$$|f_R(R_{sol})| \sim \frac{R_{sol}}{M_{scal}^2} < 4.9 \times 10^{-11}$$

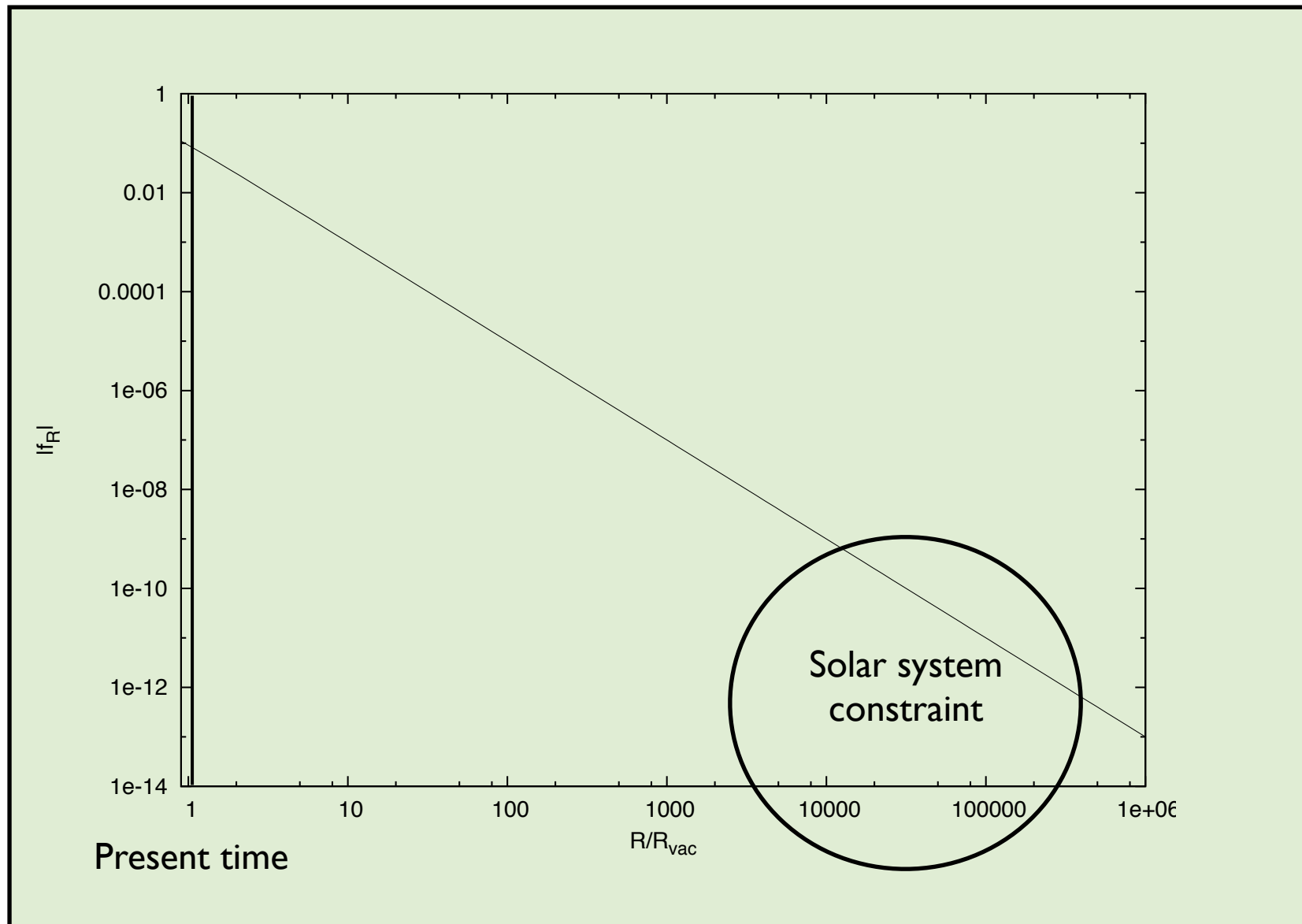


Modified gravity: Theoretical constraints





Modified gravity: Observational constraints

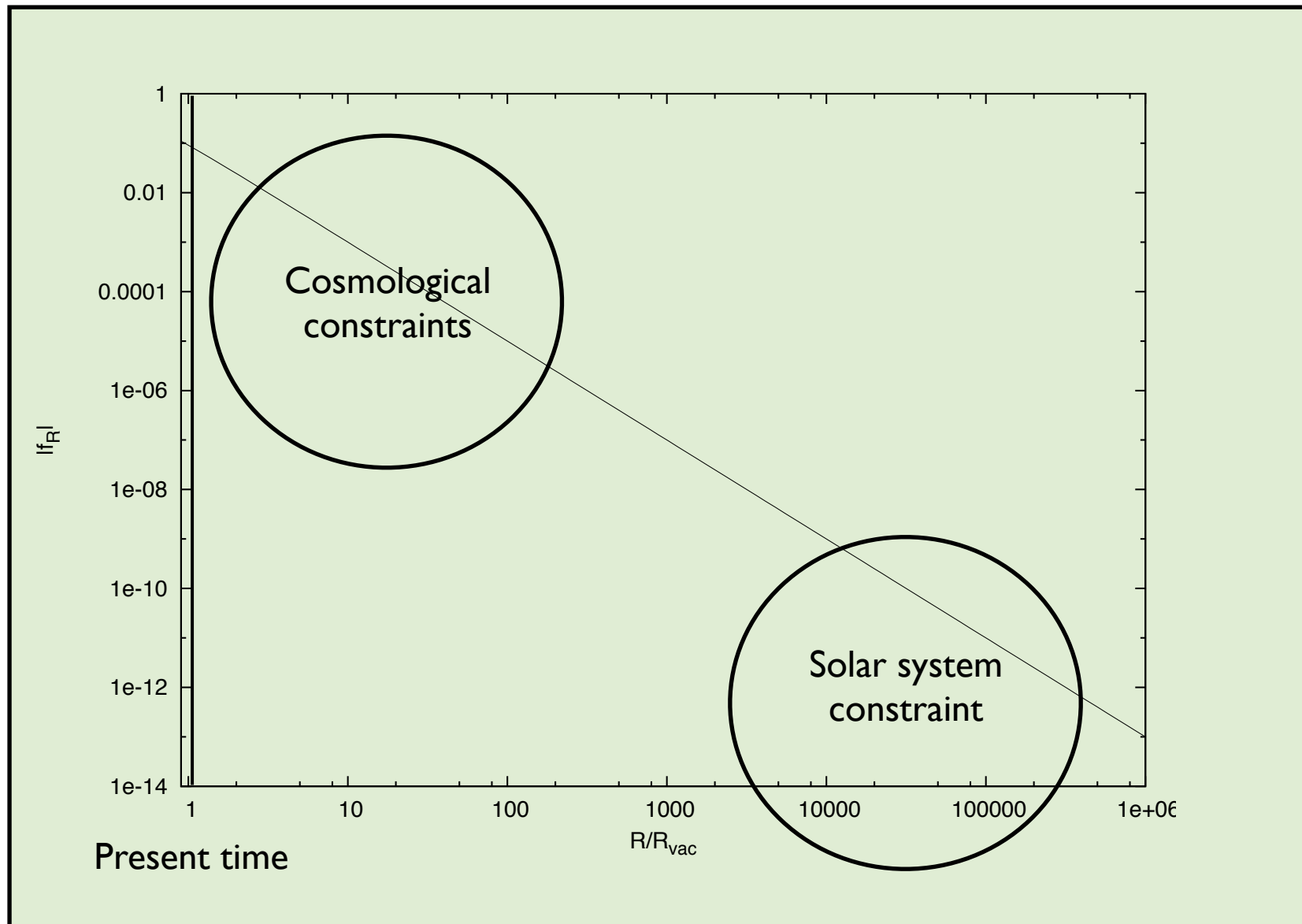


$$|f_R(R)| \sim \left(\frac{R_{vac}}{R} \right)^n$$

$$n = 2$$



Modified gravity: Observational constraints



$$|f_R(R)| \sim \left(\frac{R_{vac}}{R} \right)^n$$

$$n = 2$$



$f(R)$ models

- Constraints on $f(R)$ models;
 - Local, solar system tests of gravity
 - Cosmological signatures
 - Luminosity distance-redshift relation
 - CMB
 - Matter power spectrum



Observational signatures: Expansion history

- Background cosmology of $f(R)$ models;

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

