Recent Results in Two-Boson-Exchange Effects in the Parity-Violating Elastic Electron-Proton Scattering

Hai-Qing Zhou

Southeast University, Nanjing

Collaborators:

K. Nagata, C.W. Kao, Chung Yuan Christian U., Taiwan

S.N. Yang, National Taiwan University., Taiwan

Outline

1. **Motivation**
2. **Strange quark forms factors and parity violating *ep* scattering**
3. **Radiative corrections at zero momentum transfer**
4. **Two boson exchange corrections at finite momentum transfer (hadronic model)**
5. **Two boson exchange corrections by other methods**
6. **Summary**

Motivation: to extract s quark form factor precisely

1.

The experimental data of parity-violation *ep* scattering indicates the non-zero strangeness in the nucleon, the precise data calls for precise theoretical corrections.

2.

The importance of two boson exchange effects has been indicated in the unpolarized parity-conserving elastic *ep* scattering.
(to extract the electromagnetic form factors of proton).

Strange quark forms factors

the strange quark form factors of proton are defined by the following current matrix elements:

$$< P(p')|\bar{s}\gamma_\mu s|P(p)> = \bar{u}(p')[F_1^s\gamma_\mu + F_2^s\frac{i\sigma_{\mu\nu}}{2M}q^\nu]u(p)$$

$$< P(p')|\bar{s}\gamma_\mu\gamma_5 s|P(p)> = \bar{u}(p')[G_A^s\gamma_\mu\gamma_5 + G_P^s\frac{1}{2M}q_\mu]u(p)$$

Parity violating *ep* scattering

the parity violating elastic electron-proton scattering

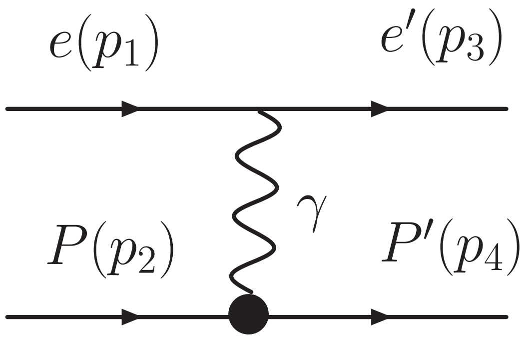
$$e(p_1, R/L) + P(p_2) \rightarrow e(p_3) + P(p_4)$$

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

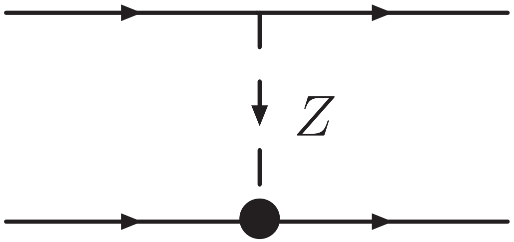
A_{PV} is about 10^{-6} at low momentum tranfer

precise experiment

Relation: tree level, one boson exchange diagrams



(a)



(b)

$$M^{(a)} = \frac{e^2}{Q^2} L^{\mu,\gamma} H_\mu^\gamma$$

$$M^{(b)} = -\frac{eg}{Q^2 + M_z^2} L^{\mu,Z} H_\mu^Z$$

Relation at tree level: current matrix elements

$$L^{\mu,\gamma} = \bar{u}(p_3, m_e)\gamma^\mu \bar{u}(p_1, m_e) \quad L^{\mu,\gamma} = \bar{u}(p_3, m_e)\gamma^\mu (1 - 4\sin^2\theta_W + \gamma_5)\bar{u}(p_1, m_e)$$

$$H_\mu^\gamma = < P(p_4) | J_\mu^\gamma | P(p_2) > \quad H_\mu^\gamma = < P(p_4) | J_\mu^Z | P(p_2) >$$

$$< P(p') | J_\mu^\gamma | P(p) > = \bar{u}(p')[F_1^{\gamma,P}\gamma_\mu + F_2^{\gamma,P}\frac{i\sigma_{\mu\nu}}{2M}q^\nu]u(p)$$

$$< P(p') | J_\mu^Z | P(p) > = \bar{u}(p')[F_1^{Z,P}\gamma_\mu + F_2^{Z,P}\frac{i\sigma_{\mu\nu}}{2M}q^\nu + G_A^Z\gamma_\mu\gamma_5]u(p)$$

Relation at tree level : form factors and A_{PV}

$$A_{PV}^{1\gamma+Z} = -\frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{A_E^{OBE} + A_M^{OBE} + A_A^{OBE}}{[\varepsilon(G_E^{\gamma,P})^2 + \tau(G_M^{\gamma,P})^2]}$$

$$A_E^{OBE} = \varepsilon G_E^{Z,P} G_E^{\gamma,P}; \qquad A_M^{OBE} = \tau G_M^{Z,P} G_M^{\gamma,P}$$

$$A_A^{OBE} = -(1-4\sin^2\theta_W)\sqrt{\tau(1+\tau)(1-\varepsilon^2)}\, G_A^Z G_M^{\gamma,P}$$

$$G_E^{\gamma(Z),P} = F_1^{\gamma(Z),P} - \tau F_2^{\gamma(Z),P}$$

$$G_M^{\gamma(Z),P} = F_1^{\gamma(Z),P} + F_2^{\gamma(Z),P}$$

Relation at tree level : strange quark form factors

assuming the charge symmetry

$$G_{E,M}^{u,d,s/p} = G_{E,M}^{d,u,s/p}$$

and use quark contents of proton at tree level

$$J_\mu^{em} = \sum_{f=u,d,s} Q_f \bar{q}_f \gamma_\mu q_f ; \quad J_\mu^Z = \sum_f \bar{q}_f (g_V^f + g_A^f \gamma_5) q_f$$

Relation at tree level: strange quark ffs and A_{PV}

$$A_{PV}^{1\gamma+Z} = A_1 + A_2 + A_3$$

$$A_1 = -a[(1-4\sin^2\theta_W) - \frac{\varepsilon G_E^{\gamma,P} G_E^{\gamma,n} + \tau G_M^{\gamma,P} G_M^{\gamma,n}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}]$$

$$A_2 = a \frac{\varepsilon G_E^{\gamma,P} G_E^{s} + \tau G_M^{\gamma,P} G_M^{s}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}$$

$$A_3 = a(1-4\sin^2\theta_W) \frac{\varepsilon' G_M^{\gamma,P} G_A^{Z}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}$$

EM and axial form factors of proton and neutron can be measured from other experiments………. Then the strange quark form factors can be measured from A_{PV} .

