Λ resonances studied in $K^- p \to \pi^0 \Sigma^0$ near threshold in a chiral quark model

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Outline

1. Motivation

2. The chiral quark model

3. The role of Λ resonances in the reaction

4. Conclusions

1. Motivation

- There are puzzles for the Λ resonances. For example, are the $\Lambda(1405)$, $\Lambda(1670)$ three quark states?[*] or multi-quark structures? [†]

- The $K^- p \to \Sigma^0 \pi^0$ gives us a rather clean channel to study the Λ resonances, for there are no isospin-1 baryons contributing here.

- The recent higher precise data of the reaction $K^- p \to \Sigma^0 \pi^0$ at eight momentum beams between 514 and 750 MeV/c were reported, which provides us a good opportunity to study the properties of these low-lying Λ resonances. [‡] .
- The chiral quark model has been used to study the $\pi^- p \to \eta n$ successfully, which also can be extended to analyze the $K^- p \to \Sigma^0 \pi^0$.[§]

[*] Isgur and Karl,PRD **18**, 4187 (1978)

[†] Oset and Ramos, NPA **635**, 99 (1998); Oller and Meissner,PLB **500**, 263 (2001)

[‡] R. Manweiler *et al.*, PRC **77**, 015205 (2008)

[§] Zhong, Zhao, He and Saghai, PRC **76**, 065205 (2007)

2. The chiral quark model

- The quark-meson couplings
- The transition amplitudes
- Separation of the resonance contributions

The quark-meson couplings

- At the leading order of the chiral Lagrangian, the quark-meson coupling is given by [¶]

$$H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu^j \gamma_5^j \psi_j \vec{\tau} \cdot \partial^\mu \vec{\phi}_m. \quad (1)$$

where ψ_j represents the j-th quark field in a hadron, the ϕ_m stands for the pseudoscalar-meson octet in the SU(3) case.

- The non-relativistic form of Eq.(1) can be written as

$$H_m^{nr} = \sum_j \{ \frac{\omega_m}{E_f + M_f} \sigma_j \cdot \mathbf{P}_f + \frac{\omega_m}{E_i + M_i} \sigma_j \cdot \mathbf{P}_i - \sigma_j \cdot \mathbf{q} + \frac{\omega_m}{2\mu_q} \sigma_j \cdot \mathbf{p}'_j \} I_j \varphi_m, \quad (2)$$

where $\mathbf{P}_i, \mathbf{P}_f$ are the three-vector momenta of the initial and final baryons, respectively. The ω_m and $\mathbf{q}$ are the energy and three-vector momentum of the light meson, respectively. $\mathbf{p}'_j$ is the internal momentum for the j-th quark in the initial meson rest frame. σ_j corresponds to the Pauli spin vector of the

[¶] Z. P. Li, H. X. Ye and M. H. Lu, PRC **56**, 1099 (1997); Q. Zhao, J. S. Al-Khalili, Z. P. Li and R. L. Workman, PRC **65**, 065204 (2002)

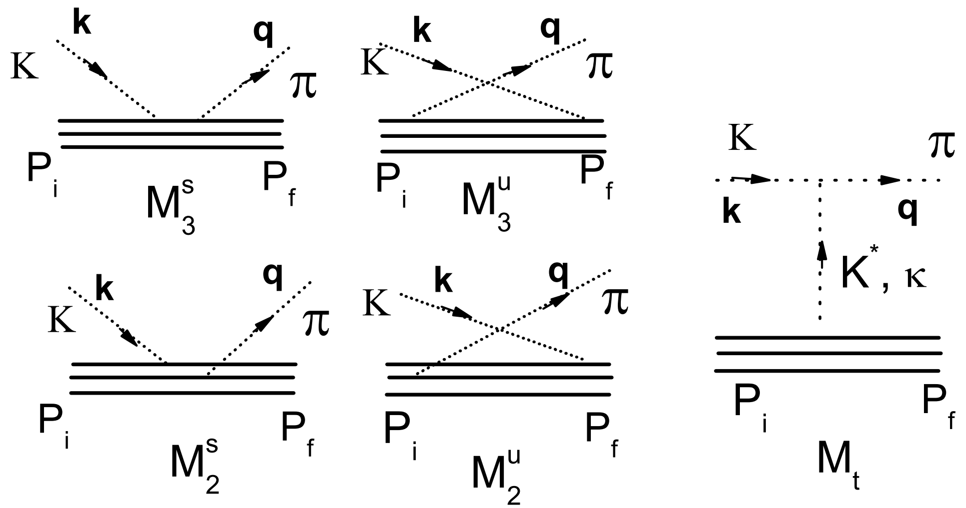
j-th quark in a baryon.The isospin operator I_j in Eq. (2) is expressed as

