Lorentz Invariance Violation:
from theoretical and phenomenological perspectives

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In collaboration with Zhi Xiao, Lijing Shao, Shimin Yang, Zhou Lingli
Triumphs of Special Relativity

- One of the foundations of modern physics.
- Proved to be valid at very high precision.

Lorentz Invariance: a basic theoretical foundation of relativity.

So why we seek for Lorentz Violation?
Planck’s **God-Given Unit System**
(Planck, 1899)

\[ c, \ G, \ \hbar, \ k_B, \text{ and } 1/4\pi\varepsilon_0 \]

units of length, mass, time, and temperature that would, independently of special bodies and substances, necessarily retain their significance for all times and all cultures, even extraterrestrial and extrahuman ones, and which may therefore be designated as natural units of measure. (Planck 1899, pp. 479–480)

Basic units of the universe: Planck Units

\[ l_P = \sqrt{\frac{G\hbar}{c^3}} = 1.61624(8) \times 10^{-35} \text{ m} \]

\[ t_P \equiv \sqrt{\frac{G\hbar}{c^5}} \simeq 5.4 \times 10^{-44} \text{ s} \]

\[ M_P = \sqrt{\frac{\hbar c}{G}} = 1.22089(6) \times 10^{19} \ \frac{\text{GeV}}{c^2} = 2.17644(11) \times 10^{-8} \text{ kg} \]

\[ E_P \equiv \sqrt{\frac{\hbar c^5}{G}} \simeq 2.0 \times 10^9 \text{ J} \]

\[ T_P \equiv \sqrt{\frac{\hbar c^5}{Gk_B^2}} \simeq 1.4 \times 10^{32} \text{ K} \]
The typical scale of quantum gravity is Planck mass

\[ M_P = \sqrt{\frac{\hbar c}{G}} = 1.22089(6) \times 10^{19} \ \text{GeV} \frac{\text{GeV}}{c^2} = 2.17644(11) \times 10^{-8} \ \text{kg} \]

Lorentz Violation might be a relic probe for quantum gravity
Why Lorentz violation?

- Quantum Gravity (QG)?
  - spacetime foam [Ellis et al.’08, PLB]
  - loop gravity [Alfaro et al.’00, PRL]
  - torsion in general gravity [Yan’83, TP]
  - vacuum condensate of antisymmetric tensor fields in string theory [Kostelecky & Samuel’89 & ’91, PRL]
  - double special relativity [Amelino-Camelia’02, Nature & ’02 IJMPD]
Various theories on Lorenz violation

- **Effective Field Theory**
  - **Standard Model Extension**
    - an explicitly introduction of condensation of background tensor field
    \[ \mathcal{L}_{LV} \sim \frac{\lambda}{M_{\text{Planck}}^k} < T > \bar{\psi} \Gamma (i \partial)^k \chi \]

- **Non EFT**
  - **Double Special Relativity with two universal invariants:**
    - photon limiting velocity \( c \),
    - Planck length scale \( l_{\text{Planck}} = 1.616 \times 10^{-33} \text{cm} \)
  - **Stringy spacetime foam model**

- **Dynamical critical exponent of space and time scaling**
  - \( t \rightarrow \lambda^z t, \quad \vec{r} \rightarrow \lambda \vec{r} \)
  - Lorentz symmetry emergent at low energies as \( z \rightarrow 1 \)
The total Lagrangian

\[ \mathcal{L} = \mathcal{L}_{SM} + \delta \mathcal{L}, \]  

where \( \delta \mathcal{L} \) denotes tiny LV parts.

