Ground-state baryon spectrum with the higher order hyperfine interactions

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Peking University, 2009.4.20

Outline

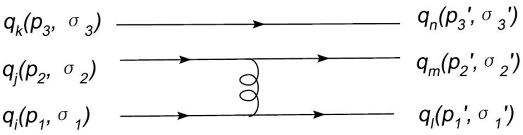
Motivation

- Several models for describing the baryon mass spectrum:

-The SU(6) model
-The Bag model
-The Skyrme model
-The quark model
-The large N_c baryon model
-The chiral dynamical model
-The lattice QCD
- ...

Motivation

- The quark model still remains a basic and indispensable tool in understanding hadron spectroscopy for its intuition and simplicity as a guide-line to other approaches. The quark model has been proved quite fruitful on the study of baryon spectrum, decays, and moments.
  S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. 45, 241 (2000).
- Quark-quark potential describes the strong interaction
  -the long range potential (the confinement potential)
  -the short range potential from one-gluon exchange
  (the perturbation term of α_s order, spin-dependent)



Motivation

- The short range potential includes:
  -a Coulomb term
  -the hyperfine interactions
  -the spin-orbit interactions
- The hyperfine interactions are important
  Δ-N mass split, L=0, Δ^0(udd)(1232) S=3/2, n(udd)(939) S=1/2
  A. De Rújula, H. Georgi, and S. L. Glashow, PRD 12, 147, (1975)
- Little experimental evidence for spin-orbit interaction. For example, consider some states...
  L=1, S=3/2,different J, $N_{3/2^-}(1675)$, $N_{5/2^-}(1670)$...

Motivation

- The hyperfine interactions bring mainly baryon (with same flavors) mass split
- Before, the hyperfine interactions consider one-gluon exchange
- What about baryon masses with higher order hyperfine interactions?

  - ? masses better if considering higher order (α_s^2) from two-gluon exchange

  S. N. Gupta and S. F. Radford, PRD 24,2309,(1981)

The quark model with higher order interactions

- The baryon Hamiltonian in the nonrelativistic quark model is N. Isgur and G. Karl, PRD 19, 2653,(1979)

$$H = \sum_i m_i + H_0 + H_{\rm hyp}, \quad (1)$$

$$H_0 = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} V_{\rm si}^{ij}, \quad (2)$$

$$H_{\rm hyp} = \sum_{i<j} H_{\rm hyp}^{ij}, \quad (3)$$

-$V_{\rm si}^{ij}$: the spin-independent potential
-$H_{\rm hyp}^{ij}$: the hyperfine interaction, spin-dependent

The quark model with Higher order interactions

- The potential $V_{\rm si}^{ij}$ has the form: $V_{\rm si}^{ij} = -\frac{2\alpha_s}{3r_{ij}} + bL_{min} + c$, in practice, $V_{\rm si}^{ij}$ is usually written: $V_{\rm conf}^{ij} = \frac{1}{2}Kr_{ij}^2 + U(r_{ij})$,
  -a harmonic-oscillator potential + an unspecified term
- The hyperfine interaction $H_{\rm hyp}^{ij}$ is

$$H_{\rm hyp}^{ij} = H_{\rm hyp}^{ij}(\alpha_s) + H_{\rm hyp}^{ij}(\alpha_s^2), \qquad (4)$$

-we add the higher order α_s^2 interaction in the usual hyperfine interaction.

The quark model with Higher order interactions

- The hyperfine interaction of α_s order derived from the one-gluon exchange process is

$$H^{ij}_{\rm hyp}(\alpha_s) = \frac{2\alpha_s}{3m_i m_j}[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij}) + \frac{1}{r^3_{ij}}(\frac{3\vec{S}_i\cdot\vec{r}_{ij}\ \vec{S}_j\cdot\vec{r}_{ij}}{r^2_{ij}} - \vec{S}_i\cdot\vec{S}_j)], \quad (5)$$

-where m_i is the constituent quark mass of the ith quark, $\vec{S}_i$ is the spin, and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the separation distance between a pair of quarks. The first term is called the Fermi contact term, and the second term is called the tensor term.
A. De Rújula, H. Georgi, and S. L. Glashow, PRD 12, 147, (1975)