$$I_j = \begin{cases} a_j^\dagger(u)a_j(s) & \text{for } K^+, \\ a_j^\dagger(s)a_j(u) & \text{for } K^-, \\ a_j^\dagger(d)a_j(s) & \text{for } K^0, \\ a_j^\dagger(s)a_j(d) & \text{for } \bar{K}^0, \\ a_j^\dagger(u)a_j(d) & \text{for } \pi^+, \\ a_j^\dagger(d)a_j(u) & \text{for } \pi^-, \\ \frac{1}{\sqrt{2}}[a_j^\dagger(u)a_j(u) - a_j^\dagger(d)a_j(d)] & \text{for } \pi^0, \end{cases} \tag{3}$$

where $a_j^\dagger(u,d,s)$ and $a_j(u,d,s)$ are the creation and annihilation operators for the u, d and s quarks.

The transition amplitudes

- The Feynman diagrams for the $K^-p \to \Sigma^0\pi^0$ reaction :



The s-channel amplitudes

- The s-channel amplitudes can be expressed as

$$\mathcal{M}_s = \sum_j \langle N_f | H_\pi | N_j \rangle \langle N_j | \frac{1}{E_i + \omega_K - E_j} H_K | N_i \rangle, \tag{4}$$

one can then express the s-channel amplitudes by operator expansions:

$$\mathcal{M}_s = \sum_j \langle N_f | H_\pi | N_j \rangle \langle N_j | \sum_n \frac{1}{\omega_K^{n+1}} (\hat{H} - E_i)^n H_K | N_i \rangle \ , \tag{5}$$

where n is the principle harmonic oscillator quantum number. Note that for any operator $\hat{\mathcal{O}}$, one has

$$(\hat{H} - E_i)\hat{\mathcal{O}} | N_i \rangle = [\hat{H}, \ \hat{\mathcal{O}}] | N_i \rangle, \tag{6}$$

a systematic expansion of the commutator between the NRCQM Hamiltonian $\hat{H}$ and the vertex coupling H_K and H_π can thus be carried out. Details of this treatment can be found in Refs [‖].

[‖] Z. P. Li, PRD **48**, 3070 (1993); **50**, 5639 (1994); **52**, 4961 (1995); C **52**, 1648 (1995)

- The s-channel amplitude in the harmonic oscillator basis is expressed as

$$\mathcal{M}^s = \sum_n (\mathcal{M}_3^s + \mathcal{M}_2^s) e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2}, \tag{7}$$

  where α is the oscillator strength, and $e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2}$ is a form factor in the harmonic oscillator basis. $\mathcal{M}_3^s$ ($\mathcal{M}_2^s$) corresponds to the amplitudes for the outgoing meson and incoming meson absorbed and emitted by the same quark (different quarks). Because of the isospin selection rule, the π and K^- can not couple to the same quark. Thus, the contribution of $\mathcal{M}_3^s$ vanishes and only $\mathcal{M}_2^s$ contributes to the s-channel.

- The transition amplitude is

$$
\begin{aligned}
\mathcal{M}_s &= \Big\{ g_{s2}\mathbf{A}_{out}\cdot\mathbf{A}_{in}\sum_{n=0}(-2)^{-n}\frac{F_s(n)}{n!}\mathcal{X}^n \\
&+ g_{s2}\left(-\frac{\omega_K}{6\mu_q}\mathbf{A}_{out}\cdot\mathbf{q}-\frac{\omega_\pi}{3m_q}\mathbf{A}_{in}\cdot\mathbf{k}+\frac{\omega_\pi}{m_q}\frac{\omega_K}{2\mu_q}\frac{\alpha^2}{3}\right) \\
&\times\sum_{n=1}(-2)^{-n}\frac{F_s(n)}{(n-1)!}\mathcal{X}^{n-1}+g_{s2}\frac{\omega_\pi\omega_K}{18m_q\mu_q}\mathbf{k}\cdot\mathbf{q} \\
&\sum_{n=2}\frac{F_s(n)}{(n-2)!}(-2)^{-n}\mathcal{X}^{n-2}+g_{v2}i\sigma\cdot(\mathbf{A}_{out}\times\mathbf{A}_{in}) \\
&\sum_{n=0}(-2)^{-n}\frac{F_s(n)}{n!}\mathcal{X}^n+g_{v2}\frac{\omega_\pi\omega_K}{18m_q\mu_q}i\sigma\cdot(\mathbf{q}\times\mathbf{k}) \\
&\times\sum_{n=2}(-2)^{-n}\frac{F_s(n)}{(n-2)!}\mathcal{X}^{n-2}\Big\}e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2}, \qquad (8)
\end{aligned}
$$

