Strategies for baryon resonance analysis

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Analyticity and Unitarity

Pole and Non-Pole T-Matrix

$$T = T^P + T^{NP}$$

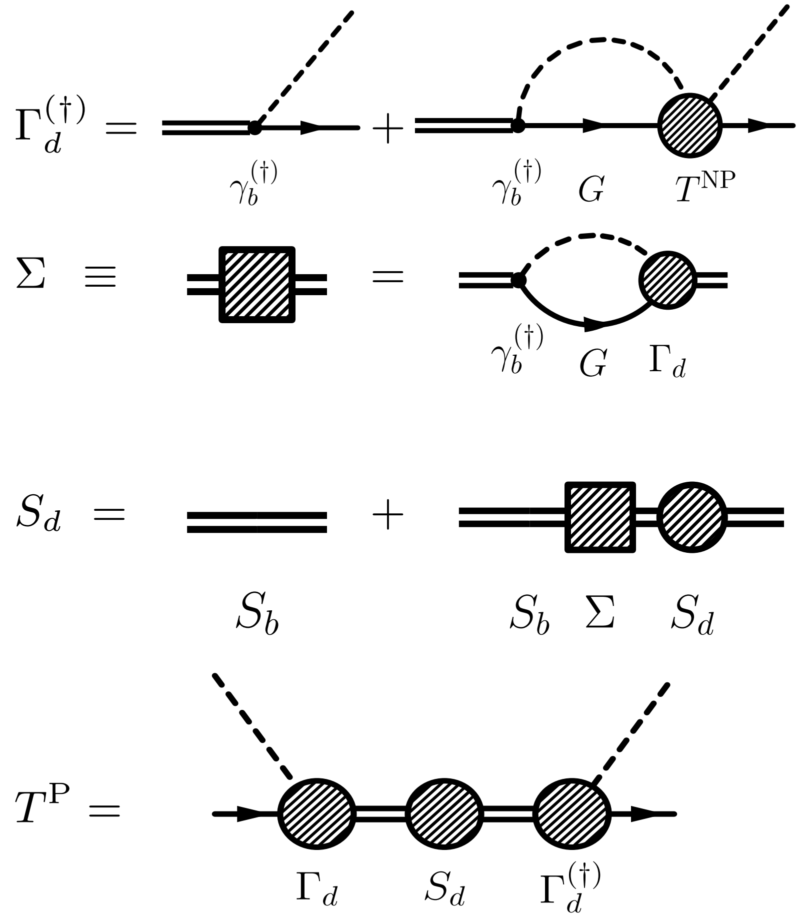
$$T = \frac{a_{-1}}{Z-Z_0} + a_0 + O(Z-Z_0)$$

$$a_{-1} = \frac{\Gamma_d \Gamma_d^{(\dagger)}}{1-\frac{\partial}{\partial Z}\Sigma}$$