take QED as example

\[ \delta \mathcal{L}_{QED} = \delta \mathcal{L}_{\text{photon}} + \delta \mathcal{L}_{\text{electron}}, \]  

where

\[ \delta \mathcal{L}_{\text{photon}} \supset -\frac{1}{4} (k_F)^{\kappa \lambda \mu \nu} F^{\kappa \lambda} F^{\mu \nu} + \frac{1}{2} (k_{AF})_{\kappa} \epsilon^{\kappa \lambda \mu \nu} A_\lambda F_{\mu \nu}, \]  

\[ \delta \mathcal{L}_{\text{electron}} \supset \frac{1}{2} \bar{\psi} (\tilde{c}^{(\nu \mu)} \gamma_\nu + \tilde{d}^{\nu \mu} \gamma_5 \gamma_\nu + \frac{1}{2} \tilde{g}^{\lambda \nu \mu} \sigma_{\lambda \nu}) \overset{\leftrightarrow}{D_\mu} \psi \]

\[ -\bar{\psi} (\tilde{b}_\mu \gamma_5 \gamma^\mu + \frac{1}{2} \tilde{H}_{\mu \nu} \sigma^{\mu \nu}) \psi. \]
Effective Field Theory

- Lorentz violation—conflict with covariance?
- Particle (active) Lorentz rotation

Graph is obtained from T. Katori (MIT)

Where the background field is unaltered under particle Lorentz rotation, as an \textbf{tensor condensate}. 
Lorentz violation-conflict with covariance?

Observer (passive) Lorentz rotation

Graph is obtained from T. Katori (MIT)
Lorentz violation—conflict with covariance?

\[
\bar{\psi}(x)(a^\nu \gamma_\nu)\psi(x) \rightarrow [U \bar{\psi}(x)U^{-1}][U a^\nu U^{-1}] U\gamma_\nu U^{-1}][U \psi(x)U^{-1}] \\
= [\bar{\psi}(\Lambda x) S^{-1}](a^\nu [\Lambda^\rho_\nu \gamma_\rho])[S \psi(\Lambda x)] = \bar{\psi}(\Lambda x)(a^\nu \Lambda^\rho_\nu \gamma_\rho)\psi(\Lambda x)
\]

So Lorentz violation can be incorporated in a covariant form.

Lorentz invariance breaks;
But Lorentz covariance works.
So we call it Lorentz invariance violation (LV or LIV)
Consequences of Lorentz violation

- Could provide explanation of neutrino oscillation without neutrino mass
  S. Yang and B.-Q. Ma, IJMPA 24(09)5861, arXiv:0910.0897
  Z. Xiao and B.-Q. Ma, IJMPA 24(09)1539

- Modified dispersion relation could increase threshold energy of photo-induced meson production of the proton: an increase of GZK cutoff energy
  Z. Xiao and B.-Q. Ma, IJMPA 24(09)1539

- Modified dispersion relation may cause time lag of photons with different energies when they propagate in space from far-away astro-objects
  Z. Xiao and B.-Q. Ma, PRD 80 (2009) 116005, arXiv:0909.4927,
Lorentz violation in three-family neutrino oscillations

The general equation for neutrino oscillation probabilities

- The Lagrangian density for neutrino sector

\[
L = \frac{1}{2} \bar{\nu}_A \gamma^\mu \bar{\partial}_\mu \nu_B \delta_{AB} + \frac{1}{2} i c^\mu_{\bar{\nu}} \bar{\nu}_A \gamma^\mu \bar{\partial}^\nu \nu_B - a^\mu_{\bar{\nu}} \bar{\nu}_A \gamma^\mu \nu_B
\] (3.1)

- From eq.(3.1), we figure out the Hamiltonian density

\[
H = (\bar{\nu}_A \gamma^\mu \nabla \delta_{AB} - i c^\mu_{\bar{\nu}} \bar{\nu}_A \gamma^\mu \nabla_i ) \nu_B + a^\mu_{\bar{\nu}} \bar{\nu}_A \gamma^\mu \nu_B
\] (3.2)

