Study of spin sum rules

(and the strong coupling constant at large distances)

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Moments of structure functions, sum rules and QCD coupling

Moments of structure functions are objects as inclusive as one could imagine:

- Sum on all the processes.
- Sum on transfered energy ν (or x, or W).

What does this talk have to do with N* ? ☹

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$\Rightarrow$ simplifications + global constrain on exclusive physics.

Ex. Gerasimov-Drell Hearn sum rules:

$$\int_{\nu_{thr}}^{\infty} (\sigma_A - \sigma_P) \frac{d\nu}{\nu} \propto \text{(target anomalous magnetic moment)}^2$$

photoproduction cross sections

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photoproduction cross sections

Sum rules are dominated by resonances photo-productions (or electro-productions at low and average Q^2.)

Moments of spin structure functions: used to define an effective QCD coupling at low Q^2.

} Okay, I do see the interest for N^* ☺

Moments of spin structure functions and spin sum rules

N^{th}-moments: $\begin{cases} \int g_1 x^{n-1} dx \\ \int g_2 x^{n-1} dx \end{cases}$

First moments: Γ_1, Γ_2

- Γ_1^N: $\begin{cases} \text{Ellis-Jaffe sum rule (large } Q^2) \\ \text{Gerasimov-Drell-Hearn (GDH) sum rule } (Q^2=0) \end{cases}$
- Γ_1^{p-n}: Bjorken sum rule (large Q^2)
- Γ_2^N: Burkhardt–Cottingham (BC) sum rule (any Q^2)
- ....

- d_2 "sum rule"
- Spin polarizability sum rules γ_0 δ_{LT}

} No low-x extrapolation

Moments of spin structure functions and spin sum rules

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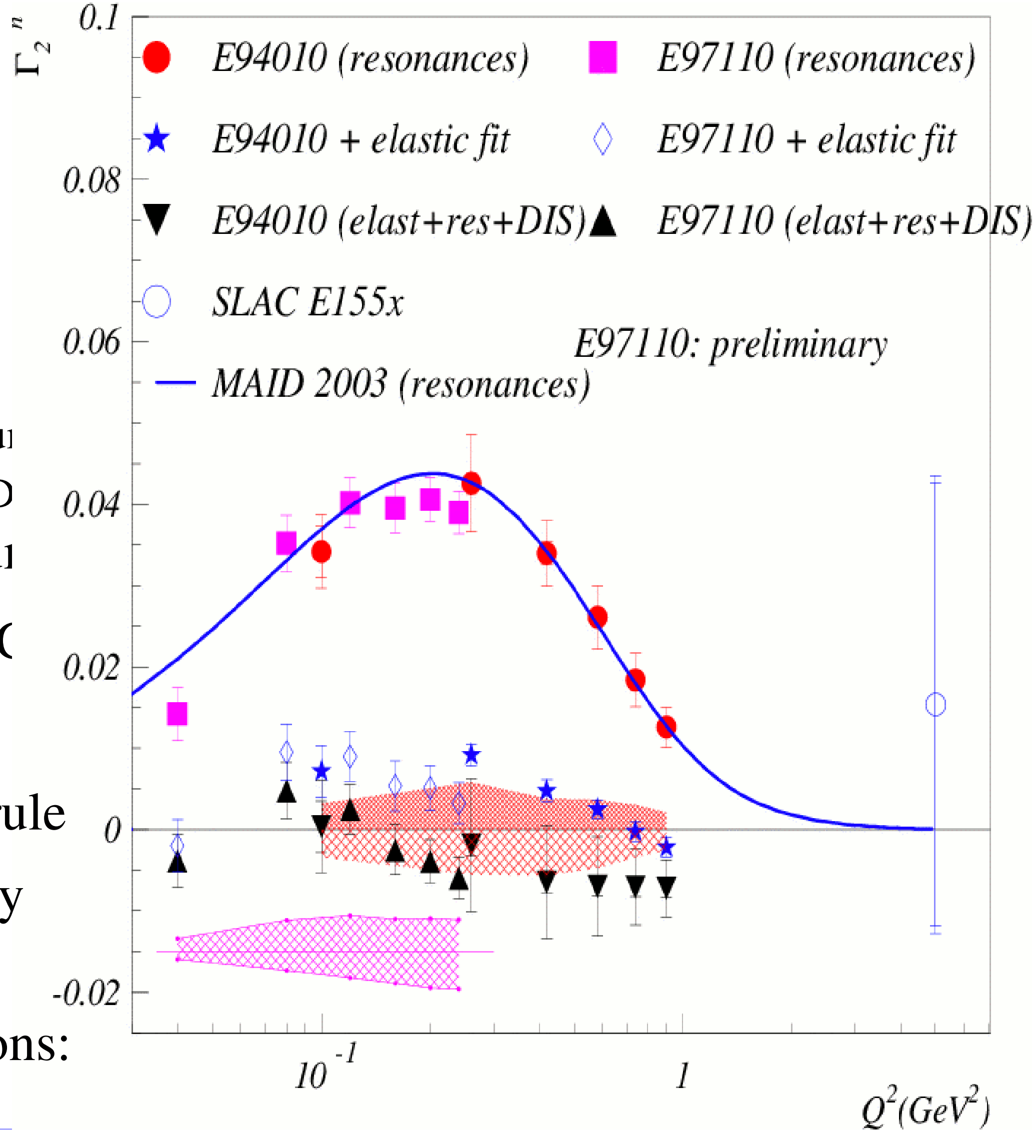


⋆ Γ_1^N: { Ellis-Jaffe su / Gerasimov-D

⋆ Γ_1^{p-n}: Bjorken su

⋆ Γ_2^N: Burkhardt–C

$\int_0^1 g_2 dx = 0 \Rightarrow$ If BC Sum rule is valid, then interplay between elastic and resonance contributions:

The generalized Bjorken Sum Rules

Bjorken sum rule (large Q^2):

$$\int g^p_1 - g^n_1 \, dx = \Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{M^2}{9Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon triplet axial charge (Bjorken limit)

pQCD radiative corrections

Higher Twists (+rad. corr.)

Fundamental test of the pQCD Q^2-evolution and OPE in the spin sector

Individual nucleon:

$$\int g_1^N dx = \left(\pm 12 g_A + \frac{a_8}{36}\right)\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{a_0}{9}\left(1 - \frac{\alpha_s}{\pi} - 1.10\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \text{Higher Twists}$$

Octet axial charge

singlet axial charge

(Assuming SU(3) symmetry and no strange quark polarization leads to the (violated) Ellis-Jaffe sum rule)

Here: $\overline{\text{MS}}$ (no gluon contribution to Γ_1) and a_0 is Q^2-independent

The generalized Gerasimov-Drell-Hearn sum

Original GDH sum rule ($Q^2 = 0$):

$$\boxed{\int_{\nu_{thr}}^{\infty} (\sigma_A - \sigma_P) \frac{d\nu}{\nu} = \frac{-4\alpha\pi^2 S\kappa^2}{M^2}}$$

σ_A, σ_P : photoproduction cross sections

κ: anomalous magnetic moment

S: Spin

Generalized GDH sum: $Q^2 > 0$:

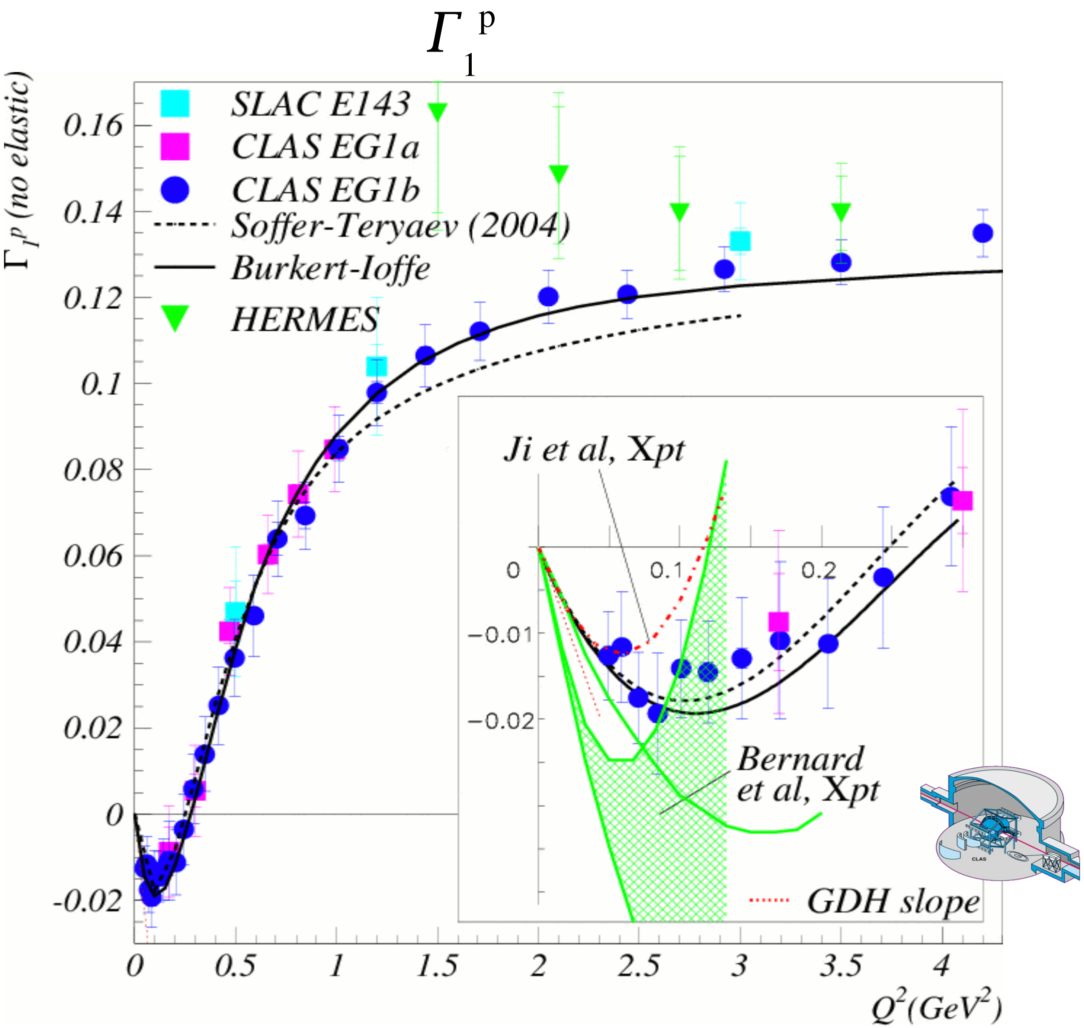
photoproduction → electroproduction $\quad \sigma_A - \sigma_P = f(g_1, g_2)$

One possible generalization: $\boxed{\frac{8}{Q^2}\int g_1 dx = S_1(0,Q^2)}$ (Ji and Osborne, 1999)

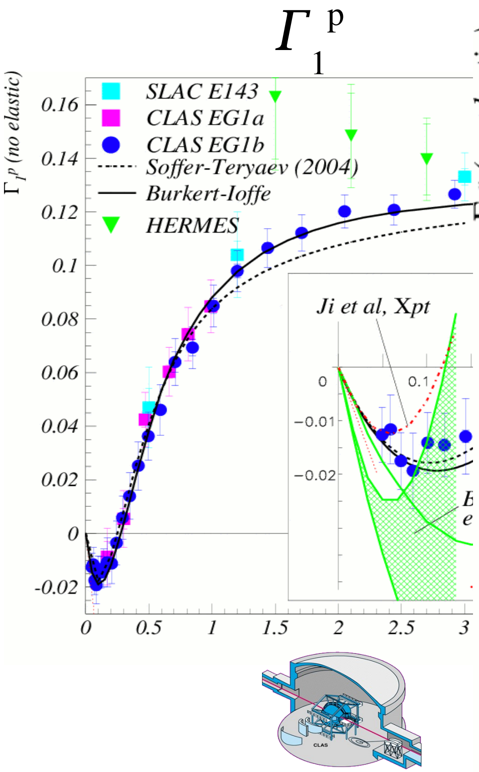
$S_1(\nu,Q^2)$: spin dependent Compton amplitude

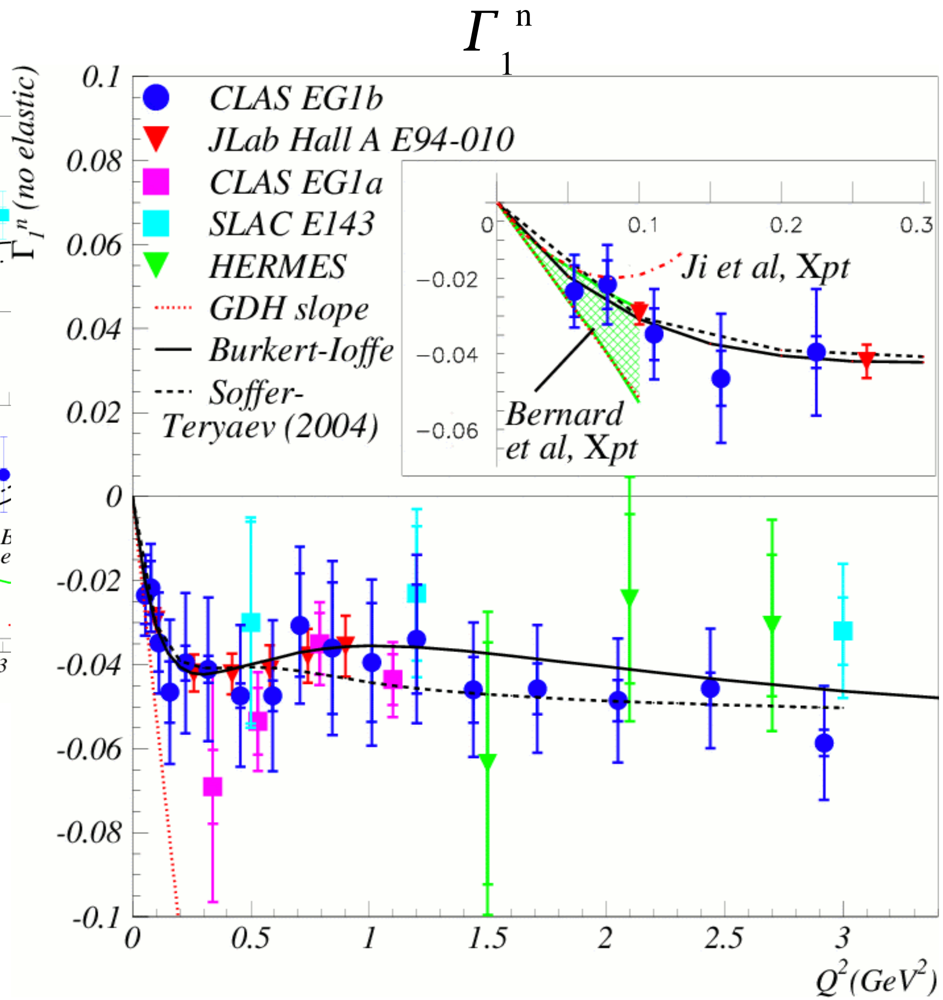
Connection allows to study the pQCD sum rules at any Q^2.