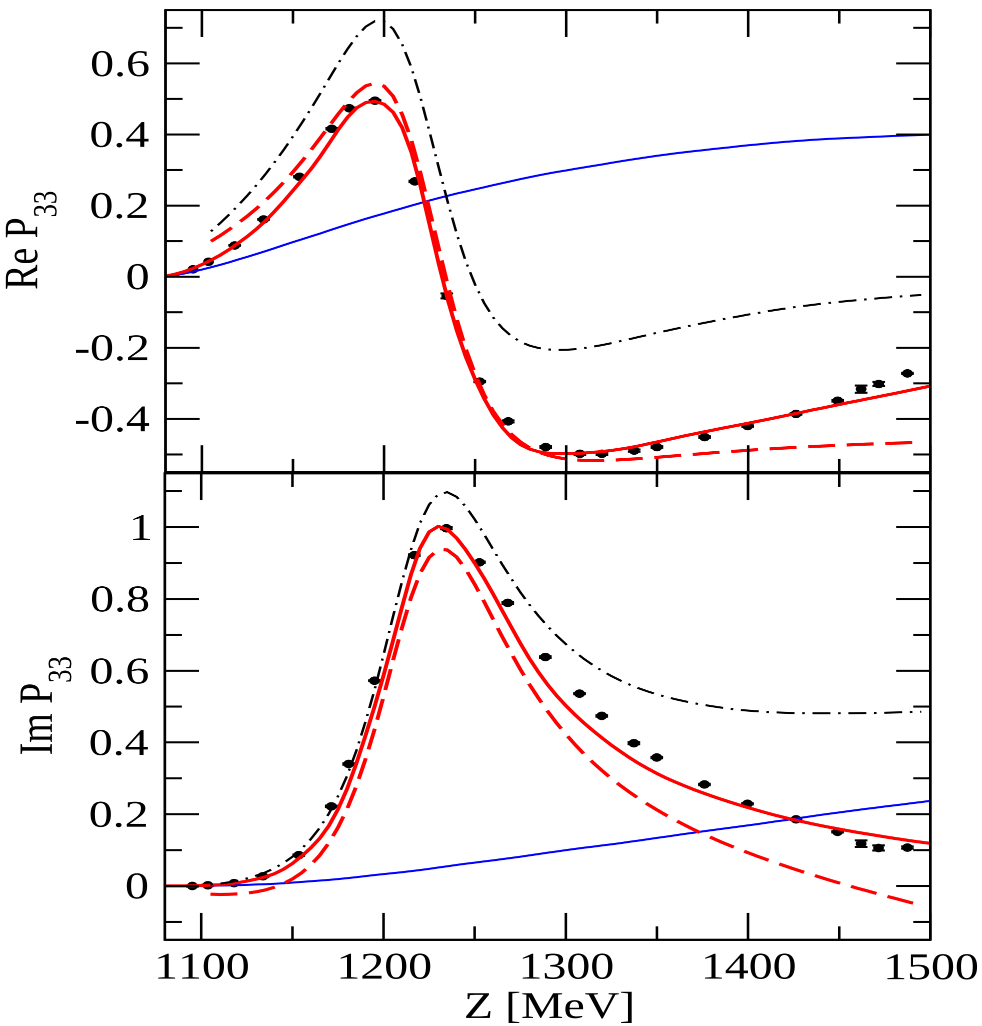
$$a_0 = T^{NP} + a_0^P$$

$$a_0^P = \frac{a_{-1}}{\Gamma_d \Gamma_d^{(\dagger)}} *$$

$$* \left(\frac{\partial}{\partial Z}(\Gamma_d \Gamma_d^{(\dagger)}) + \frac{a_{-1}}{2}\frac{\partial^2}{\partial Z^2}\Sigma\right)$$



Poles and background P_{33}

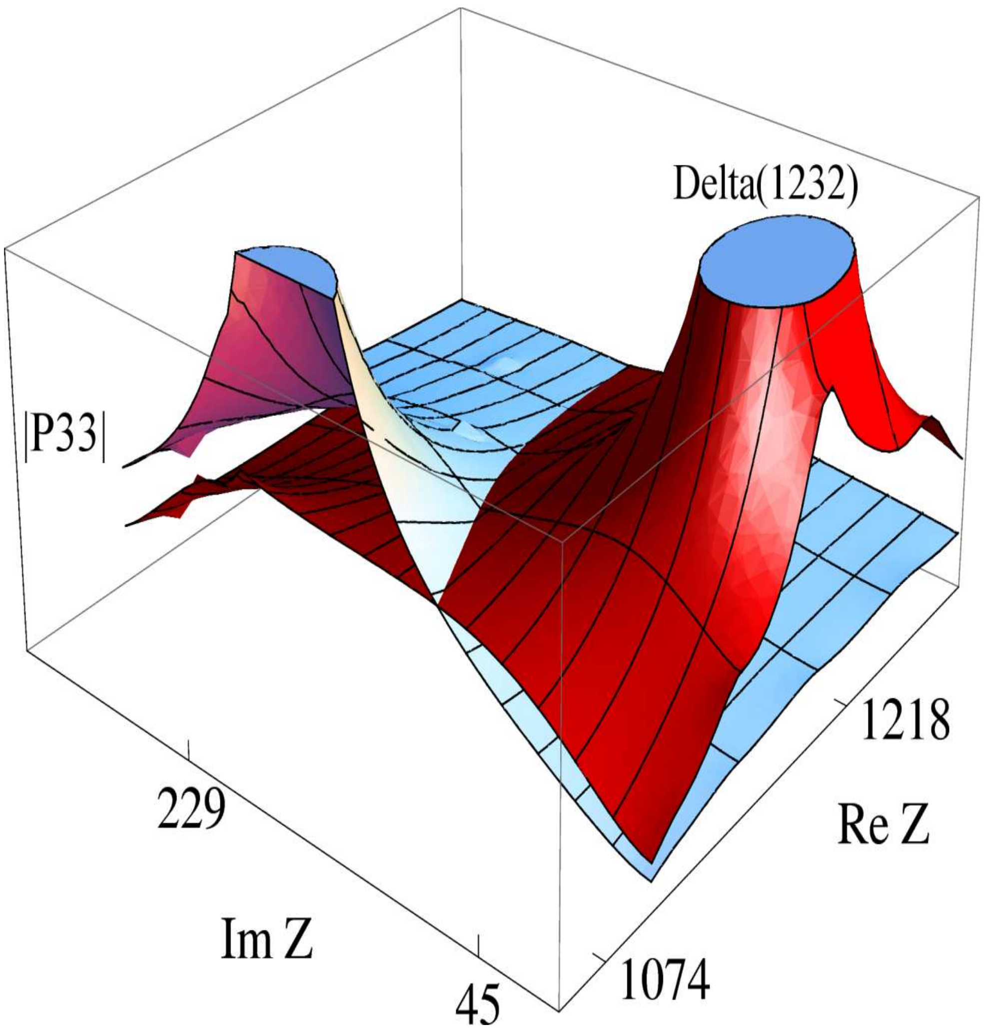


Vicinity of Pole:

$$T(Z) \sim \frac{a_{-1}}{Z-Z_0} + T^{NP}(Z)$$

$$T(Z) \sim \frac{a_{-1}}{Z-Z_0} + a_0$$

Second Riemann sheet: P_{33}



T^{NP}

$T^{P} + T^{NP}$

Toy model I

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(1) $$\epsilon_1^0 = E_1 - \frac{i}{2}\Gamma_1; \epsilon_2^0 = E_2 - \frac{i}{2}\Gamma_2$$

(2) $$h_{11} = \epsilon_1^0; h_{12} = b$$

(3) $$h_{21} = b; h_{22} = \epsilon_2^0$$

(4) $$\epsilon_{1,2} = (\epsilon_1^0 + \epsilon_2^0)/2 \pm D^{\frac{1}{2}}$$

(5) $$D = (\epsilon_1^0 - \epsilon_2^0)^2/4 + b^2$$

(6) $$(\epsilon_1 - \epsilon_2)/2 = \sqrt{D} = e + ig$$

(7)

Change bare couling $b \to \bar{b}$:

$$(\bar{\epsilon_1} - \bar{\epsilon_2})^2 - (\epsilon_1 - \epsilon_2)^2 = \bar{e}^2 - e^2 - \bar{g}^2 + g^2 + 2i * (\bar{e}\bar{g} - eg)$$

Toy model II

Change bare couling: $b \to \bar{b}$:

$$(\bar{\epsilon_1} - \bar{\epsilon_2})^2 - (\epsilon_1 - \epsilon_2)^2 = \bar{e}^2 - e^2 - \bar{g}^2 + g^2 + 2i * (\bar{e}\bar{g} - eg)$$

In case the level shift is real:

$$\bar{e}\bar{g} = eg$$

$$\bar{e} \le e \to \bar{g} \ge g$$

Consequence: one level is pushed deep into the complex plane.

P.v.Brentano, Phys. Rep.

Consequences

Warning for theories generating resonances dynamically:
Dynamically generated pole interacts with bare resonances
Quantitative reproduction of data essential!
Focus on model independent quantities.
Baz, Perelmov, Zeldovich,1970
One channel, vicinity of pole $k_0 = k_1 + ik_2$:

$$S(k) = \frac{(k-k_0^*)(k+k_0)}{(k-k_0)(k+k_0^*]} exp(2i\phi(k))$$

Meeting point:
Poles, residues, zeroes, branch points.
=>Michael Döring

Tool: Gauss plot

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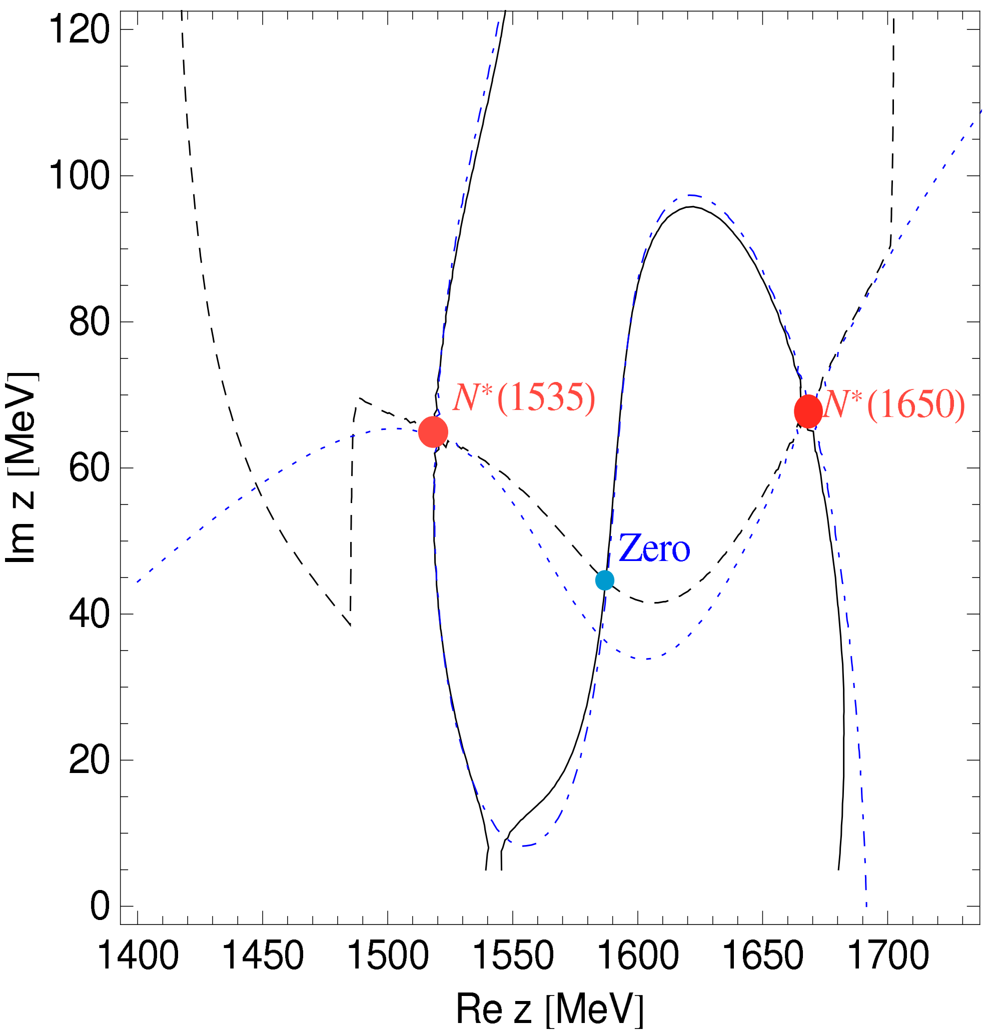

Re[T(z)]=0

Im[T(z)]=0

$$\frac{1}{x-iy} = \frac{x+iy}{x^2+y^2}$$

$$T^{[2]}(Z) = \frac{a_{-1}(1535)}{Z-Z_0(1535)} + \frac{a_{-1}(1650)}{Z-Z_0(1650)}$$

Meeting the lattice

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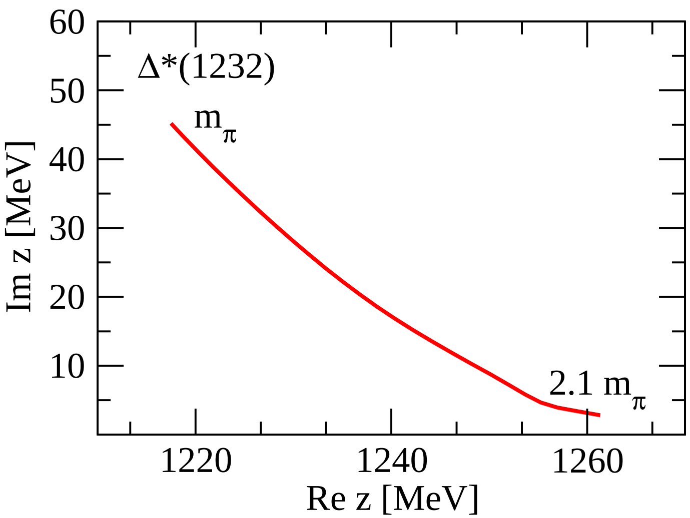
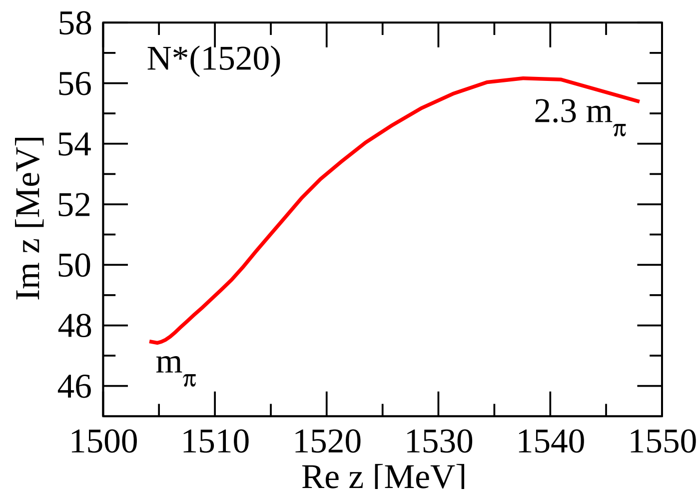
New techniques in excited mass extraction:
Fleming, Cohen, Lin, Pereyra, hep-lat 0903.2314
Meeting point:
study pion mass dependence of poles
due to final state interaction