Strange quark ffs and A_{PV} : angle dependence

$$a = G_F Q^2 / 4\pi\alpha\sqrt{2}, \qquad \varepsilon = [1 + 2(1+\tau)\tan^2\theta_L/2]^{-1},$$

$$\tau = Q^2 / 4M_N^2, \qquad \varepsilon' = \sqrt{\tau(1+\tau)(1-\varepsilon^2)},$$

angular dependence of A_{PV}

angle		remaining form factors
forward	$\varepsilon \to 1$	$G_E^s + \beta G_M^s$
backward	$\varepsilon \to 0$	$G_M^s,\ G_A^Z$

To exact ffs more precisely, one loop corrections…

Radiative corrections: zero momentum transfer

radiative corrections can be described by effective interaction

$$H_{PV}^{eHadron} = -\frac{G_F}{\sqrt{2}} \sum_i [C_{1i} \bar{e}\gamma_\mu \gamma_5 e \bar{q}_i \gamma^\mu q_i + C_{2i} \bar{e}\gamma_\mu e \bar{q}_i \gamma^\mu \gamma_5 q_i]$$

Marciano, Sirlin, PRD(1983), PRD(1984); W.-M.Yao, JPG69,1(2006))

with C_{1i} and C_{2i} matched from

$$e(p_1) + q(p_2') \rightarrow e(p_3) + q(p_4')$$

at one loop level with zero momentum transfer approximation . This approximation results in momentum-independent correction.

S quark ffs and A_{PV} : after radiative corrections

$$A_{PV}(\rho,\kappa) = A_1 + A_2 + A_3$$

$$A_1 = -a\rho[(1-4\kappa\sin^2\theta_W) - \frac{\varepsilon G_E^{\gamma,P} G_E^{\gamma,n} + \tau G_M^{\gamma,P} G_M^{\gamma,n}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}]$$

$$A_2 = a\rho \frac{\varepsilon G_E^{\gamma,P} G_E^{s} + \tau G_M^{\gamma,P} G_M^{s}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}$$

$$A_3 = a(1-4\sin^2\theta_W)\frac{\varepsilon' G_M^{\gamma,P} G_A^{Z}}{\varepsilon (G_E^{\gamma,P})^2 + \tau (G_M^{\gamma,P})^2}$$

is used to extract the strange quark form factors from experiment data in HAPPEX, A4.

$$C_{1u} = \rho(-\frac{1}{2} + \frac{4}{3}\kappa\sin^2\theta_W); \quad C_{1d} = \rho(\frac{1}{2} - \frac{2}{3}\kappa\sin^2\theta_W)$$

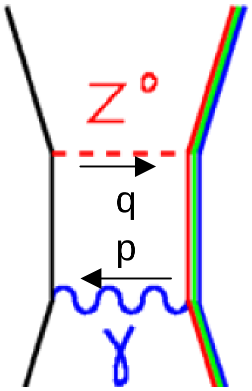
PDG values:

$\rho = 0.9876$, $\kappa = 1.0026$

Zero momentum transfer: 2 boson exchange case

Marciano, Sirlin PRD (1984)

p = q = k $\rightarrow$ $Q^2 = (p-q)^2 \equiv 0$



$$\Delta\rho = \frac{\alpha}{2\pi}4(1-4s^2)\left[\ln\left(\frac{m_z^2}{M^2}\right)+\frac{3}{2}\right],$$

$$\Delta\kappa = \frac{\alpha}{2\pi s^2}\left(\frac{9}{4}-4s^2\right)(1-4s^2)\left[\ln\left(\frac{m_Z^2}{M^2}\right)+\frac{3}{2}\right]$$

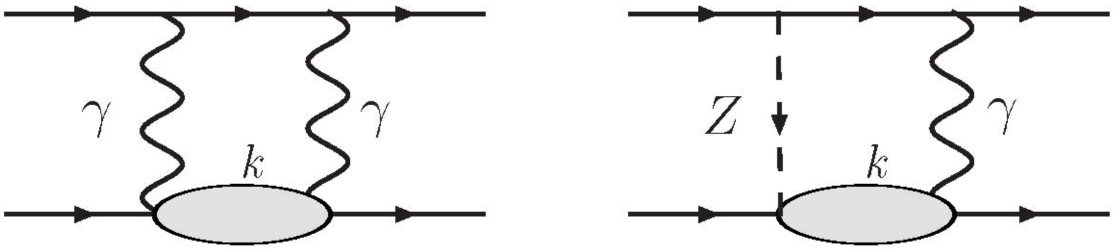
$$\Delta\rho = -3.7\times10^{-3}$$

$$\Delta\kappa = -5.3\times10^{-3}$$

Among the one loop correction, the box diagrams are most interesting due to its angular dependence. It has been indicated in the un-polarized elastic electron-proton scattering case.

TBE corrections: finite momentum transfer

when go beyond zero momentum transfer approximation, hadronic structure should be considered (model dependent)



cross diagrams are implied

Simple hadronic model: take the intermediate states as nucleon and Delta(1232)………,and use effective interactions.

P. G. Blunden et al.PRL(2003), S. Kondratyuk et al, PRL(2005).

Effective vertices : in hadronic model

effective vertices

$$\Gamma_{NN\gamma} = ie[F_1\gamma_\mu + \frac{iF_2}{2M_N}\sigma_{\mu\rho}q_R^\rho]$$

$$\Gamma_{NNZ} = -ig[G_1\gamma_\mu + \frac{iG_2}{2M_N}\sigma_{\mu\rho}q^\rho + G_A\gamma_\mu\gamma_5]$$

$$\Gamma_{\gamma\Delta\to N} = -i\sqrt{\frac{2}{3}}\frac{e}{M_N^2}[g_1(g_{\mu\alpha}\not{k}\not{q} - k_\mu\gamma_\alpha\not{q} - \gamma_\mu\gamma_\alpha k\cdot q + \gamma_\mu\not{k}q_\alpha) +$$

$$g_2(k_\mu q_\alpha^\mu - g_{\mu\alpha}k\cdot q) +$$

$$\frac{g_3}{M_N}(q\cdot q(k_\mu\gamma_\alpha - g_{\mu\alpha}\not{k}) + q_\mu(q_\alpha\not{k} - \gamma_\alpha k\cdot q))]$$

q : momentum of incoming photon

k : momentum of incoming Delta(1232)