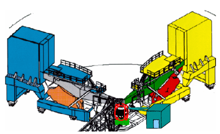
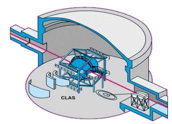
Results on sum rules



Results on sum rules





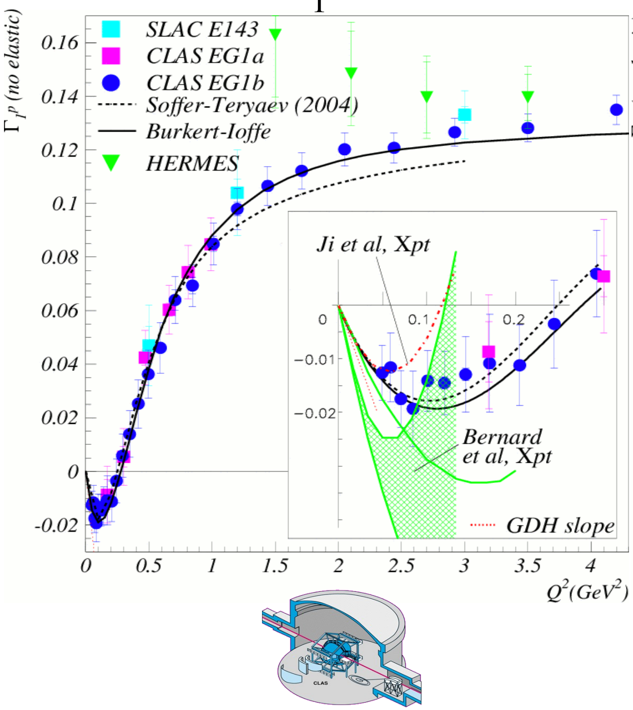
Results on sum rules

Bjorken sum: $\int g_1^{\,p} - g_1^{\,n} dx$

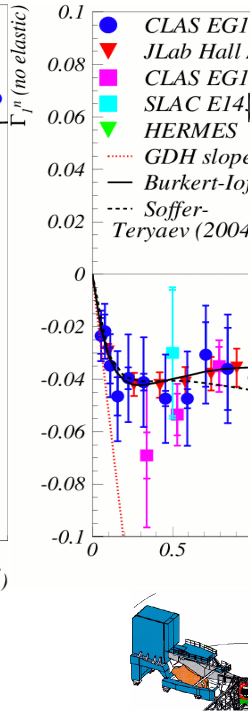
Δ contribution suppressed

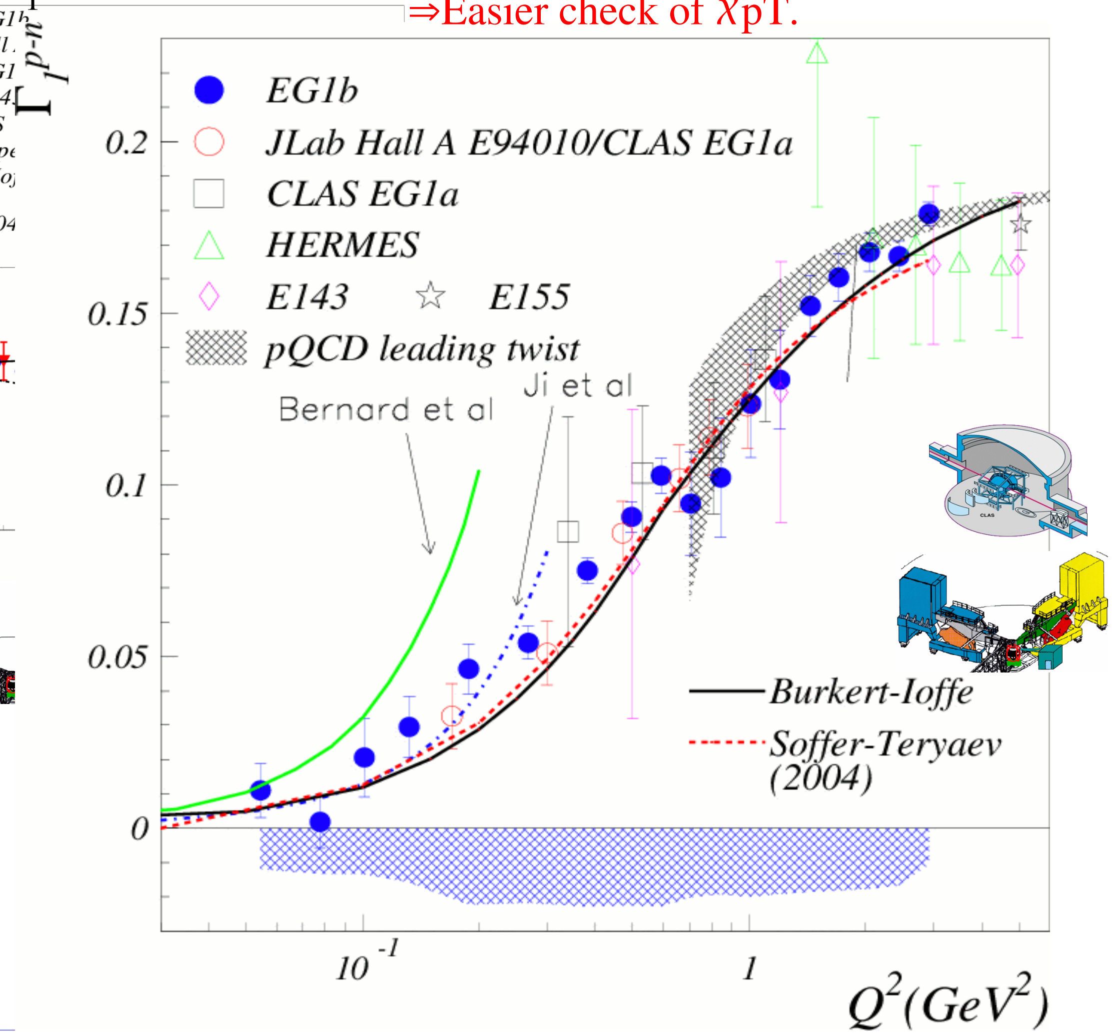
$\Rightarrow$Easier check of χpT.