$$\begin{aligned} 3H^2 &= 8\pi G(\rho_c + \rho_r) + 3(H^2 + \dot{H})f_R - \frac{f}{2} - 3H\dot{f}_R \\ -2\dot{H} - 3H^2 &= 8\pi G P_r + \ddot{f}_R + 2H\dot{f}_R + \frac{f}{2} - (\dot{H} + 3H^2)f_R \\ \dot{\rho}_c + 3H\rho_c &= 0 \\ \dot{\rho}_r + 4H\rho_r &= 0 \end{aligned}$$

General Relativity

- Fourth order field equations. To simplify, we only consider the parameter range for which the ‘quasi-static approximation’ holds

$$f_{RR}(R_{GR}) \ll H_{GR}^2$$

$$f(R) = -\frac{R_{vac}}{2} \frac{\lambda(R/R_{vac})^n}{1 + \lambda(R/R_{vac})^n}$$



Observational signatures: Expansion history

- Background equations in the quasi-static approximation

$$3H^2 = 8\pi G(\rho_c + \rho_r + \rho_\Lambda) + \varpi_0 f_{RR} H^4$$

Model dependent constant of
order unity

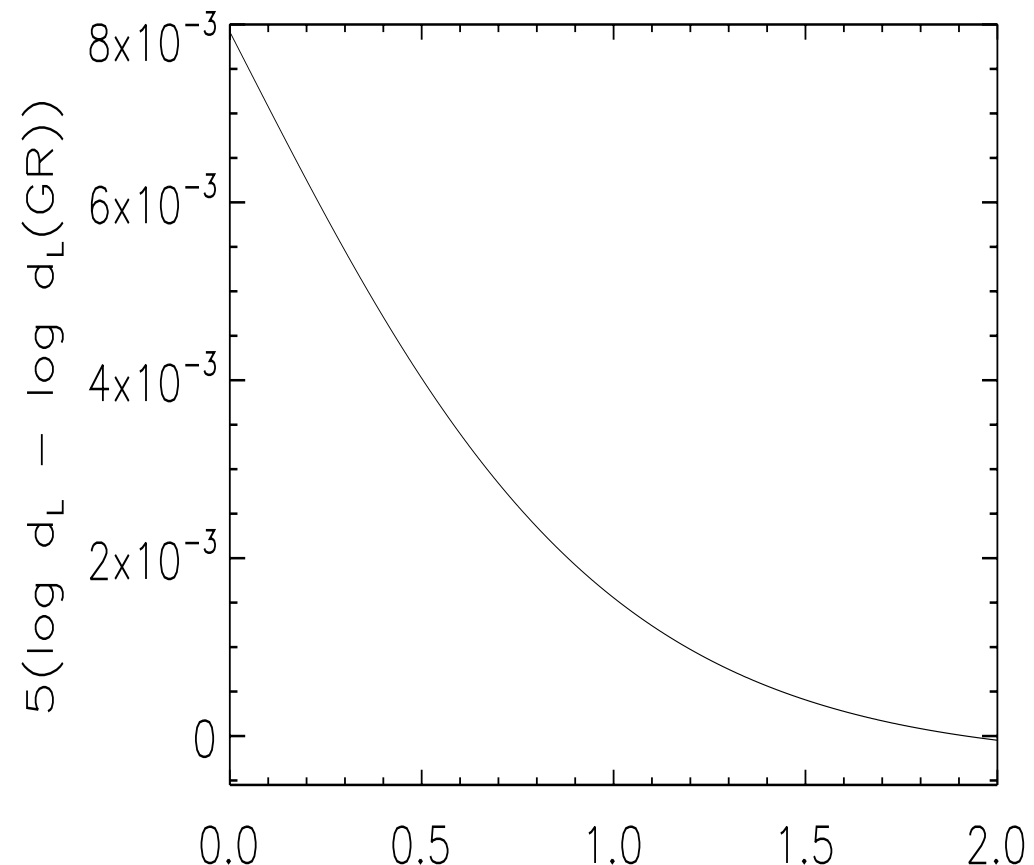
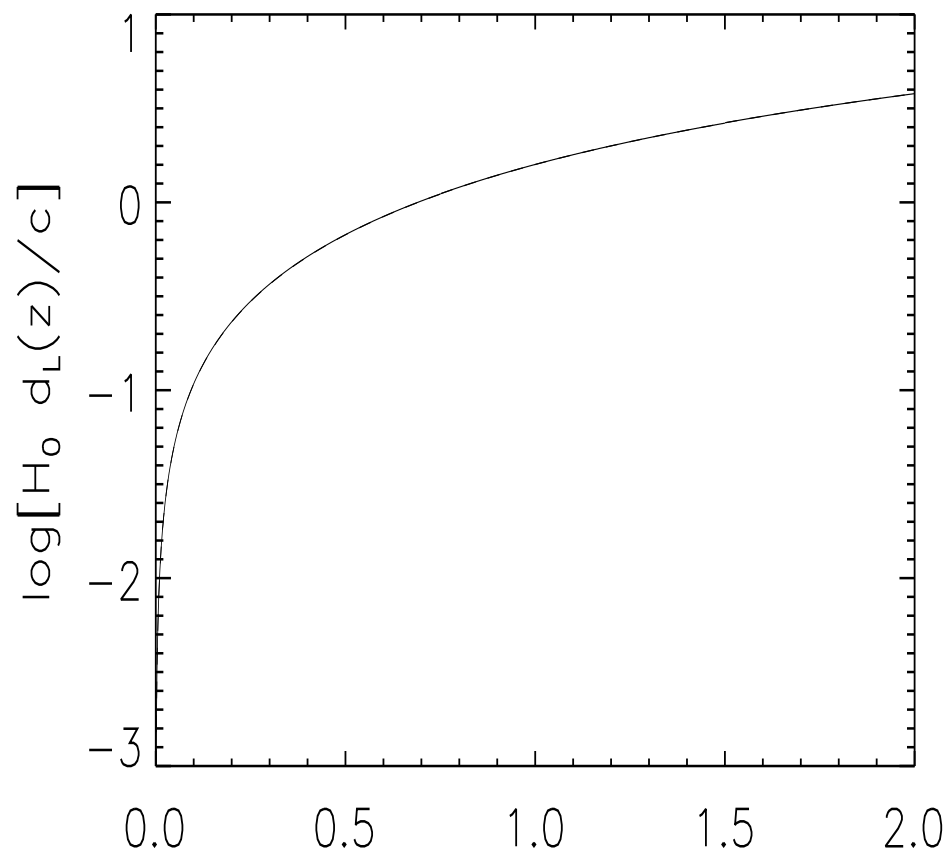
$$d_L(z) = (1+z) \int_0^z \frac{cdz'}{H(z')}$$