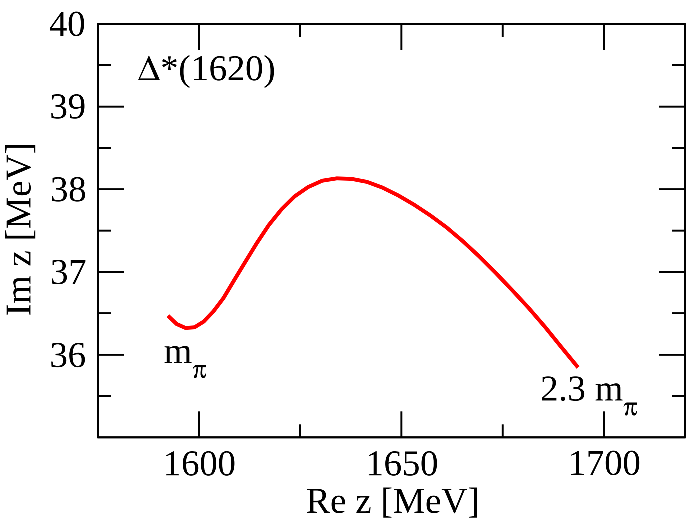
Pole path



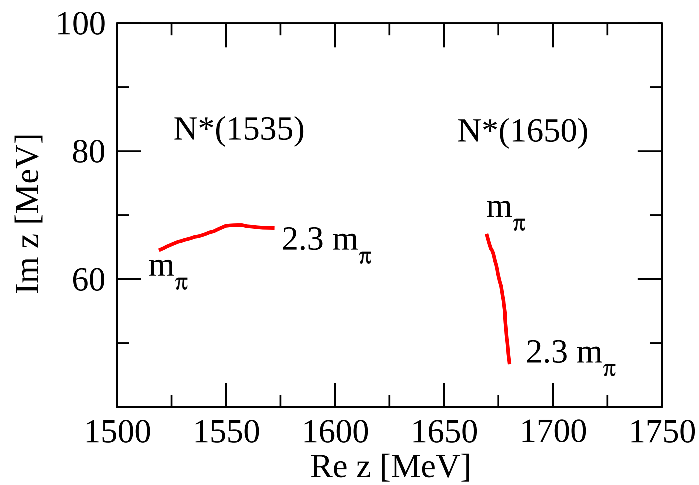
Δ*(1232)
m_π
2.1 m_π
Im z [MeV]
Re z [MeV]
60
50
40
30
20
10
1220
1240
1260



N*(1520)
m_π
2.3 m_π
Im z [MeV]
Re z [MeV]
58
56
54
52
50
48
46
1500
1510
1520
1530
1540
1550



Δ*(1620)
m_π
2.3 m_π
Im z [MeV]
Re z [MeV]
40
39
38
37
36
1600
1650
1700



N*(1535)
N*(1650)
m_π
2.3 m_π
m_π
2.3 m_π
Im z [MeV]
Re z [MeV]
100
80
60
1500
1550
1600
1650
1700
1750

Pion mass dependence

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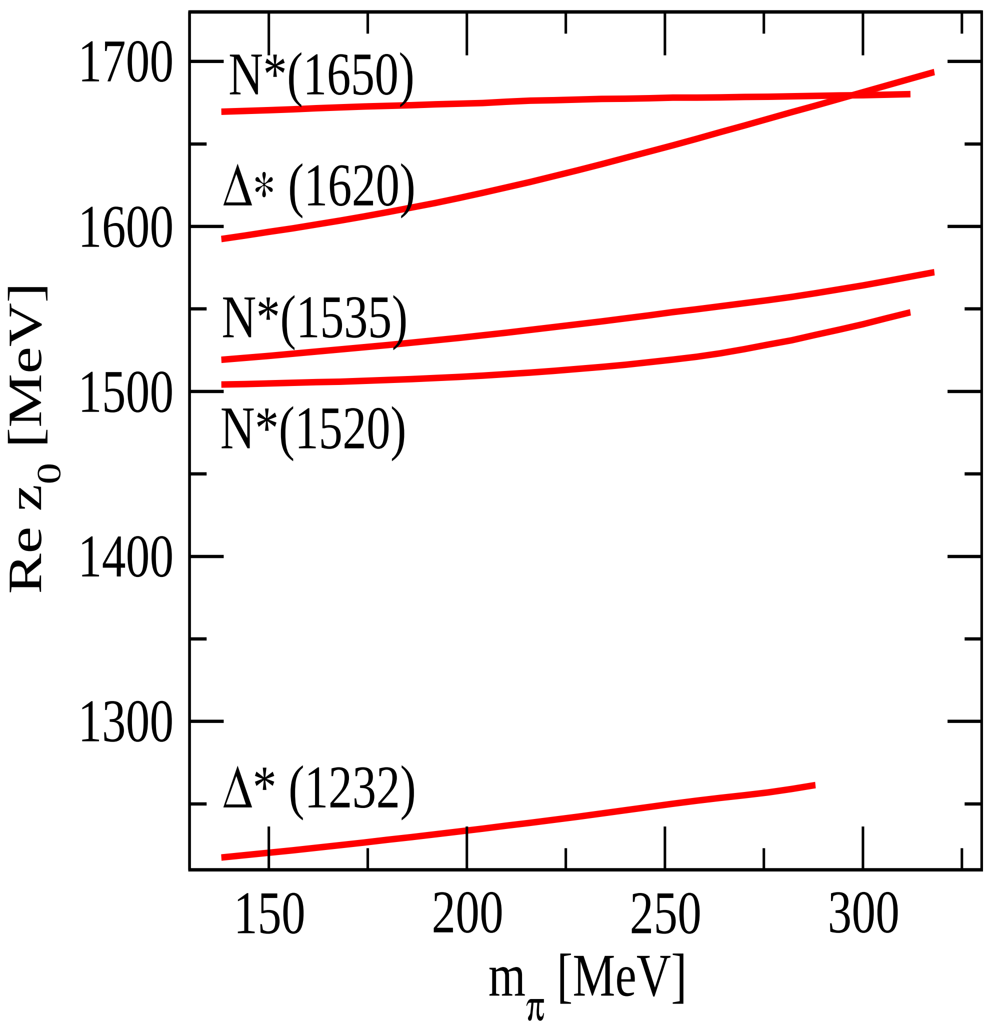