$\Gamma_1^{\,p}$

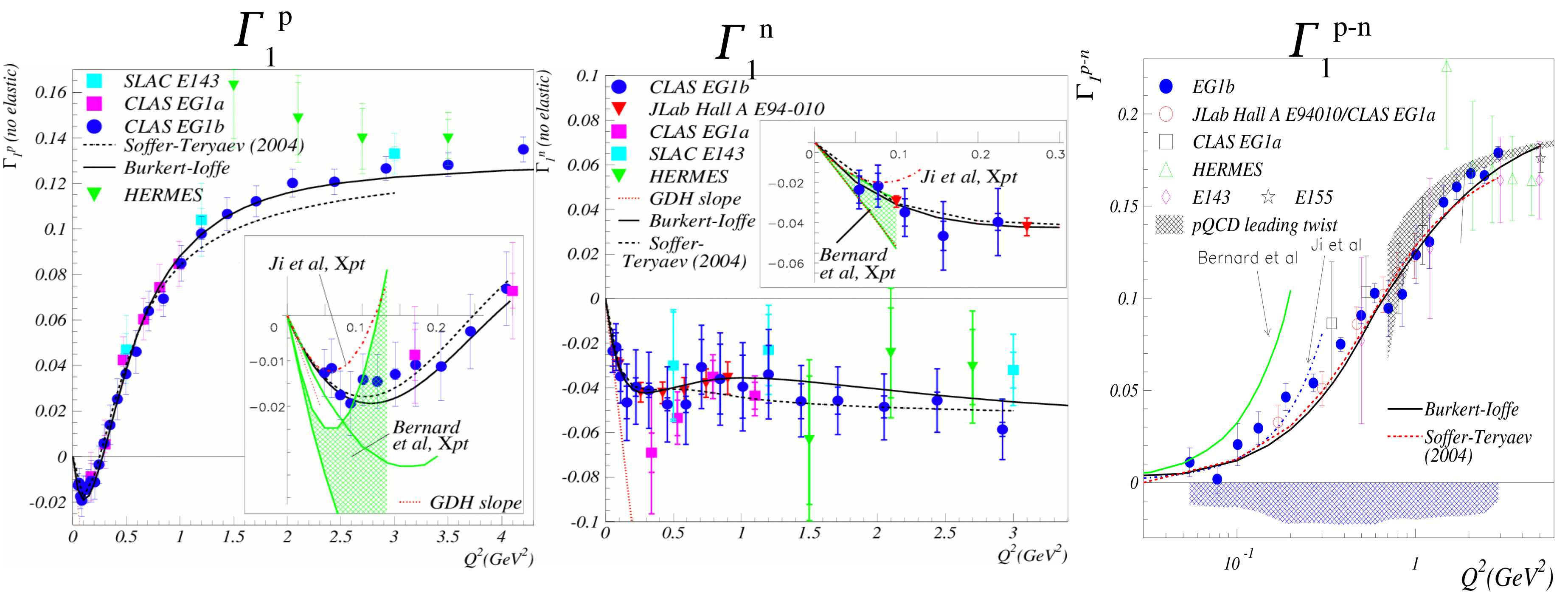


$\Gamma_1^{\,n}$





Results on sum rules

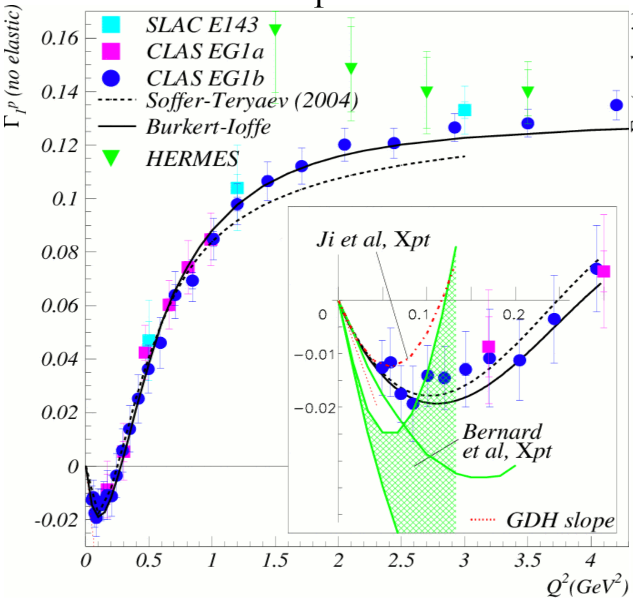


- **Models (MAID, Burkert-Ioffe, Soffer-Teryaev) are doing very well**

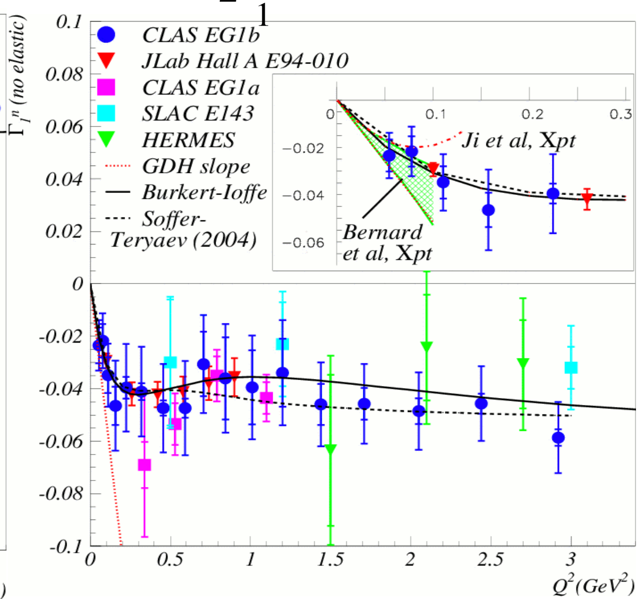
Spin structure studies in the pQCD →npQCD transition region

Smooth transition, nothing special happens near Λ_{QCD}^2.

