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Quintom Cosmology and Lorentz Violation

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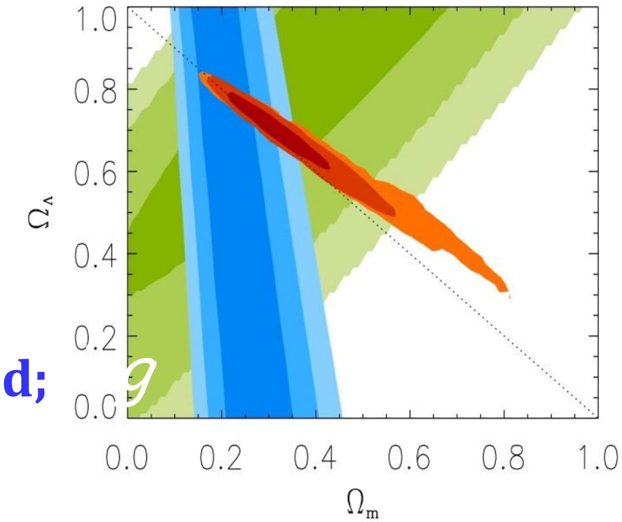
Outline

- Introduction of Quintom cosmology
  - A No-Go theorem of dark energy models
  - Model building
- Realizing Quintom scenario by virtue of Lorentz violation
  - Spontaneous Lorentz violation
  - Breaking Lorentz symmetry by hand
- Early universe implication: Quintom bounce
- Conclusions

Dark Energy:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\ddot{a} > 0 \quad \rightarrow \quad \rho + 3p < 0 \quad w = p/\rho < -1/3$$



Features: negative pressure; almost not clustered;

Candidates:

I Cosmological constant (or vacuum Energy)

$$w_\Lambda = -1 \qquad \rho_\Lambda = -p_\Lambda = \frac{\Lambda}{8\pi G} \simeq (2 \times 10^{-3} eV)^4$$

$$\rho_{ob}/\rho_{th} \sim 10^{-120}$$

cosmological constant problem!

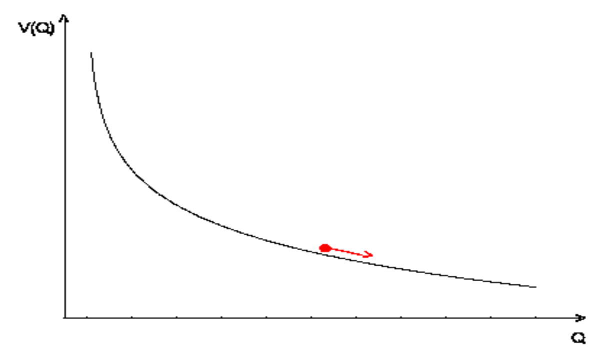
II Dynamical Fields:

Quintessence: $\mathcal{L} = \frac{1}{2}\partial_\mu Q \partial^\mu Q - V(Q)$

$-1 \leq w_Q \leq 1$

Phantom: $\mathcal{L} = -\frac{1}{2}\partial_\mu P \partial^\mu P - V(P)$

$w_P \leq -1$



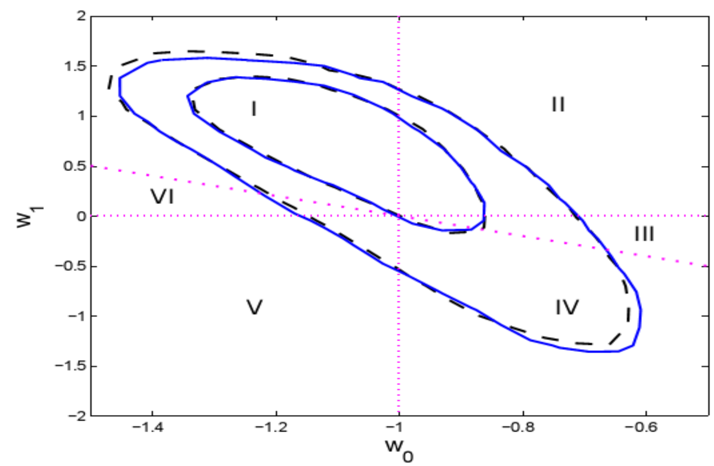
Determining the equation of state of dark energy with cosmological observations

Parametrization

$$w_{DE}(a) = w_0 + w_1(1 - a)$$

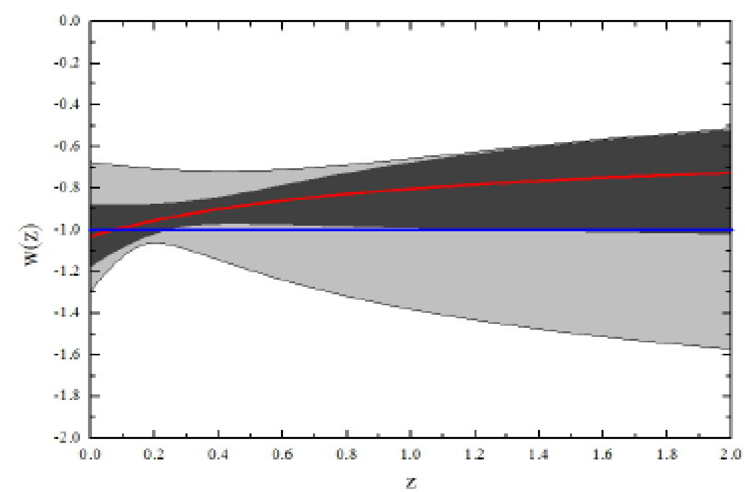
Classification of DE

- Cosmological Constant: w=-1
- Quintessence: w>-1
- Phantom: w<-1
- K-essence: w>-1 or <-1
- Quintom: w crosses -1
- Modified Gravity; …



Current status

CC fits the observation very well;
Marginal evidence favors quintom;
Need more accurate data …



Feng, Wang & Zhang, PLB 607:35,2005;
Huterer & Cooray, PRD 71:023506,2005;
Xia, Zhao, Feng & Zhang PRD 73:063521, 2006

NO-GO Theorem

- For theory of dark energy in the 4D Friedmann-Roberston-Walker universe described by a single perfect fluid or a single scalar field with a lagrangian of $L = L(\phi, \partial_\mu \phi \partial^\mu \phi)$, which minimally couples to Einstein Gravity, its equation of state cannot cross over the cosmological constant boundary.

