Coupled-channel effects in radiative charmonium transitions

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Based on:
Outline

• Introduction
• Nonrelativistic effective field theory
• Radiative charmonium transitions
• Pion mass dependence
• Summary
Introduction
**Introduction**

- Many *XYZ* states are close to or above open-charm thresholds.
- Good candidates of hadronic molecules:
  - $X(3872): D \bar{D}^*$
  - $Z_c(3900): D \bar{D}^*$
  - $Y(4260): D \bar{D}_1$
- Hadronic molecules: generated from nonperturbative hadron interactions (bound state, virtual state or resonance).
- Question: What is the role of perturbative meson loops?
- **Digression:** Q: Can a hadronic molecule be identified?  
  **A:** Yes, for an *S*-wave loosely bound state.
Introduction

Digression: S-wave loosely bound state

Suppose the physical state $\psi$ contains a two-hadron continuum state $h_1 h_2 = q$ and something else $\psi_0$.

The time-independent Schrödinger Equation

$$(\hat{H}_0 + \hat{V}) \psi = -E_B \psi$$

Here $H_0$ is the free Hamiltonian, $\hat{H}_0 q = q^2/(2\mu)$, and $E_B > 0$ is the binding energy. Multiplying by $q$, we get

$$\langle q | \psi \rangle = -\frac{\langle q | \hat{V} | \psi \rangle}{E_B + q^2/(2\mu)}$$

The probability of finding the physical state in the continuum state is

$$\lambda^2 = \int d^3q \langle q | \psi \rangle^2 = \int d^3q \frac{|\langle q | \hat{V} | \psi \rangle|^2}{[E_B + q^2/(2\mu)]^2}$$
Introduction

Digression: S-wave loosely bound state

Denoting \( g_{NR}^2(q) = |\langle q|\hat{V}|\psi\rangle|^2 \), we have

\[
\lambda^2 = 4\mu^2 \int d\Omega_q \int_0^\infty dq q^2 \frac{g_{NR}^2(q)}{(q^2 + 2\mu E_B)^2}
\]

If the binding energy is very small, so that the binding momentum \( \sqrt{2\mu E_B} \ll 1/R \) with \( R \) the range of forces, we have an expansion

\[
g_{NR}^2(q) = q^{2L} g_{NR}^2(0) + \mathcal{O}(R\sqrt{2\mu E_B})
\]

here \( L \) is the orbital angular momentum.

The integral is only convergent for an S-wave interaction with \( L = 0 \). Therefore,

the probability of finding the physical state in the S-wave two-hadron state with a small binding energy is related to the coupling constant \( g_{NR}(0) \)


\[
\lambda^2 \sim \frac{4\pi^2 \mu^2}{\sqrt{2\mu E_B}} g_{NR}^2(0)
\]
Introduction

Digression: S-wave loosely bound state

From nonrelativistic quantum mechanics to relativistic QFT:

\[ g = (2\pi)^{3/2} \sqrt{2m_1} \sqrt{2m_2} \sqrt{2(m_1 + m_2)} g_{NR}(0) \]

Here, \( g \) is the coupling constant in the relativistic Lagrangian

\[ \mathcal{L} = g \psi^\dagger h_1 h_2 + h.c. \]

Therefore, the coupling constant contains the structure information

\[ g^2 \approx 16\pi \lambda^2 (m_1 + m_2)^2 \sqrt{\frac{2E_B}{\mu}} \leq 16\pi (m_1 + m_2)^2 \sqrt{\frac{2E_B}{\mu}} \]

Bound from above! And the maximum is for a pure bound state.
Introduction

Coupled-channel effects

- Multipole
- Coupled-channel

Required by unitary

F. K. Guo (Uni. Bonn)

Zhou, Kuang (1991)


Introduction

Coupled-channel effects

<table>
<thead>
<tr>
<th>Initial meson</th>
<th>$J/\psi(1^3S_1)$</th>
<th>$\psi'(2^3S_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Final meson</strong></td>
<td>$\eta_c(1^1S_0)$</td>
<td>$\eta_c'(2^1S_0)$</td>
</tr>
<tr>
<td>$\Gamma_{M1}^{NR}$ (keV)</td>
<td>2.9</td>
<td>0.21</td>
</tr>
<tr>
<td>$\Gamma_{M1}^{GI}$ (keV)</td>
<td>2.4</td>
<td>0.17</td>
</tr>
<tr>
<td>$\Gamma_{IML}$ (keV)</td>
<td>$0.08^{+0.13}_{-0.06}$</td>
<td>$0.02^{+0.02}_{-0.01}$</td>
</tr>
<tr>
<td>$\Gamma_{all}$ (keV)</td>
<td>$1.58^{+0.37}_{-0.37}$</td>
<td>$0.08^{+0.03}_{-0.03}$</td>
</tr>
<tr>
<td>$\Gamma_{exp}$ (keV)</td>
<td>$1.58 \pm 0.37$ [4]</td>
<td>$0.143 \pm 0.027 \pm 0.092$ [34]</td>
</tr>
<tr>
<td>$\Gamma_{LQCD}$ (keV)</td>
<td>2.51 $\pm 0.08$</td>
<td>—</td>
</tr>
</tbody>
</table>

Li, Zhao (2011)

Coupled-channel effects are particularly important for $\psi' \rightarrow \gamma \eta_c$. 
Introduction
Coupled-channel effects

- **Other evidences from charmonium hadronic transitions**
  - Non-DDbar decays of the psi(3770) Liu, Zhang, Li (2008); Li, Zhao (2009)
  - Rho-pi puzzle Wang, Li, Zhao (2012)
  - Upsilon(nS) hadronic transitions Lipkin, Tuan (1988); Meng, Chao (2008)
  - Light quark mass ratio from $\psi' \rightarrow J/\psi \pi^0/\eta$ Guo, Hanhart, Meißner (2009)
  - …