$$\dot{\rho}_r + 4H\rho_r = 0$$

$$\dot{\rho}_c + 3H\rho_c = 0$$

$$f(R) = -\frac{R_{vac}}{2} \frac{\lambda(R/R_{vac})^n}{1 + \lambda(R/R_{vac})^n}$$

$$\lambda \sim 10^3$$





Observational signatures: Growth history

- Perturbation equations (scalar perturbations only, Newtonian gauge) Bean et al. 2006

$$ds^2 = a^2 \left[-(1 + 2\psi) d\tau^2 + (1 - 2\phi) \gamma_{ij} dx^i dx^j \right]$$

$$\delta_c'' + H \delta_c' + k^2 \psi - 3\phi'' - 3H\phi' = 0$$

$$\delta_\gamma'' + \frac{1}{3} k^2 \delta_\gamma + \frac{4}{3} k^2 \psi - 4\phi'' = 0$$

Fluid perturbation
equations are unchanged

$$(1 + f_R)(\psi - \phi) + f_{RR} \delta R = -\frac{3a^2}{2k^2} 8\pi G \Sigma_i (\rho_i + p_i) \sigma_i$$

$$(1 + f_R) \left[2k^2 \phi + 6H(\phi' + H\psi) \right] + 3f_{RR} H' \delta R - (k^2 f_{RR} + 3Hf_{RR}') \delta R - 3Hf_{RR} \delta R' + f_R' (6H\psi + 3\phi') = -8\pi G a^2 \Sigma_i \rho_i \delta_i$$

$$\delta R = \frac{2}{a^2} \left[-6 \frac{a''}{a} \psi - 3H\psi' + k^2 \psi - 9H\phi' - 3\phi'' - 2k^2 \phi \right]$$



Modified gravity: Evolution of perturbations

- To solve these equations, we use the quasi-static limit

- At background level, the $f(R)$ function satisfies

$$f_{RR} \ll H^2, \dot{H}$$

- The dominant contributions arise from terms involving

$$f_{RR}^{-1} = 3M^2 \quad \frac{k^2}{a^2}$$

- Approximate equations

$$k^2 \phi = -4\pi G Q(a, k) a^2 \delta_c \quad Q = \frac{3a^2 M^2 + 2k^2}{3a^2 M^2 + 3k^2}$$

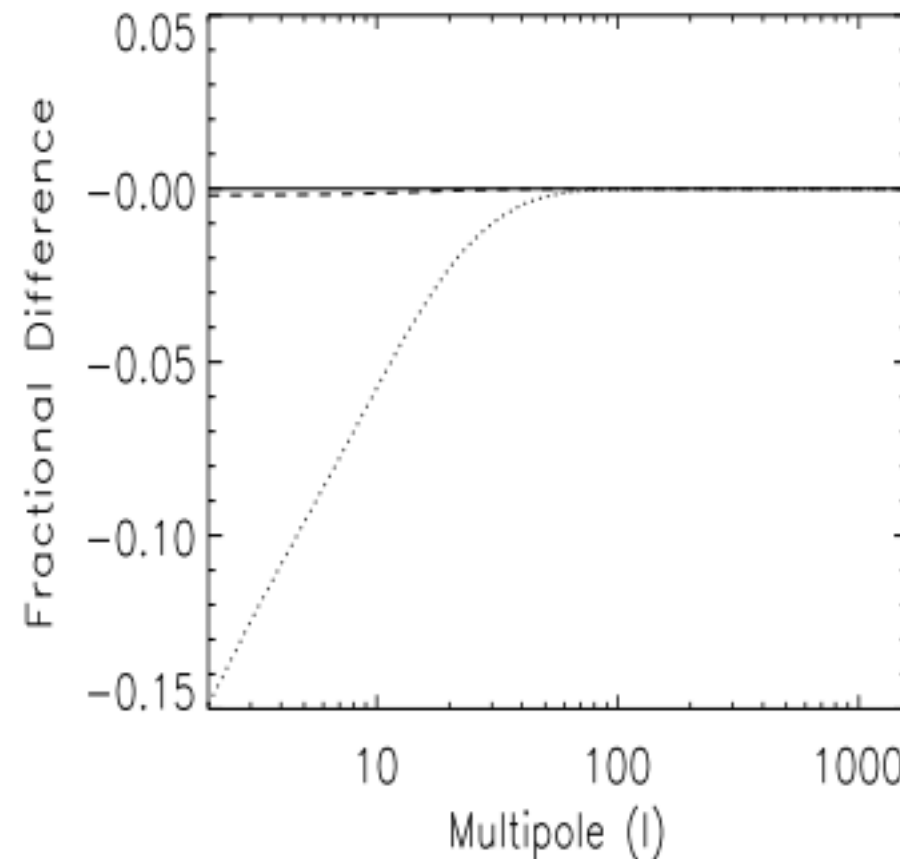
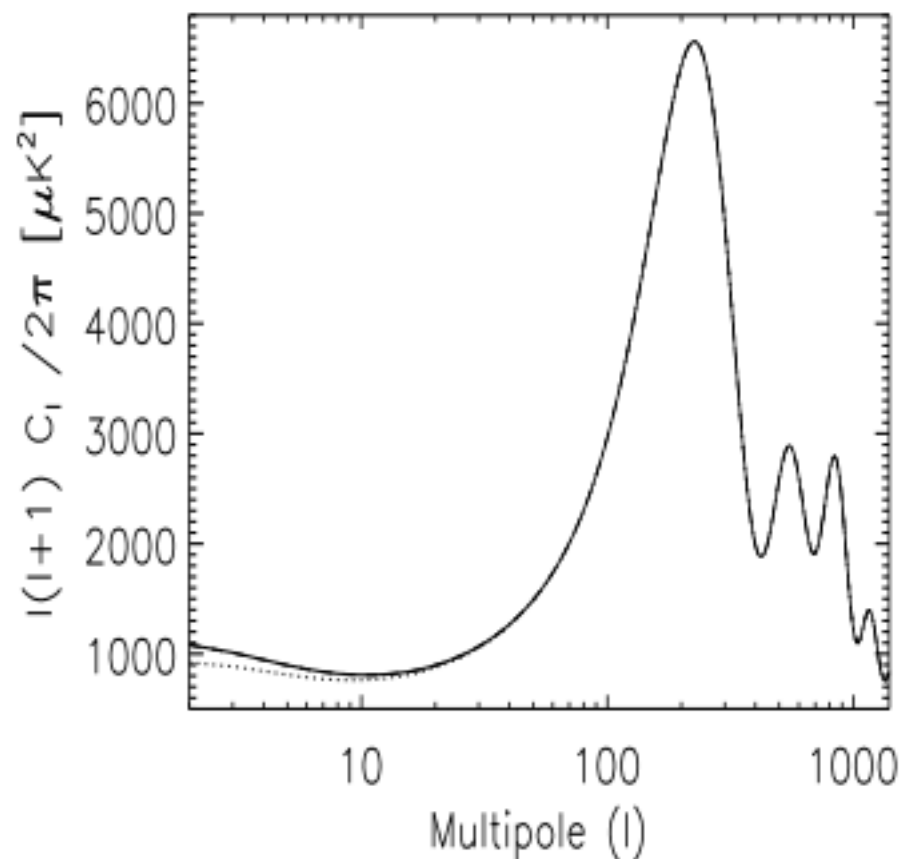
$$\psi = [1 + \eta(a, k)] \phi \quad \eta = \frac{2k^2}{3a^2 M^2 + 2k^2}$$