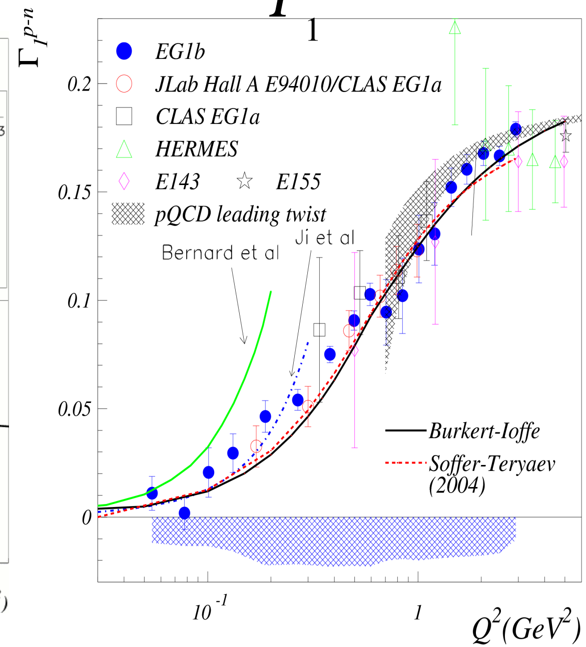
Γ_1^p



Γ_1^n

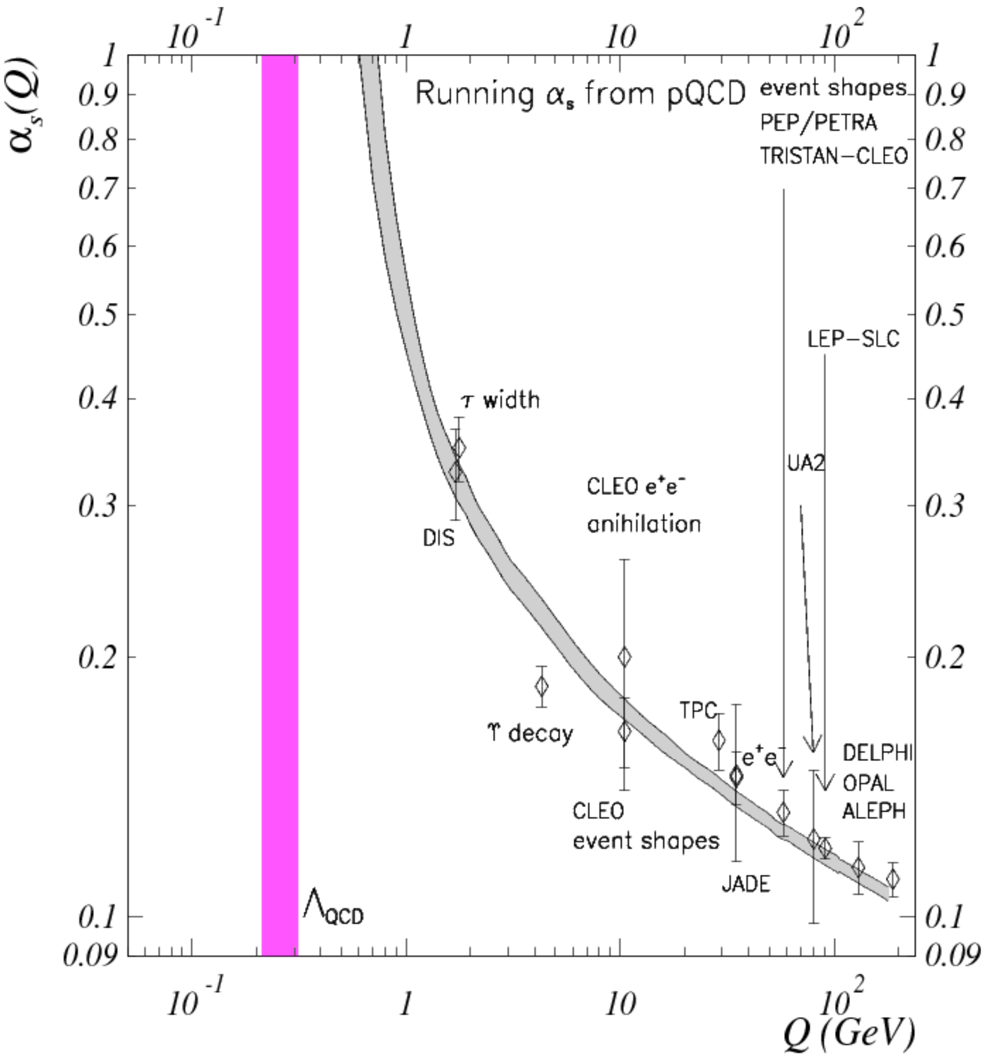


Γ_1^{p-n}



⇒ Can be used to extrapolate the definition of QCD coupling to low Q^2.

The strong coupling constant from pQCD



α_s (Q) is well defined in pQCD at large Q^2.

Can be extracted from data

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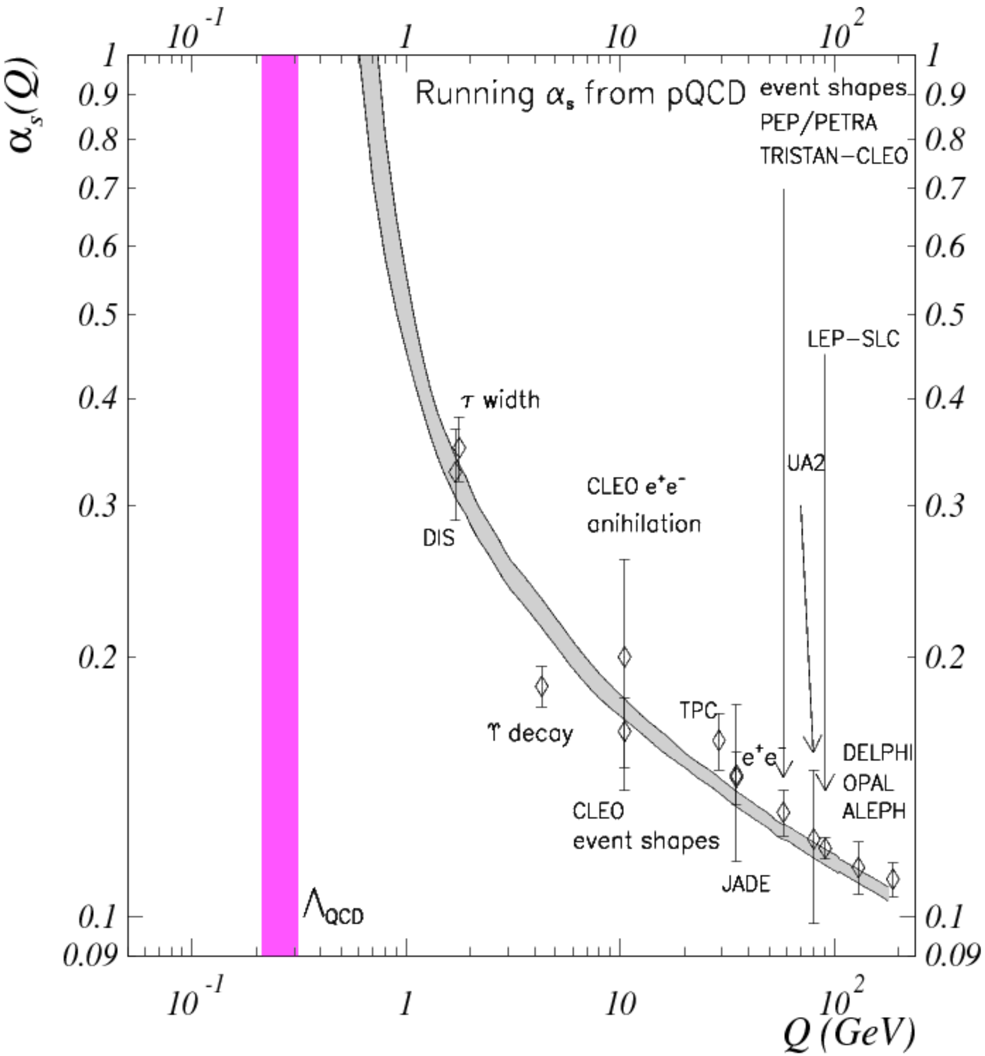

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$$\int g^p_1 - g^n_1 \, dx = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right)$$

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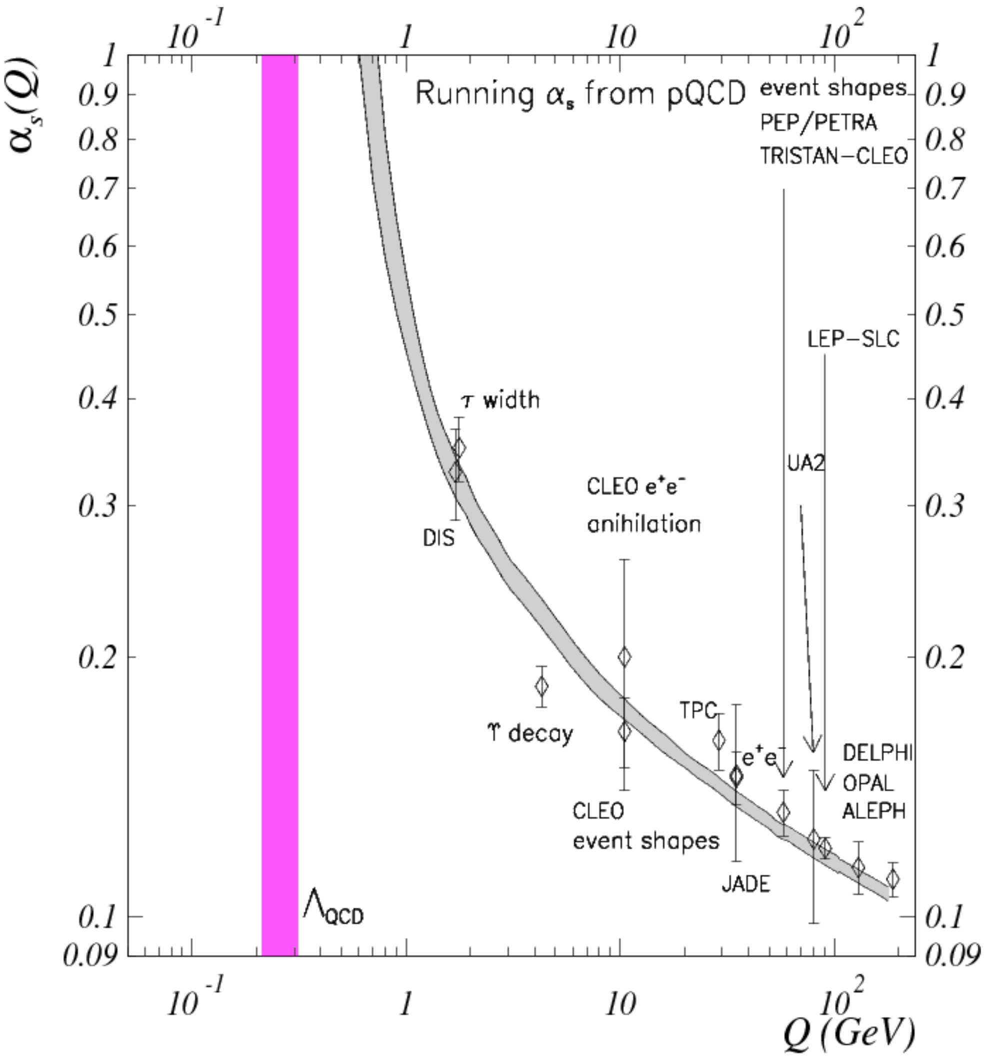

α_s (Q) is well defined in pQCD at large Q^2.
Can be extracted from data (e.g. Bjorken Sum Rule).

