Inequalities on spin observables
Application to photoproduction and other reactions

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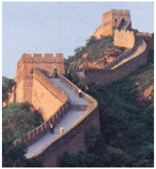

Collaboration with

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- Jacques Soffer (Temple U.)
- Oleg Teryaev (Dubna)

Outline

1. Introduction
2. Exclusive reactions
   - Simple case: elastic $\pi + \mathrm{N} \to \pi + \mathrm{N}$
   - Crossed reaction: $\bar{\mathrm{p}}\mathrm{p} \to \pi\pi$
   - Spin observables in $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$
   - Spin observables in pseudoscalar photoproduction
   - Vector-meson photoproduction
3. Inclusive reactions
   - Inclusive hadronic reactions
   - Quark distribution functions
4. Outlook

Introduction-1

- Spin observables are necessary to test the details of the dynamics,
- For instance, vector mesons anticipated (Breit) from the onset of spin-orbit forces in proton–proton scattering,
- For instance, helicity rules in QCD.
- Typical scenario: one or two observables are measured first,
- Question naturally arises: which new observable will provide the best improvement of knowledge,
- Corollary: If two or three observables are measured independently, is it possible to test whether they are compatible, without performing a full amplitude analysis?

Introduction-2

- If X, Y, Z, etc. are typical spin observables, with standard normalisation $-1 \leq X \leq +1$
- domain for $\{X, Y\}$ often smaller than the square $[-1, +1]^2$
- domain for $\{X, Y, Z\}$ *very* often smaller than the cube $[-1, +1]^3$
- Explicit inequalities are obtained relating two or three spin observables, for instance $X^2 + Y^2 \leq 1$, $\quad X^2 + Y^2 + Z^2 \leq 1$.
- Also triangles, tetrahedrons and even exotic-looking shapes,

πN scattering (1)

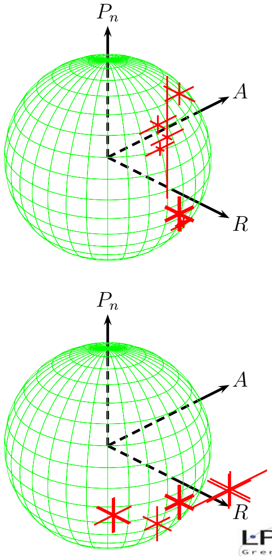
$$\mathcal{M} = -2m\bar{u}(\tilde{p}')\left[-\mathcal{A} + i\gamma.\frac{\tilde{q}+\tilde{q}'}{2}\mathcal{B}\right]u(\tilde{p})$$
$$= 8\pi\sqrt{s}\,\chi_f^\dagger\,(f + ig\,\boldsymbol{\sigma}.\boldsymbol{n})\,\chi_i\ ,$$

leading to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = I_0 = |f|^2 + |g|^2\ ,$$
$$I_0 P_n = I_0 A_n = 2\,\Im\mathrm{m}(fg^*)\ ,$$
$$I_0 A = (|f|^2 - |g|^2)\cos\vartheta + 2\,\Re\mathrm{e}(fg^*)\sin\vartheta\ ,$$
$$I_0 R = (|f|^2 - |g|^2)\sin\vartheta - 2\,\Re\mathrm{e}(fg^*)\cos\vartheta\ .$$

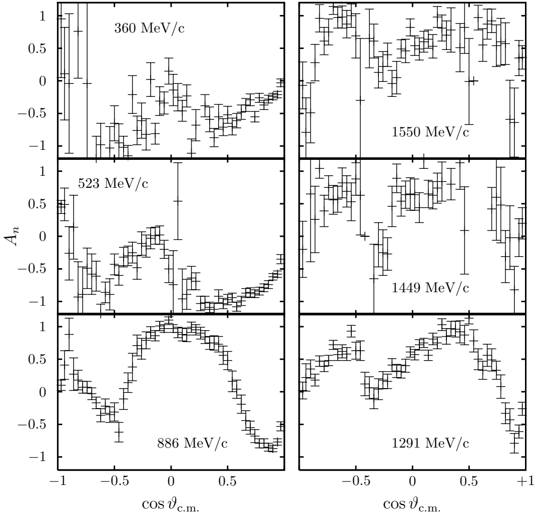
$$\boxed{P_n^2 + A^2 + R^2 = 1\ .}$$

either ignored in the analysis (and checked after) or used in the analysis.