We have defined

$$\mathbf{A}_{in} \equiv -\left(1+\omega_K \mathcal{K}_i - \frac{\omega_K}{6\mu_q}\right)\mathbf{k}, \quad (9)$$

$$\mathbf{A}_{out} \equiv -\left(1+\omega_\pi \mathcal{K}_f - \frac{\omega_\pi}{3m_q}\right)\mathbf{q}, \quad (10)$$

$$\mathcal{K}_i \equiv 1 \quad /(E_i + M_i), \quad (11)$$

$$\mathcal{K}_f \equiv 1 \quad /(E_f + M_f) \quad (12)$$

$$\mathcal{X} \equiv \frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2} \quad (13)$$

$$F_s(n) \equiv \frac{M_n}{P_i\cdot k - nM_n\omega_h}, \quad (14)$$

- the g-factors, g_{s2} and g_{v2}, in the s-channel are defined as

$$g_{s2} \equiv \langle N_f | \sum_{i \neq j} I_i^{\pi} I_j^{K} \sigma_i \cdot \sigma_j | N_i \rangle / 3, \tag{15}$$

$$g_{v2} \equiv \langle N_f | \sum_{i \neq j} I_i^{\pi} I_j^{K} (\sigma_i \times \sigma_j)_z | N_i \rangle / 2, \tag{16}$$

which can be derived from the quark model in the $SU(6) \otimes O(3)$ limit.

The u-channel amplitudes

- Following the same procedure in the s-channel, we obtain the amplitude for the u-channel is expressed as

$$
\begin{aligned}
\mathcal{M}_u \;=\; & -\Big\{\mathbf{B}_{in}\cdot\mathbf{B}_{out}\sum_{n=0}\left[g^u_{s1}+(-2)^{-n}g^u_{s2}\right]\frac{F_u(n)}{n!}\mathcal{X}^n \\
& +\left(-\frac{\omega_\pi}{3m_q}\mathbf{B}_{in}\cdot\mathbf{k}-\frac{\omega_K}{3m_q}\mathbf{B}_{out}\cdot\mathbf{q}+\frac{\omega_K}{2\mu_q}\frac{\omega_\pi}{m_q}\frac{\alpha^2}{3}\right) \\
& \times\sum_{n=1}[g^u_{s1}+(-2)^{-n}g^u_{s2}]\frac{F_u(n)}{(n-1)!}\mathcal{X}^{n-1} \\
& +\frac{\omega_\pi\omega_K}{18m_q\mu_q}\mathbf{k}\cdot\mathbf{q}\sum_{n=2}\frac{F_u(n)}{(n-2)!}[g^u_{s1}+(-2)^{-n}g^u_{s2}]\mathcal{X}^{n-2} \\
& +i\sigma\cdot(\mathbf{B}_{in}\times\mathbf{B}_{out})\sum_{n=0}\left[g^u_{v1}+(-2)^{-n}g^u_{v2}\right]\frac{F_u(n)}{n!}\mathcal{X}^n \\
& -\frac{\omega_\pi\omega_K}{18m_q\mu_q}i\sigma\cdot(\mathbf{q}\times\mathbf{k})\sum_{n=2}[g^u_{v1}+(-2)^{-n}g^u_{v2}] \\
& \times\frac{F_u(n)}{(n-2)!}\mathcal{X}^{n-2}+i\sigma\cdot\left[-\frac{\omega_\pi}{3m_q}(\mathbf{B}_{in}\times\mathbf{k})-\frac{\omega_K}{6\mu_q}(\mathbf{q}\times\mathbf{B}_{out})\right] \\
& \sum_{n=1}\left[g^u_{v1}+(-2)^{-n}g^u_{v2}\right]\mathcal{X}^{n-1}\frac{F_u(n)}{(n-1)!}\Big\}\times e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2},
\end{aligned}
\tag{17}
$$

- The g factors in the u-channel are determined by

$$g^u_{s1} \equiv \langle N_f | \sum_j I^K_j I^\pi_j | N_i \rangle, \quad (18)$$

$$g^u_{s2} \equiv \langle N_f | \sum_{i \neq j} I^K_i I^\pi_j \sigma_i \cdot \sigma_j | N_i \rangle / 3, \quad (19)$$

$$g^u_{v1} \equiv \langle N_f | \sum_j I^K_j I^\pi_j \sigma^z_j | N_i \rangle, \quad (20)$$

$$g^u_{v2} \equiv \langle N_f | \sum_{i \neq j} I^K_i I^\pi_j (\sigma_i \times \sigma_j)_z | N_i \rangle / 2. \quad (21)$$

The numerical values of these factors can be derived in the $SU(6) \otimes O(3)$ symmetry limit.