$$\ddot{\delta}_c + 2H\dot{\delta}_c - 4\pi G \rho_c \theta(a, k) \delta_c = 0 \quad \theta = \frac{4k^2 + 3a^2 M^2}{3k^2 + 3a^2 M^2}$$



Cosmological constraints: CMB

- CMB angular power spectrum;



$$\lambda \sim 10^3$$

$$f(R) = -\frac{R_{vac}}{2} \frac{\lambda (R/R_{vac})^n}{1 + \lambda (R/R_{vac})^n}$$

- Very low modified gravity signal in the angular power spectrum, even on large scales.

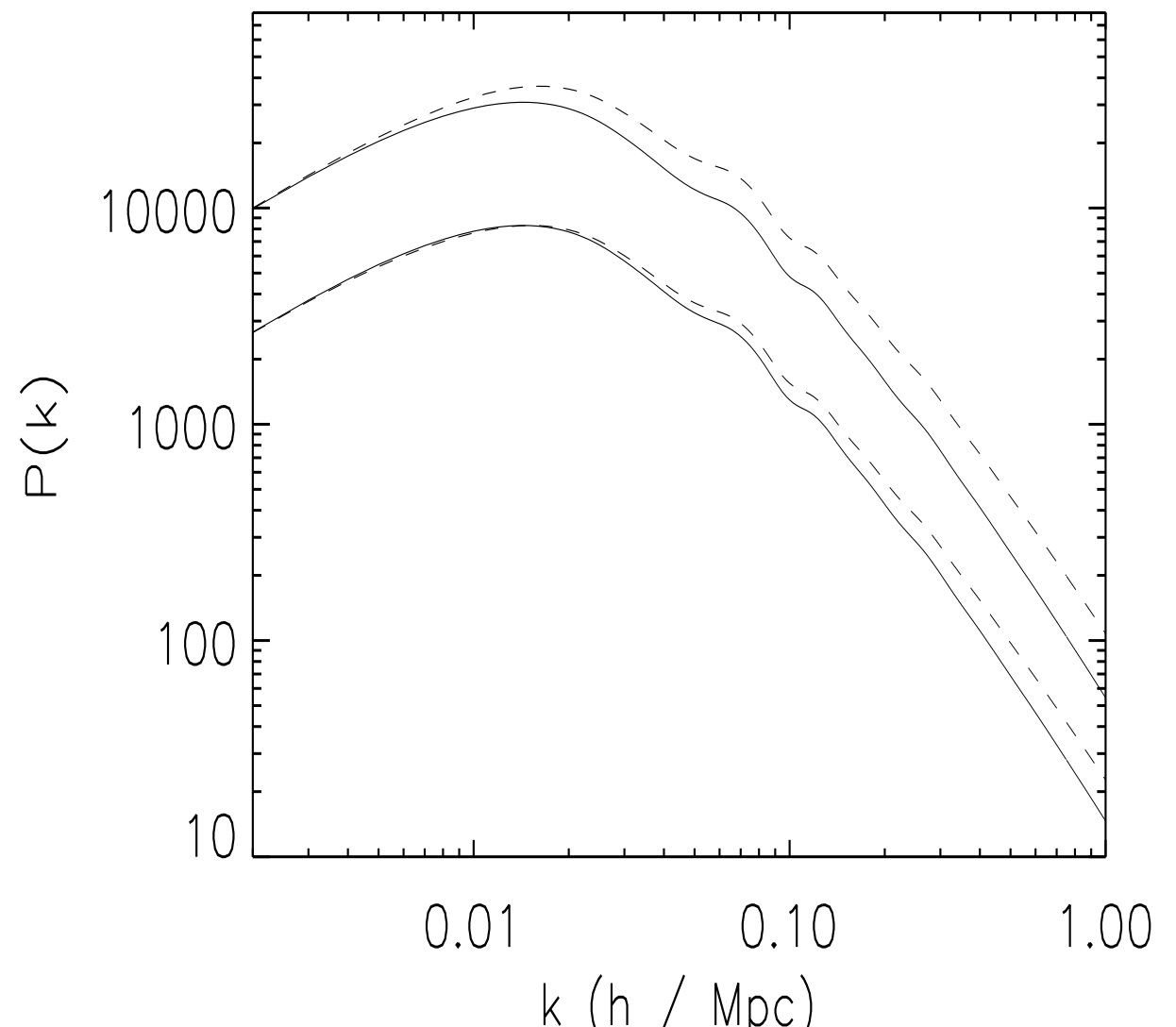


Modified gravity: Perturbations

- The power spectrum is modified at late times
- $P(z, k) = P_{\text{GR}}(z = 10, k) \left(\frac{\delta_c(z, k)}{\delta_c(z = 10, k)} \right)^2$
- $f(R)$ models generically lead to increased power at 'intermediate' scales