$\bar{\mathrm{p}}\mathrm{p} \to \pi\pi$

Large spin effects observed in the crossed reaction (LEAR)



$\bar{\mathrm{p}}\mathrm{p} \to \pi\pi$

Similar identity

$$A_n^2 + A_{mm}^2 + A_{ml}^2 = 1$$

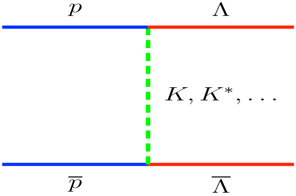
as for πN

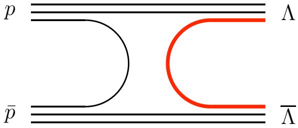
Hence $A_{mm} = A_{ml} = 0$ when $|A_n| = 1$

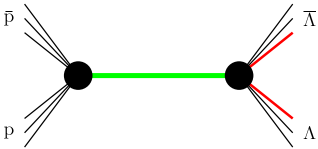
Similar results for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\mathrm{K}}\mathrm{K}$

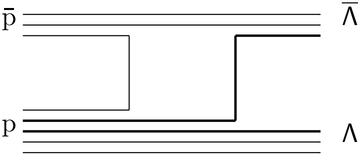
Motivations for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$

One of the best measured reactions at low-energy. Motivations: disentangle among





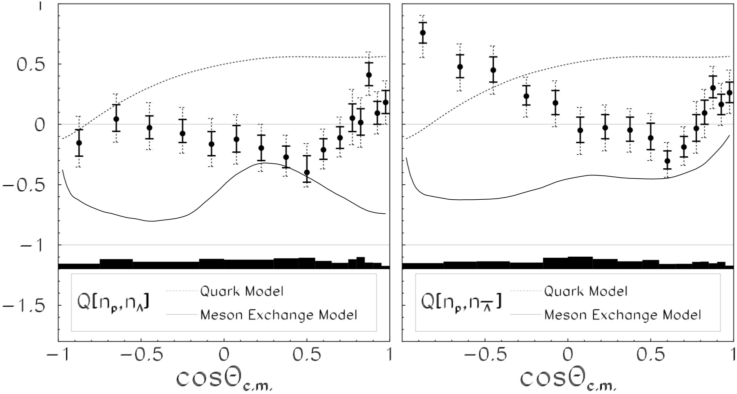




Results with a polarised target

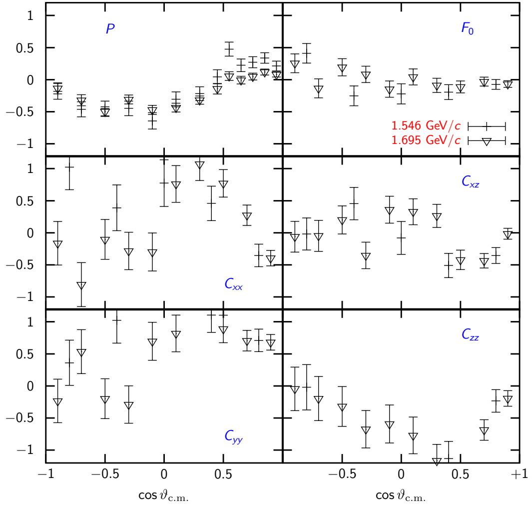
Last runs of PS185: the analysis led to D_{nn} and K_{nn} and several other observables.

Results of Paschke et al. at 1.637 GeV/c vs. some popular models



Similar results at 1.525 GeV/c. Considered as disappointing.

Inequalities



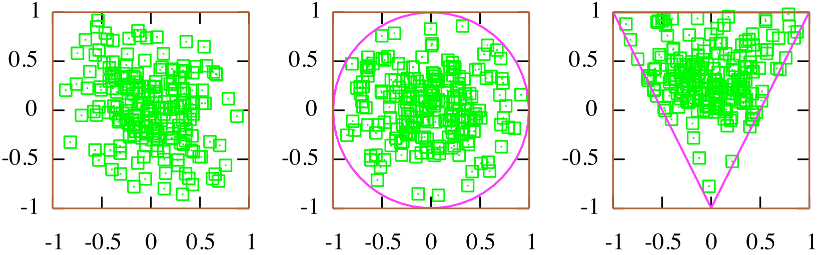
But could have been anticipated on the basis of previous results without polarised target

$$C_{zz}^2 + D_{nn}^2 \le 1$$

and similar inequalities.