The quark model with Higher order interactions

- $H^{ij}_{\rm hyp}(\alpha_s^2)$ derived from two-gluon exchange is
  S. N. Gupta and S. F. Radford, PRD 24,2309,(1981)

$$H^{ij}_{\rm contact}(\alpha_s^2) = \frac{16\pi\alpha_s}{9m_im_j}\vec{S}_i\cdot\vec{S}_j\{[\frac{\alpha_s}{12\pi}(26+9\ln 2)]\delta^3(\vec{r}_{ij})$$

$$-\frac{\alpha_s}{24\pi^2}(33-2n_f)\vec{\nabla}^2[\frac{\ln(\mu_{\rm GR}r_{ij})+\gamma_{\rm E}}{r_{ij}}]+\frac{21\alpha_s}{16\pi^2}\vec{\nabla}^2[\frac{\ln(\sqrt{m_im_j}r_{ij})+\gamma_{\rm E}}{r_{ij}}]\},$$

$$H^{ij}_{\rm tensor}(\alpha_s^2) = \frac{2\alpha_s}{3m_im_j}\frac{1}{r_{ij}^3}[\frac{3(\vec{S}_i\cdot\vec{r}_{ij})(\vec{S}_j\cdot\vec{r}_{ij})}{r_{ij}^2}-\vec{S}_i\cdot\vec{S}_j]\cdot\{\frac{4\alpha_s}{3\pi}+$$

$$\frac{\alpha_s}{6\pi}(33-2n_f)[\ln(\mu_{\rm GR}r_{ij})+\gamma_{\rm E}-\frac{4}{3}]-\frac{3\alpha_s}{\pi}[\ln(\sqrt{m_im_j}r_{ij})+\gamma_{\rm E}-\frac{4}{3}]\}, \quad (6)$$

- where $\mu_{\rm GR}$ is a renormalization scale, the subscript $\rm GR$ refers to the renormalization scheme and n_f is the number of effective quark flavors.

Calculate method

- Calculate the baryon masses
  Solve the equation

$$H|\Psi\rangle = E|\Psi\rangle \tag{7}$$

-step1. Approximate solutions, Ψ
-step2. Calculate $\langle\alpha|H|\beta\rangle$
-step3. Diagonalize the complete hamiltonian matrices to obtain masses

Approximate solutions

- A baryon wave function is described in terms of a totally antisymmetric color wave function C_A, multiplying a symmetric combination of flavor Φ_{flavor}, space functions ψ_{space} and spin wave functions χ_{spin}.

$$|\alpha >= C_A \sum \Phi\psi\chi = C_A \Psi^S \tag{8}$$

-Ψ^S is $SU(6) \otimes O(3)$ symmetric.

Approximate solutions

- $\chi^{\sigma}_{SS_z}$, the spin wave functions, from coupling three spins, $\frac{1}{2} \otimes \frac{1}{2} \otimes \frac{1}{2} = (1 \oplus 0) \otimes \frac{1}{2} = (\frac{3}{2}_S \oplus [\frac{1}{2}]_\rho) \oplus [\frac{1}{2}]_\lambda$
- Φ^{σ}_{B}, the flavor wave functions found from coupling three flavors $3 \otimes 3 \otimes 3 = 10_S \oplus 8_\rho \oplus 8_\lambda \oplus 1_A$
- $\psi^{\sigma}_{NLL_z}(\rho, \lambda)$, the spatial wave functions, the harmonic oscillator eigenfunctions
- where, $\sigma = S, A$ is totally symmetric and antisymmetric, $\sigma = \rho, \lambda$ is antisymmetric and symmetric under interchange of the first two quarks respectively.

Approximate solutions

- Using the following rules for combining C_A, χ^σ, Φ^σ, ψ^σ to obtain the baryon states function.

$$S = \frac{1}{\sqrt{2}}(M^\rho M^\rho + M^\lambda M^\lambda), A = \frac{1}{\sqrt{2}}(M^\rho M^\lambda - M^\lambda M^\rho), \quad (9)$$

$$M^\rho = \frac{1}{\sqrt{2}}(M^\rho M^\lambda + M^\lambda M^\rho), M^\lambda = \frac{1}{\sqrt{2}}(M^\rho M^\rho - M^\lambda M^\lambda). \quad (10)$$

$$e.g. \quad |N_8{}^2S_S\frac{1}{2}^+\rangle = C_A\frac{1}{\sqrt{2}}(\chi^\rho_+\phi^\rho + \chi^\lambda_+\phi^\lambda)\psi^S_{00}, \quad (11)$$

The states are labeled by $|X_{N_3}{}^{2S+1}L_\sigma J^P\rangle$, where X = baryons, S is the total quark spin, $L = S, P, D...$ is the total orbital angular momentum, σ = S, M or A is the permutational symmetry (symmetric, mixed symmetry, or antisymmetric respectively) , and J^P is the state's total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$ and parity.