Separation of the resonance contributions

- All the resonances with the same number n are degenerate to each other. We need separate out the single resonance contribution out.

  In s channel, for n=0, only the Λ contributing to the amplitude, which can be written as

$$\mathcal{M}_{\Lambda}^{s} = \mathcal{O}_{\Lambda}\frac{2M_{\Lambda}}{s-M_{\Lambda}^{2}}e^{-(\mathbf{k}^{2}+\mathbf{q}^{2})/6\alpha^{2}}, \qquad (22)$$

with

$$\mathcal{O}_{\Lambda} = g_{s2}\mathbf{A}_{out}\cdot\mathbf{A}_{in} + g_{v2}i\sigma\cdot(\mathbf{A}_{out}\times\mathbf{A}_{in}), \qquad (23)$$

where M_{Λ} is the Λ-hyperon mass.

- For n=1, the amplitude can be written as

$$\mathcal{M}_R^s = \frac{2M_R}{s - M_R^2 + iM_R\Gamma_R}\mathcal{O}_R e^{-(\mathbf{k}^2+\mathbf{q}^2)/6\alpha^2}. \tag{24}$$

Here S and D waves involve in the reaction. According to the roles of S and D waves, firstly we can separate out the amplitudes for them:

$$\mathcal{O}_S = -\frac{1}{2}g_{s2}\left(|\mathbf{A}_{out}|\cdot|\mathbf{A}_{in}|\frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^2} - \frac{\omega_K}{6\mu_q}\mathbf{A}_{out}\cdot\mathbf{q} - \frac{\omega_\pi}{3m_q}\mathbf{A}_{in}\cdot\mathbf{k} + \frac{\omega_\pi\omega_K}{2m_q\mu_q}\frac{\alpha^2}{3}\right), \tag{25}$$

$$\mathcal{O}_D = -\frac{1}{2}g_{s2}|\mathbf{A}_{out}|\cdot|\mathbf{A}_{in}|(3\cos^2\theta - 1)\frac{|\mathbf{k}||\mathbf{q}|}{9\alpha^2} - \frac{1}{2}g_{v2}i\sigma\cdot(\mathbf{A}_{out}\times\mathbf{A}_{in})\frac{\mathbf{k}\cdot\mathbf{q}}{3\alpha^2}. \tag{26}$$

In the NRCQM, the $n = 1$ shell contains three different SU(6) representations:

1 . [**70**,^{2} **1**]: $\Lambda(1405)S_{01}$ and $\Lambda(1520)D_{03}$ (no counterparts in the nucleon spectrum)

2. [**70**,^{2} **8**]: $\Lambda(1670)S_{01}$ and $\Lambda(1690)D_{03}$ (partners of the nucleon resonances $S_{11}(1535)$ and $D_{13}(1520)$)

3. [**70**,^{4} **8**]:$\Lambda(1800)S_{01}$ and $\Lambda(1830)D_{05}$ (the contributions of [**70**,^{4} **8**] are forbidden in $K^-p \to \Sigma^0\pi^0$ due to the so-called "Λ-selection rule" **)

Q. Zhao and F. E. Close,PRD **74, 094014 (2006);N. Isgur, G. Karl and R. Koniuk,PRL**41**, 1269 (1978);A. J. G. Hey, P. J. Litchfield and R. J. Cashmore,NPB **95**, 516 (1975)

