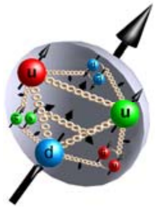


The Nucleon Spin:

Transversity and Preztelosity

?

Bo-Qiang Ma (马伯强)

PKU (北京大学物理学院)

Nstar 2009, April 19-22, 2009, IHEP, Beijing

It has been 20 years

of the proton "spin crisis" or "spin puzzle"

- **Spin Structure:**

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.020$$

$$\Downarrow$$

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

spin "crisis" or "puzzle": where is the proton's missing spin?

The first stage of experiments

- **Non-zero strange spin constribution**

$$\Delta u = 0.750$$

$$\Delta d = -0.511$$

$$\Delta s = -0.218$$

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.020$$

A large strange spin contribution?

The Ellis-Jaffe sum rule & Its violation

$$A_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$

- **Neutron beta decay and isospin symmetry**

$$\Delta u - \Delta d = \frac{G_A}{G_V} = 1.261$$

- **Strangeness changing hyperon decay and SU(3) symmetry**

$$\Delta u + \Delta d - 2\Delta s = 0.675$$

- **The assumption of zero strange spin constribution** $\Delta s = 0$

The Ellis-Jaffe sum $A_1^p = \int_0^1 dx g_1^p(x) = 0.198$

However, what EMC measured $A_1^p = \int_0^1 dx g_1^p(x) = 0.126$

A previous global fit:
SU(3) symmetry+measured g_1^p g_1^n

$$\Delta u = 0.83 \pm 0.03$$

$$\Delta d = -0.43 \pm 0.03$$

$$\Delta s = -0.10 \pm 0.03$$

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

The second stage of experiments.

The third stage of experiments:

g_1^p g_1^n +semi-inclusive DIS process

$$\Delta u = 0.599 \pm 0.022 \pm 0.065$$

$$\Delta d = -0.280 \pm 0.026 \pm 0.057$$

$$\Delta s = 0.028 \pm 0.033 \pm 0.009$$

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.347 \pm 0.024 \pm 0.040$$

HERMES Collaboration, PRL92 (2004) 012005.

The Proton "Spin Crisis"

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$\Delta u = \tfrac{4}{3}; \qquad \Delta d = -\tfrac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Many Theoretical Explanantions

- The sea quarks of the proton are largely negatively polarized
- The gluons provide a significant contribution to the proton spin

It was though that the spin "crisis" cannot be understood within the quark model: " the lowest uud valence component of the proton is estimated to be of only a few percent."   R.L. Jaffe and Lipkin, PLB266(1991)158

The proton spin crisis

& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.

B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

The Notion of Spin

- Related to the space-time symmetry of the Poincaré group
- Generators $P^{\mu} = (H, \vec{P}),$ space-time translator

$J^{\mu\nu}$ infinitesimal Lorentz transformation

$\vec{J}$ $\quad J^{k} = \frac{1}{2}\varepsilon_{ijk} J^{ij}$ angular momentum

$\vec{K}$ $\quad K^{k} = J^{k0}$ boost generator

Pauli-Lubanski vertor $w_{\mu} = \frac{1}{2} J^{\rho\sigma} P^{\nu} \varepsilon_{\nu\rho\sigma\mu}$

Casimir operators: $P^2 = P^{\mu} P_{\mu} = m^2$ mass

$w^2 = w^{\mu} w_{\mu} = s^2$ spin

The Wigner Rotation

for a rest particle $(m,\vec{0}) = p^{\mu} \qquad (0,\vec{s}) = \mathrm{w}^{\mu}$

for a moving particle $\mathrm{L}(p)p = (m,\vec{0}) \qquad (0,\vec{s}) = \mathrm{L}(p)w/m$

$\mathrm{L}(p)$ = ratationless Lorentz boost

Wigner Rotation

$$\vec{s}, p_{\mu} \rightarrow \vec{s'}, p'_{\mu}$$

$$\vec{s'} = R_w(\Lambda, p)\vec{s} \qquad p' = \Lambda p$$

$R_w(\Lambda, p) = \mathrm{L}(p')\Lambda \mathrm{L}^{-1}(p)$ a pure rotation

E.Wigner, Ann.Math.40(1939)149

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame

Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^{+} + m)\chi^{\uparrow}(F) - q^{R}\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^{+} + m)\chi^{\downarrow}(F) + q^{L}\chi^{\uparrow}(F)].$$

What is Δ q measured in DIS

- Δ q is defined by $\Delta q \, s_\mu = \langle p,s|\bar{q}\gamma_\mu\gamma_5 q|p,s\rangle$

$$\Delta q = \langle p,s|\bar{q}\gamma^+\gamma_5 q|p,s\rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 \mathrm{d}x\left[q^\uparrow(x) - q^\downarrow(x)\right]$$

- Using conventional Dirac spinors

$$\Delta q = \int \mathrm{d}^3\vec{p} M_q\left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p})\right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin or quark spin in the infinite momentum frame, not that in the rest frame of the proton

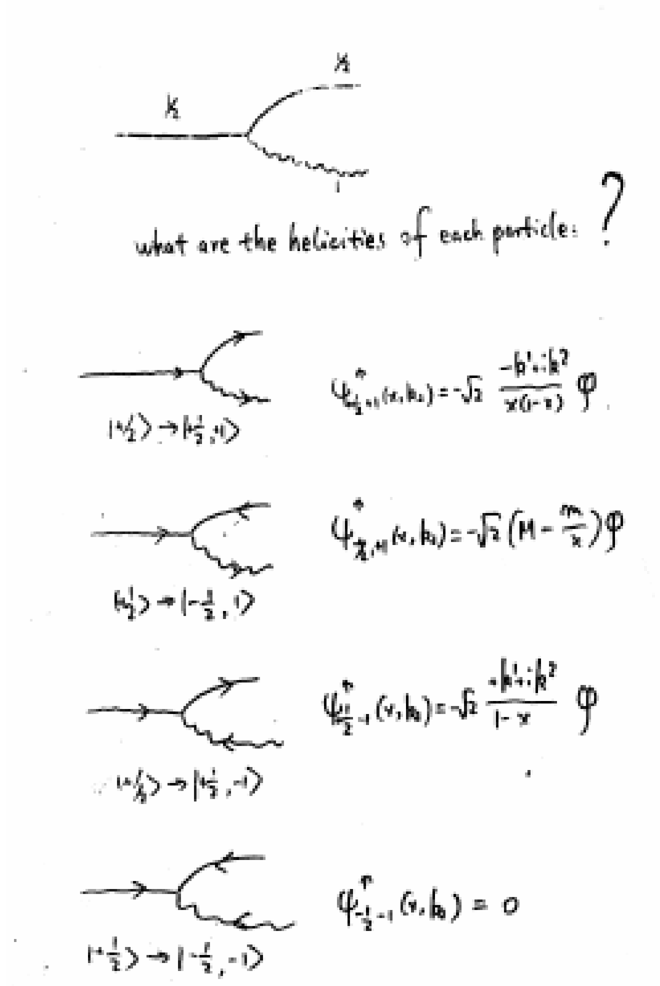
A general consensus

The quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

H.-Y.Cheng, hep-ph/0002157，
Chin.J.Phys.38:753,2000

A QED Example of Relativistic Spin Effect

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311



The lowest spin states of a composite system must contain the orbital angular momentum contribution.

$$\Delta s_{\text{non-rel}} + L_{\text{non-rel}} = \Delta s_{\text{rel}} + L_{\text{rel}}$$

Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame

$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the rest frame}$$

does not mean that

$$\sum_q \vec{s}_q = \vec{S}_p \quad \text{in the infinite momentum frame}$$

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

The Spin Distributions in Quark Model

The spin distribution probabilities in the quark-diquark model

$$u_V^{\uparrow} = \frac{1}{18}; \quad u_V^{\downarrow} = \frac{2}{18}; \quad d_V^{\uparrow} = \frac{2}{18}; \quad d_V^{\downarrow} = \frac{4}{18};$$

$$u_S^{\uparrow} = \frac{1}{2}; \quad u_S^{\downarrow} = 0; \quad d_S^{\uparrow} = 0; \quad d_S^{\downarrow} = 0. \qquad (7)$$

Naive Quark Model:

$$\Delta u = \frac{4}{3}; \qquad \Delta d = -\frac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Relativistic Effect due to Melosh-Rotation

$$\Delta u_v(x) = u_v^{\uparrow}(x) - u_v^{\downarrow}(x) = -\frac{1}{18} a_V(x) W_V(x) + \frac{1}{2} a_S(x) W_S(x);$$

$$\Delta d_v(x) = d_v^{\uparrow}(x) - d_v^{\downarrow}(x) = -\frac{1}{9} a_V(x) W_V(x).$$

from

$$a_S(x) = 2u_v(x) - d_v(x);$$

$$a_V(x) = 3d_v(x).$$

We obtain

$$\Delta u_v(x) = [u_v(x) - \frac{1}{2} d_v(x)] W_S(x) - \frac{1}{6} d_v(x) W_V(x);$$

$$\Delta d_v(x) = -\frac{1}{3} d_v(x) W_V(x).$$

Relativistic SU(6) Quark Model

Flavor Symmetric Case

Relativistic Correction: $M_q = 0.75$

$\Delta u = \frac{4}{3} M_q = 1; \quad \Delta d = -\frac{1}{3} M_q = -0.25; \quad \Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s = 0.75$

$F_2^n(x)/F_2^p(x) \geq \frac{2}{3}$ for all x

Relativistic SU(6) Quark Model

Flavor Asymmetric Case

Relativistic Correction: $M_u \approx 0.6$; $M_d \approx 0.9$

$\Delta u = \frac{4}{3} M_u = 0.8$; $\Delta d = -\frac{1}{3} M_d = -0.3$; $\Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.5$

$F_2^n(x)/F_2^p(x) \to \frac{1}{4}$ at large x

B.-Q.Ma, Phys. Lett. **B 375** (1996) 320.

Relativistic SU(6) Quark Model

Flavor Asymmetric Case + Intrinsic Sea

For Intrinsic $d\bar{d}$ Sea ($\sim$ 15%): $\Delta d_{\rm sea} \approx -0.07$

For Intrinsic $s\bar{s}$ Sea ($\sim$ 5%): $\Delta s_{\rm sea} \approx -0.03$

Thus: $\Sigma = \Delta u + \Delta d + \Delta s + \Delta d_{\rm sea} + \Delta s_{\rm sea} \approx 0.4$

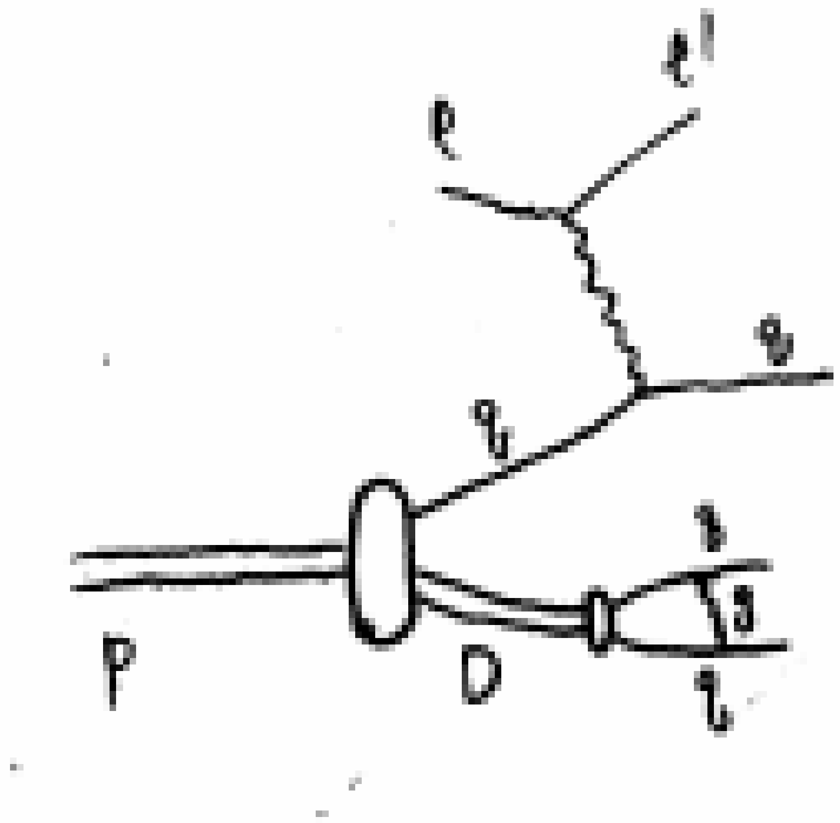
S. J. Brodsky and B.-Q.Ma, Phys. Lett. B 381 (1996) 317.