- Key points to the proof:
  - For a single perfect fluid, its sound speed square is of form,

$$c_s^2 \equiv \left.\frac{\delta p}{\delta \rho}\right|_S = (w - \frac{w'}{3H(1+w)})$$

    which would be divergent when w=-1;
  - For a single scalar field, there is a general dispersion relation for its perturbations:

$$\omega^2 = c_s^2 k^2 - \frac{z''}{z} - 3c_s^2 (H' - H^2) \qquad z \equiv \sqrt{\phi'^2 |\rho_{,X}|}$$

    which would also be divergent when w=-1.

NO-GO Theorem

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Feng, Wang & Zhang, Phys. Lett. B 607:35, 2005;

Vikman, Phys. Rev. D 71:023515, 2005;

Hu, Phys. Rev. D 71:047301, 2005;

Caldwell & Doran, Phys. Rev. D 72:043527, 2005;

Zhao, Xia, Li, Feng & Zhang, Phys. Rev. D 72:123515, 2005;

Kunz & Sapone, Phys. Rev. D 74:123503, 2006;

……

Xia, **CYF**, Qiu, Zhao, & Zhang, Int.J.Mod.Phys.D17:1229,2008

Quintom Model-building

- See the review:

  CYF, Saridakis, Setare & Xia, Phys.Rept.493:1,2010

- Gauss-Bonnet Modified gravity
  - Cai, Zhang & Wang Commun.Theor.Phys.44:948,2005
- DGP Braneworld DE
  - Zhang & Zhu, Phys.Rev.D75:023510,2007
- Vector DE
  - Picon, JCAP 0407:007,2004;
  - Wei & Cai, Phys.Rev.D73:083002,2006
- Interactive DE
  - Wang, Gong & Abdalla, Phys.Lett.B624:141,2005
- Effective Field Theory (EFT) approach

Why we use EFT to study DE?
EFT may do not solve the problem of DE completely, but can tell us generic features and potential observational signals of DE.

The earliest Quintom model: Double-field

Action: $S=\int d^4x\sqrt{-g}[\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1-\frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2-V(\phi_1,\phi_2)]$

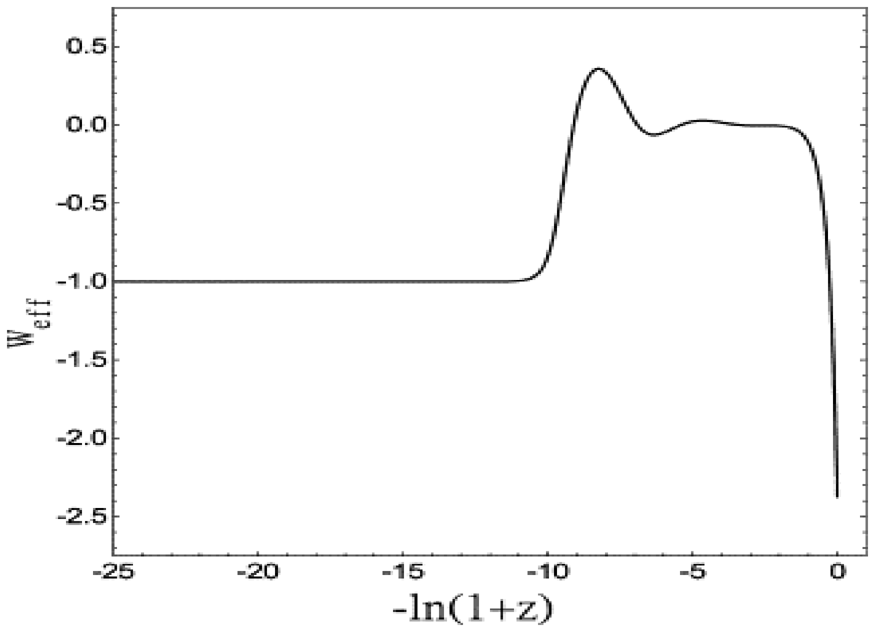
Potential: $V(\phi_1,\phi_2)=V_0\left[\exp(-\frac{\lambda}{M_p}\phi_1)+\exp(-\frac{\lambda}{M_p}\phi_2)\right]$

➢Benefits:

- w can cross -1;
- Easily fit to observations;
- With specific potentials it has tracking behavior;
- Classical perturbations are well defined.

➢Problem:

- If quantized, it is unstable because of a ghost.



Feng, Wang & Zhang, PLB 607:35, 2005

Quintom Model from EFT: Lee-Wick model

Lagrangian: $$\mathcal{L} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \frac{c}{2M^2}\Box\phi\Box\phi - V(\phi)$$