- The separated amplitudes for the S- and D-wave can thus be re-written as

$$\mathcal{O}_S = [g_{S_{01}(1405)} + g_{S_{01}(1670)}]\mathcal{O}_S, \quad (27)$$

$$\mathcal{O}_D = [g_{D_{03}(1520)} + g_{D_{03}(1690)}]\mathcal{O}_D, \quad (28)$$

where the factor g_R ($R = S_{01}(1405)$, etc) represents the resonance transition strengths in the spin-flavor space, and is determined by the matrix element $\langle N_f|H_\pi|N_j\rangle\langle N_j|H_K|N_i\rangle$. Their relative strengths can be explicitly determined by the following relations

$$\frac{g_{S_{01}(1405)}}{g_{S_{01}(1670)}} = \frac{\langle N_f|I_3^\pi\sigma_3|S_{01}(1405)\rangle\langle S_{01}(1405)|I_3^K\sigma_3|N_i\rangle}{\langle N_f|I^\pi\sigma_3|S_{01}(1670)\rangle\langle S_{01}(1670)|I_3^K\sigma_3|N_i\rangle}, \quad (29)$$

$$\frac{g_{D_{03}(1520)}}{g_{D_{03}(1690)}} = \frac{\langle N_f|I_3^\pi\sigma_3|D_{03}(1520)\rangle\langle D_{03}(1520)|I_3^K\sigma_3|N_i\rangle}{\langle N_f|I^\pi\sigma_3|D_{03}(1690)\rangle\langle D_{03}(1690)|I_3^K\sigma_3|N_i\rangle}. \quad (30)$$

- Various g and g_R factors defined in this work and extracted in the symmetric quark model.:

factor	value	factor	value
g^u_{s1}	1/2	g^s_t	$\sqrt{2}/2$
g^u_{s2}	2/3	g^v_t	$-\sqrt{2}/6$
g^u_{v1}	-1/6	$g_{S_{01}(1405)}$	3/2
g^u_{v2}	-1	$g_{S_{01}(1670)}$	-1/2
g_{s2}	2/3	$g_{D_{03}(1520)}$	3/2
g_{v2}	1	$g_{D_{03}(1690)}$	-1/2

3. The role of Λ resonances in the reaction

- The parameters
- The roles of Λ resonances
- The backgrounds

The parameters

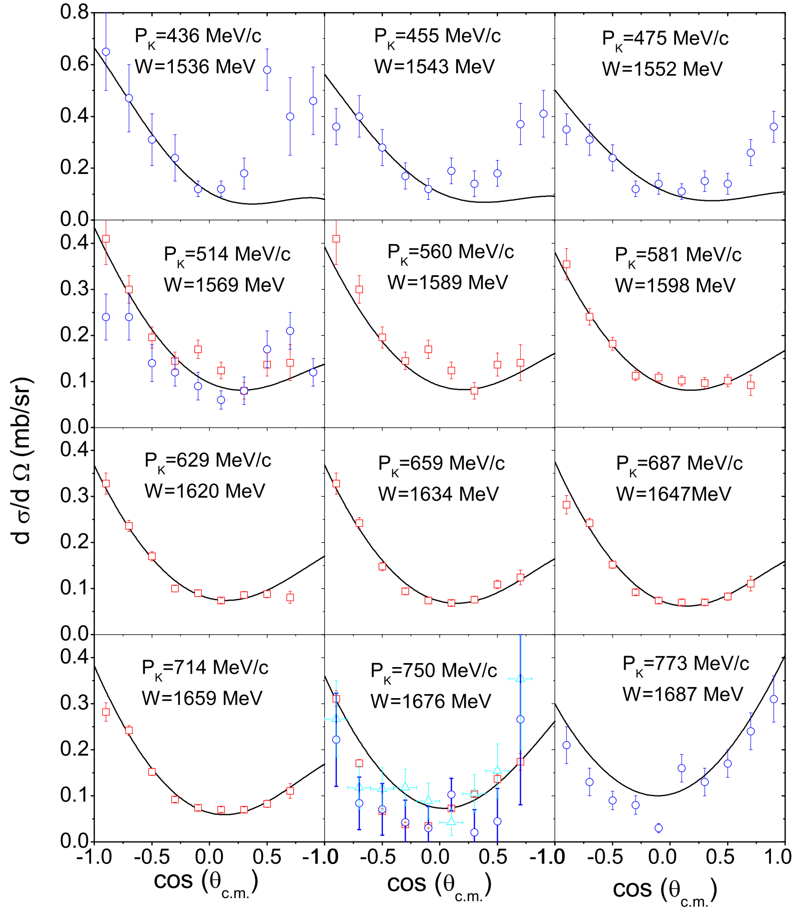
- Breit-Wigner masses M_R (MeV) and widths Γ_R (MeV) for the resonances in the s-channel. States in the $n = 2$ shell are treated as degenerate to n:

resonance	M_R	Γ_R	M_R (PDG)	Γ_R (PDG)
$S_{01}(1405)$	1420	48	1406 ± 4	50 ± 2
$S_{01}(1670)$	1697	65	1670 ± 10	$25 \sim 50$
$D_{03}(1520)$	1520	8	1520 ± 1	16 ± 1
$D_{03}(1690)$	1685	63	1690 ± 5	60 ± 10
n=2	1850	100		