More detailed discussions, see, B.-Q.Ma, J.-J.Yang, I.Schmidt, Eur.Phys.J.A12(2001)353

Understanding the Proton Spin "Puzzle" with a New "Minimal" Quark Model

Three quark valence component could be as large as 70% to account for the data

A relativistic quark-diquark model



A relativistic quark-diquark model

- The unpolarized distribution of quark q in hadron h can be written as

$$q(x) = c_q^S a_S(x) + c_q^V a_V(x),$$

where $a_D(x)$ is

$$a_D(x) \propto \int [\mathrm{d}^2\mathbf{k}_\perp] |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V),$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$\phi(x, \mathbf{k}_\perp) = A_D \exp\left\{-\frac{1}{8\alpha_D^2}\left[\frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}\right]\right\},$$

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [\mathrm{d}^2 \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

The Melosh-Wigner rotation

in pQCD based parametrization of quark helicity distributions

"The helicity distributions measured on the light-cone are related by a Wigner rotation (Melosh transformation) to the ordinary spin S_i^z of the quarks in an equal-time rest-frame wavefunction description. Thus, due to the non-collinearity of the quarks, one cannot expect that the quark helicities will sum simply to the proton spin."

S.J.Brodsky, M.Burkardt, and I.Schmidt,

Nucl.Phys.B441 (1995) 197-214, p.202

pQCD counting rule

$$q_{\mathrm{h}}^{\pm} \propto (1-x)^{p}$$

$$p = 2n - 1 + 2\,|\Delta s_z| \qquad\qquad \Delta s_z = s_q - s_N$$

- **Based on the minimum connected tree graph of hard gluon exchanges.**
- **"Helicity retention" is predicted -- The helicity of a valence quark will match that of the parent nucleon.**

Parameters in pQCD counting rule analysis

In leading term

$$q_{\mathrm{i}}^{+} = \frac{\tilde{A}_{q_i}}{B_3} x^{-\frac{1}{2}} (1-x)^3$$

$$q_{\mathrm{i}}^{-} = \frac{\tilde{C}_{q_i}}{B_5} x^{-\frac{1}{2}} (1-x)^5$$

Baryon	q_1	q_2	$\tilde{A}_{q_1}$	$\tilde{C}_{q_1}$	$\tilde{A}_{q_2}$	$\tilde{C}_{q_2}$
p	u	d	1.375	0.625	0.275	0.725

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

Two different sets of parton distributions

- SU(6) quark-diquark model

$$\Delta u_v(x)=[u_v(x)-\frac{1}{2}d_v(x)]W_S(x)-\frac{1}{6}d_v(x)W_V(x),$$
$$\Delta d_v(x)=-\frac{1}{3}d_v(x)W_V(x).$$

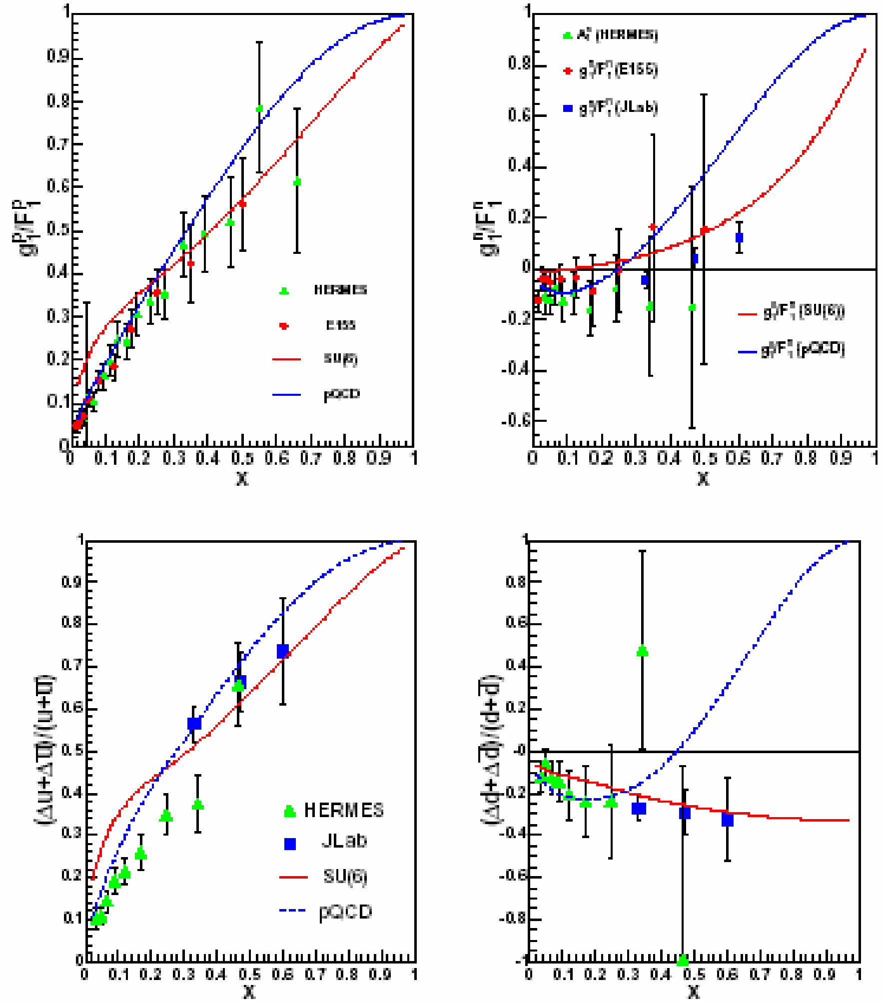
- pQCD based counting rule analysis

$$\begin{aligned}
u_v^{\mathrm{pQCD}}(x) &= u_v^{\mathrm{para}}(x),\\
d_v^{\mathrm{pQCD}}(x) &= \frac{d_v^{\mathrm{th}}(x)}{u_v^{\mathrm{th}}(x)}u_v^{\mathrm{para}}(x),\\
\Delta u_v^{\mathrm{pQCD}}(x) &= \frac{\Delta u_v^{\mathrm{th}}(x)}{u_v^{\mathrm{th}}(x)}u_v^{\mathrm{para}}(x),\\
\Delta d_v^{\mathrm{pQCD}}(x) &= \frac{\Delta d_v^{\mathrm{th}}(x)}{u_v^{\mathrm{th}}(x)}u_v^{\mathrm{para}}(x),
\end{aligned}$$

- CTEQ5 set 3 as input.

Different predictions in two models

- Helicity distribution
  - SU(6) quark-diquark model: $\Delta u(x)/u(x) \to 1$ as $x \to 1$. $\Delta d(x)/d(x) \to -\frac{1}{3}$ as $x \to 1$.
  - pQCD based counting rule analysis: $\Delta u(x)/u(x) \to 1$ as $x \to 1$. $\Delta d(x)/d(x) \to 1$ as $x \to 1$.



$W^{\pm}$ production at RHIC

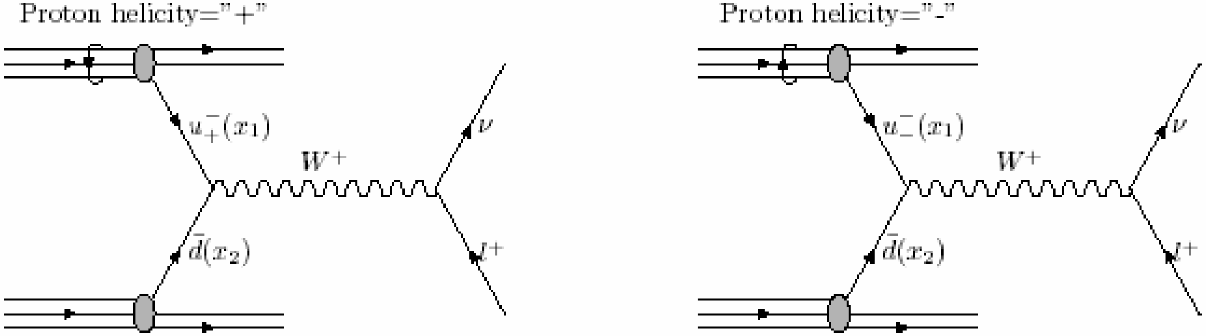
- Parity-violating asymmetry

$$A_L = -\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad A_L = -\frac{1}{P} \times \frac{N'_+ - N'_-}{N'_+ + N'_-},$$

- The maximum parity violation of W bosons.
- $u\bar{d} \to W^+$ and $\bar{u}d \to W^-$.
- At LO, the parity-violating asymmetry will approach $\Delta q(x)/q(x)$ when the rapidity of $W^{\pm}$, y_W, is large.

C. Bourrely, J. Soffer, Nucl. Phys. B423(1994) 329

One of the possible leading order production of W^+ production.



$$A_L^{W^+} = \frac{u_-^-(x_1)\bar{d}(x_2) - u_+^-(x_1)\bar{d}(x_2)}{u_-^-(x_1)\bar{d}(x_2) + u_+^-(x_1)\bar{d}(x_2)} = \frac{\Delta u(x_1)}{u(x_1)}$$

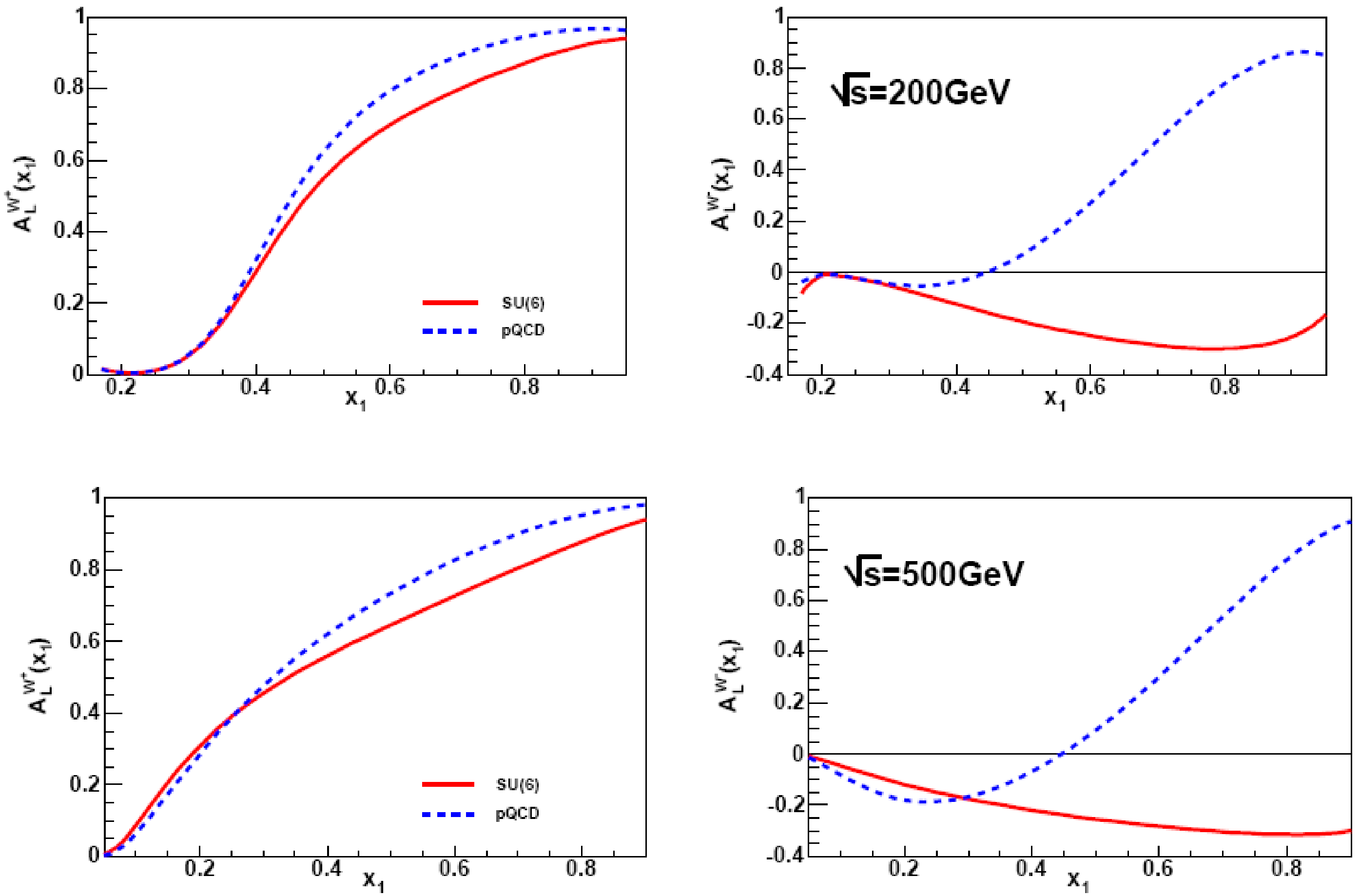
$$A_L^{W^+} = \frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$A_L^{W^-} = \frac{\Delta d(x_1)\bar{u}(x_2) - \Delta\bar{u}(x_1)d(x_2)}{d(x_1)\bar{u}(x_2) + \bar{u}(x_1)d(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}} e^{y_W}$$