Equivalent Lagrangian:

$$\mathcal{L} = -\frac{1}{2}\nabla_\mu\psi\nabla^\mu\psi + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi - V(\psi-\chi) - \frac{M^2}{2c}\chi^2$$

where we have redefine,

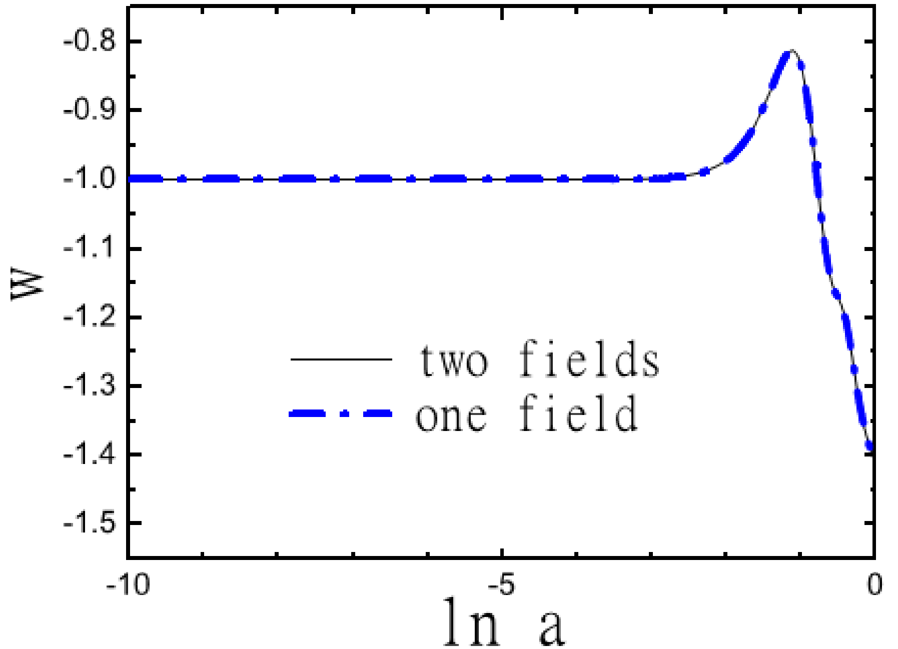
$$\chi = \frac{c}{M^2}\Box\phi \,,$$
$$\psi = \phi + \chi \,.$$

➢Progress:

- Perturbatively well-defined;

➢Problem:

- Incomplete in ultraviolet limit.



Li, Feng & Zhang, JCAP 0512:002,2005

Quintom Model from EFT: Spinor Quintom

Action: $$S_\psi = \int d^4x\, e\, [\frac{i}{2}(\bar{\psi}\Gamma^\mu D_\mu\psi - D_\mu\bar{\psi}\Gamma^\mu\psi) - V]$$

The vierbein algebra yields:

$$\bar{\psi}\psi \propto a^{-3}$$
$$w = -1 + \frac{V'\bar{\psi}\psi}{V}$$

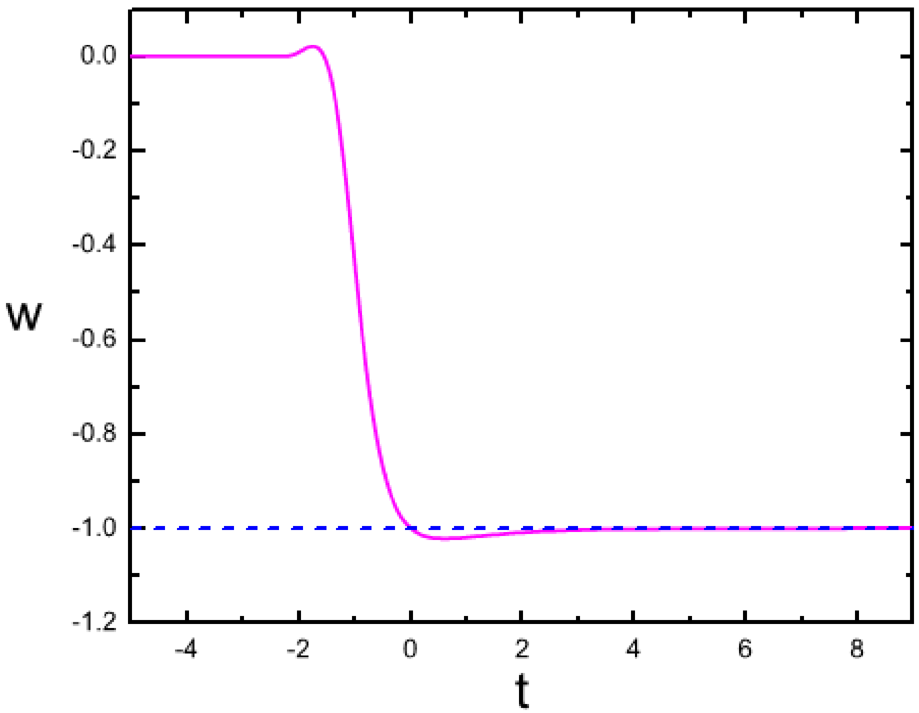
➢Progresses:

- No ghost;
- With certain potential, it has the similar behavior of Chaplygin gas, namely

$$V = \sqrt{V_0(\bar{\psi}\psi - b)^2 + c}$$

➢Weak point:

- An effectively negative mass when w is below -1.



CYF & Wang, CQG25:165014,2008

Realizing quintom scenario by virtue of Lorentz violation

- Spontaneous Lorentz violation

  Ghost condensation

- Breaking Lorentz symmetry by hand

  Horava-Lifshitz type models

Ghost Condensation

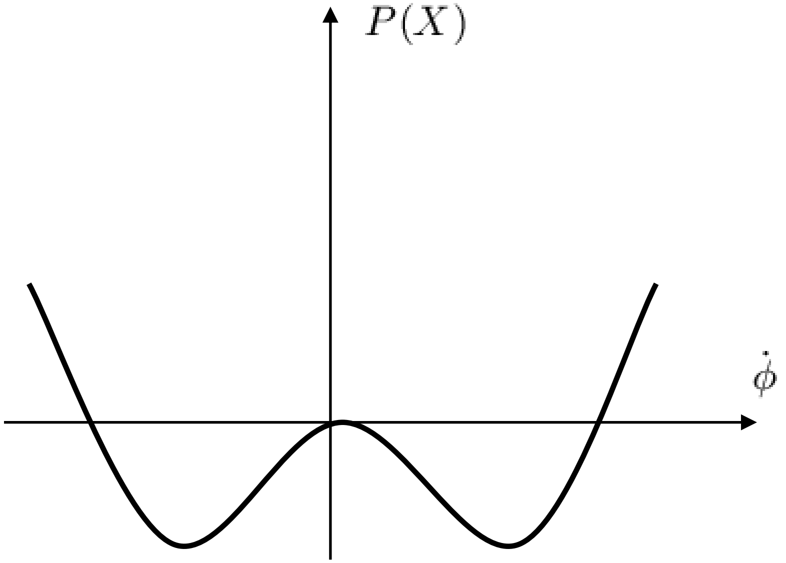
- The action takes $\mathcal{L} = P(X)$
  where we define $X = \partial_\mu \phi \partial^\mu \phi$