Calculate $\langle\alpha|H|\beta\rangle$

- U and $H_{\rm hyp}$ differ from zero

$$H = H_{si} + H_{sd}, \tag{12}$$

where

$$H_{si} = \sum(m_i + \sum_i \frac{p_i^2}{2m_i}) + \sum_{i<j}(\frac{1}{2}Kr_{ij}^2 + U(r_{ij})), \tag{13}$$

$$H_{sd} = \sum(H^{ij}_{\rm hyp}(\alpha_s) + H^{ij}_{\rm hyp}(\alpha_s^2)), \tag{14}$$

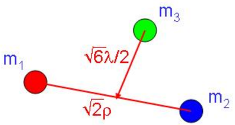
- Calculate $H_{\alpha\beta} = \langle\alpha|H|\beta\rangle = \langle\alpha|H_{si}|\beta\rangle + \langle\alpha|H_{sd}|\beta\rangle$
  -calculate $\langle\alpha|H_{si}|\beta\rangle$, spin-independent.
  -calculate $\langle\alpha|H_{sd}|\beta\rangle$, spin-dependent, baryon state mixing.

Calculate $\langle\alpha|H|\beta\rangle$

- The $\langle\alpha|H_{si}|\beta\rangle$ and $\langle\alpha|H^{ij}_{\rm hyp}(\alpha_s)|\beta\rangle$ have been obtained.
  N. Isgur and G. Karl, PRD, 20, 1191, (1979)
  N. Isgur and G. Karl, PRD 19, 2653,(1979)
  K.-T. Chao, N. Isgur, and G. Karl, PRD,23,155,(1981)
- We calculate $\langle\alpha|H^{ij}_{\rm hyp}(\alpha_s^2)|\beta\rangle$.
  Calculation methods from
  S. Capstick and N. Isgur, PRD, 34, 2809, (1986)
  S. N. Gupta and S. F. Radford, PRD 24,2309,(1981)
  S. N. Gupta and S. F. Radford, PRD, 25, 3430, (1982)
  J. Eiglsperger, arXiv:0707.1269 [hep-ph]
  D. M. Brink and G. R. Satchler, *Angular Monmentum* (Oxford Univ. Press, London, England, 1962); D. A. Varshalovich, A. N. Moskalev, and V. K. Khersonskii, *Quantum Theory of Angular Momentum* (World Scientific, Singapore, 1988)

Calculate $\langle\alpha|H_{\rm hyp}|\beta\rangle$

- The method of calculation $\langle\alpha|H^{ij}_{\rm hyp}(\alpha_s^2)|\beta\rangle$.
  -In the $S=0$ and $S=-3$ sectors, the states are completely symmetric in flavor, spin, and space under interchange of any two quarks
  $$\langle\alpha|H_{\rm hyp}|\beta\rangle = 3\langle\alpha|H^{12}_{\rm hyp}|\beta\rangle, \quad (15)$$
  -In the $S=-1$ and $S=-2$ sectors, $m_1=m_2=m_d$, $m_3=m_s$, the states are always symmetric under exchanger of quarks one and two
  $$\langle\alpha|H^{13}_{\rm hyp}|\beta\rangle = \langle\alpha|H^{23}_{\rm hyp}|\beta\rangle,$$
  and the hyperfine matrix elements become
  $$\langle\alpha|H_{\rm hyp}|\beta\rangle = \langle\alpha|H^{12}_{\rm hyp}+2H^{13}_{\rm hyp}|\beta\rangle$$



Calculate $\langle\alpha|H_{\rm hyp}|\beta\rangle$

- The $H^{12}_{\rm hyp}(r_{12})$ matrix elements are straight-forward $r_{12}=\sqrt{2}\rho$.
- The $H^{13}_{\rm hyp}(r_{13})$ matrix elements, change variables

$$\vec{\rho}'=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_3),\vec{\lambda}'=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_3-2\vec{r}_2), \tag{18}$$

$$\langle\alpha|H^{13}_{\rm hyp}|\beta\rangle=\sum_{\alpha',\beta'}\langle\alpha|\alpha'\rangle\langle\alpha'|H^{13}_{\rm hyp}|\beta'\rangle\langle\beta'|\beta\rangle, \tag{19}$$

-the calculation of the H^{13} part is identical to the H^{12} calculation.

-the matrix $T_{\alpha\alpha'}=\langle\alpha|\alpha'\rangle$ gives us the transformation between the two bases.

Calculate $\langle\alpha|H_{\rm hyp}|\beta\rangle$

- Object: study the ground-state baryons
  -take into account effects in $H_{\rm hyp}$ by calculating the mixing between the ground states N=0 and the positive-parity excited states N=2.
- Spatial wave function used:
  -N=0, spatial wave function: S_S;
  -N=2, spatial wave function: S'_S, S_M, D_S, D_M, P_A.
- Parameters: the values $\alpha_s = 0.7$, $x = 0.616$, $\alpha = 0.38$, $m_d = 0.275$ GeV, $\mu_{d\rm GR} = 0.333$ GeV, and $\delta = 270$ MeV.
  ($\delta \equiv \frac{4\alpha_s\alpha^3}{3\sqrt{2\pi}m_d^2}$, compare with the one-gloun exchange $\alpha_s = \pi/2$, $m_d = 0.33$ GeV, δ=260 MeV or 300 MeV)

The calculated ground-state masses

- Diagonalize the complete hamiltonian matrices to obtain the baryon masses. The calculated ground-state masses:

Table: The ground-state baryons (in units of MeV).