$$x_2 = \frac{M_W}{\sqrt{s}} e^{-y_W}$$

X. Chen, Y. Mao, B.-Q. Ma, NPA A759(2008)188



The Melosh-Wigner rotation
is not the whole story

- **The role of sea is not addressed**
- **The role of gluon is not addressed**

It is important to study the roles played by the sea quarks and gluons. Thus more theoretical and experimental researches can provide us a more completed picture of the nucleon spin structure.

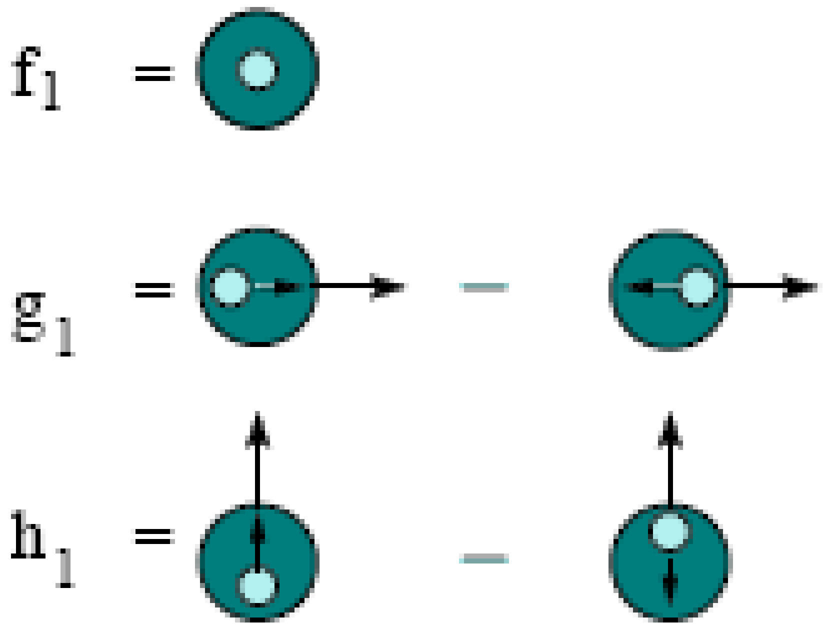
Chances: New Research Directions

- New quantities：Transversity, Generalized Parton Distributions, Collins Functions, Silver Functions, Boer-Mulders Functions

- Hyperon Physics：The spin structure of Lambda and Sigma hyperons

What is transversity?

Three fundamental quantities of quark distributions



The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle\, p, \uparrow | \bar{q}_{\lambda} \gamma^{\perp} \gamma^{+} q_{-\lambda} | p, \downarrow \rangle$$

$$\delta q(x) = \int [\mathrm{d}^2 \boldsymbol{k}_{\perp}] \tilde{M}_q(x, \boldsymbol{k}_{\perp}) \Delta q_{\mathrm{RF}}(x, \boldsymbol{k}_{\perp})$$

$$\tilde{M}_q(x, \boldsymbol{k}_{\perp}) = \frac{(k^{+} + m)^2}{(k^{+} + m)^2 + \boldsymbol{k}_{\perp}^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

Transversity with Melosh-Wigner rotation in the quark-diquark model

$$\delta u_v(x) = \left[u_v(x) - \frac{1}{2} d_v(x) \right] \hat{W}_S(x) - \frac{1}{6} d_v(x) \hat{W}_V(x),$$

$$\delta d_v(x) = -\frac{1}{3} d_v(x) \hat{W}_V(x),$$

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The transversity in pQCD, in similar to helicity distributions

$$\delta q(x) = \frac{\tilde{A}_q}{B_3} x^{(-1/2)} (1-x)^3 - \frac{\tilde{C}_q}{B_5} x^{(-1/2)} (1-x)^5$$

Baryon	q_1	q_2	$\tilde{A}_{q1}$	$\tilde{C}_{q1}$	$\tilde{A}_{q2}$	$\tilde{C}_{q2}$	$\hat{A}_{q1}$	$\hat{C}_{q1}$	$\hat{A}_{q2}$	$\hat{C}_{q2}$
p	u	d	1.375	0.625	0.275	0.725	1.52	0.48	0.305	0.695

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

Transversity in two models

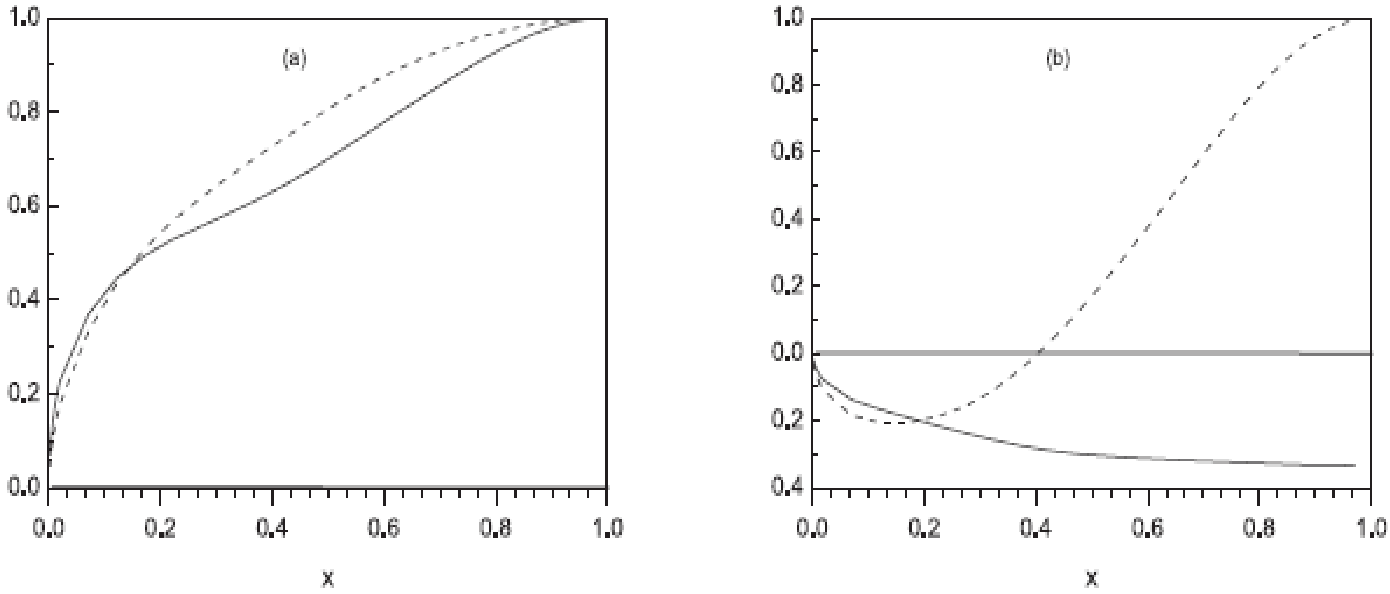
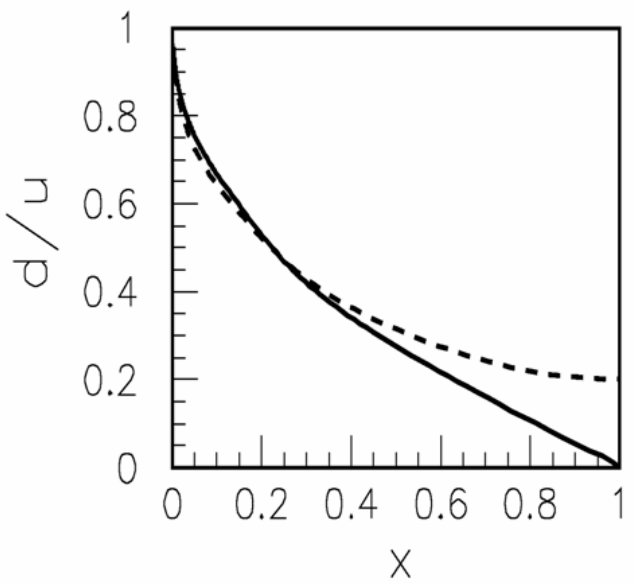
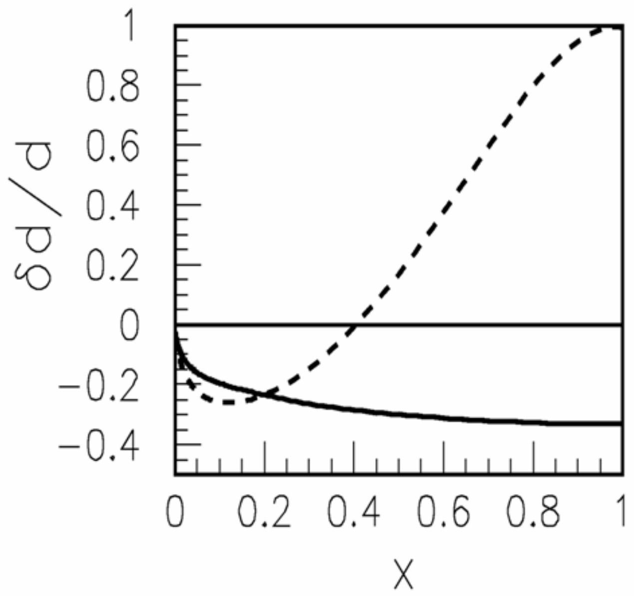


图 3.1 $\delta u/u$ (a) 和$\delta d/d$ (b) 的曲线示意图，$Q^2 = 2$ GeV^2, 实线代表的是quark-diquark 模型，虚线代表的是pQCD 理论.