- For simplicity, we take $P(X) = M^{-4}(X - M^4)^2$

  - Ghost instability around $\dot{\phi} = 0$
  - Ghost condensates at

$$P_{,X} = 0$$

$$\Rightarrow \langle \dot{\phi}^2 \rangle = M^4$$



Arkani-Hamed, Cheng, Luty, Mukohyama, JHEP 0405:074,2004
Arkani-Hamed, Creminelli, Mukohyama, Zaldarriaga, JCAP 0404:001,2004

Ghost Condensation

- In the flat FRW universe, the EoM is given by

$$\partial_t [a^3 P_{,X} \dot{\phi}] = 0 \quad \Longrightarrow \quad P_{,X} \dot{\phi} \propto a^{-3}$$

This solution shows that the universe approaches to the state of condensation along with the cosmic expansion.

- Time translational symmetry is broken in this theory, and thus Lorentz violation spontaneously. It can be viewed as a Higgs phase of gravity.

- The ghost condensation accompanied with other canonical scalar field, can lead to the equation of state of the universe across the line w=-1. Thus a quintom scenario is achieved.

Ghost Condensation

- Unsettled issues:

- For example, in the simplest case $P(X) = M^{-4}(X - M^4)^2$. We have the sound speed parameter:

$$c_s^2 \equiv \frac{P_{,X}}{\rho_{,X}} = \frac{X - M^4}{3X - M^4}$$

- Then we find this theory allows superluminal propagation of information in certain case.

- Also it may yields negative sound speed square which implies perturbative instability.

- This model does not admit a Hamiltonian formulation. This is because the Legendre transform is multi-valued. Therefore its quantization is still unclear.

Horava-Lifshitz type models

- Inspired by condensed matter physics, a non-relativistic field theory with well-defined Hamiltonian formulation is possible.

- We work on a d+1 dimensional space-time, with one time coordinate and d spatial coordinates. The theories of Horava-Lifshitz type exhibit fixed points with anisotropic scaling governed by a dynamical critical exponent: $t \to b^z t \; , \quad x^i \to b x^i$

  with $[t] = -z$ and $[x^i] = -1$.

- At the starting point, this theory abandoned the Lorentz symmetry.

Horava, Phys. Rev. D 79, 084008 (2009) ;
Lifshitz, Zh. Eksp. Teor. Fiz. 11, 255 (1941).

Horava-Lifshitz type models

- Specifically, we consider a scalar field minimally coupling to gravity sector in this theory.

- The full action of HL gravity for z=3 is given by,

$$S_g = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k - \right.$$
$$\left. - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left[\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\},$$

where K_{ij} is the extrinsic curvature, C^{ij} is the cotton tensor.

- The full action of HL scalar for z=3 is given by,

$$S_\sigma = \int dt d^3x \sqrt{g} N \left[\frac{3\lambda - 1}{4} \frac{\dot{\sigma}^2}{N^2} + h_1 h_2 \sigma \nabla^2 \sigma - \frac{1}{2} h_2^2 \sigma \nabla^4 \sigma + \frac{1}{2} h_3^2 \sigma \nabla^6 \sigma - V_\sigma(\sigma) \right]$$

Horava-Lifshitz type models

- Effectively, the dark energy sector can be expressed by

$$\rho_{DE} \equiv \frac{3\lambda - 1}{4}\dot{\sigma}^2 + V_\sigma(\sigma) - \frac{3\kappa^2\mu^2k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)}$$

$$p_{DE} \equiv \frac{3\lambda - 1}{4}\dot{\sigma}^2 - V_\sigma(\sigma) - \frac{\kappa^2\mu^2k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda - 1)}$$

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Kinetic Potential Dark radiation

Cosmological constant

Horava-Lifshitz type models

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Kinetic Potential

Dark radiation

Cosmological constant

- Quintessence part: the σ field

- Ghost part: dark radiation + CC

  Dark radiation exists when the spatial curvature is not flat

Horava-Lifshitz type models

- The combination of Quintessence part and ghost part leads to a quintom scenario.

- The -1 crossing can take place at early universe, so that gives rise to bouncing and cyclic solutions. (Refer to Robert Brandenberger's talk)

Brandenberger, Phys.Rev.D80:043516,2009;
CYF, Saridakis, JCAP 0910:020,2009.

- Benefits on renormalization:

Gravity is marginally renormalizable, since the propagator takes

$$\sim \frac{1}{k^6}$$

- Unclear issue: Strong coupling problem for HL gravity.