Effective vertexes : in hadronic model

$$\Gamma_{Z\Delta\to N} = -i\frac{g}{M_N^2}4\cos\theta_W[\{\tilde{g}_1(g_{\mu\alpha}\not{k}\not{q}^R - k_\mu\gamma_\alpha\not{q} - \gamma_\mu\gamma_\alpha k\cdot q + \gamma_\mu\not{k}q_\alpha) +$$

$$\tilde{g}_2(k_\mu q_\alpha^\mu - g_{\mu\alpha}k\cdot q) +$$

$$\frac{\tilde{g}_3}{M_N}(q\cdot q(k_\mu\gamma_\alpha - g_{\mu\alpha}\not{k}) + q_\mu(q_\alpha\not{k} - \gamma_\alpha k\cdot q))\} +$$

$$\{h_1(g_{\mu\alpha}k\cdot q - k_\mu q_\alpha) + \frac{h_2}{M_N^2}(q_\mu q_\alpha\not{k}\not{q} - k\cdot q q_\mu\gamma_\alpha\not{q}) +$$

$$h_3(k\cdot q\gamma_\alpha\gamma_\mu - \not{k}\gamma_\mu q_\alpha) + h_4(g_{\mu\alpha}\not{k}\not{q} - k_\mu\gamma_\alpha q_\alpha)\}]$$

parameters from other experiments

TBE corrections to A_{PV} : results in hadronic model

define the corrections as

different interference

$$\delta \equiv \frac{A_{PV}^{\gamma+Z+2\gamma+\gamma Z} - A_{PV}^{\gamma+Z}}{A_{PV}^{\gamma+Z}}$$

$$\approx \underbrace{\delta_N^{\gamma(2\gamma)} + \delta_N^{Z(2\gamma)} + \delta_N^{\gamma(\gamma Z)}}_{\text{N intermediate}} + \underbrace{\delta_\Delta^{\gamma(2\gamma)} + \delta_\Delta^{Z(2\gamma)} + \delta_\Delta^{\gamma(\gamma Z)}}_{\text{Delta intermediate}}$$

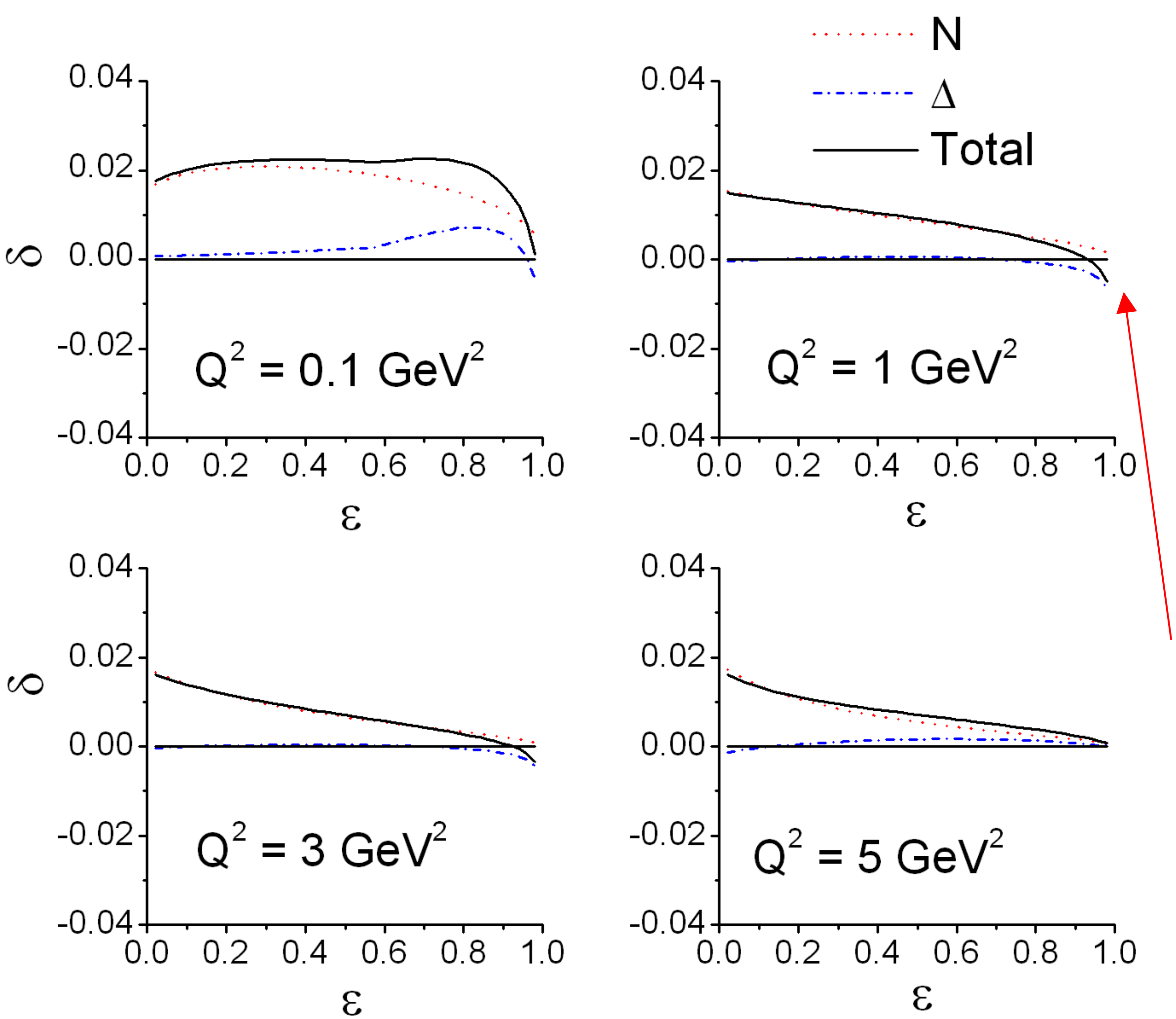
N intermediate

Delta intermediate

the IR divergence is removed using the standard method of Mo & Tasi.

L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41, 205 (1969).