State (J^P)	Mass	Mass*	$M_{\rm exp}$	ΔM	ΔM*
$N\frac{1}{2}^+$	939	940	939	0	-1
$\Lambda\frac{1}{2}^+$	1113	1110	1116	3	6
$\Sigma\frac{1}{2}^+$	1192	1190	1193	1	3
$\Xi\frac{1}{2}^+$	1317	1325	1318	-1	-7
$\Delta\frac{3}{2}^+$	1231	1240	1232	1	-8
$\Sigma^*\frac{3}{2}^+$	1383	1390	1385	2	-5
$\Xi^*\frac{3}{2}^+$	1526	1530	1530	4	0
$\Omega\frac{3}{2}^+$	1673	1675	1672	-1	-3

$\Delta M = \mathrm{Mass} - M_{\rm exp}$, $\sum(\Delta M)^2 = 33\ \mathrm{MeV}^2$ and $193\ \mathrm{MeV}^2$ in Ref.*.

*N. Isgur and G. Karl, PRD,20,1191,(1979)

Corrections

- The corrections to the Gell-Mann–Okubo mass formula (GMO) and Gell-Mann's equal spacing rule (GME).

$$\frac{M_N + M_\Xi}{2} = \frac{3M_\Lambda + M_\Sigma}{4} + \delta_{\mathrm{GMO}}, \qquad (20)$$

$$\begin{aligned} M_{\Sigma^*} - M_\Delta &= M_{\Xi^*} - M_{\Sigma^*} + \delta_{\mathrm{GME1}} \\ &= M_\Omega - M_{\Xi^*} + \delta_{\mathrm{GME2}}, \end{aligned} \qquad (21)$$

-When $\delta_{\mathrm{GMO}} = \delta_{\mathrm{GME1}} = \delta_{\mathrm{GME2}} = 0$, equations (20) and (21) are the standard mass formulas which do not consider the H_{hyp} term.
- However, in the real world, $\delta_{\mathrm{GMO}} = -6.75$ MeV, $\delta_{\mathrm{GME1}} = 8$ MeV and $\delta_{\mathrm{GME2}} = 11$ MeV.

Corrections

Table: The corrections to GMO and GME in MeV.

	δ_{GMO}	δ_{GME1}	δ_{GME2}
δ_{exp}	-6.75	8	11
δ^*	2.5	10	5
δ_{our}	-4.75	9	5

- the corrections to the GMO improve significantly.
- the correction to the Gell-Mann's equal spacing rule is getting better slightly. (the calculated Ξ^* mass is not very good)
- It implies that the correction to the GMO is mainly due to the hyperfine interactions of α_s^2 order.

Summary and discussion

Taking into account the higher order hyperfine interactions

- The predicted ground-state masses are closer to the experimental data.
- The correction to the Gell-Mann–Okubo mass formula for the baryon-octet is distinctly improved.
- The effects from higher order hyperfine interaction shoule be considered in the quark model.

Discussion

- However, the values of the order α_s^2 matrix elements are in the range $0.4 - 10$ MeV, which are one order less than those matrix elements $2 - 170$ MeV obtained by leading order $O(\alpha_s)$. After diagonalizing the complete hamiltonian matrices, the order $O(\alpha_s^2)$ produces the calculated ground state mass shifts of $7 - 10$ MeV compared with that of the order $60 - 200$ MeV.
  Is the change value of few MeV reasonable and real? And how to understand?
- The higher order hyperfine interaction may be applied in heavy-quark baryon.

Thanks

Appendix

The light baryons belong to the soft regime. It has been shown, through a solution of Schwinger-Dyson equation, that when decreasing Q^2, α_s does not increase continuously but instead it saturates at a critical value $\alpha_s(0) \approx 0.7 \pm 0.3$. In our study regime, we assume $\alpha_s = 0.7$. The effective mass range of the constituent quark is $m_d \simeq 200 - 350$ MeV, and we suppose $m_d = 0.275\,\mathrm{GeV}$ here and the parameter $\alpha \approx 0.38$ to satisfy the value $\delta = 270$ MeV. By the similar relation that $\mu_{c\mathrm{GR}} \propto m_c$ for charmonium, we suppose that the renormalization scale $\mu_{d\mathrm{GR}}$ $=0.333$ GeV for baryons with $m_1 = m_2 = m_d$, and $\mu_{s\mathrm{GR}} = 0.333/x$ GeV for baryons with $m_1 = m_2 = m_s$, where we take $x = 0.616$. Th number of effective quark flavors is $n_f = 2$, which is the number of quarks with mass less than the energy scale μ_{GR}.