Quintom Bounce: Basic Idea

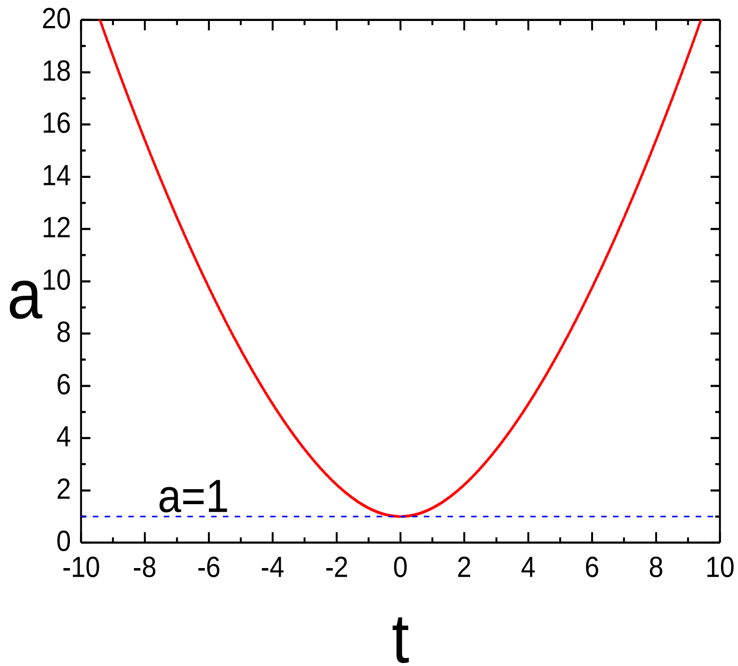
The expanding of the universe is transited from a contracting phase; during the transition, the scale factor of the universe is at its minimum but non-vanishing, thus the singularity problem can be avoided.

Contracting: $H < 0$ Expanding: $H > 0$

Bounce point: $H = 0$ Around it: $\dot{H} > 0$

$$\dot{H} = -4\pi G\rho(1+w) \quad \Rightarrow \quad w < -1$$

Transition to the observable universe (radiation dominant, matter dominant, …)
So w needs to cross -1, and Quintom is required!



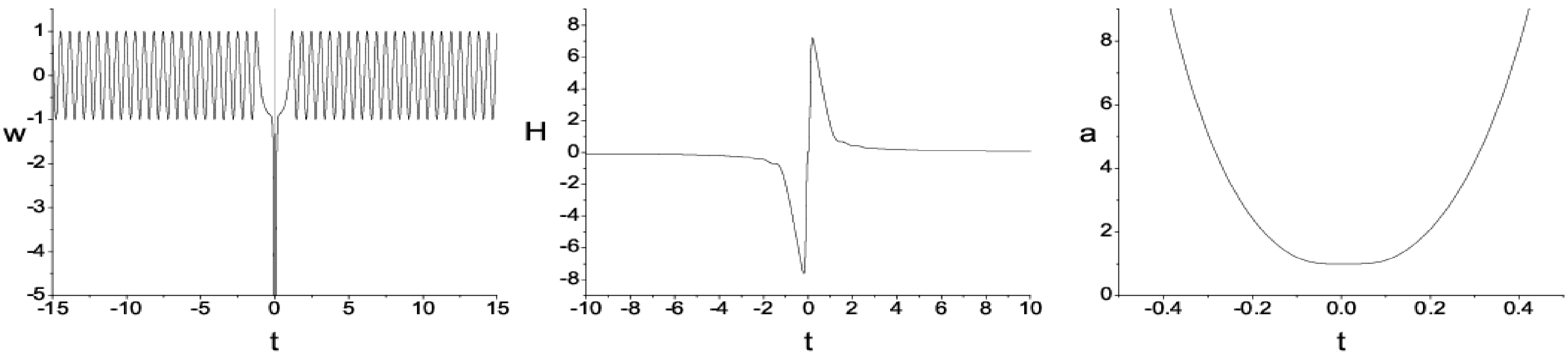
CYF, Qiu, Piao, Li & Zhang, JHEP0710:071, 2007

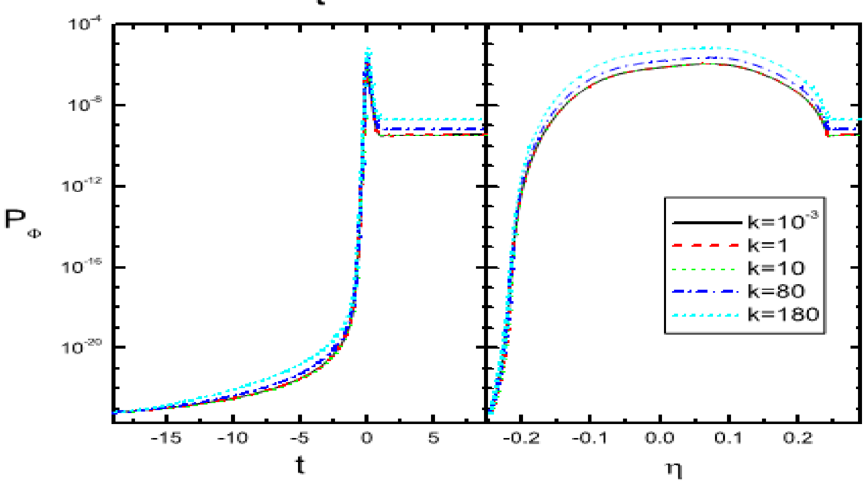
Lee-Wick Bounce

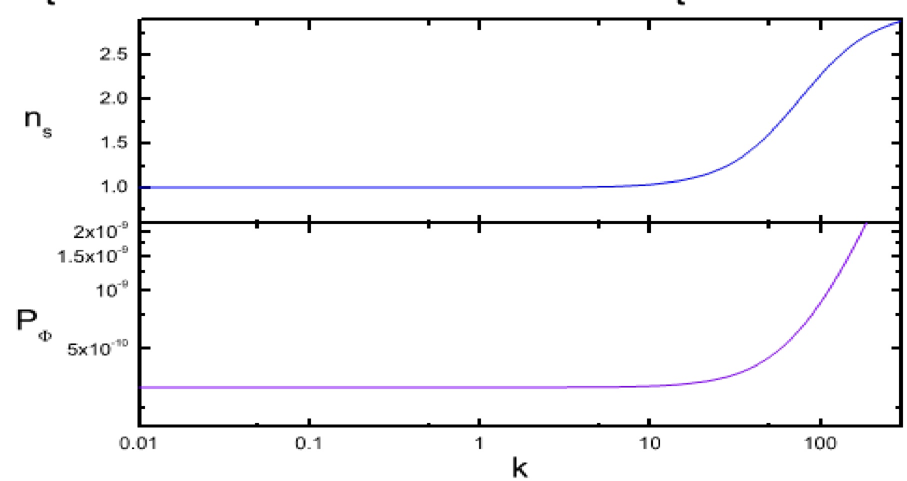
The simplest Lagrangian: **CYF**, Qiu, Brandenberger & Zhang,PRD80:023511,2009.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$$

Plots of background evolution and perturbation:







An alternative to inflation!