TBE corrections to A_{PV} : results in hadronic model

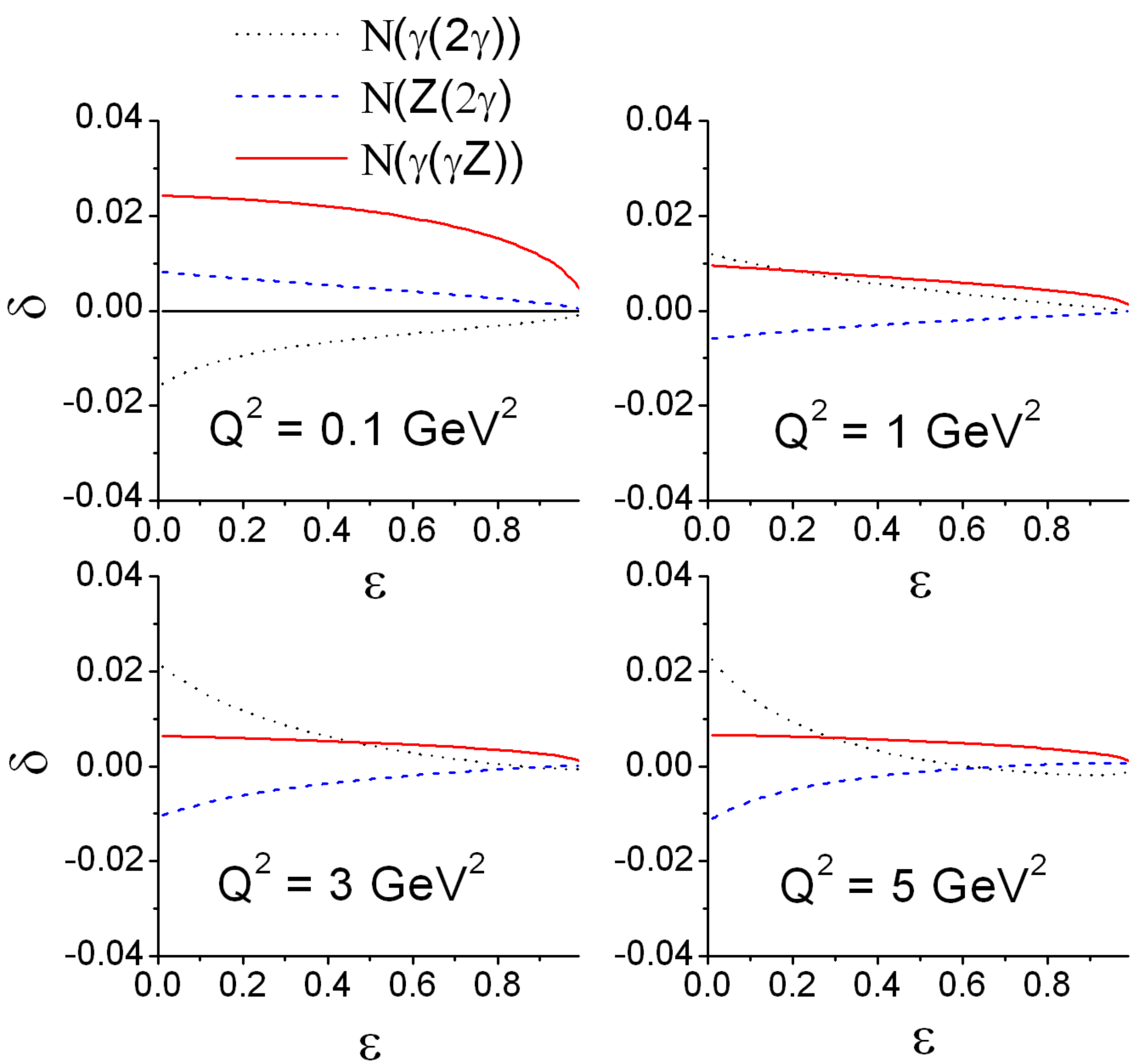


depends on the angle and Q^2 strongly.

about 2% at small ε where N gives the main contribution.

relatively smaller at large ε where Delta gives main contribution.

TBE corrections to A_{PV} : results in detail

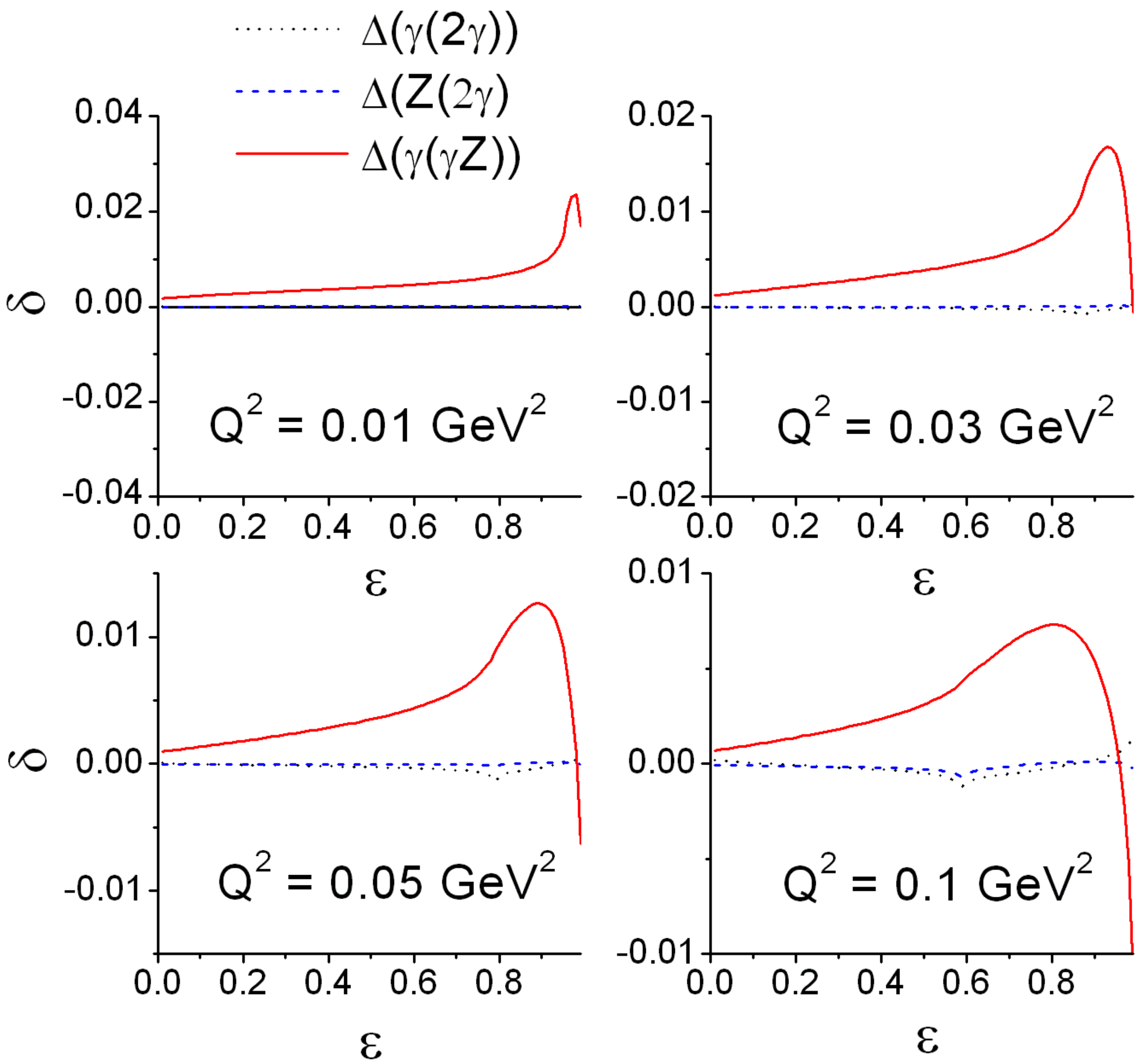


large in small ε
small in large ε

usually,the contribution from $\gamma(2\gamma)$ is cancelled by $Z(2\gamma)$, this leads the small total 2 γcorrection.