At low Q^2 (~GeV^2) , pQCD cannot be used to define α_s : *If* pQCD is trusted, $\alpha_s \rightarrow \infty$ for $Q \rightarrow \Lambda_{QCD}$.

Definition of effective QCD couplings

G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

Prescription:
Define effective couplings (in the DIS domain) from a perturbative series truncated to the first term in α_s.

Generalized Bjorken sum rule:

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$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6}\, g_A\left(1 - \frac{\alpha_{s,g1}}{\pi}\right)}$$

$\alpha_{s,g1} \triangleq \alpha_s^{eff}$ extracted from Γ_1^{p-n}

By doing so we obtain a coupling constant that is:

- Free of divergence.
- Not renormalization scheme dependent.
- Analytic when crossing quark thresholds.

But that is:

- Process dependent

⇒There is a priori a different α_s^{eff} for each different process.

<u>However</u> these α_s^{eff} can be related, so they are not useless quantities.

"Commensurate scale relations"

S.J. Brodsky & H.J Lu, PRD 51 3652 (1995)

S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J Lu, PLB 372 133 (1996)

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Extrapolating Grumberg's prescriptions to low Q^2:

Fold pQCD gluon emission + higher twists (i.e. QCD final state interactions) in the effective coupling

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[aside]

Folding the dynamics due to forces (here HT) into an effective parameter so that the particle is treated as free is common, e.g. effective masses of electrons in a crystal in quantum electronics:

In a crystal under a force F, the force effect is folded into an effective mass:

$$F_i = m_{ij}^* \gamma_j$$

Near an energy extremum E_0, e^- are described as free particles:

$$E = E_0 + \frac{1}{2} \langle m^* \rangle v^2$$

<u>Properties:</u>

- m_{ij}^* is a tensor (depending on the e^- energy) because the lattice is not isotropic and the total acceleration γ_j ,depends on the lattice force
- m_{ij}^* depends on material;
- Near an energy max. $m_{ii}^* < 0$;
- Holes have also effective masses of sign opposite to the e^-;
- m_{ij}^* determines the quantum states density, the speed of electric signals, surfaces of isoenergy,...

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⇒We should not be shocked if effective couplings depends on reactions, or may be negative.

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- m_{ij}^* determines the quantum states density, the speed of electric signals, surfaces of isoenergy,...

Advantages of extracting $\alpha_{s,g1}$ from the Bjorken Sum Rule

- Bjorken sum: simple Q^2-dependence.

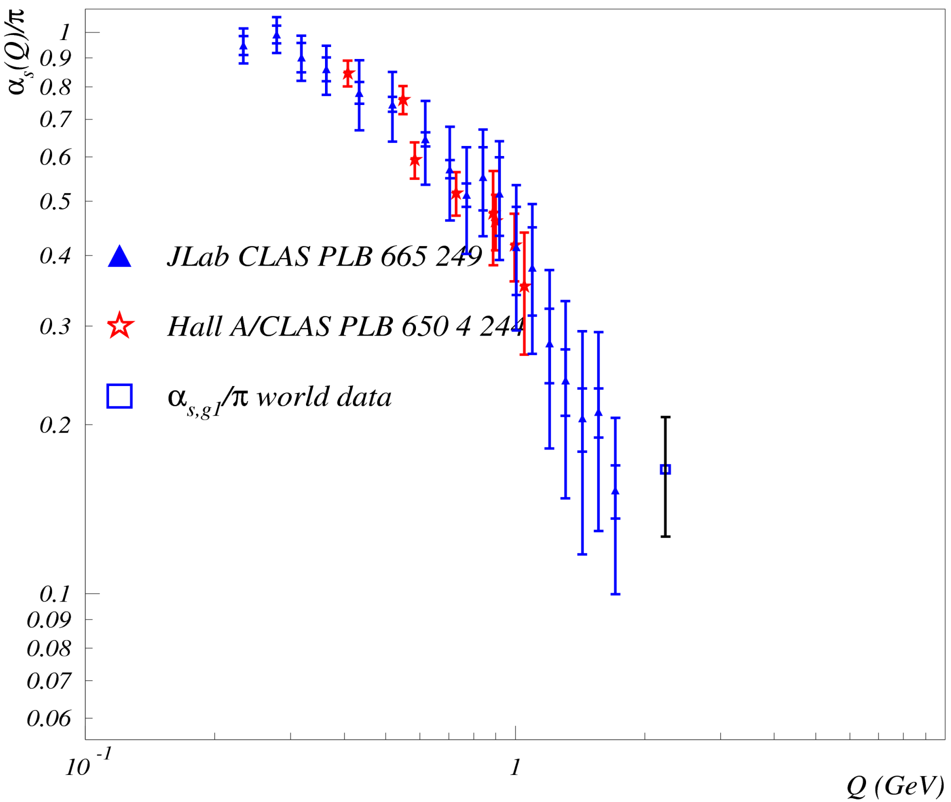
- Data exist at low, intermediate, and high Q^2.

- Sum rules (generalized GDH and Bjorken sum rules) complement the data in the unmeasured regions $Q^2 \rightarrow 0$ and $Q^2 \rightarrow \infty$.

  $\Rightarrow$ We can obtain $\alpha_{s,g1}$ at any Q^2.

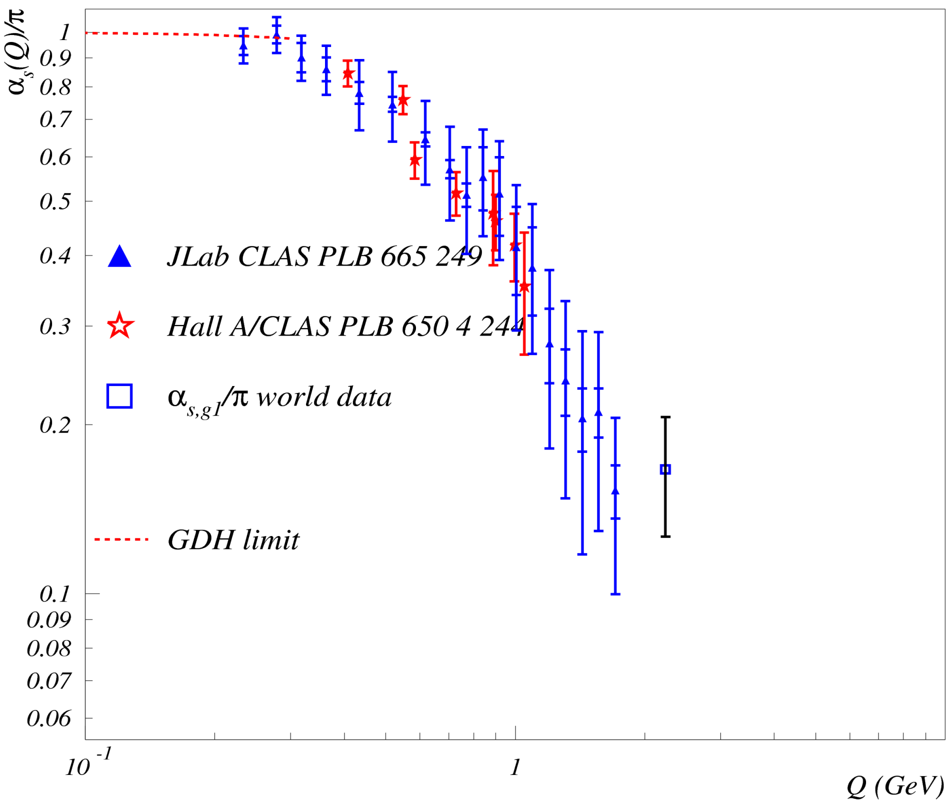
- Coherent contribution partly suppressed in the Bjorken sum. $\Rightarrow$ Definition of $\alpha_{s,g1}$ may be closest to α_s^{pQCD} definition ? Argument is stronger if global duality works (excluding the Δ and the elastic contributions).