Refer to Brandenberger's talk

Summary

- ✓Review on Quintom Cosmology:
  - Unique signal: w across -1;
  - No-Go Theorem and Model-building;
- ✓Realizing quintom scenario in the theories with Lorentz violation:
  - Ghost condensation
  - Horava-Lifshitz type model
- ✓Application of Quintom in early universe:
  - bounce cosmology;
  - scale-invariant spectrum.

THANK YOU!

Happiness only real when shared!

from the novel “Into the wild”

Perturbations of quintom dark energy

- A self-consistent data-fitting method should include the perturbations, even for a cosmological constant;
- Difficulties of DE perturbations when w crosses -1:

$$\dot{\delta}_i = -(1+w_i)(\theta_i - 3\dot{\Phi}) - 3\mathcal{H}(\frac{\delta P_i}{\delta \rho_i} - w_i)\delta_i \quad ,$$

$$\dot{\theta}_i = -\mathcal{H}(1-3w_i)\theta_i - \frac{\dot{w}_i}{1+w_i}\theta_i + k^2(\frac{\delta P_i/\delta \rho_i}{1+w_i}\delta_i - \sigma_i + \Psi)$$

δ ,θ are the density and velocity fluctuation respecitvely

$$w \to -1, \dot{w} \neq 0 \;\Rightarrow\; \delta, \theta \to \infty$$

- This trouble can be solved in a double-field quintom model;

$$\dot{\delta}_i = -(1+w_i)(\theta_i - 3\dot{\Phi}) - 3\mathcal{H}(1-w_i)\delta_i - 3\mathcal{H}\frac{\dot{w}_i + 3\mathcal{H}(1-w_i^2)}{k^2}\theta_i$$

$$\dot{\theta}_i = 2\mathcal{H}\theta_i + \frac{k^2}{1+w_i}\delta_i + k^2\Psi \ .$$

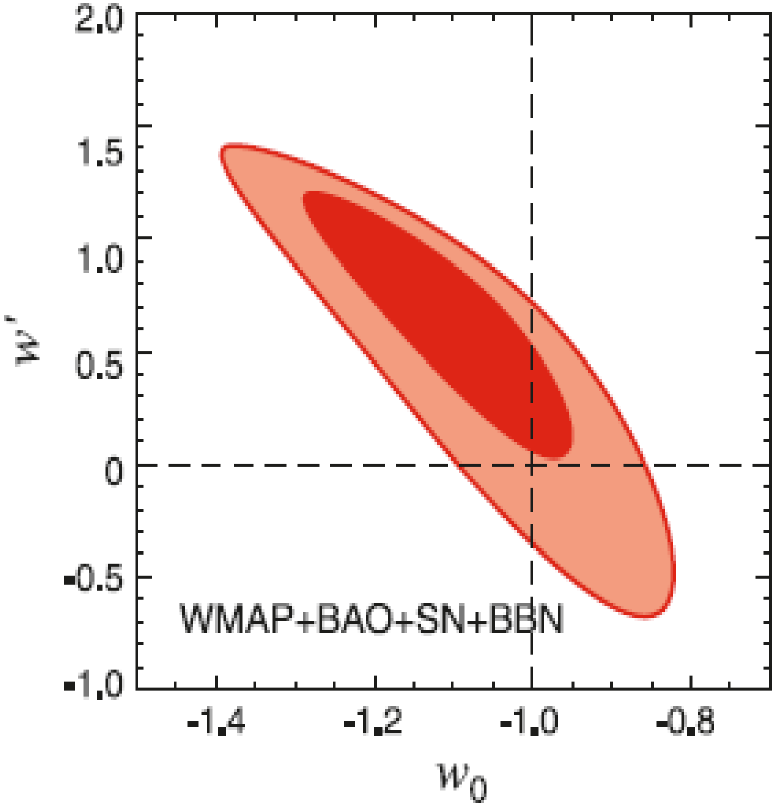
$$w_{quintom} = \frac{\sum_i P_i}{\sum_i \rho_i} \qquad \delta_{quintom} = \frac{\sum_i \rho_i \delta_i}{\sum_i \rho_i} \qquad \theta_{quintom} = \frac{\sum_i (\rho_i + p_i)\theta_i}{\sum_i (\rho_i + P_i)}$$

The perturbations are continuous and well-defined when w crosses -1

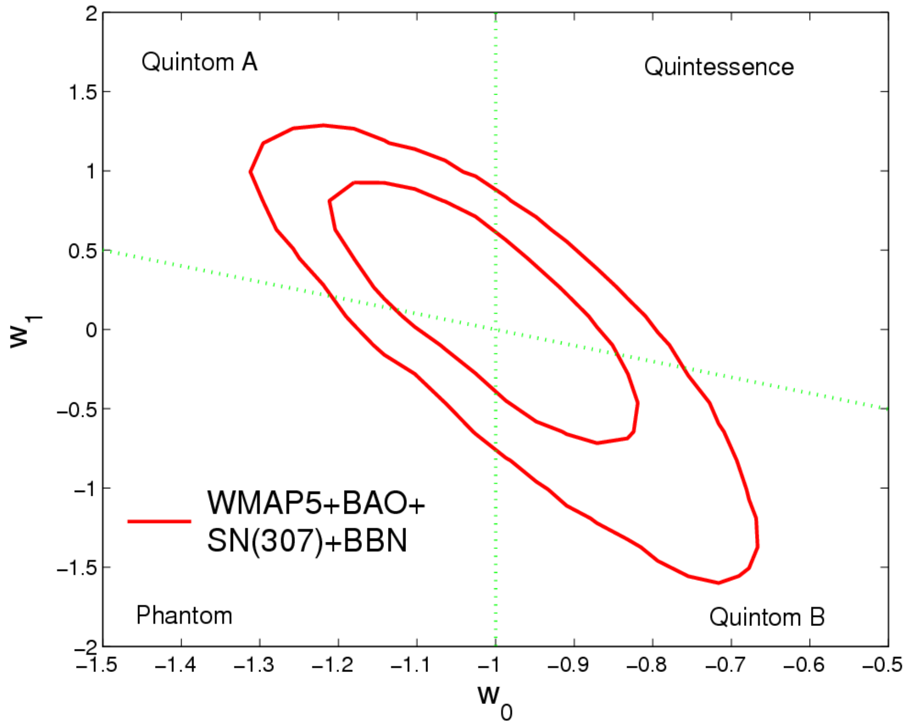
- Also see the review:

  CYF, Saridakis, Setare & Xia, Phys.Rept.493:1,2010

Constraints on the EoS of dark energy



WMAP5 result (without perturbations)
E. Komatsu et al., arXiv:0803.0547



With perturbations
Xia, Li, Zhao, Zhang, PRD78:083524,2008

Status: 1) Cosmological constant fits data well;
2) Dynamical model not ruled out;
3) Best fit value of EoS:
slightly w across -1 → Quintom model

Quintom examples in string theory

Descriptions:

- A rolling tachyon is effectively described by a non-local cubic string field theory, which corresponds to a slowly decaying D3-brane;
- In a local approximation, this model contains quintom degrees of freedom.

The model:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + \gamma^4 \left(\frac{1}{2} \phi F(\Box) \phi - V(\phi) \right) \right]$$

where

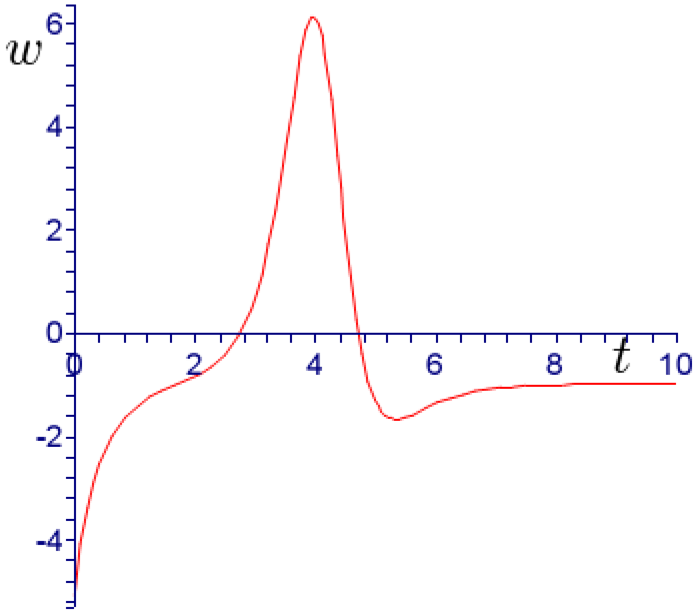
$$F(\Box) = (1 - 4\xi^2 \alpha \Box) e^{\alpha \Box}$$

- At low energy limit, it behaves as a double-field quintom;

  Aref'eva, Koshelev & Vernov, PRD72:064017,2005

- While at high energy scale, the model contains multiple quintom degrees of freedom.

  Mulryne & Nunes, PRD78:063519,2008



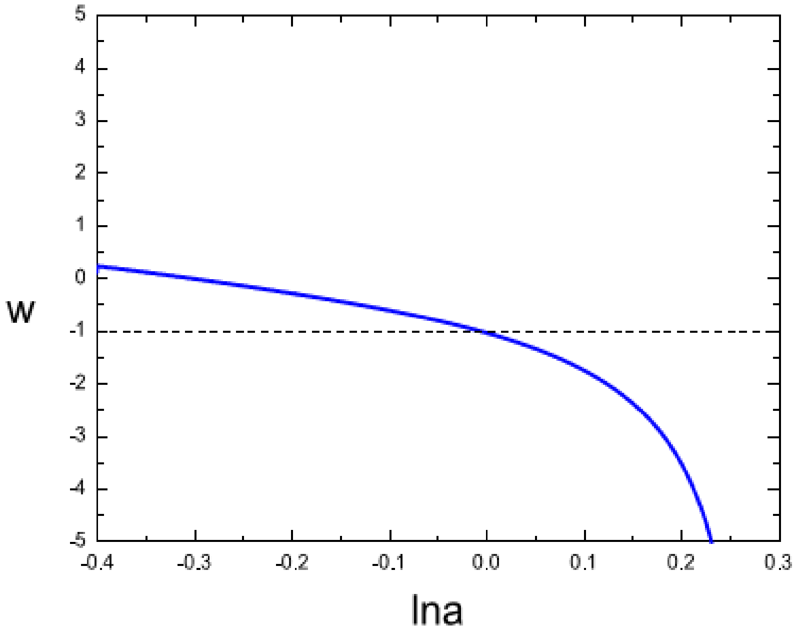
Quintom examples in string theory

An equivalent scenario with a generalized DBI action:

$$S = \int d^4x\sqrt{-g}\left[-V(\phi)\sqrt{1-\alpha'\nabla_\mu\phi\nabla^\mu\phi+\beta'\phi\Box\phi}\right].$$

with the potential: $V(\phi) = V_0 e^{-\lambda\phi^2}$

This is an action including higher derivatives, but of a non-perturbative form. (beta term involves two scalars and two derivatives, the same as alpha term)
The quantization of this class of models is still an open issue.