H-Q. Zhou et al. PRL (2007)

TBE corrections to A_{PV} : results in detail

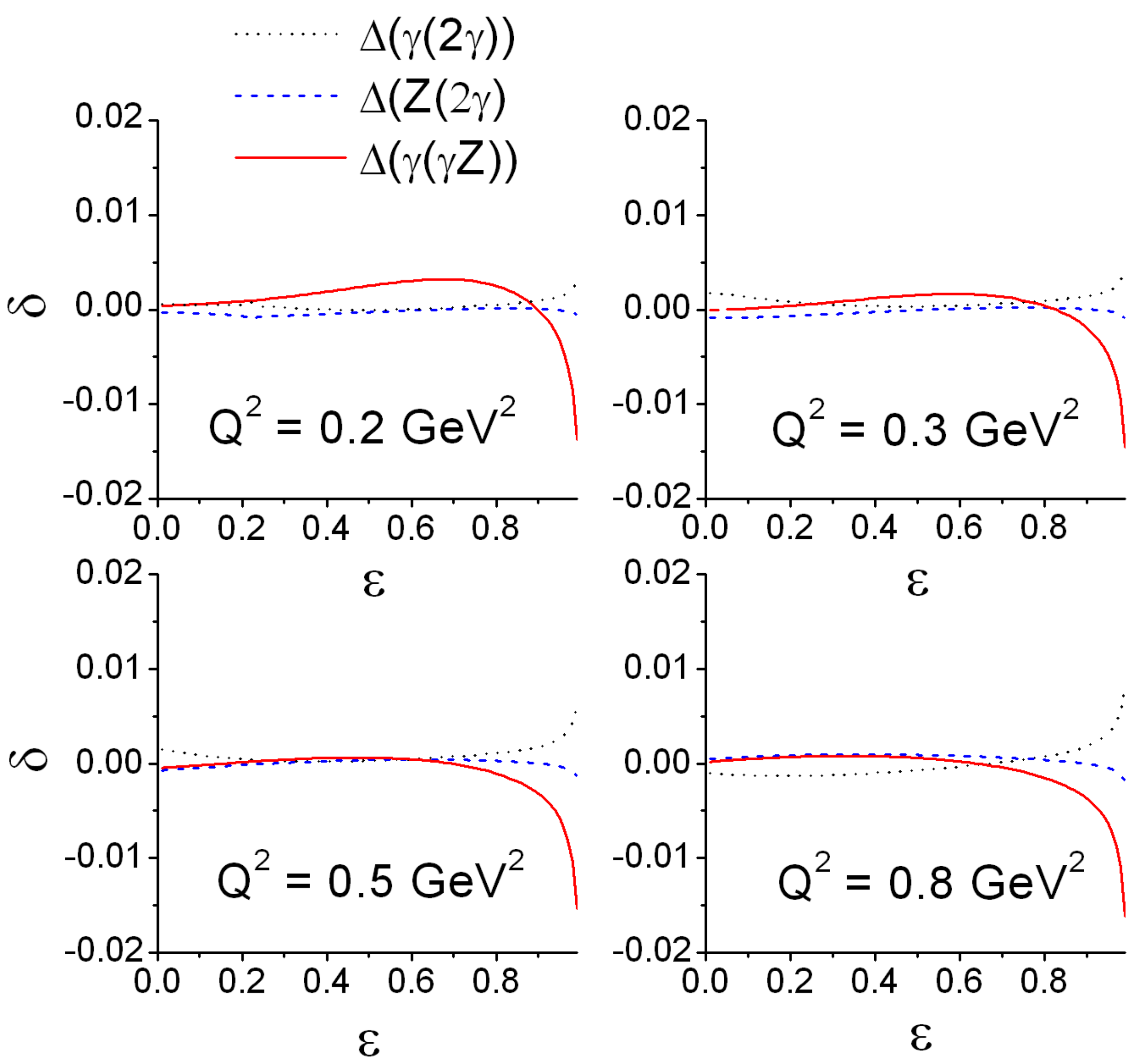


the main contribution is from $\gamma(\gamma Z)$.

relatively large at small Q^2 and large epsilon.