SU(6) quark-diquark model VS pQCD based analysis

Ma, Schmidt and Yang, PRD 65, 034010 (2002)





solid curve for SU(6) and dashed curve for pQCD

Collins asymmetry in semi-inclusive production

$$A_{UT}^{Collins} = \frac{1}{|S_\perp|}\frac{d\sigma_{UT}^{Collins}}{d\sigma_{UU}}$$ After integration over specific weighting functions

$$A_T(x,y,z) = -\frac{(1-y)\sum_q e_q^2 \delta q(x) H_1^{\perp(1)q}(z)}{(1-y+y^2/2)\sum_q e_q^2 q(x) D_1^q(z)}$$

$q(x)$ unpolarized quark distrtion

$\delta q(x)$ transversity

$D_1(x)$ unpolarized fragmentation function

$H_1^{\perp(1)q}(x)$ Collins function

Two sets of Collins functions

Set I

$$\delta\hat{q}_{fav}^{\pi(1/2)}(z) = C_f z(1-z)\hat{u}^{\pi^+}(z) \quad \delta\hat{q}_{unfav}^{\pi(1/2)}(z) = C_u z(1-z)\hat{u}^{\pi^+}(z)$$

$$C_f = -0.29 \pm 0.04 \qquad C_u = 0.33 \pm 0.04$$

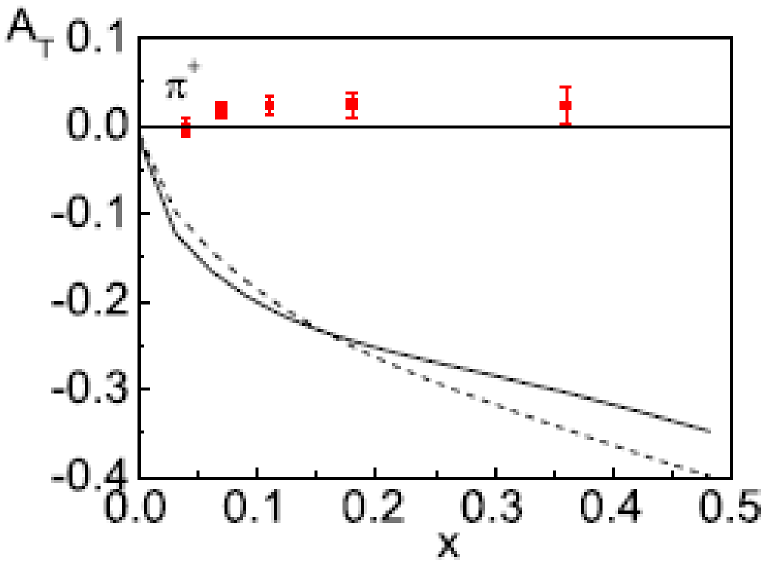
Set II

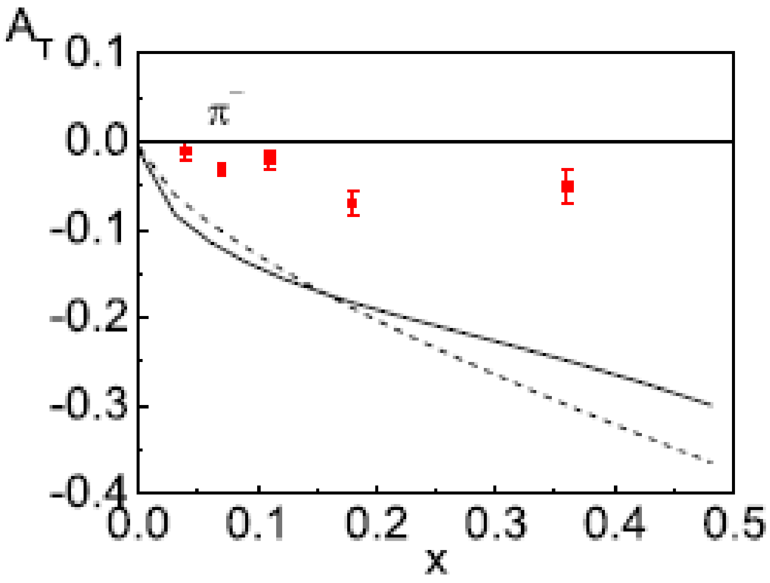
$$\delta\hat{q}_{fav}^{\pi(1/2)}(z) = C_f z(1-z)\hat{u}^{\pi^+}(z) \quad \delta\hat{q}_{unfav}^{\pi(1/2)}(z) = C_u z(1-z)\hat{d}^{\pi^+}(z)$$

$$C_f = -0.29 \pm 0.02 \qquad C_u = 0.56 \pm 0.07$$

W. Vogelsang and F. Yuan, Phys. Rev. D72 (2005).

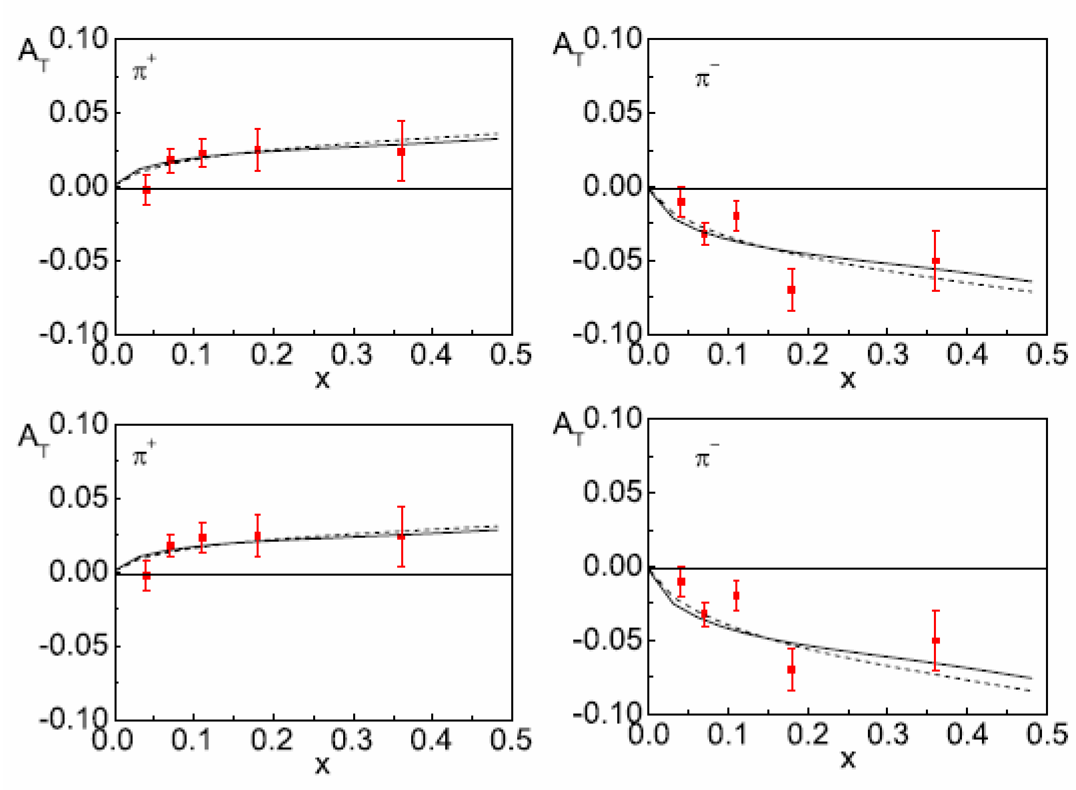
Prediction for HERMES with only favored fragmentation





Y. Huang, J. She, and B.-Q. Ma, Phys. Rev. D76 (2007) 034004.

Including unfavored fragmentation in HERMES condition



set I

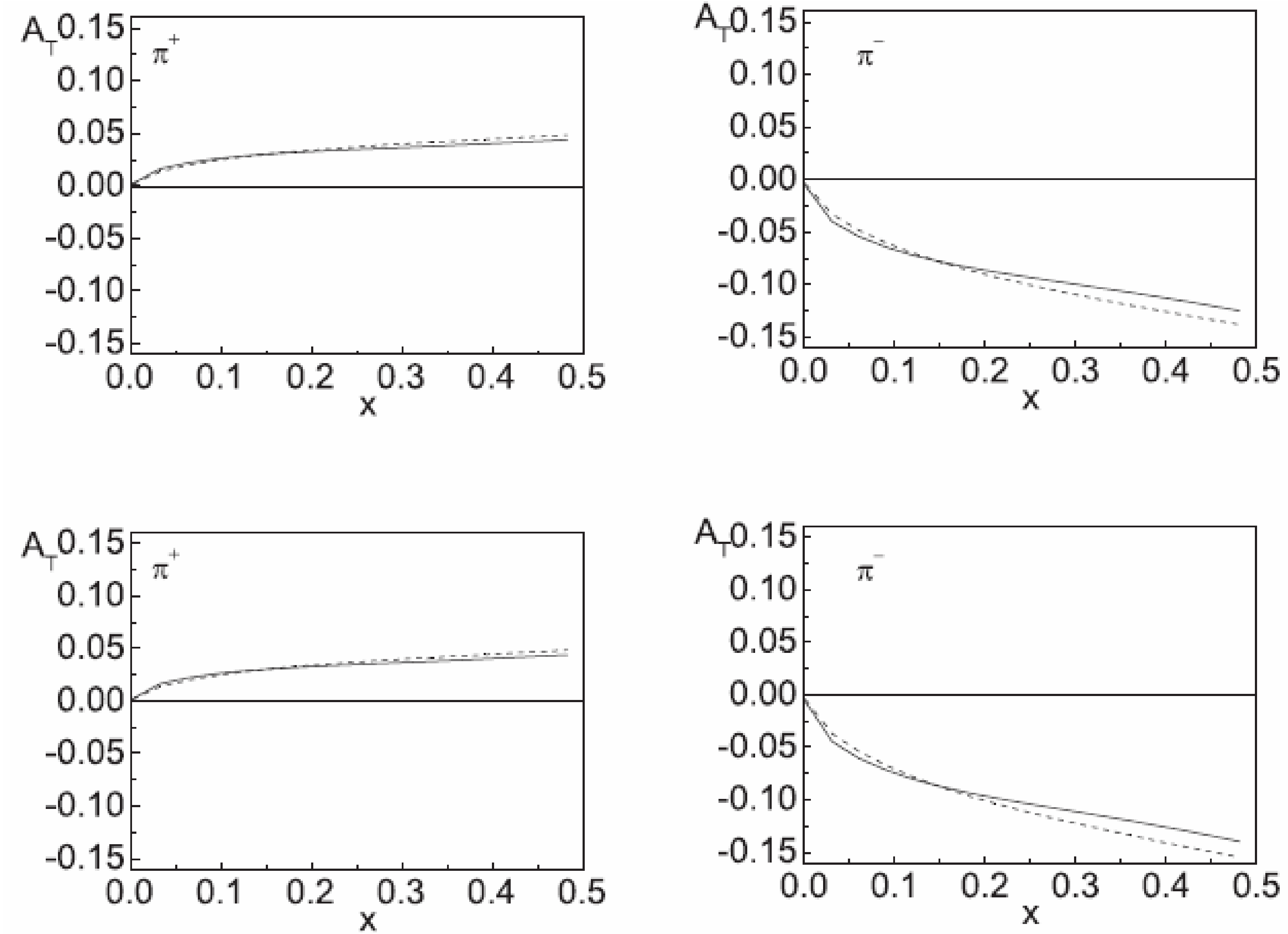
set II

Y. Huang, J. She, and B.-Q. Ma, Phys. Rev. D76 (2007) 034004.

Prediction in JLab condition (proton target)