$\alpha_{s,g1}$ from the Bjorken Sum data



$$\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{s,g1}}{\pi}\right)$$

Low Q^2 limit



Bjorken and Gerasimov-Drell-Hearn sums are related:

$\Rightarrow$ $Q^2 = 0$ constraints:

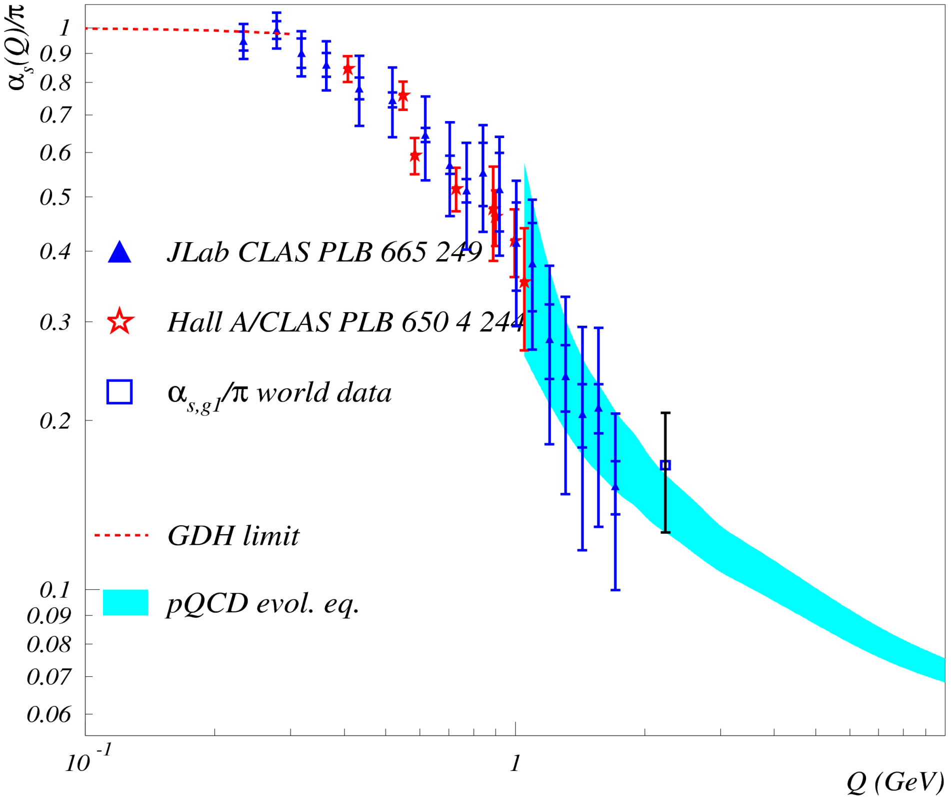
$$\Gamma_1^{p-n} = \frac{Q^2}{16\alpha\pi^2}\left(GDH^p - GDH^n\right)$$

$$\boxed{\Rightarrow \begin{cases} \alpha_{s,g1} = \pi \\ \dfrac{d\alpha_{s,g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^{\,2}}{M_n^{\,2}} - \dfrac{\kappa_p^{\,2}}{M_p^{\,2}}\right) \end{cases}}$$

Q^2=0

Large Q^2 limit

$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s^{pQCD}}{\pi} - 3.58\left(\frac{\alpha_s^{pQCD}}{\pi}\right)^2 - \ldots\right] = \frac{g_A}{6}\left(1 - \frac{\alpha_{s,g1}}{\pi}\right)$$



$$\boxed{\Rightarrow \alpha_{s,g1} = \alpha_s^{pQCD}}$$

$Q^2 \to \infty$

$\Rightarrow$ We know $\alpha_{s,g1}$ at any Q^2.

First experimental indication of *conformal behavior* (i.e. no Q^2-dependence) of α_s at low Q^2.

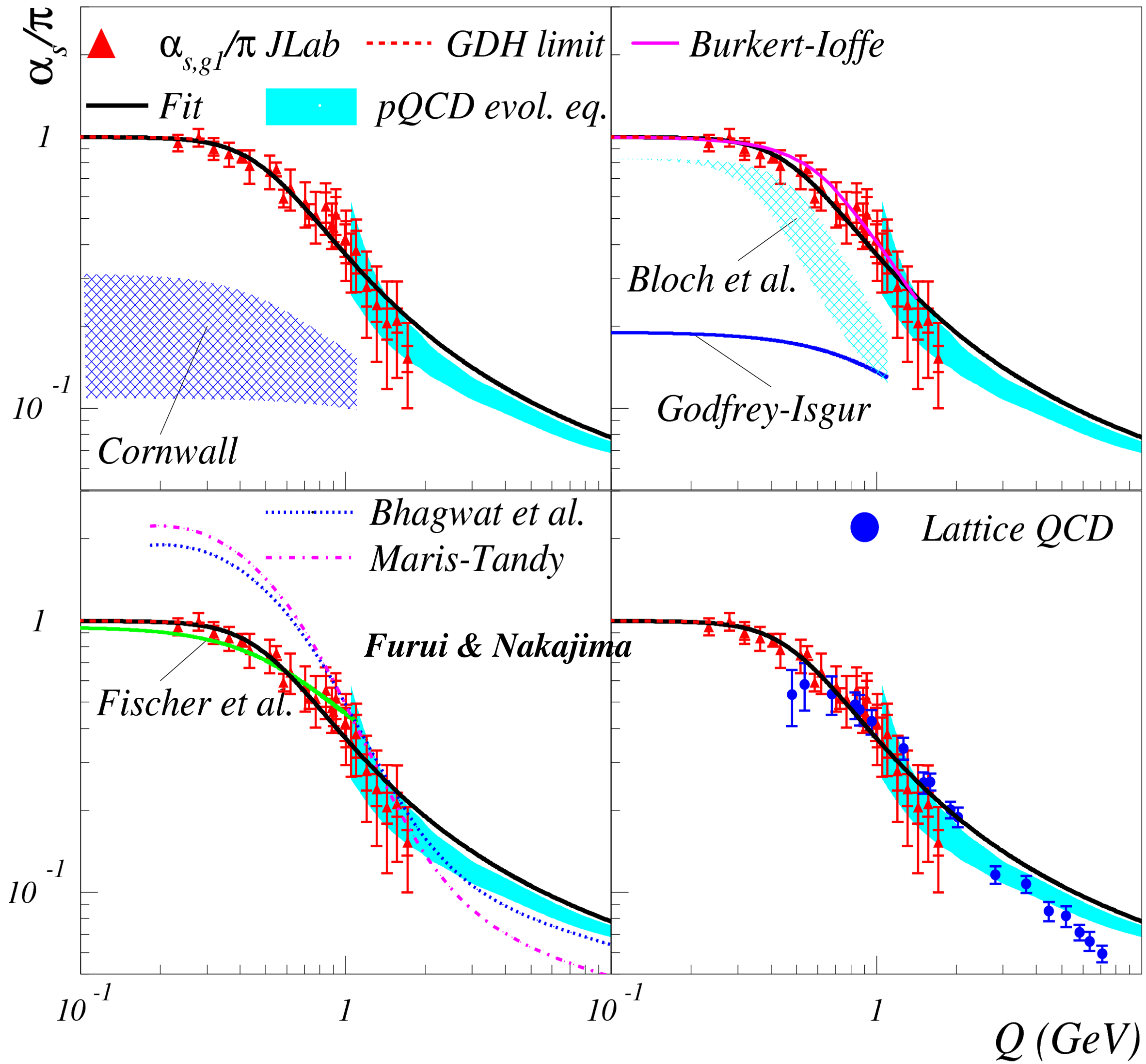
We have extrapolated a prescription defined in the DIS domain.