$$f(R) = -\frac{R_{\text{vac}}}{2} \frac{\lambda (R / R_{\text{vac}})^n}{1 + \lambda (R / R_{\text{vac}})^n}$$

$$\lambda \sim 10^3$$



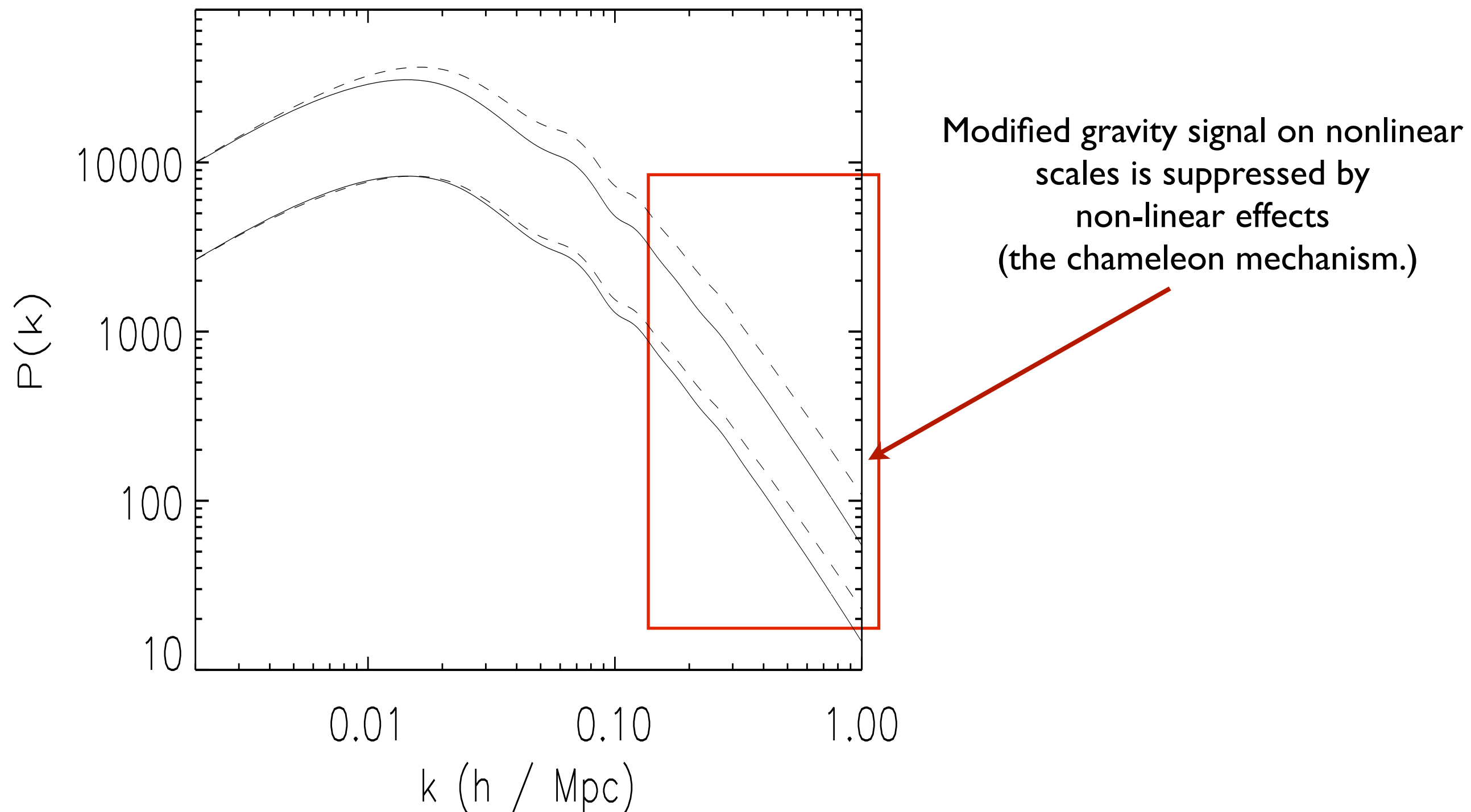


Modified gravity: Constraints

- The growth history is far more sensitive to modified gravity than the expansion history
- Weak lensing will provide the most stringent constraints* (cluster number counts are exponentially sensitive to the amplitude of the matter power spectrum...)
- How well can future missions constrain these models?
- Euclid:
 - Full extra galactic sky survey with 1.2m telescope at L2
 - Optimised for weak gravitational lensing
- Measurements will be precise enough to reconstruct the growth history in several redshift bins, allowing us to reconstruct the metric potentials and hence constrain this class of modified gravity models.
- To maximize the constraining power of future surveys, we must also construct the non-linear power spectrum.



Modified gravity: Weak lensing





Modified gravity: Non-linear regime

- Use higher order perturbation theory for the mildly nonlinear regime

$$\Phi_a = \begin{pmatrix} \delta(\tau, \mathbf{k}) \\ -\theta(\tau, \mathbf{k}) \end{pmatrix} \quad \tau = \log[a]$$

$$\frac{\partial \Phi_a(\tau, \mathbf{k})}{\partial \tau} + \Omega_{ab} \Phi_b(\tau, \mathbf{k}) = \int \frac{d^3 \mathbf{k}_1 d^3 \mathbf{k}_2}{(2\pi)^3} \delta^D(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) \gamma_{abc}(\tau, \mathbf{k}) \Phi_b(\mathbf{k}_1, \tau) \Phi_c(\mathbf{k}_2, \tau)$$

$$\Omega_{ab} = \begin{pmatrix} 0 & -1 \\ -\frac{4\pi G \rho_m}{H^2} \left(1 + \frac{k^2}{3a^2 \Pi(a, \mathbf{k})}\right) & 2 + \frac{\dot{H}}{H^2} \end{pmatrix}$$

$$\Pi(a, \mathbf{k}) = \left(\frac{k^2}{a^2} + \frac{M^2}{3} \right)$$

$$\Phi_a = \Phi_a^{(1)} + \Phi_a^{(2)} + \Phi_a^{(3)} + \dots$$

$$P_{ab}(\mathbf{k}, \tau) = P_{ab}^{(11)}(\mathbf{k}, \tau) + P_{ab}^{(22)}(\mathbf{k}, \tau) + P_{ab}^{(13)}(\mathbf{k}, \tau) + \dots$$

$$\gamma_{112} = \frac{1}{2} \left(1 + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{|\mathbf{k}_1|^2} \right)$$

$$\gamma_{121} = \frac{1}{2} \left(1 + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{|\mathbf{k}_2|^2} \right)$$

$$\gamma_{222} = \frac{1}{2} \frac{\mathbf{k}_1 \cdot \mathbf{k}_2 |\mathbf{k}_1 + \mathbf{k}_2|^2}{|\mathbf{k}_1|^2 |\mathbf{k}_2|^2}$$

$$\gamma_{211} = -\frac{1}{12H^2} \left(\frac{8\pi G \rho_m}{3} \right)^2 \frac{(k_1 + k_2)^2}{a^2} \frac{M_2(a)}{\Pi(\tau, \mathbf{k}_{12}) \Pi(\tau, \mathbf{k}_1) \Pi(\tau, \mathbf{k}_2)}$$