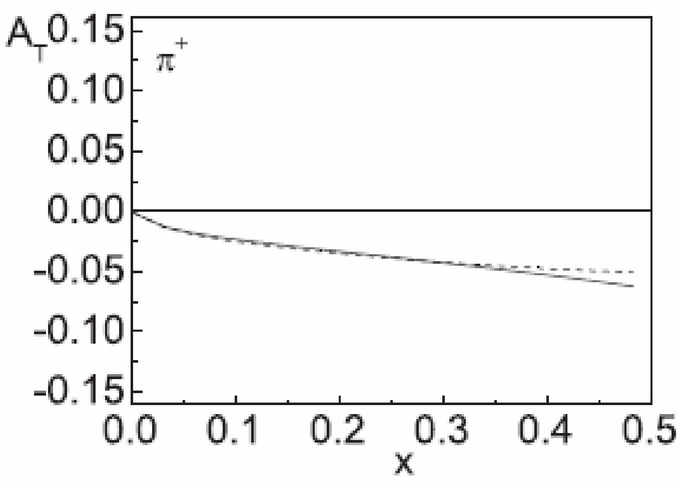
Set I

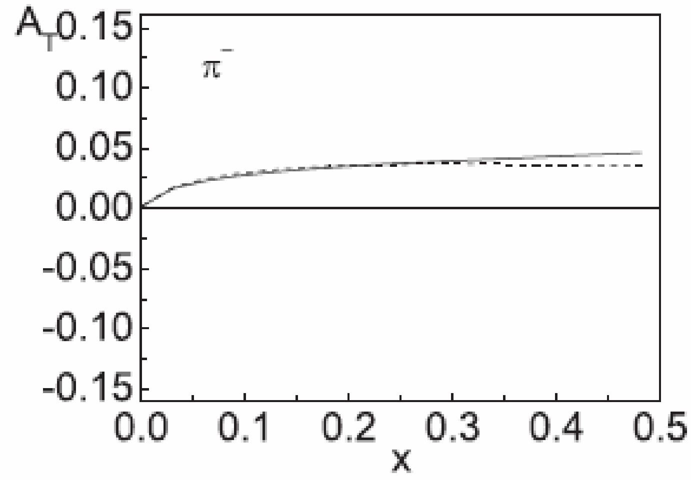
Set II



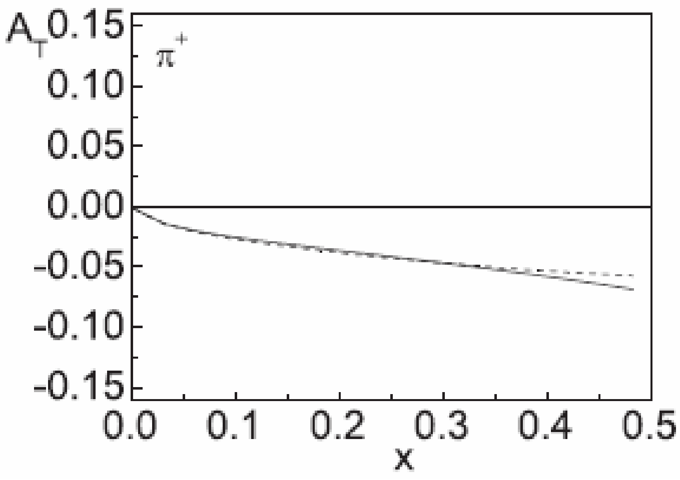
Prediction in JLab condition (neutron target)

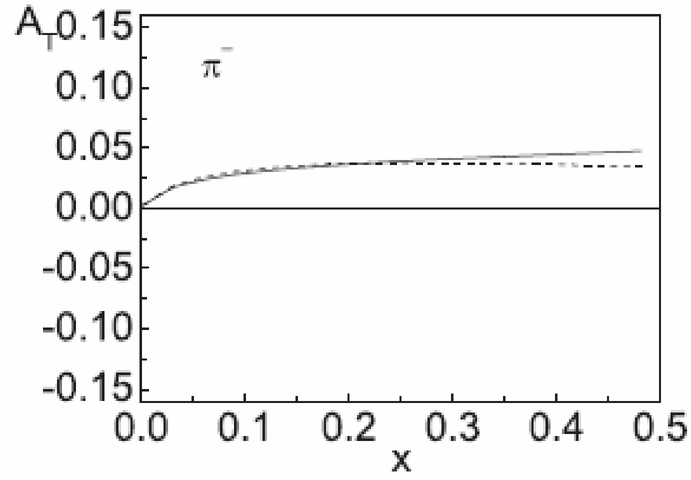
Set I





Set II





Transversity

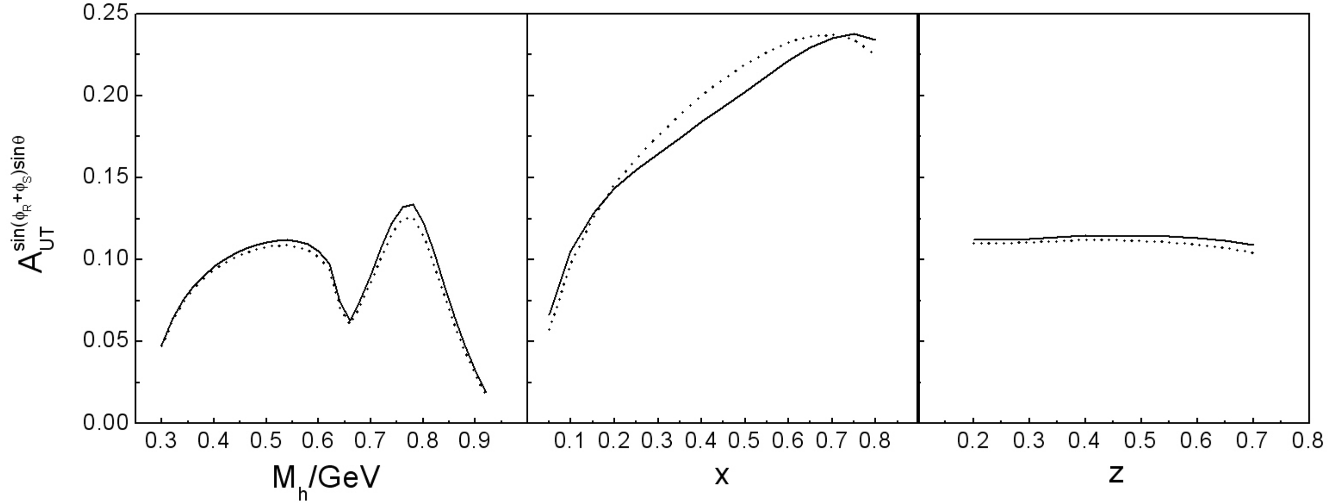
from two pion interference fragmentation

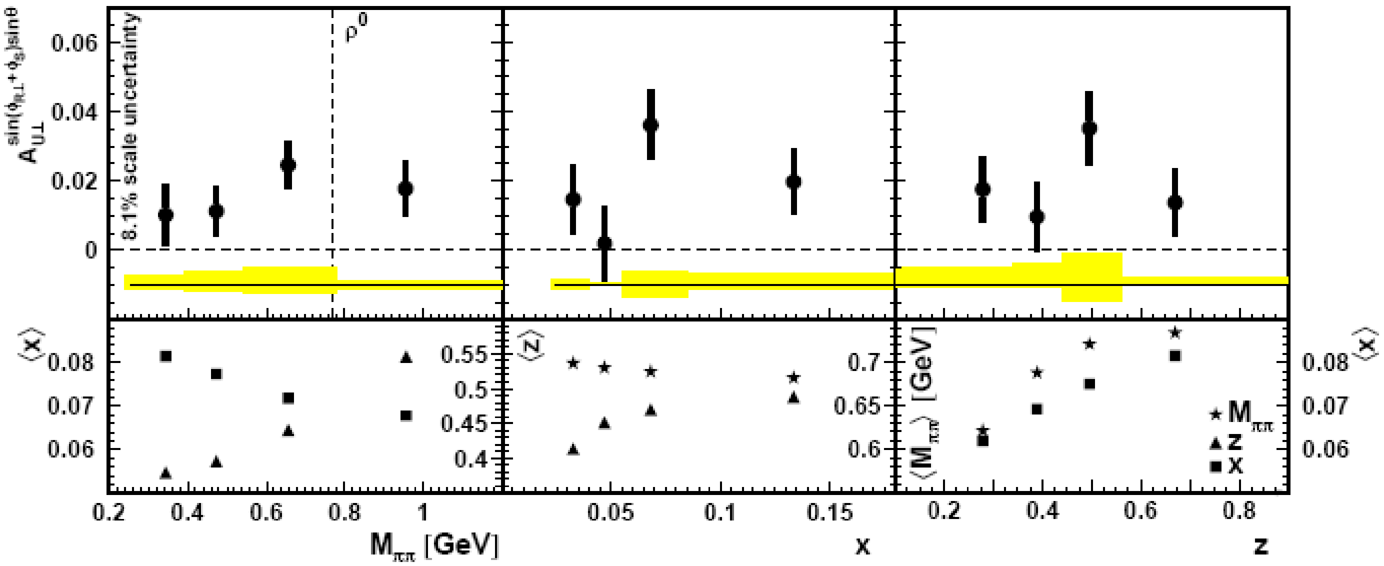
$$A_{UT}^{\langle 2\sin(\phi_R+\phi_S)/\sin\theta\rangle} \sim -\frac{\sum_a e_a^2 \delta f^a(x) \int d\zeta \frac{|\vec{R}|}{M_h} H_1^{\sphericalangle a}(z,\zeta,M_h^2)}{\sum_a e_a^2 f^a(x) \int d\zeta D_1^a(z,\zeta,M_h^2)}$$

New fragmentation functions are introduced: the dihadron FFs, including the chiral odd interference FF.

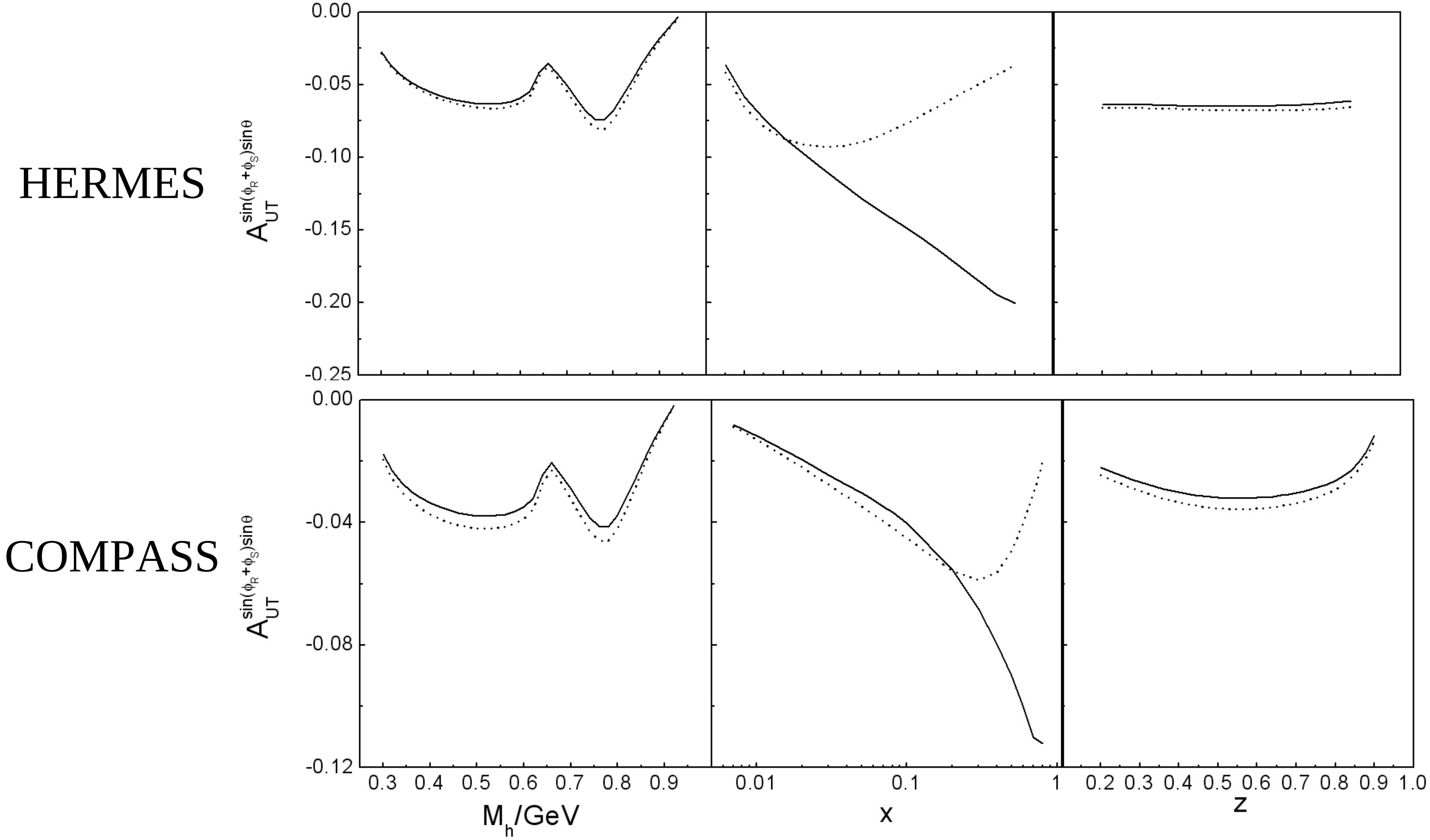
- Jaffe, Jin and Tang, PRL 80, 1166 (1998)
- Radici, Jakob and Bianconi, PRD, 65, 074031 (2002)
- Bacchetta and Radici, PRD 74, 114007 (2006)