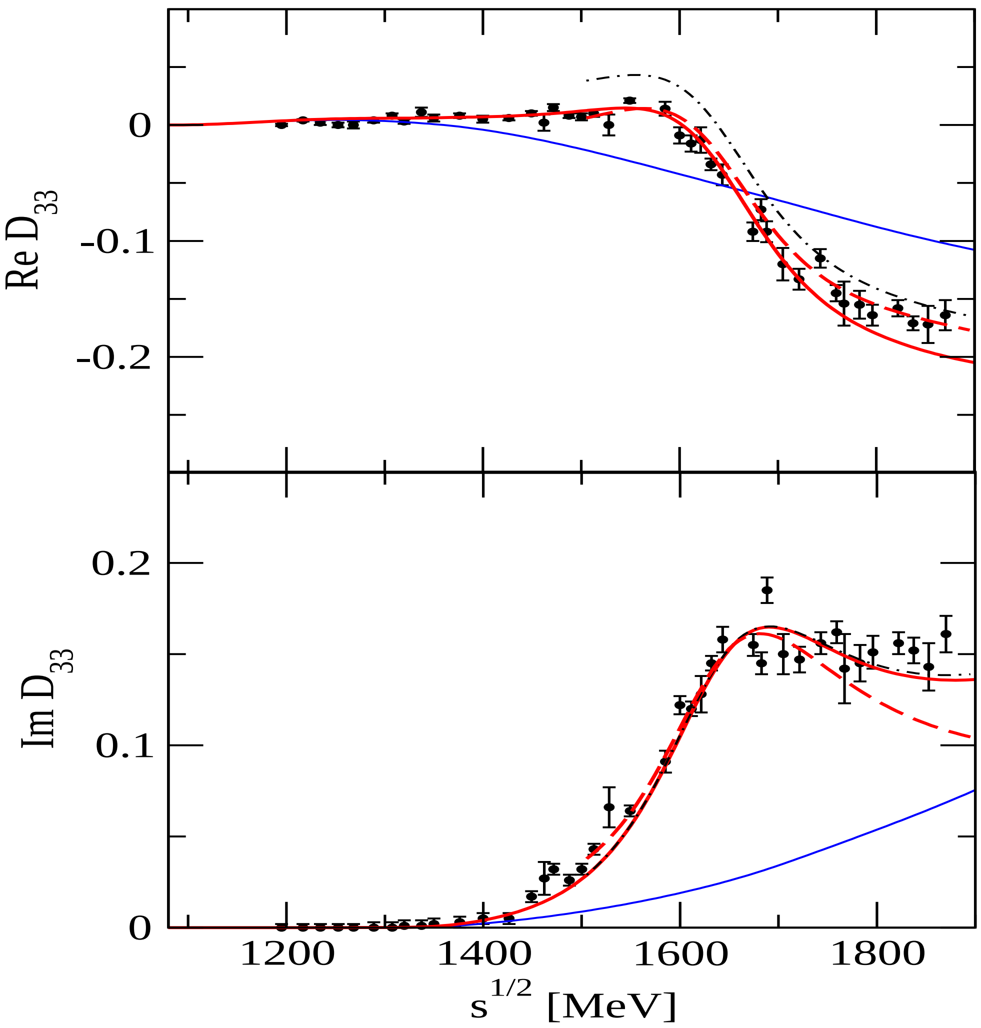
Energies above 2 GeV

Too much freedom for theorists??