Higher order vertex
functions



Modified gravity: Non-linear regime

- Use higher order perturbation theory for the mildly nonlinear regime; closure relations

Taruya, Hiramatsu (2007)

Koyama, Taruya, Hiramatsu (2009)

$$\begin{aligned} (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') R_{ab}(|\mathbf{k}|, \tau, \tau') &= \langle \Phi_a(\mathbf{k}, \tau), \Phi_b(\mathbf{k}', \tau') \rangle \quad \tau > \tau' \\ (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') G_{ab}(|\mathbf{k}|, \tau, \tau') &= \left\langle \frac{\delta \Phi_a(\mathbf{k}, \tau)}{\delta \Phi_b(\mathbf{k}', \tau')} \right\rangle \quad \tau > \tau' \end{aligned} \quad \Phi_a = \begin{pmatrix} \delta(\tau, \mathbf{k}) \\ -\theta(\tau, \mathbf{k}) \end{pmatrix}$$

$$\begin{aligned} \Lambda_{ab}(\mathbf{k}, \tau) R_{bc}(|\mathbf{k}|, \tau, \tau') &= 0 & \Lambda_{ab} &= \delta_{ab} \frac{\partial}{\partial \tau} + \Omega_{ab}(\mathbf{k}, \tau) \\ \Lambda_{ab}(\mathbf{k}, \tau) G_{bc}(|\mathbf{k}|, \tau, \tau') &= 0 & \Gamma_{abcd} &= \delta_{ac} \delta_{bd} \frac{\partial}{\partial \tau} + \delta_{ac} \Omega_{bd}(k, \tau) + \delta_{bd} \Omega_{ac}(k, \tau). \end{aligned}$$

$$\Gamma_{abcd}(\mathbf{k}, \tau) P_{cd}(|\mathbf{k}|, \tau) = \int_{\tau_i}^{\tau} d\tau'' M_{as}(k, \tau, \tau'') R_{bs}(k, \tau, \tau'') + \int_{\tau_i}^{\tau} N_{as}(k, \tau, \tau'') G_{bs}(k, \tau, \tau'')$$

$$\gamma_{112}(\mathbf{k}_1, \mathbf{k}_2, \tau) = \frac{1}{2} \left(1 + \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{|\mathbf{k}_1|^2} \right)$$

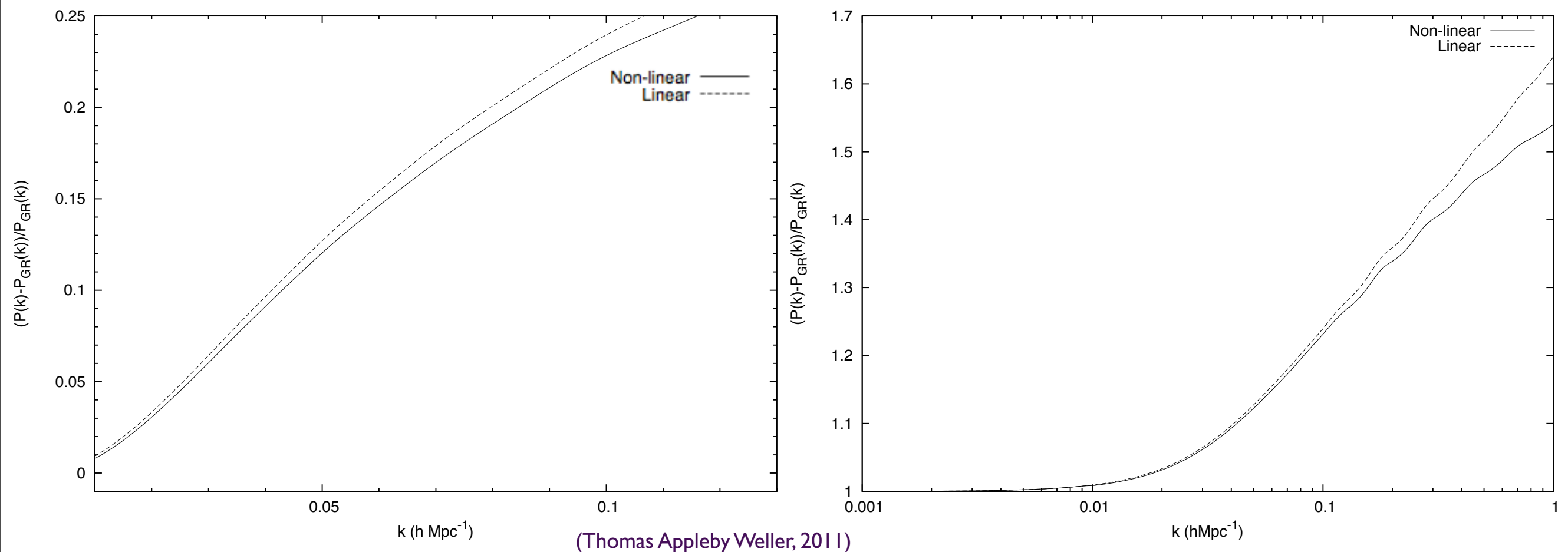
$$\gamma_{211}(\mathbf{k}_1, \mathbf{k}_2, \tau) = -\frac{1}{12H^2} \left(\frac{8\pi G \rho_m}{3} \right)^2 \frac{(k_1 + k_2)^2}{a^2} \frac{M_2(a)}{\Pi(\tau, \mathbf{k}_{12}) \Pi(\tau, \mathbf{k}_1) \Pi(\tau, \mathbf{k}_2)}$$

- Extra terms; backreaction of the metric perturbations on the $f(R)$ functional form (the chameleon mechanism!)



Modified gravity: Non-linear regime

- We observe the effect of the chameleon mechanism in the mildly non-linear regime



- Can use the mildly non-linear regime to calibrate fully non-linear fitting formulas.