- The other input parameters used in this work are grouped as below

$$\begin{array}{ll} \alpha = 0.4\text{GeV}, & m_u = m_d = 330\text{MeV}, \\ m_s = 450\text{MeV}, & f_K = 160 MeV \\ f_\pi = 132 MeV, & \delta = 1.557 \\ G_v a = 38, & g_{\kappa K \pi} = 4. \end{array}$$

- The differential cross sections for $P_K = 475 \sim 775$ MeV/c (i.e. $W = 1536 \sim 1687$ MeV):



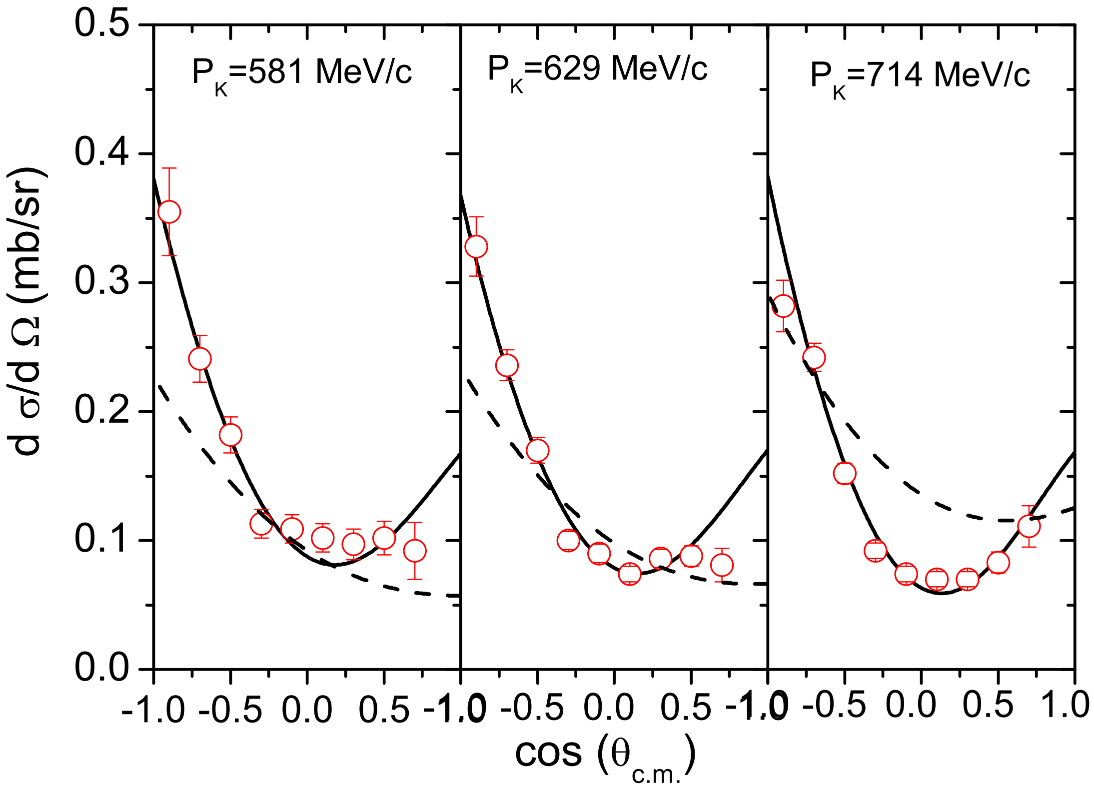
The roles of the resonances

- The configuration mixing between $[\mathbf{70},^2\mathbf{1}]$ and $[\mathbf{70},^2\mathbf{8}]$ in $\Lambda(1405)$ and $\Lambda(1670)$.
  We empirically introduce a mixing angle between $[\mathbf{70},^2\mathbf{1}]$ and $[\mathbf{70},^2\mathbf{8}]$ within the physical states $S_{01}(1405)$ and $S_{01}(1670)$, i.e.

$$|S_{01}(1405)\rangle = \cos(\theta)|\mathbf{70},^2\mathbf{1}\rangle - \sin(\theta)|\mathbf{70},^2\mathbf{8}\rangle, \quad (31)$$