- **Nonrelativistic effective field theory (NREFT) for coupled-channel effects** Guo et al PRL103(2009)082003; PRD83(2011)034013
NREFT
Power counting

Nonrelativistic \(2 M_D - M_{c\bar{c}} \ll M_D\) \(D\)-meson velocity \(v_D \ll 1\)

three-momentum \(p \sim O(v)\); energy \(E = \frac{p^2}{2m} \sim O(v^2)\)

propagator \(\frac{1}{E - p^2/(2m)} \sim O(v^{-2})\)

Examples

- **S-wave charmonium**

  \[
  O\left(\frac{v^5}{(v^2)^2}v^2\right) = O(v^3)
  \]

- **P-wave charmonium**

  \[
  O\left(\frac{v^5}{(v^2)^2}\right) = O(v)
  \]

F. K. Guo (Uni. Bonn) Radiative charmonium transitions
Radiative charmonium transitions

Hindered M1 transitions between P-wave states

- M1: radiative transitions with a flip of the heavy quark spin
- Hindered: vanishing in the nonrelativistic limit
- Quark model predictions
  \[ \Gamma_{M1} \propto |\langle \psi_f | \psi_i \rangle|^2 E_\gamma^3 \]
- Leading order amplitude

Brambilla, Jia, Vairo (2006)

\[ \sim E_\gamma \frac{v_c}{m_c} \]

Relativistic corrections

P-wave Spin-symmetry breaking
Radiative charmonium transitions
Hindered M1 transitions between P-wave states

- **Leading order loop**

- **The loop integral is convergent**

\[
I(q) = i \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m_1^2 + i\epsilon)((P - l)^2 - m_2^2 + i\epsilon)[(l - q)^2 - m_3^2 + i\epsilon]}
\]

\[
\mathcal{A}^{(a)} \sim \frac{v^5}{(v^2)^3} \frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c v}
\]
Radiative charmonium transitions
Hindered M1 transitions between P-wave states

- Other loops-I

\[ \mathcal{A}_{(b)} \sim \frac{\nu^5}{(\nu^2)^2} \frac{E_\gamma}{m_c} = \nu \frac{E_\gamma}{m_c} \]

- Gauge invariant by itself

- Suppressed by \( \nu^2 \)

\[ \mathcal{A}_{(a)} \sim \frac{\nu^5}{(\nu^2)^3} \frac{E_\gamma}{m_c} = \frac{E_\gamma}{m_c \nu} \]

\[ F_{\mu\nu} \]
Radiative charmonium transitions
Hindered M1 transitions between P-wave states

- Other loops-II

\[ \mathcal{A} \sim \frac{(v^5)^2}{(v^2)^5} \frac{g^2}{(4\pi)^2} \frac{E_\gamma}{m_c} \frac{M_D^2}{F_\pi} \]

One more loop than (a)  For matching dimensions

\[ \mathcal{A} \sim \frac{E_\gamma}{m_c} \left( \frac{g M_D}{\Lambda} \right)^2 \sim \frac{E_\gamma}{m_c} \]

- Suppressed compared with (a) by a factor of \( v \sim 0.4 \)
Radiative charmonium transitions
Hindered M1 transitions between P-wave states

- **Results**

\[ \Gamma(\chi'_c \rightarrow \gamma h_c) = (10.7 \pm 4.3) \frac{(g_1 g'_1)^2}{\text{GeV}^{-2}} \text{keV} \]

- Coupling constants \( g_1^{(')} \) are unknown

- If we take model values \( g_1 \sim -4 \text{ GeV}^{-1/2}, |g'_1| \sim 1 \text{ GeV}^{-1/2} \)


\[ \Gamma(\chi'_c \rightarrow \gamma h_c) = \mathcal{O}(170 \text{ keV}) \gg 1.3 \text{ keV} \text{ predicted in a quark model} \]
Radiative charmonium transitions
Hindered M1 transitions between P-wave states

Guo, Meißner, PRL 108 (2012) 112002
Radiative charmonium transitions
Suggestions for lattice (I)

- Quenched lattice QCD is not unitary and hence not physical. But its results correspond to the case without the coupled-channel effects.

- Calculating the same widths using both quenched and unquenched lattice would be necessary to understand the coupled-channel effects.

- Existing lattice calculations: both quenched and unquenched e.g. Dudek et al (2009); Chen et al (2011) …
Pion mass dependence
Is it important for heavy quarkonia?

- $D$ mesons contain light quarks, thus will introduce pion mass dependence into charmonium systems

- Pion-mass dependence of $M_D$

\[ M_D = \hat{M}_D + h_1 \frac{M_{\pi}^2}{\hat{M}_D} + O(M_{\pi}^3) \]

$h_1 = 0.44$ determined from the SU(3) mass difference
Pion mass dependence

Results

Guo, Meißner, PRL 109 (2012) 062001
Pion mass dependence
Suggestions for lattice (II)

- Pion mass dependence of some heavy quarkonium radiative transitions could be both very strong and nonanalytic, especially for $P$-wave states
- It is better to have lattice results close to the physical pion mass for comparing with the experimental data
Summary

- **Digression:** There exists a model-independent relation between the coupling constant and binding energy for an S-wave loosely bound state.

- Coupled-channel effects in some charmonium transitions can be studied using NREFT.

- **Coupled-channel effects could introduce strong pion-mass dependence in heavy quarkonium systems.**

- Both quenched and unquenched lattice simulations are very useful.