Prediction on the proton target





Prediction on neutron target



J. She, Y.Huang, and B.-Q. Ma, Phys. Rev. D77 (2008) 014035.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1} \right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Three QCD spin sums for the proton spin

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x \psi^\dagger \vec{x} \times (-i\nabla)\psi + \int d^3x \vec{E}^a \times \vec{A}^a + \int d^3x E^{ai}\vec{x} \times \nabla A^{ai} \equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D})\psi + \int d^3x \vec{x} \times (\vec{E} \times \vec{B}) \equiv \vec{S}_q + \vec{L}_q + \vec{J}_g,$$

$$\vec{J}_{QCD} = \int d^3x \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x \psi^\dagger \vec{x} \times (-i\vec{D}_{pure})\psi + \int d^3x \vec{E}^a \times \vec{A}^a_{phys} + \int d^3x E^{ai}\vec{x} \times \nabla A^{ai}_{phys} \equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,$$

X.-S.Chen, X.-F.Lu, W.-M.Sun, F.Wang, T.Goldman, PRL100(2008)232002

Spin and orbital sum in light-cone formalism

$$\frac{1}{2}M_q + M_L = \frac{1}{2}$$

$$M_q(x,k_\perp) = \frac{(k^+ + m)^2 - k_\perp^2}{(k^+ + m)^2 + k_\perp^2} \qquad M_L(x,k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

$$\frac{1}{2}\Delta q(x) + L_q(x) = \frac{1}{2}\Delta q_{QM}(x)$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Relations of quark distributions

$$\Delta q_{QM}(x) + \Delta q(x) = 2\,\delta q(x)$$

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

$$\frac{1}{2}\Delta q(x) + L_q(x) = \frac{1}{2}\Delta q_{QM}(x),$$

$$\Delta q(x) + L_q(x) = \delta q(x),$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

The Melosh-Wigner Rotation in "Pretrelosity"

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) \; .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretrelocity} = \Delta \mathrm{q} - \delta \mathrm{q} = -\mathrm{L_q}$$

$$\mathrm{Pretrelocity} = -\int [\mathrm{d}^2\mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q}_{\mathrm{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

"Pretrel" or "Brezel"

“Pretrel” or “Brezel”

“Mahua(麻花)”: the Chinese Preztel

What is “Pretrelosity” ?

- Pretzelosity: one of the eight leading twist transverse dependent parton distributions (TMDs).
- The quark-quark correlator up to the leading twist

$$
\begin{aligned}
\Phi(x, \mathbf{p}_\perp) = \frac{1}{2}\{ & f_1 \not{n}_+ - f_{1T}^\perp \frac{\epsilon_\perp^{ij} p_\perp^i S_\perp^j}{M_N} \not{n}_+ \\
& +(S_\parallel g_{1L} + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} g_{1T}) \gamma_5 \not{n}_+ + h_{1T} \frac{[\not{S}_\perp, \not{n}_+]\gamma_5}{2} \\
& +(S_\parallel h_{1L}^\perp + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M_N} h_{1T}^\perp) \frac{[\not{p}_\perp, \not{n}_+]\gamma_5}{2M_N} + i h_1^\perp \frac{[\not{p}_\perp, \not{n}_+]}{2M_N} \}. (7)
\end{aligned}
$$

P.J. Mulders and R.D. Tangerman, Nucl. Phys. **B 461**, 197 (1996), Erratum-ibid. **B 484**, 538 (1997). K. Goeke, A. Metz, and M. Schlegel, Phys. Lett. **B 618**, 90 (2005).

What is “Pretrelosity” ?

$$\frac{p_\perp^x p_\perp^y}{M_N^2} h_{1T}^\perp(x, p_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{i(xP^+\xi^- - \mathbf{p}_\perp \cdot \boldsymbol{\xi}_\perp)} \times \langle PS^y | \bar{\psi}(0) i\sigma^{1+}\gamma_5 \psi(0, \xi^-, \xi_\perp) | PS^y \rangle, \quad (12)$$

$|PS^y\rangle$: the hadronic state with a polarization in the y direction.

- Some properties of pretzelosity:
  1. It is chiral-odd, and needs a chiral-odd partner in the SIDIS.
  2. There is no gluon analog of pretzelosity.
  3. In a large class of models, it is the difference of helicity and transversity, and hence a measure for relativistic effects.

H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

A Simple Relation

- The difference of helicity and transversity is the first moment of pretzelosity.

$$h_{1T}^{\perp(1)qv}(x, \mathbf{p}_\perp) \equiv \frac{p_\perp^2}{2M_N^2} h_{1T}^{\perp qv}(x, \mathbf{p}_\perp) = g_1^{qv}(x, \mathbf{p}_\perp) - h_1^{qv}(x, \mathbf{p}_\perp),$$

- This relation has already been obtained in
  H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355. B. Pasquini, S. Cazzaniga and S. Boffi, Phys. Rev. **D 78**, 034025 (2008).
- But this relation is not fully satisfied in
  A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. **D 78**, 074010 (2008).

Connection with Quark Orbital Angular Momentum

- The rotation factor for $\vec{x} \times -i\nabla$ is $\frac{p_\perp^2}{(x\mathscr{M}_D+m_q)^2+p_\perp^2}$
  B.-Q. Ma, I. Schmidt, Phys. Rev. **D 58**, 096008 (1998).
- a simple relation between the pretzelosity and the quark orbital angular momentum

$$L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x,\mathbf{p}_\perp) = h_1^{qv}(x,\mathbf{p}_\perp) - g_1^{qv}(x,\mathbf{p}_\perp), (21)$$

or at the integration level

$$L^{qv}(x) = \int d^2\mathbf{p}_\perp L^{qv}(x,\mathbf{p}_\perp) = -h_{1T}^{\perp(1)qv}(x) = h_1^{qv}(x) - g_1^{qv}(x).$$

- A measurement of pretzelosity may reveal the information on the quark orbital angular momentum.

Preztelosity in SIDIS

- Pretzelosity can be measured through $\sin(3\phi_h - \phi_S)$ asymmetry in the SIDIS process, where the cross section can be written as

$$\frac{d^6\sigma_{UT}}{dxdyd\phi_S dzd^2\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2}\{(1-y+\frac{1}{2}y^2)F_{UU} \\ +S_\perp \sin(3\phi_h-\phi_S)(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)} + \ldots\}, \quad (23)$$

with $F_{UU} = \mathcal{F}[\omega_1 f_1 D_1]$, $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{F}[\omega_2 h_{1T}^\perp H_1^\perp]$

- The $\sin(3\phi_h - \phi_S)$ asymmetry

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \frac{\frac{2\alpha^2}{sxy^2}(1-y)F_{UT}^{\sin(3\phi_h-\phi_S)}}{\frac{2\alpha^2}{sxy^2}(1-y+\frac{1}{2}y^2)F_{UU}}. \quad (24)$$

Quantities in Calculation

- DFs and FFs to be parametrized:

	x dependence	z dependence	TM dependence
f_1	well known	—	not so clear
$h_{1T}^{\perp}$	not known	—	not known
D_1	—	known	not so clear
$H_1^{\perp}$	—	a little known	not clear

- Theoretical understanding: non-perturbative, model calculation, cannot give the exact value so far.
- Transverse momentum dependence: not so clearly yet, usually parametrized in a Gaussian form.
- D_1 and $H_1^{\perp}$: Gaussian parametrization given by S. Kretzer, *et al.*, Eur. Phys. J. **C 22**, 269 (2001). M. Anselmino, *et al.*, arXiv:0807.0173.

Approach 0 to TMDs

- Starting with the equation

$$h_{1T}^{\perp(uv)}(x) = \left[f_1^{(uv)}(x) - \frac{1}{2}f_1^{(dv)}(x)\right]\hat{W}_S(x) - \frac{1}{6}f_1^{(dv)}(x)\hat{W}_V(x),$$
$$h_{1T}^{\perp(dv)}(x) = -\frac{1}{3}f_1^{(dv)}(x)\hat{W}_V(x), \qquad (25)$$

where $\hat{W}_D(x) = \int d^2\mathbf{p}_\perp \varphi^2(x,\mathbf{p}_\perp)W_D(x,\mathbf{p}_\perp)/\int d^2\mathbf{p}_\perp \varphi^2(x,\mathbf{p}_\perp)$

- $f_1(x)$: CTEQ6L as an input. $h_{1T}^{\perp}(x)$: from Eq. 25
- Transverse momentum dependence: Gaussian form.
- How to fit the Gaussian width? $p_{av}/k_{av} \approx 2$?
  H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

Approach 1 to TMDs

- Model calculation.

$$f_1^{(uv)}(x, \mathbf{p}_\perp) = \frac{1}{16\pi^3} \times (\frac{1}{3}\sin^2\theta\varphi_V^2 + \cos^2\theta\varphi_S^2),$$
$$f_1^{(dv)}(x, \mathbf{p}_\perp) = \frac{1}{8\pi^3} \times \frac{1}{3}\sin^2\theta\varphi_V^2.$$

$$h_{1T}^{\perp(uv)}(x, \mathbf{p}_\perp) = -\frac{1}{16\pi^3} \times (\frac{1}{9}\sin^2\theta\varphi_V^2 W_V - \cos^2\theta\varphi_S^2 W_S),$$
$$h_{1T}^{\perp(dv)}(x, \mathbf{p}_\perp) = -\frac{1}{8\pi^3} \times \frac{1}{9}\sin^2\theta\varphi_V^2 W_V.$$

- $\varphi_D(x, \mathbf{p}_\perp)$: adopting the BHL form:

$$\varphi_D(x, \mathbf{p}_\perp) = A_D \exp\{-\frac{1}{8\alpha_D^2}[\frac{m_q^2 + p_\perp^2}{x} + \frac{m_D^2 + p_\perp^2}{1-x}]\},$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Approach 2 to TMDs

- Staring with the equation (an unintegrated version)

$$
\begin{aligned}
h_{1T}^{\perp(uv)}(x,\mathbf{p}_\perp) &= \left[f_1^{(uv)}(x,\mathbf{p}_\perp) - \frac{1}{2} f_1^{(dv)}(x,\mathbf{p}_\perp)\right] W_S(x,\mathbf{p}_\perp) \\
&\quad - \frac{1}{6} f_1^{(dv)}(x,\mathbf{p}_\perp) W_V(x,\mathbf{p}_\perp), \\
h_{1T}^{\perp(dv)}(x,\mathbf{p}_\perp) &= -\frac{1}{3} f_1^{(dv)}(x,\mathbf{p}_\perp) W_V(x,\mathbf{p}_\perp). \qquad (27)
\end{aligned}
$$