Open questions:

- Does the commensurate scale relations hold at low Q^2 ?
- How does this particular effective coupling relates to theory calculations ?

Those are important questions, but we can always form $\alpha_{s,g1}$ at any Q^2 and see how it compares to calculations.

"Comparison" with theory



Fisher *et al.*
Bloch *et al.*
Maris-Tandy
Bhagwat *et al.*
Cornwall

} Schwinger -Dyson

Godfrey-Isgur: Constituant Quark Model

Furui & Nakajima: Lattice

“Comparison” with theory

⇒Similar behavior. Hint of connection between these various quantities?

Establishing such connection would help understanding quark confinement:

“The question of light-quark confinements can be translated into charting the <u>infrared behavior of the *universal* β-function</u>.. i.e. α_s at low Q^2.
An elemental goal of hadron physics during the next ten years must be to design a program of experiments and theory that can map out the β-function.”

C. Roberts, White Paper on N* physics. Dec. 2008

$\alpha_{s,g1}$ and the AdS/CFT correspondance

Anti de Sitter/ Conformal Field Theory correspondence (AdS/CFT, or Maldacena duality):

Anti de Sitter space: ~Space with constant negative curvature.

Conformal Field Theory: ~Field theory without scale dependence.

Correspondence: a weakly interactive, gravity-like, theory in N-dimentional anti de Sitter space can be mapped on the boundary of the anti de Sitter space (⇒N-1 dim.) into a strongly interacting, QCD-like, conformal field theory.

$\alpha_{s,g1}$ and the AdS/CFT correspondance

Important fact: Strong force is conformal at low Q^2.

$$\begin{pmatrix} \text{weakly interacting} \\ \text{theory in 5-dim} \\ \text{AdS}_5 \text{ space} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \text{Strongly coupled} \\ \text{CFT}_4 \text{ in 4-dim.} \\ \ \end{pmatrix}$$

$\uparrow$ Calculable

$\uparrow$ Can be Strong Force at low Q^2

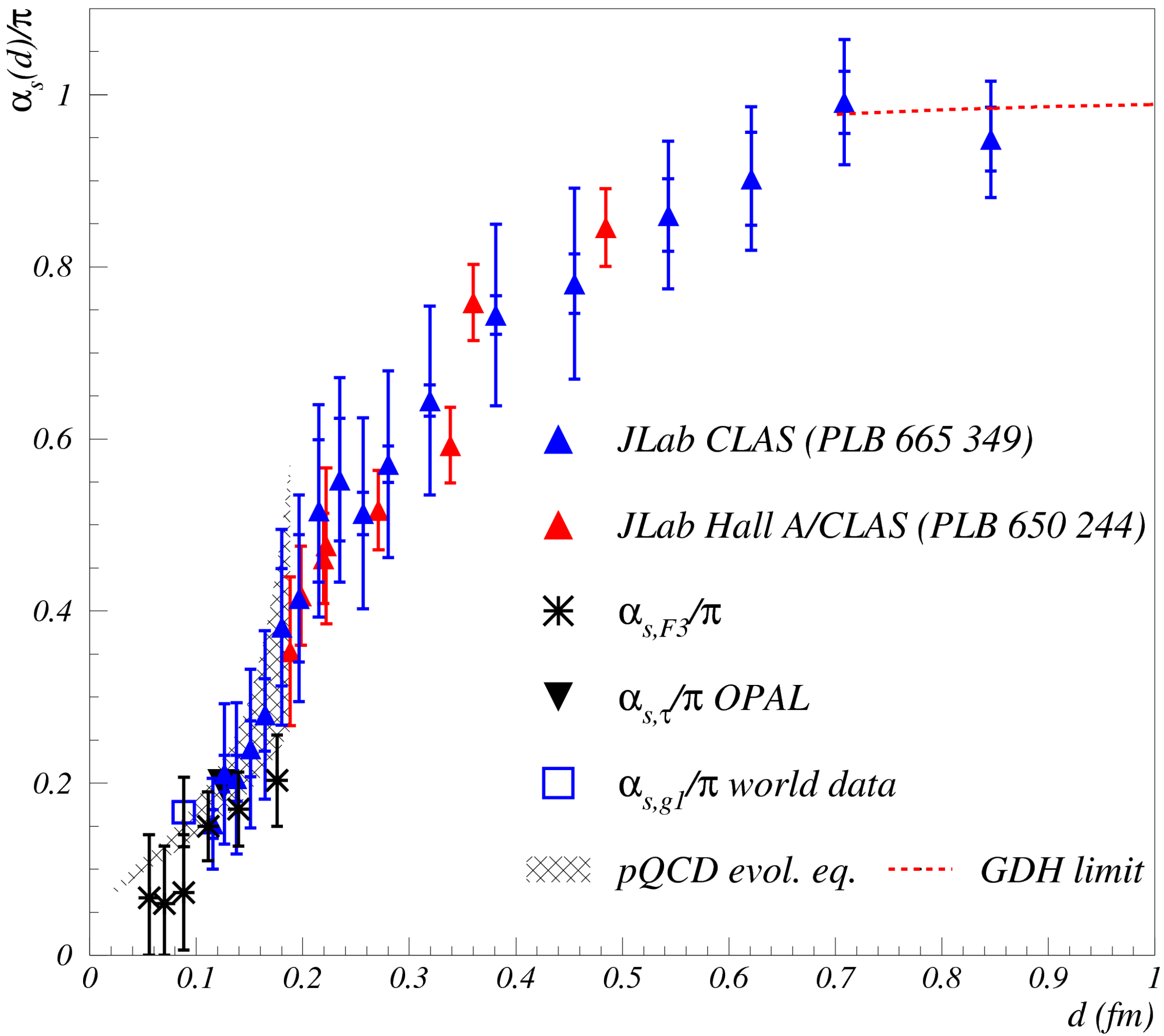
⇒May open new possibilities of QCD analytical calculations in non-perturbative domain (S. J. Brodsky, G. de Teramond,...)

PRL 94 201601 (2005); PRL 96 201601(2006)

Conclusions

- Precise data on SSF moments at low and intermediate Q^2.
- Good agreement with resonance models (MAID, Burkert-Ioffe). Not so for χpT.
- Effective QCD couplings can be defined over the whole Q^2 domain.
- Bjorken Sum is advantageous to define an effective coupling.
- Data and Sum rules allow to obtain the effective coupling at all Q^2.
- Comparison with low-Q^2 calculation shows similar features, same Q^2-dependence and similar size. In particular α_s "freezes" at low Q^2.
- QCD conformal at low Q^2 $\Rightarrow$ Application of AdS/CFT correspondence to non-perturbative QCD.

$\alpha_{s,g1}(d)$



$\alpha_s(d)/\pi$
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
d (fm)
JLab CLAS (PLB 665 349)
JLab Hall A/CLAS (PLB 650 244)
$\alpha_{s,F3}/\pi$
$\alpha_{s,\tau}/\pi$ OPAL
$\alpha_{s,g1}/\pi$ world data
pQCD evol. eq.
GDH limit