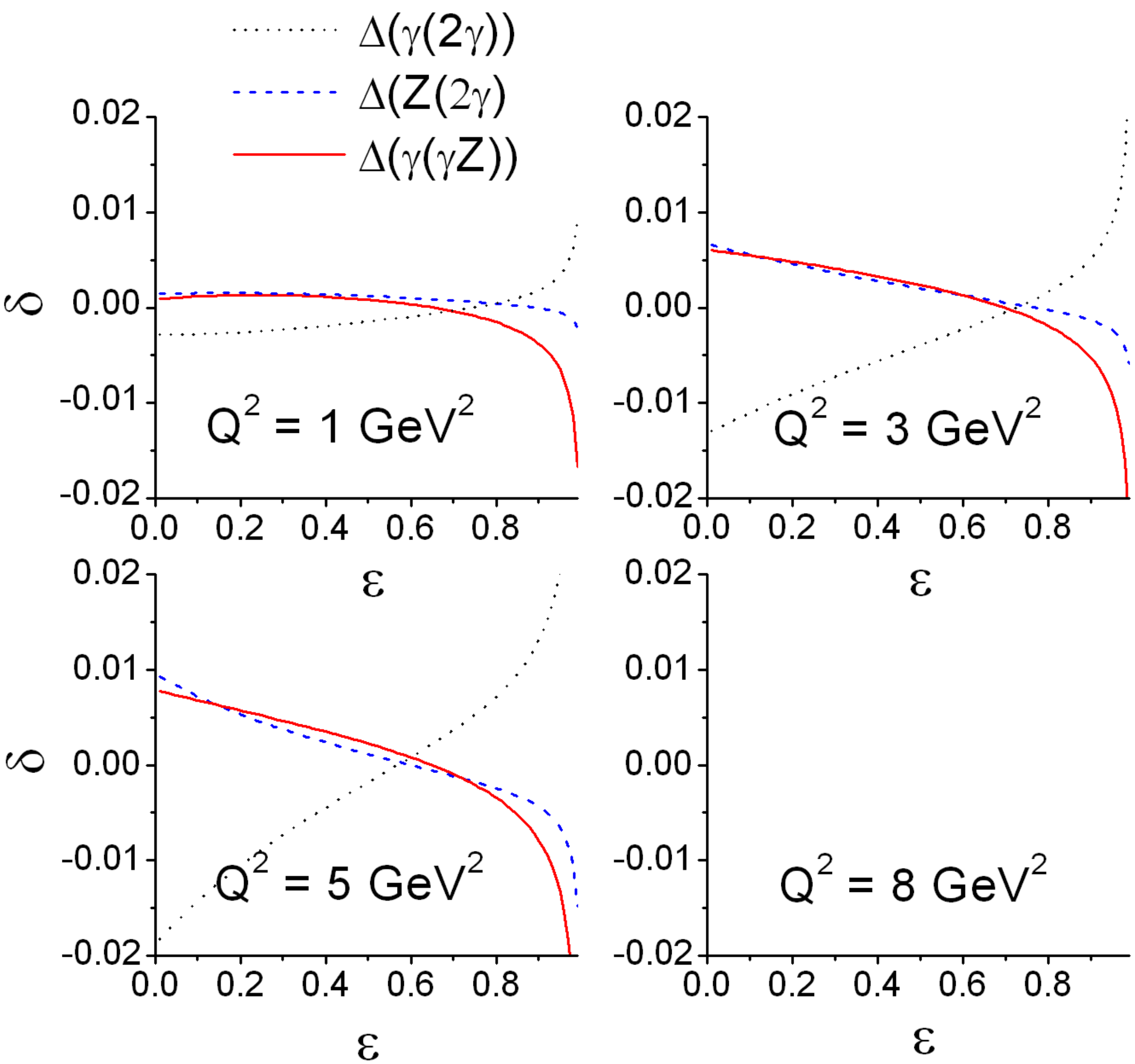
Keitaro Nagata et al. ar**Xiv:0811.3539**

TBE corrections to A_{PV} : results in detail



small in middle Q^2
and small epsilon.

TBE corrections to A_{PV} : results in detail



large cancellation leads small total correction.

Corrections to strange quark ffs: expressions

To avoid double counting: TBE corrections at zero momentum transfer should be subtracted

$$\rho' = \rho - \Delta\rho, \qquad \kappa' = \kappa - \Delta\kappa$$

Then

$$G_E^s + \beta G_M^s = \frac{\epsilon (G_E^{\gamma,p})^2 + \tau (G_M^{\gamma,p})^2}{a\rho\epsilon G_E^{\gamma,p}} [A_{PV}^{Exp} - A_1(\rho,\kappa) - A_3],$$

← **old**

$$\overline{G}_E^s + \beta \overline{G}_M^s = \frac{\epsilon (G_E^{\gamma,p})^2 + \tau (G_M^{\gamma,p})^2}{a\rho'\epsilon G_E^{\gamma,p}} \left[\frac{A_{PV}^{Exp}}{1+\delta} - A_1(\rho',\kappa') - A_3\right]$$

← **new**

Corrections to strange quark ffs: results

define $$\delta_G \triangleq \frac{\overline{G}_E^s + \beta \overline{G}_M^s}{G_E^s + \beta G_M^s} - 1$$

	Q^2	ε	$\delta_{N+\Delta}$ (%)	δ_G (%)
HAPPEX	0.477	0.974	-0.33	-25.52
	0.109	0.994	-1.15	-75.23
A4	0.23	0.83	1.07	-2.76
	0.108	0.83	1.97	-2.27

Corrections to strange quark ffs: G0 case

Q^2	ε	$\delta_{N+\Delta}(\%)$	A_{Exp}	$\delta_G(\%)$	$G_E^s + \eta G_M^s$
0.122	0.993	-1.06	-1.51	-21.7	0.01765
0.128	0.993	-1.02	-0.97	-6.49	0.05398
0.136	0.992	-0.97	-1.3	-8.42	0.04132
0.144	0.992	-0.93	-2.71	17.9	-0.0208
0.153	0.991	-0.89	-2.22	-31.0	0.01106
0.164	0.990	-0.84	-2.88	54.7	-0.00615
0.177	0.990	-0.79	-3.95	10.2	-0.03244
0.192	0.989	-0.73	-3.85	21.3	-0.01431

Corrections to strange quark ffs: G0 case

Q^2	ε	$\delta_{N+\Delta}(\%)$	A_{Exp}	$\delta_G(\%)$	$G_E^s+\eta G_M^s$
0.21	0.9875	-0.681	-4.68	12.597	-0.02285
0.232	0.986	-0.615	-5.27	13.122	-0.02012
0.262	0.984	-0.542	-5.26	170.4	-0.00137
0.299	0.9814	-0.465	-7.72	9.131	-0.02284
0.344	0.9783	-0.392	-8.4	16.495	-0.01089
0.41	0.9735	-0.304	-10.25	20.621	-0.00714
0.997	0.9197	0.0539	-37.9	4.233	-0.00919

Comparing: results by Tjon et al. arXiv:0903.2759

If take the same parameters and relations, get almost the same results for N case and PC parts of Delta(1232) case, the PV parts of Delta(1232) are under checking and are expected to be consistent,

H-Q. Zhou et al. PRL (2007),
Keitaro Nagata et al. arXiv:0811.3539

P.G. Blunden et al.PRL(2003),
S. Kondratyuk et al, PRL(2005).
J.A.Tjon et al PRL(2008).
J.A.Tjon et al. arXiv:0903.2759

(but.........)