CYF, Li, Lu, Piao, Qiu & Zhang, PLB651:1,2007

Power counting renormalizability

$$I_K \supset \int dt dx^3 \dot{\phi}^2 \qquad I_V \supset \int dt dx^3 \phi^p$$

Scaling dimension of ϕ		Condition of renormalizability
Lorentz QFT	t -> b t, x -> b x E -> b^{-1} E ϕ -> b^s ϕ 1+3-2+2s=0 -> s=-1	$\propto E^{-(1+3+ps)}$ -> p <= 4
HL QFT	t -> b^z t, x -> b x E -> b^{-z} E ϕ -> b^s ϕ z+3-2z+2s=0 -> s=(z-3)/2	$\propto E^{-(z+3+ps)/z}$ -> when z=3, s=0, any nonlinear interactions can be renormalized. Thus gravity becomes renormalizable

Quintom cosmology and the origin of the universe

Cosmology: science based on observations

CMB: WMAP, Planck

LSS: SDSS ⇒ **hot Big Bang**

SuperNovae

Theoretical Problems:

- Horizon
- Flatness
- Monopole
- Structure formation

→ **Inflation**

- And singularity

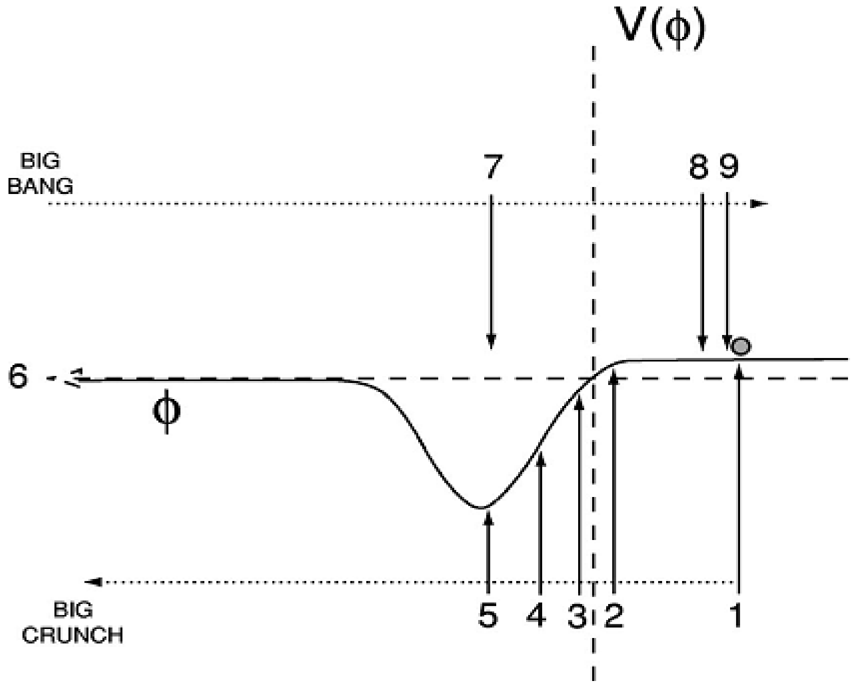
→ One of the most fundamental questions related to theories of quantum gravity

Borde and Vilenkin, PRL72:3305, 1994

Ekpyrotic model

The collision of two M branes in 5D gives rise to a nonsingular cyclic universe, and the description of EFT in 4D is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 - V(\phi) + \beta^4(\phi)(\rho_M + \rho_R) \right)$$



1 DE domination
2 decelerated expansion
3 turnaround
4 ekpyrotic contracting phase
5 before big crunch
6 a singular bounce in 4D
7 after big bang
8 radiation domination
9 matter domination

Khoury, Ovrut, Steinhardt & Turok, PRD64:123522, 2001

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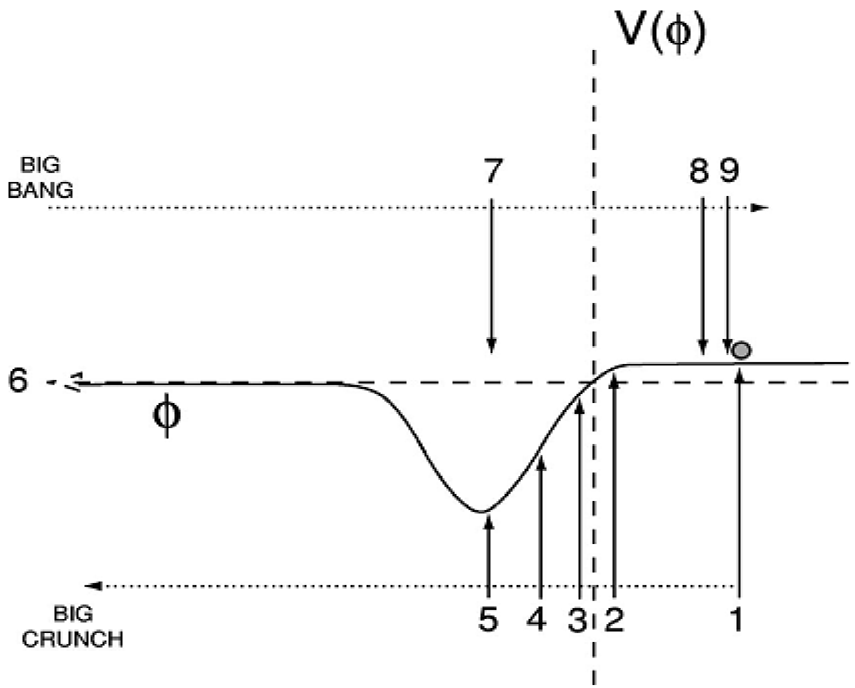

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Failure of effective field theory description, uncertainty involved in perturbations.

Nonsingular Bounce

- Pre-big-bang
  (non-perturbative effects)
- String gas cosmology
  (thermal non-local system)
- Mirage cosmology
  (braneworld)
- Loop quantum gravity
  (high-order corrections)
- New Ekpyrotic model
  (ghost condensate)

, and others …