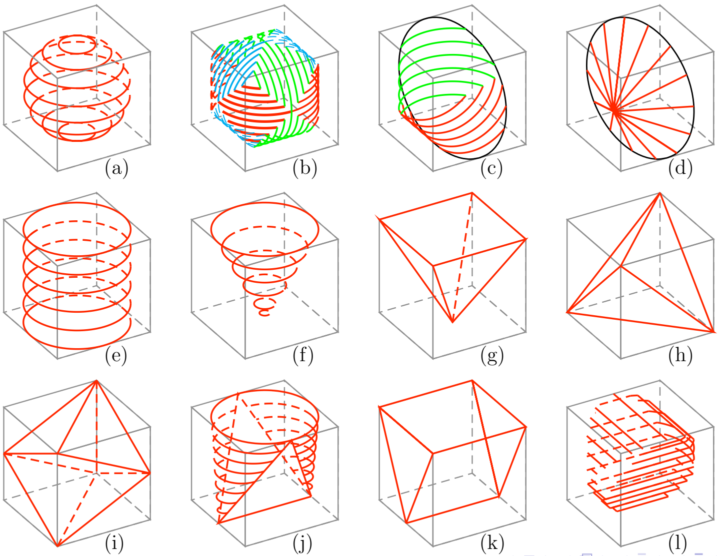
Systematics of inequalities for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$

Pair of observables Random simulations followed by rigorous derivation.



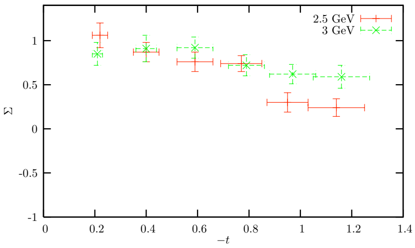
P_n vs. A_n A_n vs. D_{mm} P_n vs. C_{nn}

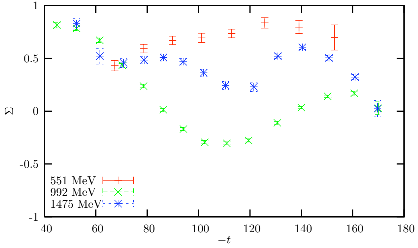
Cas of three observables of $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$



Beam asymmetry for $\gamma p \to \eta p$ and $\gamma p \to \pi^0 p$

Bussey et al. (1976) for η, GRAAL (2005) for π^0

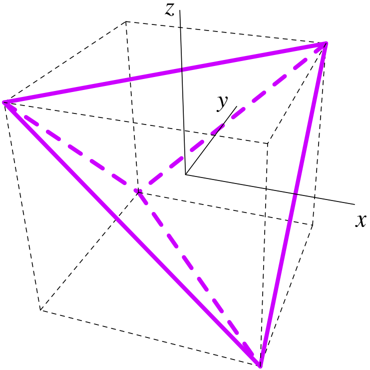




Spin observables in $\gamma\mathrm{N} \to \mathrm{K}\Lambda$

$\gamma + \mathrm{N} \to \mathrm{K} + Y$, and similar, have 4 amplitudes only (Chew et al., 1959).
All triples of observables are constrained in a domain smaller than the $[-1, +1]^3$ cube.

For instance, for
A = target asymmetry
P = polarisation of recoil baryon
Σ = beam asymmetry



Tetrahedron domain limiting the observables $x = A$, $y = P$ and $z = \Sigma$.

GRAAL–CLAS analysis

- The reaction $\gamma \mathrm{N} \to \mathrm{K}\Lambda$ recently measured by GRAAL (Grenoble) and CLAS (Jlab) at about the same energies, but with different spin observables.
- It is perhaps premature to attempt an unambiguous amplitude analysis combining both data sets, but our inequalities (Phys.Rev.C75:024002,2007, see also, Tabakin et al., Goldstein et al., etc.) can be used to check whether GRAAL and CLAS are compatible.

Some inequalities for $\gamma N \to K\Lambda$

A = target asymmetry
P = polarisation of recoil baryon
Σ = beam asymmetry
O_i = beam-recoil
C_i = target-recoil