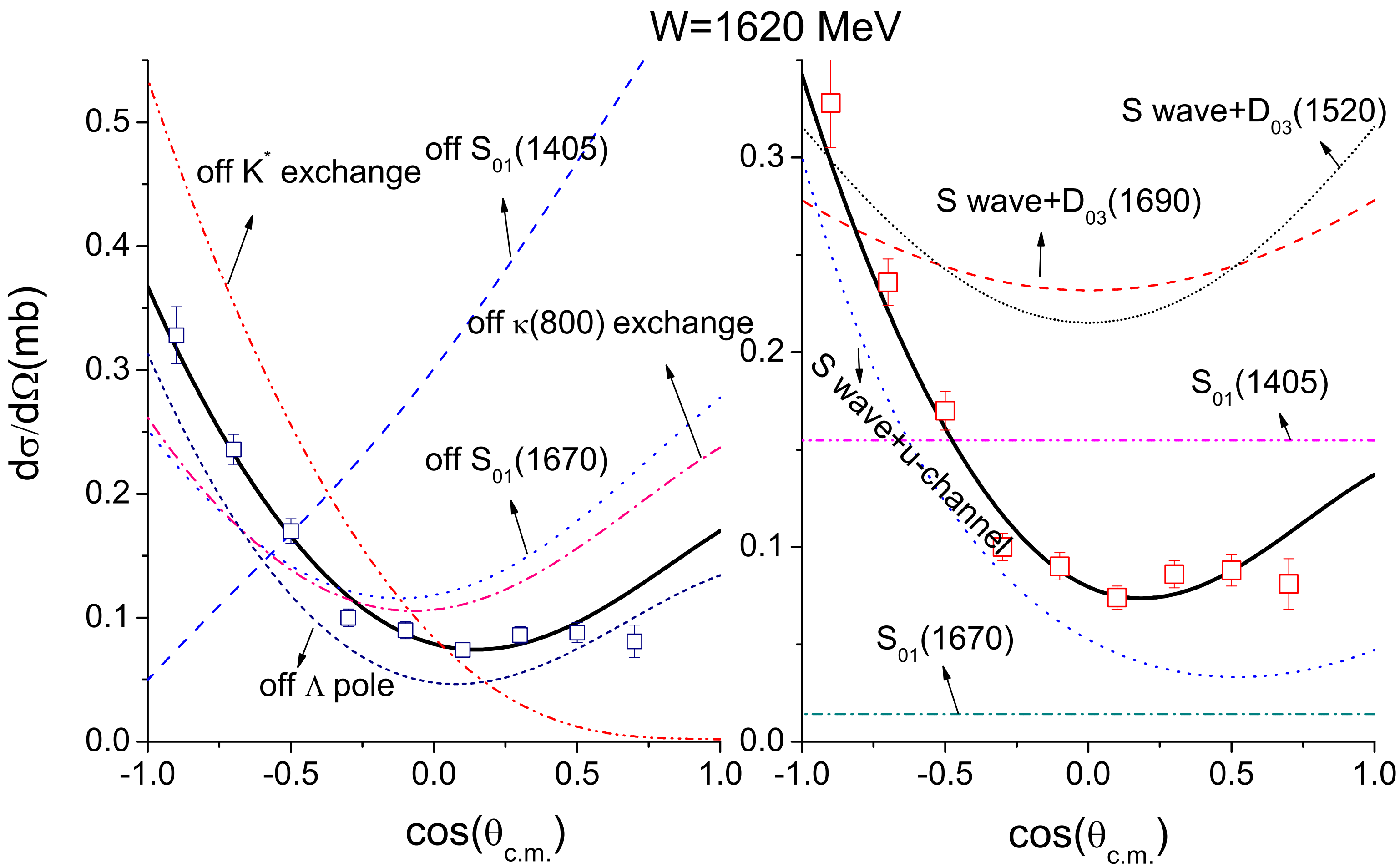
$$|S_{01}(1670)\rangle = \sin(\theta)|\mathbf{70},^2\mathbf{1}\rangle + \cos(\theta)|\mathbf{70},^2\mathbf{8}\rangle. \quad (32)$$

- The solid and dotted curves correspond to the results with and without configuration mixing respectively.
  Configuration mixing is needed. In agreement with coupled channel studies based on UχPT ††