Comparing: by different parameters arXiv:0903.2759

Q^2	ε	$\delta_{N+\Delta}(\%)$	A_{Exp}	$\delta_G(\%)$	$G_E^s+\eta G_M^s$	$\delta_{N+\Delta}(\%)$
0.122	0.993	-1.06	-1.51	-21.7	0.01765	-0.88
0.128	0.993	-1.02	-0.97	-6.49	0.05398	-0.85
0.136	0.992	-0.97	-1.3	-8.42	0.04132	-0.81
0.144	0.992	-0.93	-2.71	17.9	-0.0208	-0.79
0.153	0.991	-0.89	-2.22	-31.0	0.01106	-0.75
0.164	0.990	-0.84	-2.88	54.7	-0.00615	-0.71
0.177	0.990	-0.79	-3.95	10.2	-0.03244	-0.67
0.192	0.989	-0.73	-3.85	21.3	-0.01431	-0.63

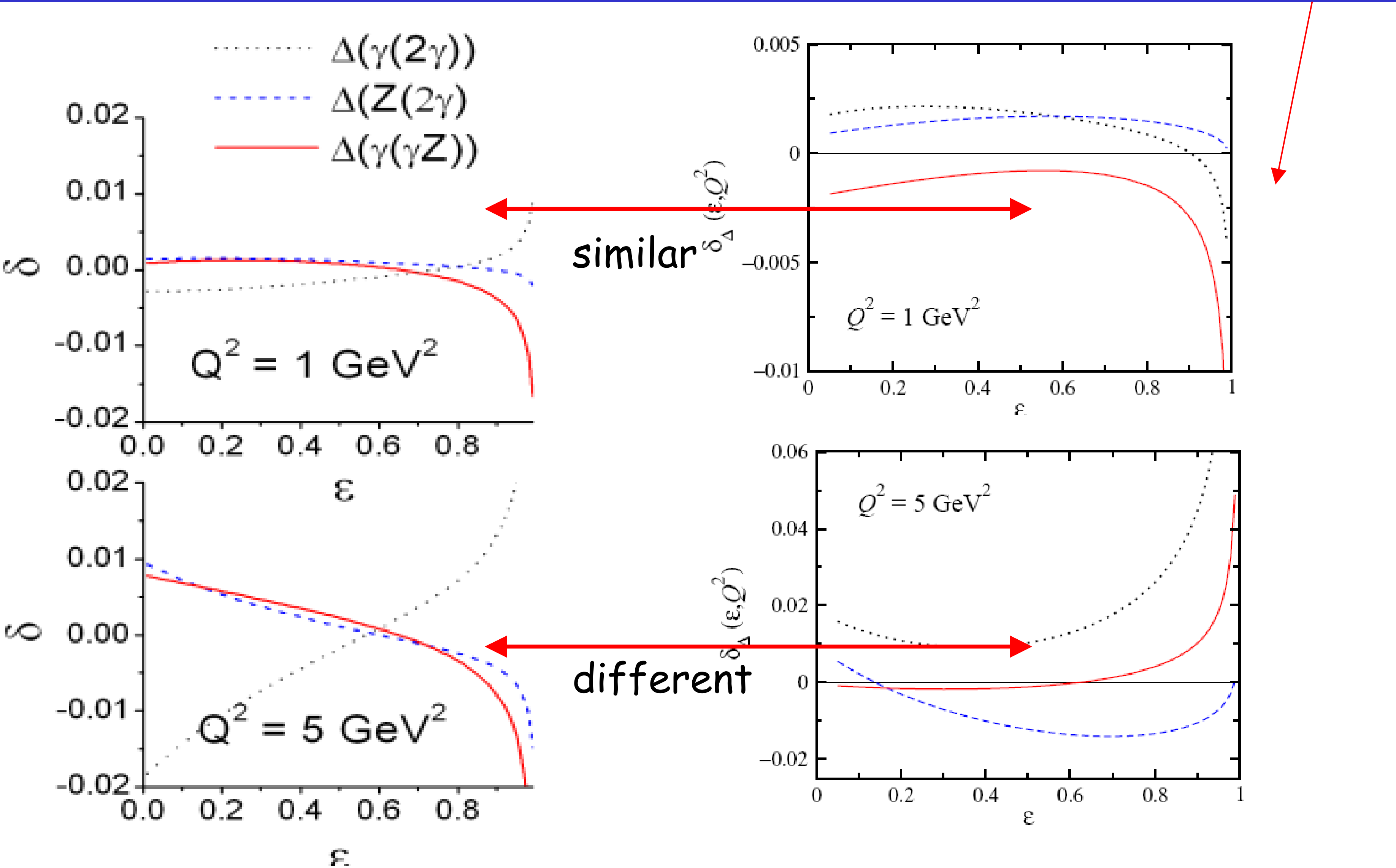
about 20% difference

Comparing: by different parameters arXiv:0903.2759

Q^2	ε	$\delta_{N+\Delta}(\%)$	A_{Exp}	$\delta_G(\%)$	$G_E^s + \eta G_M^s$	$\delta_{N+\Delta}(\%)$ (arXiv:0903.2759)
0.21	0.9875	-0.681	-4.68	12.597	-0.02285	-0.57
0.232	0.986	-0.615	-5.27	13.122	-0.02012	-0.52
0.262	0.984	-0.542	-5.26	170.4	-0.00137	-0.47
0.299	0.9814	-0.465	-7.72	9.131	-0.02284	-0.40
0.344	0.9783	-0.392	-8.4	16.495	-0.01089	-0.33
0.41	0.9735	-0.304	-10.25	20.621	-0.00714	-0.26
0.997	0.9197	0.0539	-37.9	4.233	-0.00919	-0.05