Two-body scattering focusses on forward direction.

Study high energy limit.

Poles and background D_{33}



Vicinity of Pole:

$$T(Z) \sim \frac{a_{-1}}{Z-Z_0} + T^{NP}(Z)$$

$$T(Z) \sim \frac{a_{-1}}{Z-Z_0} + a_0$$

Amplitudes for charge exchange

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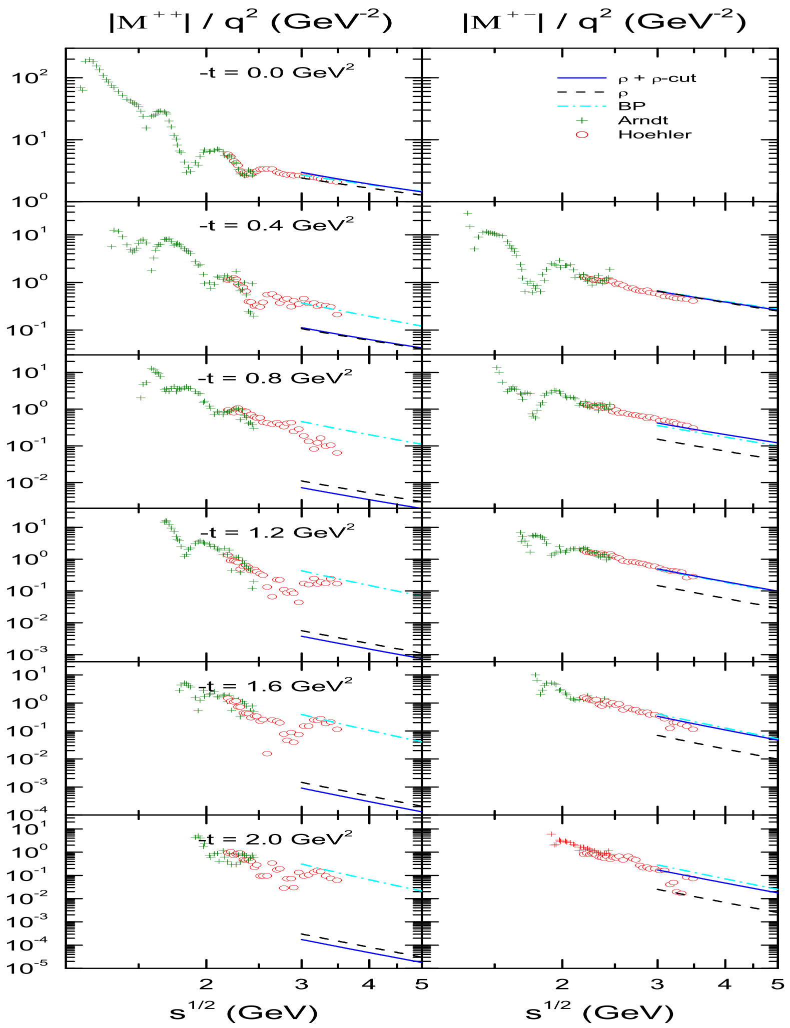

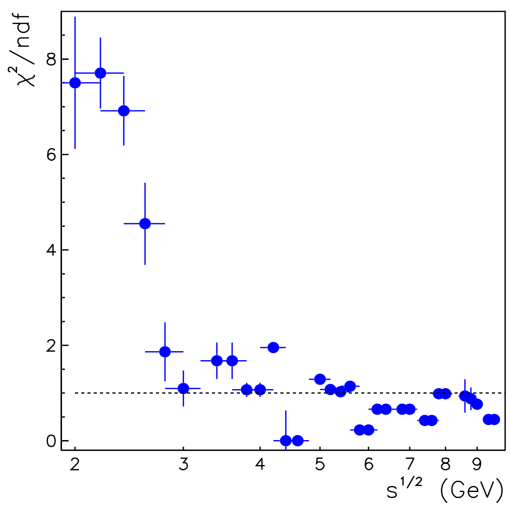

=>Fei Huang

Summary

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Jülich Lagrangian model: Analyticity and Unitarity
Combined treatment of poles and background
$T = \frac{a_{-1}}{Z-Z_0} + a_0 + O(Z - Z_0)$
model independent, good to meet experiment.