$$C_x^2 + C_z^2 + O_x^2 + O_z^2 = 1 + T^2 - P^2 - \Sigma^2 \ ,$$

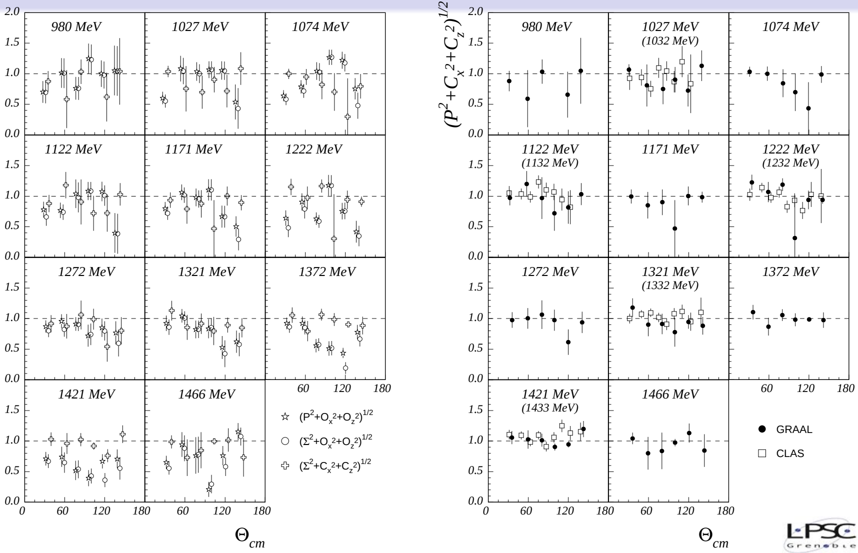
$$(P \text{ or } \Sigma)^2 + (O \text{ or } C)_x^2 + (O \text{ or } C)_z^2 \leq 1 \ ,$$

$$|T \pm P| \leq 1 \mp \Sigma \ .$$

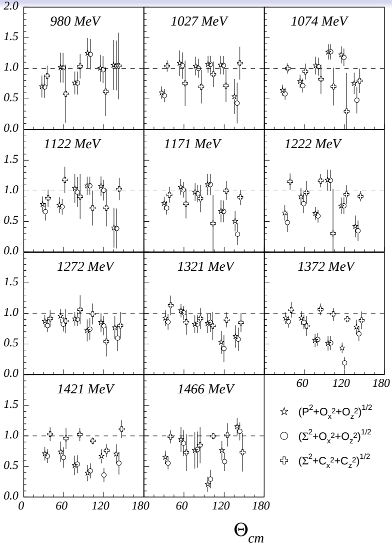
$$|O \text{ or } C|_{x,z}| \leq \min\{\sqrt{1-\Sigma^2}, \sqrt{1-P^2}\} \ ,$$

etc.

$(P^2 + O_x^2 + O_z^2)^{1/2} \leq 1$, $(\Sigma^2 + O_x^2 + O_z^2)^{1/2} \leq 1$, $(\Sigma^2 + C_x^2 + C_z^2)^{1/2} \leq 1$ and $(P^2 + C_x^2 + C_z^2)^{1/2} \leq 1$

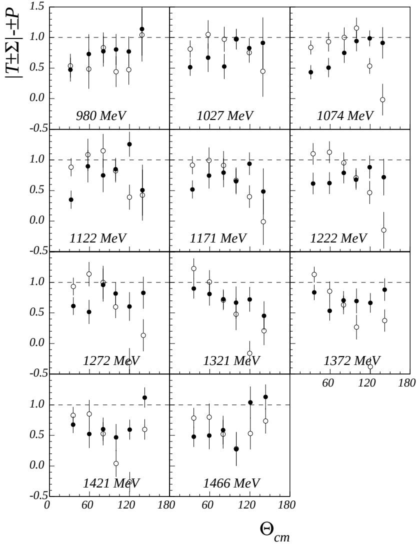


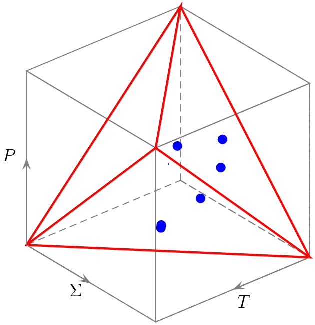
$(P^2 + O_x^2 + C_z^2)^{1/2} \leq 1$, $(\Sigma^2 + O_z^2 + C_x^2)^{1/2} \leq 1$, $(\Sigma^2 + O_x^2 + C_x^2)^{1/2} \leq 1$ and $(P^2 + O_z^2 + C_z^2)^{1/2} \leq 1$



From a combination of CLAS data (C_x, C_z) and GRAAL data (O_x, O_z)