- $f_1(x,\mathbf{p}_\perp)$: a Gaussian form

$$
f_1(x,\mathbf{p}_\perp) = f_1(x)\frac{\exp(-p_\perp^2/p_{av}^2)}{\pi p_{av}^2}, \qquad (28)
$$

with CTEQ6L parametrization for $f_1(x)$.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

$h_{1T}^{\perp(1)}(x)$ and $f_1(x)$

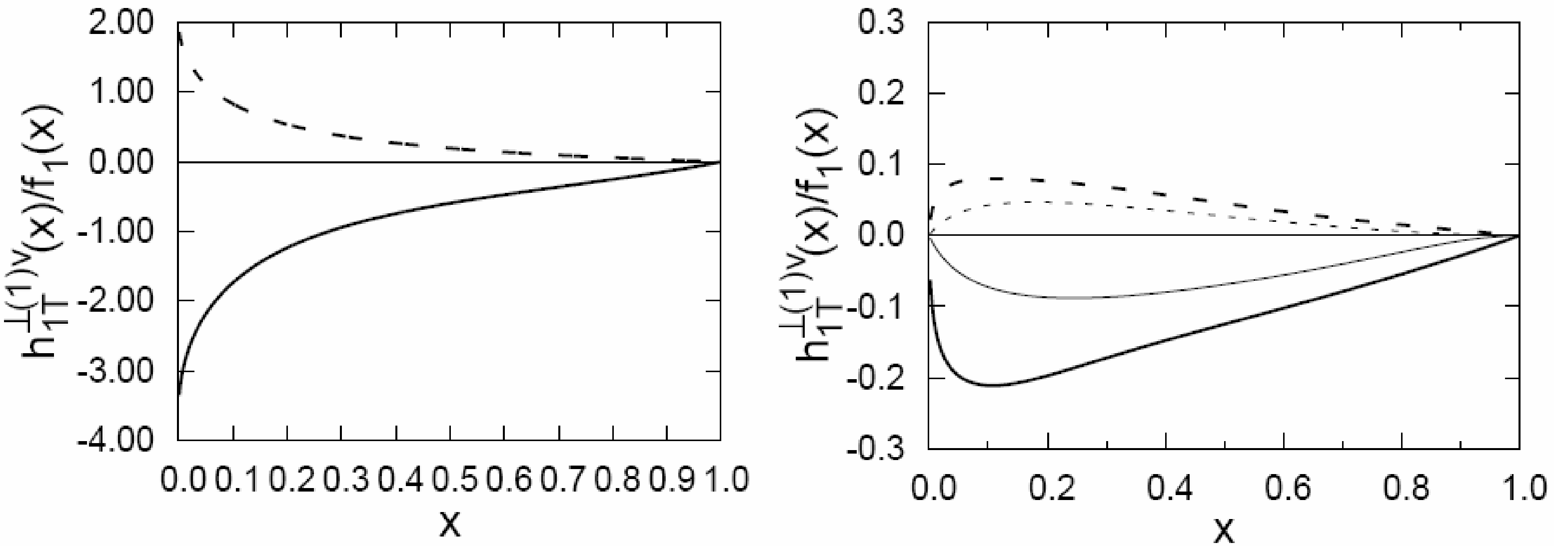


Figure: The ratio $h_{1T}^{\perp(1)(x)}/f_1(x)$. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the *u* quark, and dashed curves for the *d* quark. Only valence quarks are considered.

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

Results at HERMES kinematics.

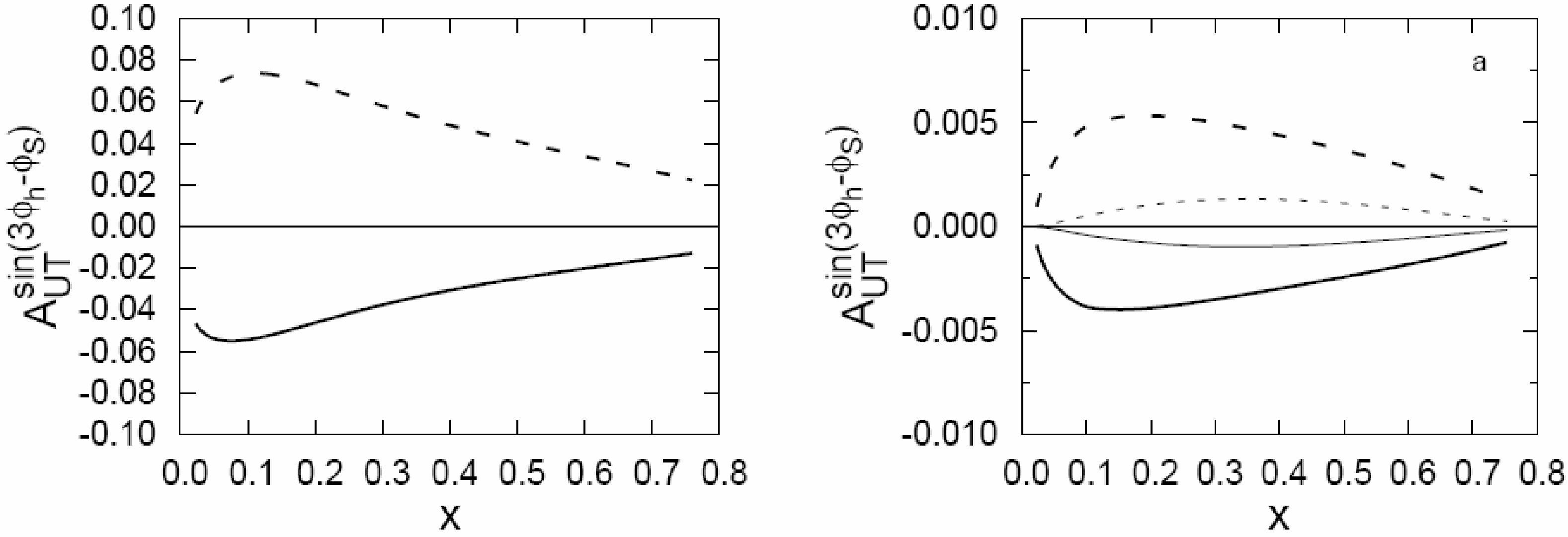


Figure: The results for HERMES kinematics with a proton target. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the π^+ production, and dashed curves for the π^- production.

Results at COMPASS kinematics.

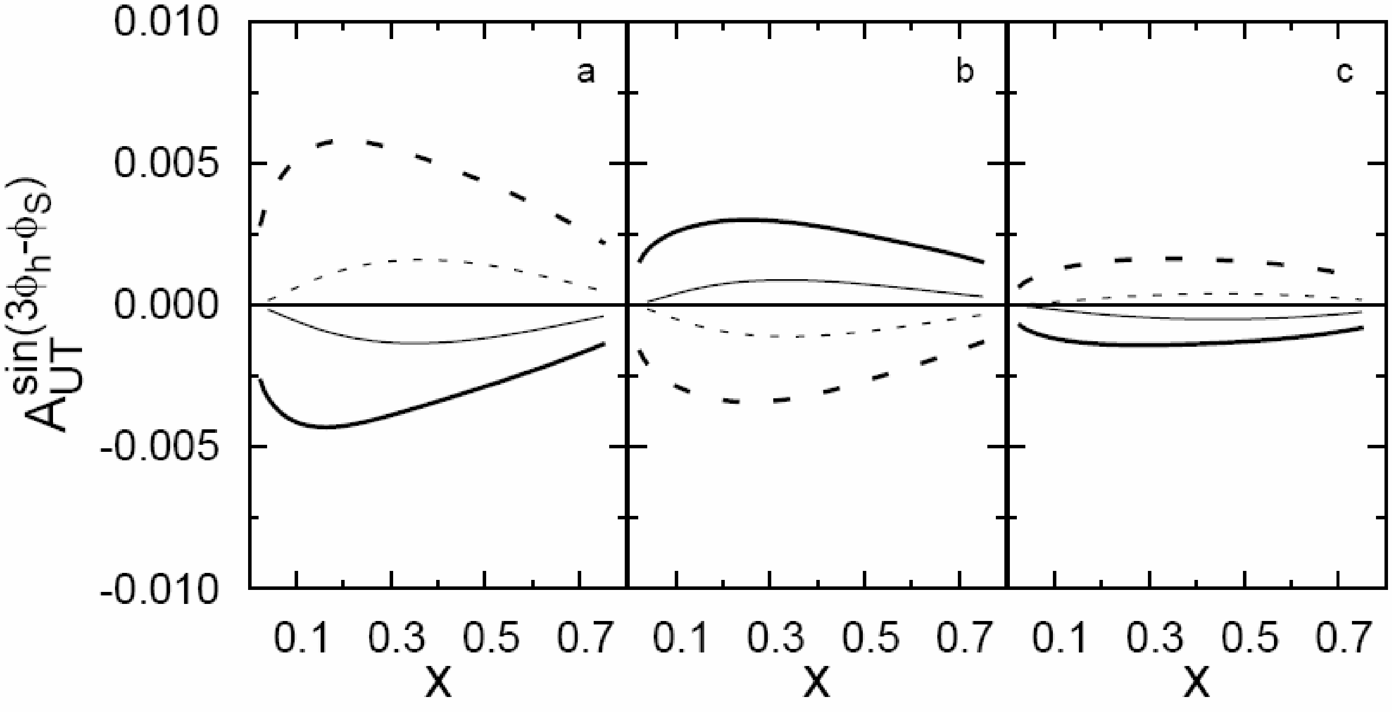


Figure: The results for COMPASS kinematics. a) proton target, b) neutron target, and c) deuteron target.

Results at JLab kinematics.

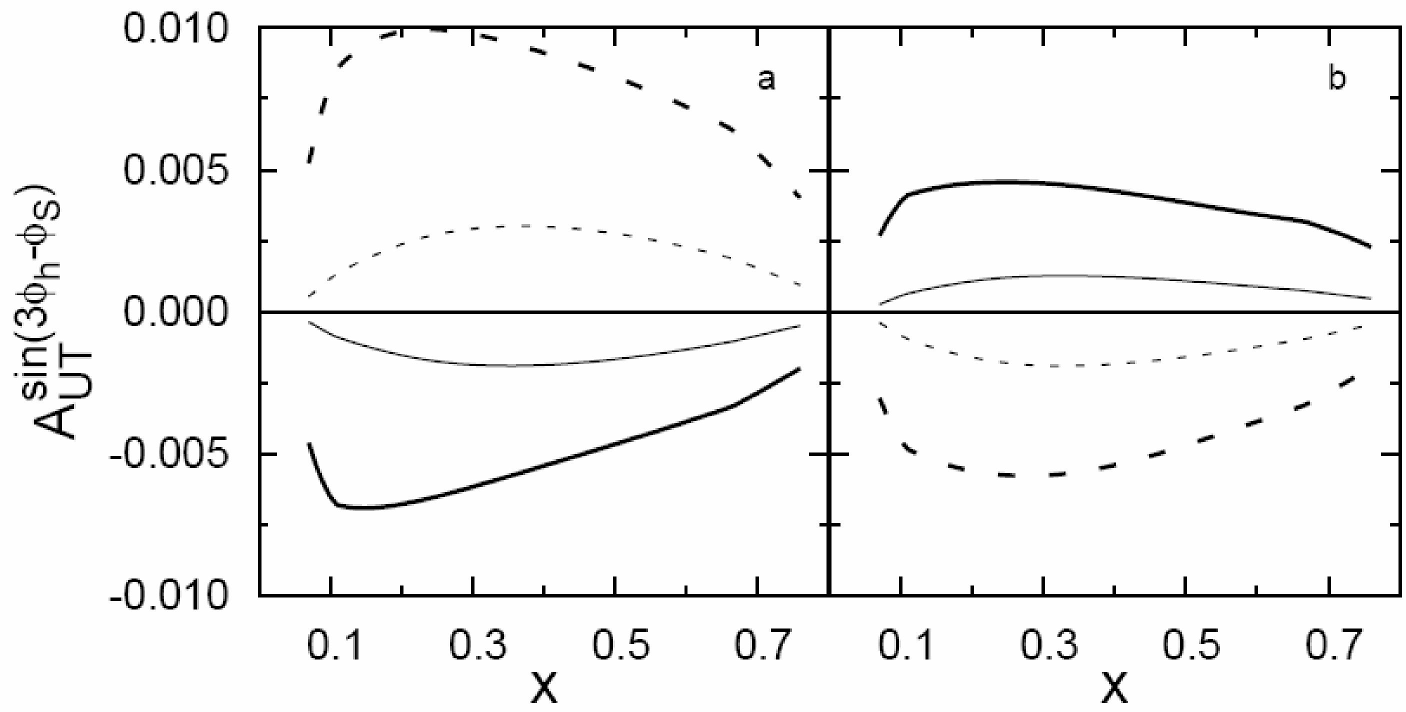


Figure: The results for JLab kinematics. a) proton target and b) neutron target.