$a_{-1} = \frac{\Gamma_d}{\sqrt{1-\frac{\partial}{\partial Z}\Sigma}} \frac{\Gamma_d^{(\dagger)}}{\sqrt{1-\frac{\partial}{\partial Z}\Sigma}}$
Factorization of residue. Bare vertex Γ_{bare} accessible only in models, differs from dressed vertex.
Meet the lattice by pion mass dependence.
Poles and residues from Juelich model:Michael Doering
High energy limit: Fei Huang

Poles and residues I

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	Re Z_0 [MeV]	-2 Im Z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1520)\,D_{13}$	1505	95	32	-18
Arndt06	1515	113	38	-5
Hohler93	1510	120	32	-8
Cutkosky79	1510± 5	114± 10	35± 2	-12$\pm$5
$\Delta(1232)\,P_{33}$	1218	90	47	-37
Arndt06	1211	99	52	-47
Hohler93	1209	100	50	-48
Cutkosky79	1210± 1	100± 2	53± 2	-47$\pm$1
$\Delta^*(1700)\,D_{33}$	1637	236	16	-38
Arndt06	1632	253	18	-40
Hohler93	1651	159	10	
Cutkosky79	1675± 25	220± 40	13± 3	-20$\pm$25

Poles and residues II

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	Re Z_0 [MeV]	-2 Im Z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$N^*(1535)\,S_{11}$	1519	129	31	-3
Arndt06	1502	95	16	-16
Hohler93	1487			
Cutkosky79	1510$\pm$50	260$\pm$80	120$\pm$40	+15$\pm$45
$N^*(1650)\,S_{11}$	1669	136	54	-44
Arndt06	1648	80	14	-69
Hohler93	1670	163	39	-37
Cutkosky79	1640$\pm$20	150$\pm$30	60$\pm$10	-75$\pm$25
$N^*(1440)\,P_{11}$	1387	147	48	-64
Arndt06	1359	162	38	-98
Hohler93	1385	164	40	
Cutkosky79	1375$\pm$30	180$\pm$40	52$\pm$5	-100$\pm$35

Poles and residues III

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	Re Z_0 [MeV]	-2 Im Z_0 [MeV]	$\|R\|$ [MeV]	θ [deg] [^{0}]
$\Delta^*(1620)\,S_{31}$	1593	72	12	-108
Arndt06	1595	135	15	-92
Hohler93	1608	116	19	-95
Cutkosky79	1600± 15	120± 20	15± 2	-110$\pm$20
$\Delta^*(1910)\,P_{31}$	1840	221	45	-153
Arndt06	1771	479	38	+172
Hohler93	1874	283	19	
Cutkosky79	1880± 30	200± 40	20± 4	-90$\pm$30
$N^*(1720)\,P_{13}$	1663	212	14	-82
Arndt06	1666	355	25	-94
Hohler93	1686	187	15	
Cutkosky79	1680± 30	120± 40	8± 12	-160$\pm$30

Background

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	T^{NP}	a_0^{P}	Ratio
$N^*(1440)\,P_{11}$	$15.3 - 7.60i$	$-10.9 + 7.92i$	0.26
$\Delta^*(1620)\,S_{31}$	$9.01 - 6.37i$	$-1.21 + 0.24i$	0.9
$\Delta^*(1910)\,P_{31}$	$4.58 - 2.76i$	$-0.78 + 0.24$	0.9
$N^*(1720)\,P_{13}$	$1.76 - 0.10i$	$0.45 - 0.56i$	1.3
$N^*(1520)\,D_{13}$	$-4.62 - 0.56i$	$3.03 + 1.23i$	0.4
$\Delta(1232)\,P_{33}$	$-16.7 - 3.57i$	$17.1 + 10.6i$	0.4
$\Delta^*(1700)\,D_{33}$	$0.80 - 0.52i$	$0.40 + 0.11i$	1.3

The high energy limit: Regge theory

$A(s,t) \to \frac{1+exp(-i\pi\alpha)}{2sin(\pi\alpha)}\phi(t)s^{\alpha}$

$\alpha(t) = \alpha(0) + \alpha' t; \alpha(0) = 0.55; \alpha' = 0.86 GeV^{-2}$

Regge trajectory: $l = \alpha(t = M^2)$

Phenomenology:$\phi(t) = \beta_0 exp(bt)$

Analytical structure:

$\phi(t) = \frac{\Phi(t)}{\Gamma(\alpha)}$

Euler products:

$sin(\pi\alpha) = \alpha * (1 - 1\alpha)(1 + 1\alpha) * ...$

$\frac{1}{\Gamma(\alpha)} = \alpha exp(\gamma\alpha) * (1 + \frac{\alpha}{1})exp(-\frac{\alpha}{1}) * ...$

rescale:

$\beta_0 exp(bt) \to exp((\gamma - a)\alpha)exp(-\frac{\alpha}{2}...$

All unphysical singularities manifestly cancelled.