Modified gravity: Non-linear regime

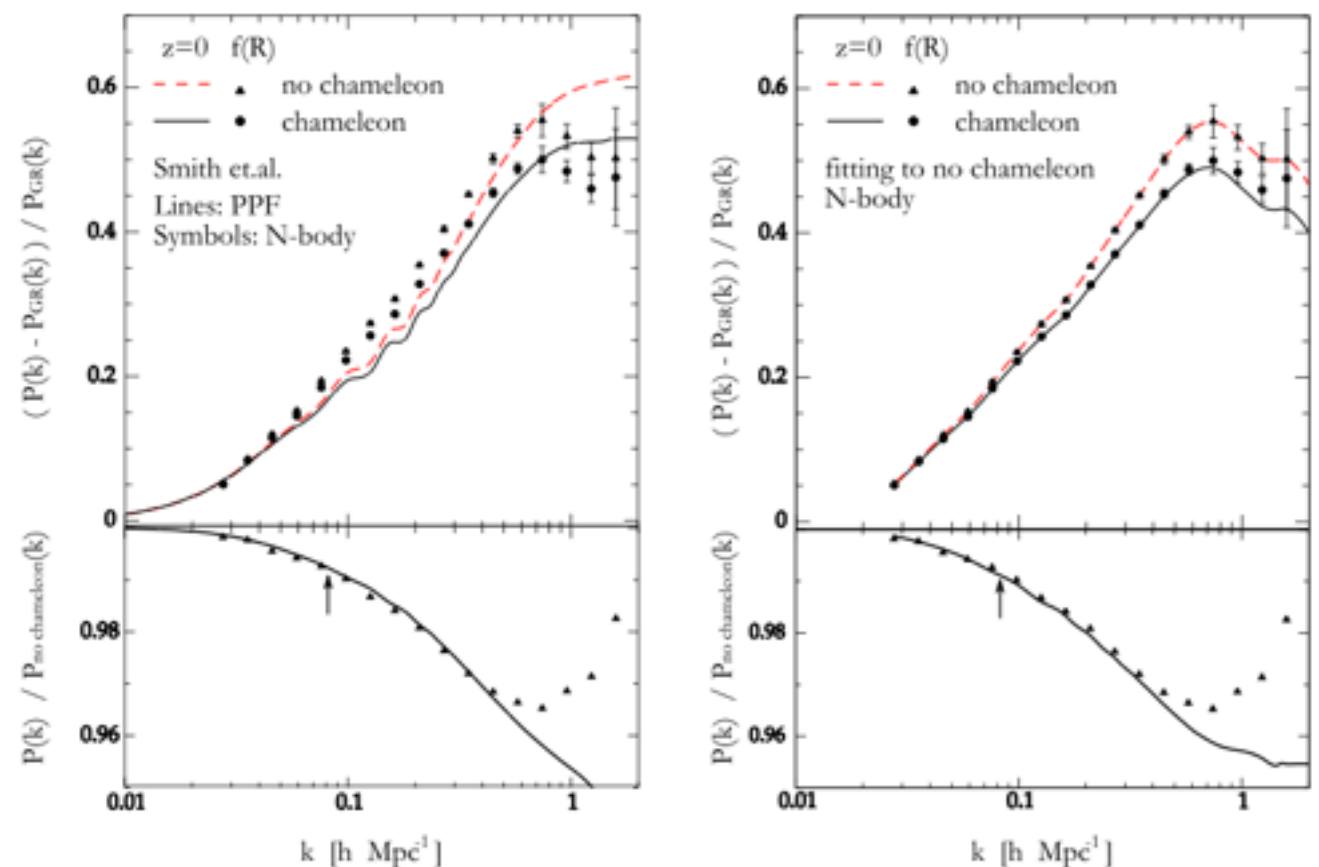
- Perturbation theory cannot be trusted above $k \sim 0.1 h \text{ Mpc}^{-1}$
- Beyond this, we must use alternative approaches (N-body simulations,...?)

- The PPF formalism,

$$P(a, k) = \frac{P_{\text{no-cham}}(a, k) + c_{\text{nl}}(a) \Sigma^2(a, k) P_{\text{GR}}(a, k)}{1 + c_{\text{nl}}(a) \Sigma^2(a, k)}$$

Hu, Sawicki (2007)

- Requires simulations to calibrate!



Koyama, Taruya, Hiramatsu (2009)



Modified gravity: Fisher matrix analysis

- How well can future missions constrain these models?
- Fisher matrix analysis
- Calculate the shear power spectrum for these models

- Take two cuts in the data
$$C_{ij}(l) = \frac{l^4}{4} \int d\chi \frac{W_i(\chi)W_j(\chi)}{\chi^6} P_{(\phi-\psi)}\left(\frac{l}{\chi}, \chi\right)$$

- Conservative $l_{cut} = 400$ (only consider the linear regime)
- Include nonlinear physics $l_{cut} \sim 10000$ (using GR fitting function; used as an indication of possible gain due to using the nonlinear regime only!)

- Parameterize the $f(R)$ function as
$$f_{RR}(a) = \frac{M_0^{-2}}{3} \left(\frac{a^{-3} + 4a_*^{-3}}{1 + 4a_*^{-3}} \right)^{-2v} \quad M_0 > H_0 \quad v > \frac{1}{2}$$

Euclid

$$F_{ij} = \sum_l \frac{\partial C}{\partial p_i} \text{Cov}^{-1} \frac{\partial C}{\partial p_j}$$

$$\sigma_z = \sigma_p(1+z), \sigma_p = 0.03$$

$$C_{ij}(l) = P_{ij} + \langle \gamma_{int}^2 \rangle \frac{\delta_{ij}}{\bar{n}_i}$$