A short summary

- Results are sensitive to different transverse momentum approaches.
- The asymmetry is not an increasing function of x.
- The asymmetry is too small, up to a maximum less than 1%. A great challenge for a direct measurement.
- Can we enhance the asymmetry? We observe that the asymmetry is an increasing function of $\mathbf{p}_\perp^2$, but $\mathbf{p}_\perp^2$ cannot be manipulated directly.
- A compromise method is to select large $P_{h\perp}$ events instead, $\mathbf{P}_{h\perp} = z(\mathbf{p}_\perp - \mathbf{k}_\perp)$. We can exclude most small $p_\perp$ events.
- We will recalculate our results with a cutoff $P_{h\perp} > 1.0\mathrm{GeV}$.

HERMES results with a cutoff.

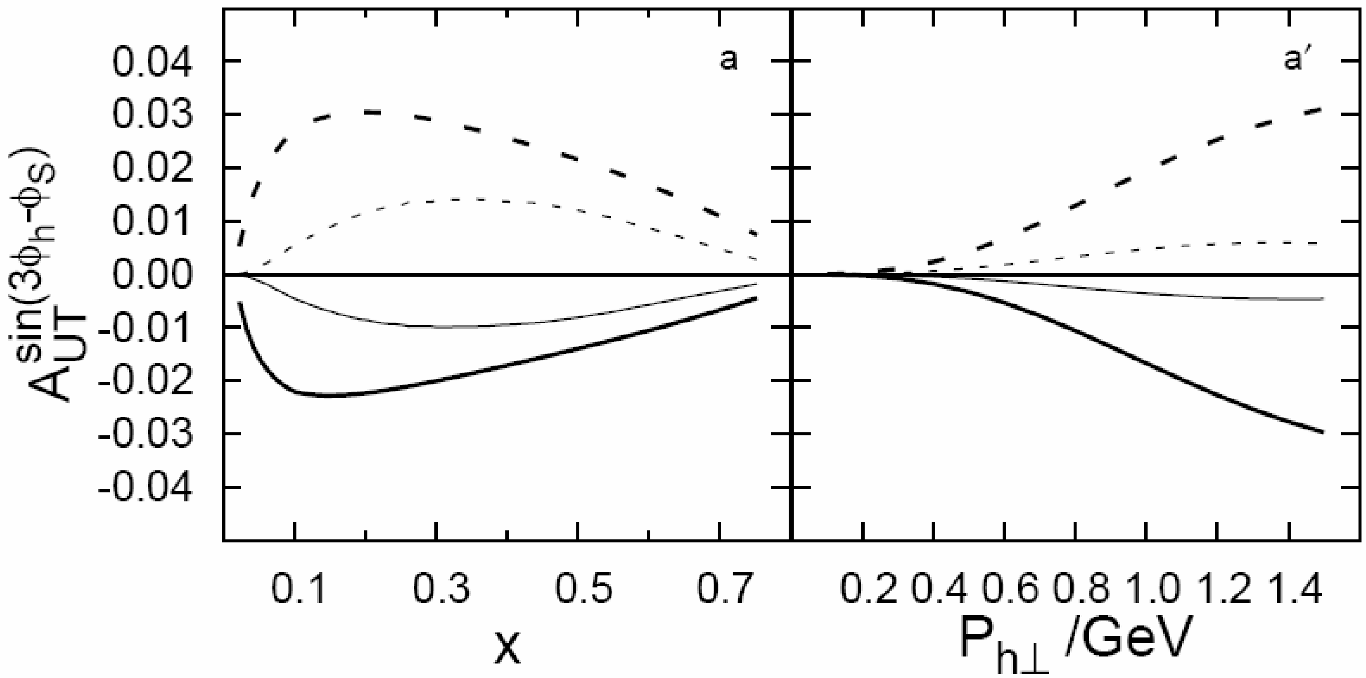


Figure: The results for HERMES kinematics on a proton target with a cutoff $P_{h\perp} > 1.0\,\mathrm{GeV}$, while the right panel shows the $P_{h\perp}$ dependence of the asymmetry after integrating all the other kinematic variables.

COMPASS results with a cutoff.

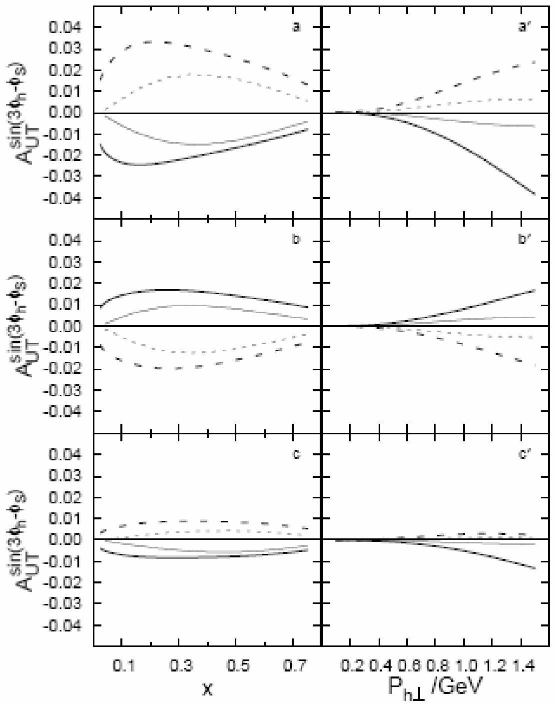


Figure: The results for COMPASS kinematics with a cutoff $P_{h\perp} > 1.0\,\mathrm{GeV}$. The upper, middle, and lower panels correspond to the proton, neutron, and deuteron target, respectively.

JLab results with a cutoff.

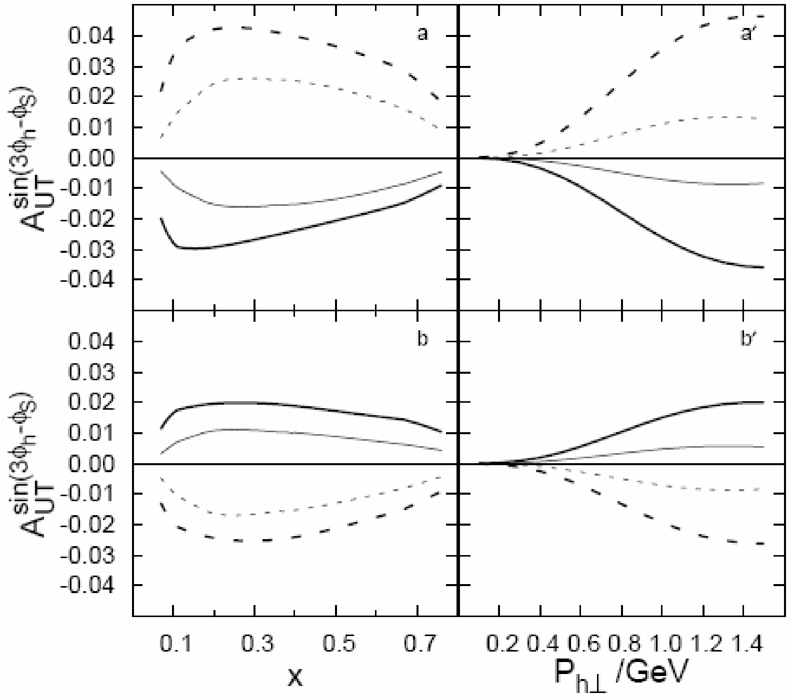


Figure: The results for JLab kinematics with a cutoff $P_{h\perp} > 1.0\,\mathrm{GeV}$. The upper and lower panels correspond to the proton and neutron target, respectively.

Caution

- The TMD factorization was proved to be valid only in the region $\Lambda_{\rm QCD} \ll P_{h\perp} \ll Q$.
- If $P_{h\perp} \sim \Lambda_{\rm QCD}$ and Q^2 is too large, a higher order pQCD correction (the gluon radiation) will be important.
- This transition point is around $P_{h\perp} \approx 1{\rm GeV}$.
- We must be careful and we assume that our results at a little larger $P_{h\perp}$ but not too larger than 1GeV are still acceptable.
- Another problem: the events will be exponentially suppressed at $P_{h\perp} \gg \Lambda_{\rm QCD}$, a challenge for the experiments to collect more data.

Conclusions

- **Relativistic effect of Melosh-Winger rotation is important in hadron spin physics.**
- **Transversity is being accessed in SIDIS and two pion inteference fragmentation processes**
- **The unfavored Collins fragmentation plays a surprising role to reproduce the data, than naively expected.**
- **The preztelosity is an important quantity for the spin-orbital correlation.**
- **New way to access quark orbital angular momentum is suggested.**

Pion Spin Structure and Form Factor

Based on collaborated works with T.Huang and Q.-X.Shen

[1] T. Huang, B.Q. Ma, and Q.X. Shen, Phys. Rev. D **49**, 1490 (1994).

[2] B. Q. Ma, Z. Phys. A **345**, 321 (1993).

[3] B.Q. Ma and T.Huang, J. Phys. G **21**, (765) (1995).

Pion Spin-Space Wave Function in Rest Frame

In the pion rest frame, the instant-form spin space wave-function of pion is

$$\chi_T = (\chi_1^{\uparrow}\chi_2^{\downarrow} - \chi_2^{\uparrow}\chi_1^{\downarrow})/\sqrt{2},$$

in which $\chi_i^{\uparrow,\downarrow}$ are the two-component Pauli spinors.

The Lowest Valence State Wave Function in Light-Cone

$$|\psi^{\pi}_{q\overline{q}}> = \psi(x, \mathbf{k}\_, \uparrow, \downarrow)|\uparrow\downarrow> + \psi(x, \mathbf{k}\_, \downarrow, \uparrow)|\downarrow\uparrow>$$

$$+\psi(x, \mathbf{k}\_, \uparrow, \uparrow)|\uparrow\uparrow> + \psi(x, \mathbf{k}\_, \downarrow, \downarrow)|\downarrow\downarrow>,$$

where

$$\psi(x, \mathbf{k}\_, \lambda_1, \lambda_2) = C_0^F(x, \mathbf{k}\_, \lambda_1, \lambda_2)\varphi(x, \mathbf{k}\_).$$

Here $\varphi(x, \mathbf{k}\_)$ is the momentum space wave function in the light-cone formalism.

The Spin Component Coefficients

The spin component coefficients C_0^F have the forms,

$$C_0^F(x,q,\uparrow,\downarrow) = w_1 w_2[(q_1^- + m)(q_2^- + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x,q,\downarrow,\uparrow) = -w_1 w_2[(q_1^- + m)(q_2^- + m) - \mathbf{q}_\perp^2]/\sqrt{2};$$

$$C_0^F(x,q,\uparrow,\uparrow) = w_1 w_2[(q_1^- + m)q_2^L - (q_2^- + m)q_1^L]/\sqrt{2};$$

$$C_0^F(x,q,\downarrow,\downarrow) = w_1 w_2[(q_1^- + m)q_2^R - (q_2^- + m)q_1^R]/\sqrt{2}.$$

C_0^F satisfy the relation

$$\Sigma_{\lambda_1,\lambda_2} C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)C_0^F(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = 1.$$