about 20% difference

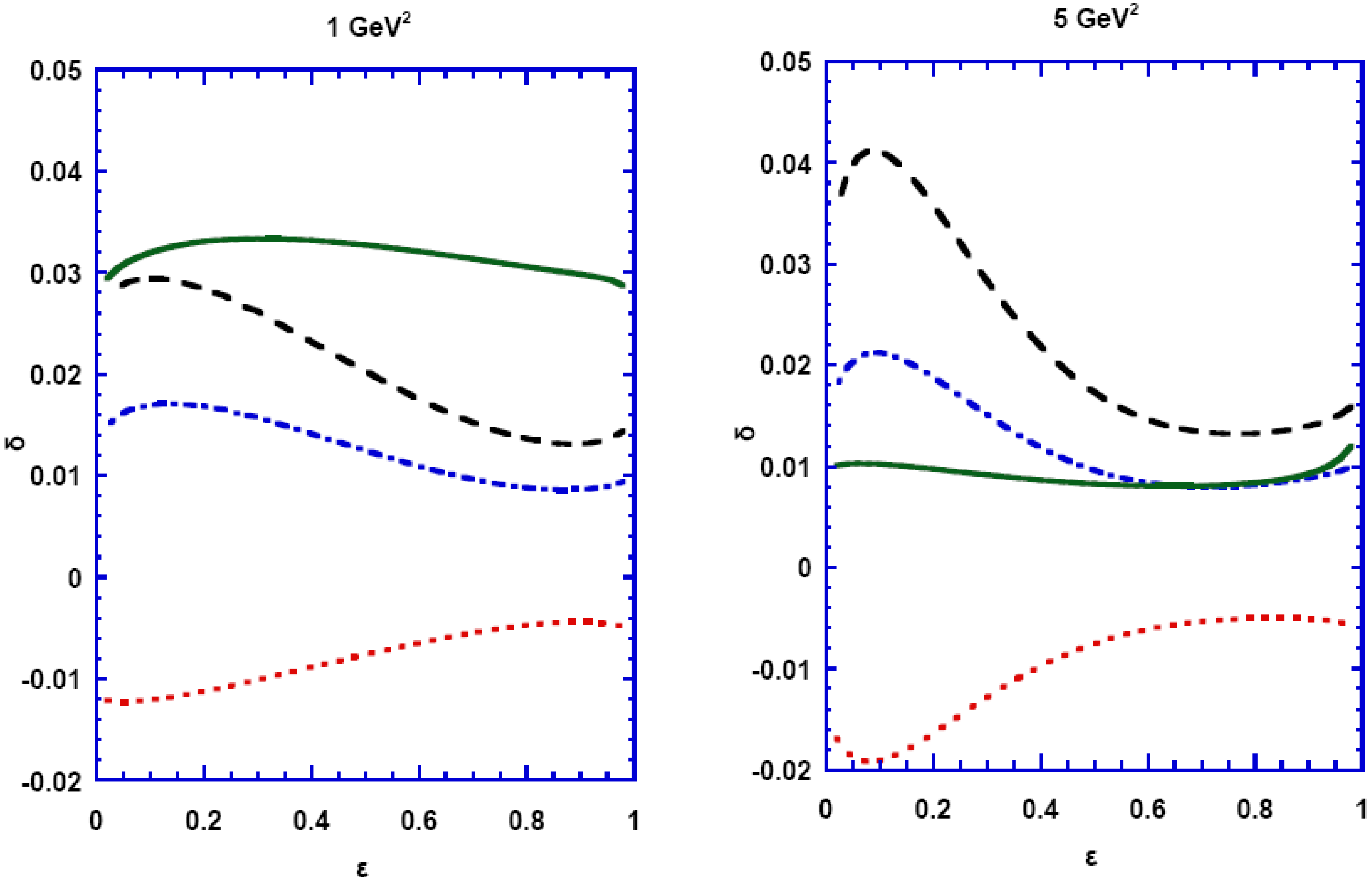
Comparing: by different parameters arXiv:0903.2759



our results. There is a minus difference in the definition of $\gamma(2\gamma)$

Box diagrams corrections in other methods

GPDs: YC.Chen et al arXiv 0903.1098



2γ :consistent

γZ :different

large momentum region in hadronic model

dash line: $\gamma(2\gamma)$, dot line: $Z(2\gamma)$, solid line: $\gamma(\gamma Z)$.

dash-dot: 2gamma

Other methods: GPDs, YC.Chen et al arXiv 0903.1098

On the contrary, the result of the hadronic model [10] shows that both of $\delta_{2\gamma}$ and $\delta_{\gamma Z}$ have very strong ε dependence and always decrease into zero when ε approaches one. Such a difference may be explained by the following: In the hadronic model the loop momentum integration is mostly from the low loop momentum because the form factors are inserted as regulators. However, in the partonic calculation the loop momentum integration is dominated by the high loop momentum. Naively one should add the results of the two calculation together. How to combine the results of these two calculations remains an open issue.

Box diagrams corrections in other methods

dispersion calculation of gamma Z correction to Qweak

M. Gorchtein and C. J. Horowitz, PRL 102, 091806 (2009)

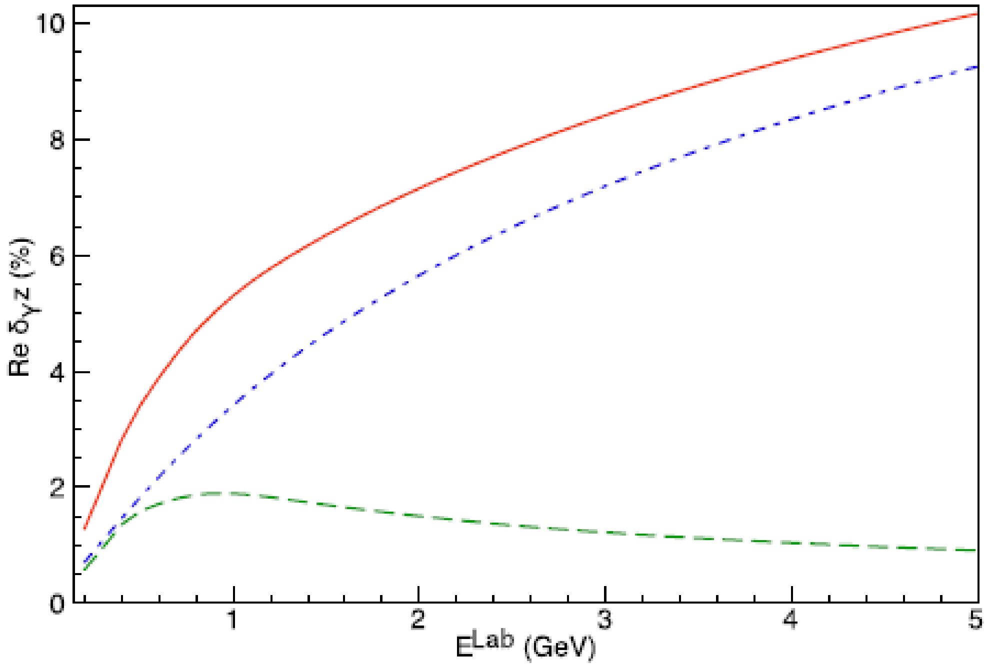


FIG. 3 (color online). Results for $\mathrm{Re}\,\delta_{\gamma Z_A}$ as a function of energy. The contributions of nucleon resonances (dashed line), the Regge part (dashed-dotted line), and the sum of the two (solid line) are shown.

has not been compared in detail

Summary

- The TPE and γ Z-exchange corrections to the PV asymmetry of elastic *ep* scattering with elastic intermediate states can reach a few percent and are compatible with the current experimental measurements of the strange effects in the proton neutral weak current. The effects on the extracted values of $G_E^s + \beta G_M^s$ are large.

- More detailed analysis and discussions combining the different methods are needed.