Test of the tetrahedron inequality $|T \pm \Sigma| \mp P \leq 1$





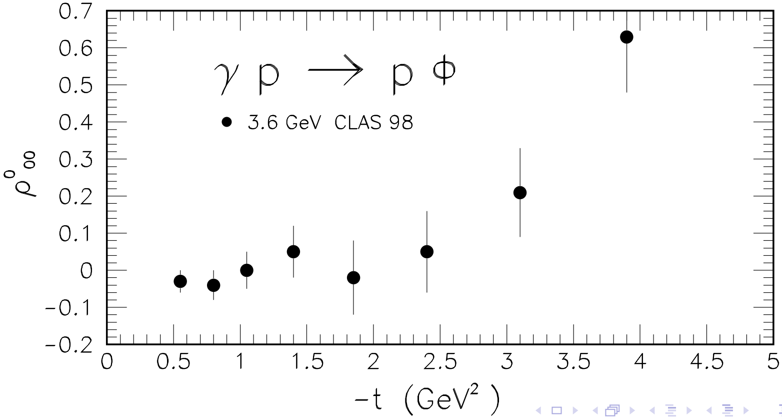
Case of 1222 MeV data.

Summary for pseudo-scalar photoproduction

- Several constraints on pairs of observables,
- All triples contrained,
- Consistency checks of GRAAL data
- Consistency checks of combined GRAAL & CLAS data

Vector-meson photoproduction

$\gamma + \mathrm{N} \to \phi + \mathrm{N}$ and similar, with 12 amplitudes. Some triples of observables are unconstrained. Note that if the vector meson is identified through its decay into two pseudoscalars, such as $\rho \to \pi\pi$ or $\phi \to \mathrm{KK}$, only the *tensor* polarisation is accessed. To get the axial polarisation, one needs other decay modes.

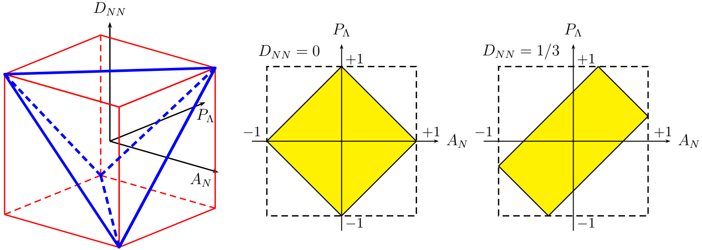


Inclusive hadronic reactions

For $a(\text{spin } 1/2) + b(\text{unpolarised}) \to c(\text{spin } 1/2) + X$, then

$$(1 \pm D_{NN})^2 \geq (A_{aN} \pm P_{cN})^2 + (D_{LL} \pm D_{SS})^2 + (D_{LS} \mp D_{SL})^2 \ .$$

in particular for $\mathrm{p}^\uparrow \mathrm{p} \to \Lambda^\uparrow X$, $\quad 1 \pm D_{NN} \geq |P_\Lambda \pm A_N|$,

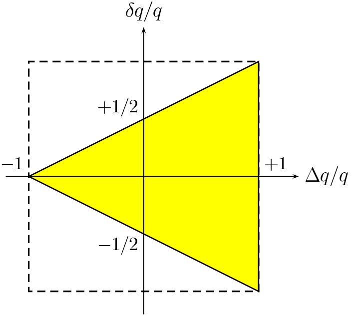


The allowed domain corresponding to the constraints (*left*). The slice of the full domain for $D_{NN} = 0$ (*middle*) and for $D_{NN} = 1/3$ (*right*).

Quark distribution function, Soffer's inequality

Let $q(x)$ be a quark distribution function,
$q_\pm(x)$ the quark distributions of definite helicity,
with $q(x) = q_+(x) + q_-(x)$ and $\Delta q(x) = q_+(x) - q_-(x)$ the usual spin-dependent distribution.
The positivity of each $q_\pm$ implies
$q(x) \geq \Delta q(x)$.
To construct the transversity distributions, one also needs the non-diagonal term in the helicity basis, δq. ($\delta q = q_\uparrow - q_\downarrow$ for a $N_\uparrow$)

The Soffer inequality
$[q + \Delta q]/2 \geq \delta q$ can be viewed as in the figure, similar the the triangle inequality on some pairs of observables for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$.



Outlook

- Rediscovery of the works by Michel, Minnaert, etc., and further development of limits on the domain allowed for spin observables (Artru, Elchikh, Soffer, etc.),
- Constraints useful when a few observables are measured
  - to see whether they are compatible
  - to determine which of the yet unknown observables has the widest range left
- Identities and inequalities on spin observables first derived by algebraic methods
- Better understood from the positivity of the density matrices describing the reaction and the crossed reactions
- Link with the theory of quantum information: a quantum state (initial spin) undergoes a quantum process (scattering), leading to a new state (final state). This is submitted to the usual restrictions, differences between pure and entangled spin states, the increase of entropy, etc.