20'000 square degrees

40 gal/arcmin²

5 Tomographic bins

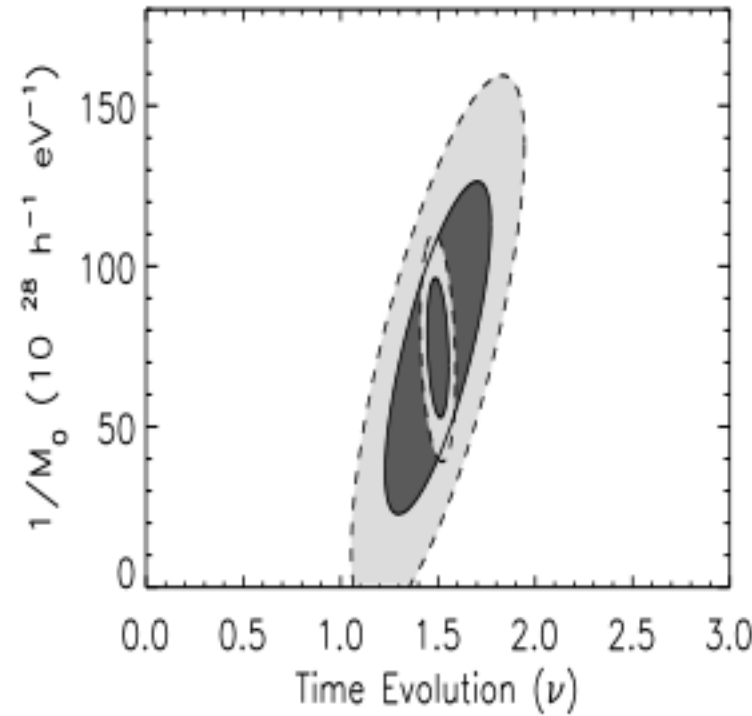
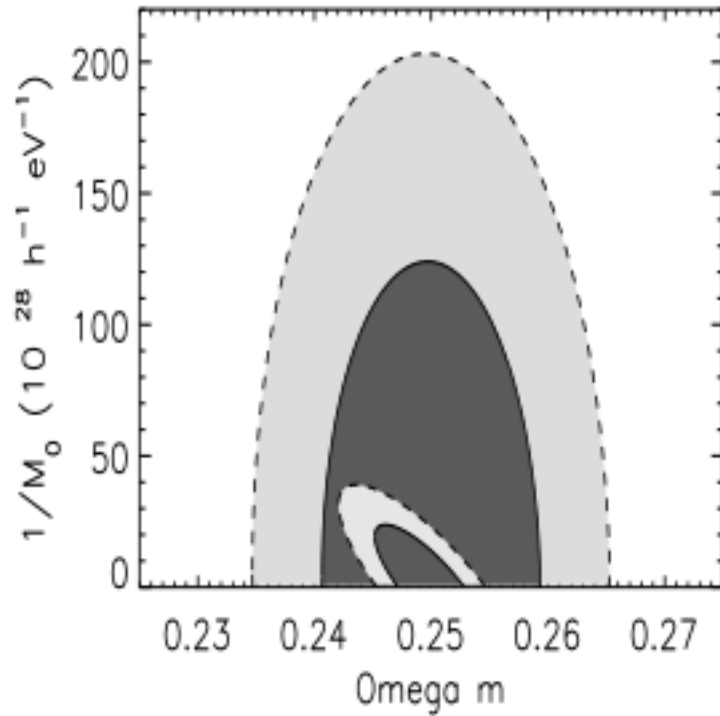
$$n(z) \propto z^\alpha \exp[-(z/z_0)^\beta] \quad z_m = 0.9$$

$$z_0 = z_m/1.412, \alpha = 2, \beta = 1.5.$$

$$\Omega_m, h, \ln(A_S), \Omega_b \text{ and } n_s$$



Modified gravity: Forecast constraints



Fiducial values

$$\Omega_{m0} = 0.25$$

$$\Omega_{b0} = 0.05$$

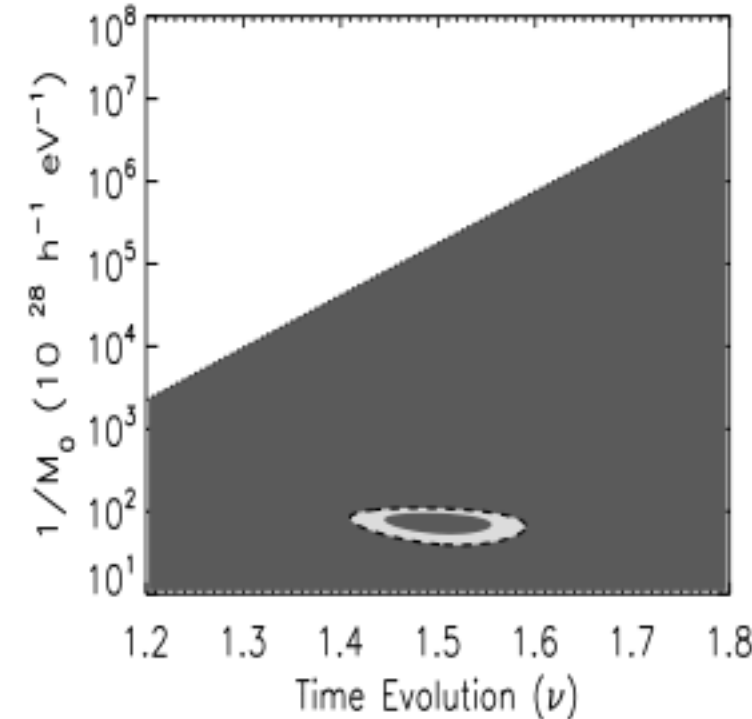
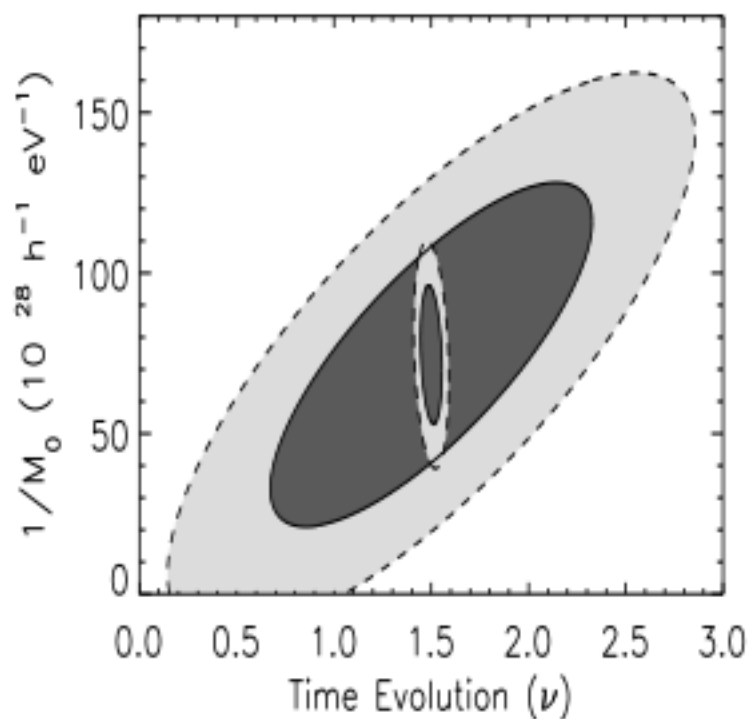
$$h = 0.7$$

$$n_s = 1.0$$

$$\log[A_s] = \log[2.34e - 9]$$

$$M_0^{-1} = 0$$

$$\nu = 1.5$$



$$\Omega_{m0} = 0.25$$

$$\Omega_{b0} = 0.05$$

$$h = 0.7$$

$$n_s = 1.0$$

$$\log[A_s] = \log[2.34e - 9]$$

$$M_0 = 10^2 H_0$$

$$\nu = 1.5$$

$$f_{RR}(a) = \frac{M_0^{-2}}{3} \left(\frac{a^{-3} + 4a_*^{-3}}{1 + 4a_*^{-3}} \right)^{-2\nu}$$



Modified gravity: Conclusion

- The expected constraints on $f(R)$ models from future surveys are at least two orders of magnitude greater than current cosmological bounds.
- These constraints will be tighter than those obtained from local, solar system tests.
- We must be careful to analyse the nonlinear regime correctly!
- The parameter space must be explored in further detail to determine whether the scalar field can roll on cosmological scales.