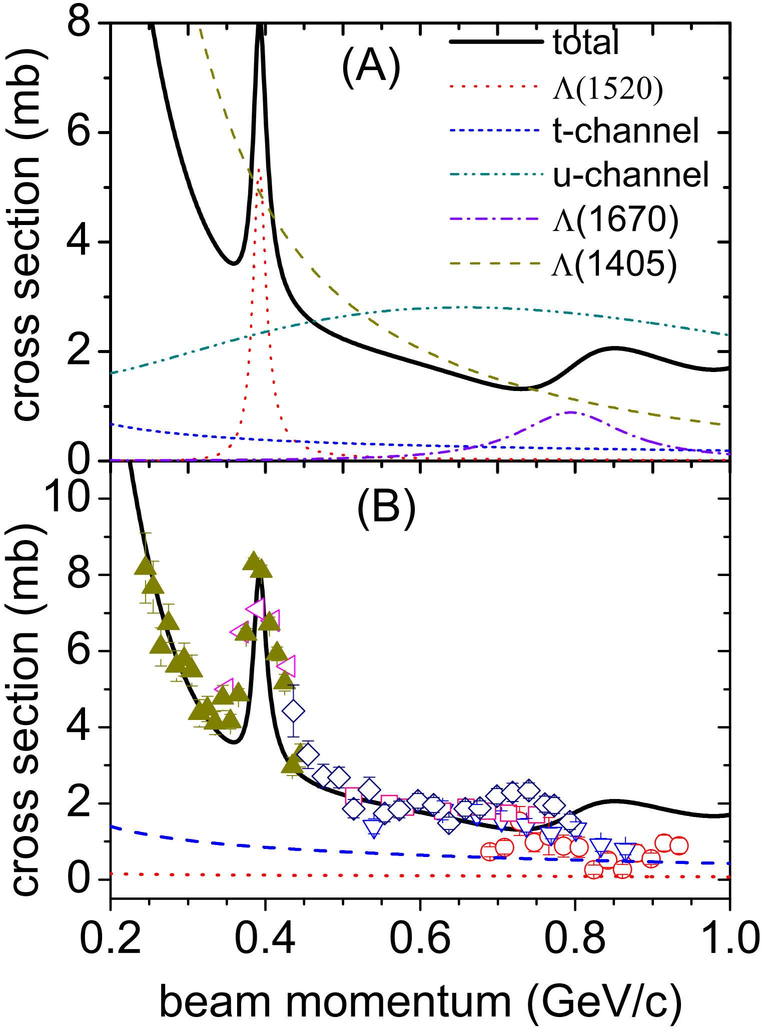


††D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, NPA **725**, 181 (2003).

- The partial wave contributions to the differential cross section



- The exclusive cross sections for different contributions.



- The $\Lambda(1405)S_{01}$ is very crucial in the reactions. It is the major contributor of the S-wave amplitude in the low-energy region. In particular, in the region of $P_K \simeq 300$ MeV/c, $\Lambda(1405)S_{01}$ dominates the amplitudes, and contributions of the other resonances are nearly invisible in the total cross section.

- Around $P_K = 400$ MeV/c, the $\Lambda(1520)D_{03}$ is responsible for the strong resonant peak in the total cross section.
- Around $P_K = 800$ MeV/c, the differential cross sections are sensitive to the $\Lambda(1670)S_{01}$. In this energy region the role of $\Lambda(1690)D_{03}$ is visible, but less important than $\Lambda(1670)S_{01}$.

The backgrounds

- The backgrounds u channel contributions are crucial in the reactions, which agrees with the UχPT predictions. [‡‡]

- The backgrounds t channel contributions are significant in the reactions, Which is dominated by the vector meson K^* ex-change.

[‡‡]J. A. Oller,EPJA **28**, 63 (2006);J. A. Oller, J. Prades and M. Verbeni,PRL **95**, 172502 (2005)

4. Conclusions

- The $\Lambda(1405)S_{01}$ dominates the reactions over the energy region considered here.
- Around $P_K \simeq 400$ MeV/c, the $\Lambda(1520)D_{03}$ is responsible for a strong resonant peak in the cross section.
- The $\Lambda(1670)S_{01}$ has obvious contributions around $P_K = 750$ MeV/c, while the contribution of $\Lambda(1690)D_{03}$ is less important in this energy region.
- The non-resonant background contributions, i.e. u-channel and t-channel, also play important roles in the explanation of the angular distributions due to amplitude interferences. In the t-channel, the K^*-exchange is more dominant over the κ-exchange.
- There exist configuration mixings within the $\Lambda(1405)$ and $\Lambda(1670)$ as admixtures of the $[\mathbf{70},^2\mathbf{1},1/2]$ and $[\mathbf{70},^2\mathbf{8},1/2]$ configurations. $\Lambda(1405)$ is dominated by $[\mathbf{70},^2\mathbf{1},1/2]$, and $\Lambda(1670)$ by $[\mathbf{70},^2\mathbf{8},1/2]$.

Thanks very much for your attentions!