- Transforming the description into quantum mechanics

\[
\hat{H} = -i \gamma^0 \bar{\nu}_A \gamma^\mu \nabla \delta_{AB} - i c^\mu_{\bar{\nu}} \gamma^0 \gamma^\mu \nabla_i + a^\mu_{\bar{\nu}} \gamma^0 \gamma^\mu
\] (3.3)
The general equation for neutrino oscillation probabilities

- Up to now, we have not detected right-handed neutrinos or left-handed anti-neutrinos, so we choose the basis vector as

\[
\begin{pmatrix}
  u_L(p) \\
v_R(p)
\end{pmatrix}
\]

(3.4)

- We could get the dynamical equation for neutrinos

\[
\left( i \frac{\partial}{\partial t} - H_{AB} \right) \begin{pmatrix} a_B \\ b_B \end{pmatrix} = 0
\]

(3.5)

- The Hamiltonian matrix for neutrino given

\[
H_{AB} = \begin{pmatrix}
|p| \delta_{AB} + c_{AB}^{\mu j} \frac{p_\mu p_j}{|p|} + a_{AB}^{\mu} \frac{p_\mu}{|p|} & 0 \\
0 & |p| \delta_{AB} + c_{AB}^{\mu j} \frac{p_\mu p_j}{|p|} + a_{AB}^{\mu} \frac{p_\mu}{|p|}
\end{pmatrix}
\]

(3.6)
The general equation for neutrino oscillation probabilities

- Diagonalizing $H_{AB}$ to get the energy spectrum for neutrinos
  \[ E = U^+ H U \] (3.7)

- The relationship between energy eigenstates $\nu_i$ and flavor eigenstates $\nu_\alpha$
  \[ |\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle \] (3.8)

- The general equation for neutrino oscillation
  \[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i > j} \Re[(U^+)_{i\alpha} (U^+)_{i\beta} (U^+)_{j\alpha} (U^+)_{j\beta}] \sin^2 \left[ \frac{\Delta E_{ij}}{2} t \right]
  + 2 \sum_{i > j} \Im[(U^+)_{i\alpha} (U^+)_{i\beta} (U^+)_{j\alpha} (U^+)_{j\beta}] \sin[\Delta E_{ij} t]
  \] (3.9)
Neutrino oscillation for Lorentz violation:

- We carried out Lorentz violation contribution to neutrino oscillation by the effective field theory for LV and give out the equations of neutrino oscillation probabilities.

- In our model, neutrino oscillations do not have drastic oscillation at low energy and oscillations still exist at high energy.

- Neutrinos may have small mass and both LV and the conventional oscillation mechanisms contribute to neutrino oscillation.

S. Yang and B.-Q. Ma, IJMPA 24 (09) 5861, arXiv:0910.0897
Z. Xiao and B.-Q. Ma, IJMPA 24 (09) 1539
Ultra-high energy cosmic rays (UHECRS)

- $E > 10^{18} - 10^{19}$ eV
- Extragalactic origin above the ankle

Energy = 50 J, the same as a well-hit tennis ball at 42 m/s.
Greisen-Zatsepin-Kuzman (GZK) cutoff energy of nucleon cosmic rays

predicted in 1966

pion production

\[ N + \gamma_{\text{CMB}} \rightarrow \pi + N \]

\[ E = \frac{S - m_{\pi}^2}{2\varepsilon_\gamma (1 - \cos \theta)} \]

threshold energy

\[ E \approx \frac{2m_N m_{\pi} + m_{\pi}^2}{4\varepsilon_\gamma} = 1.10 \times 10^{20} \text{ eV} \]

mean free path

\[ \lambda_N \sim 3 \text{ Mpc} \quad \text{GZK zone} \sim 50 \text{ Mpc} \]
The energy spectrum of cosmic rays
Proposed origins of super GZK events
---before 2007

• Z-bursts

• Decay of relic superheavy particles in galactic halo
  • Heavy primaries: i.e., iron
  • New hadrons
  • Violation of Lorentz symmetry

• Photons with photon-axion mixing
  • ...

However, new experimental results appeared: HiRes 2007.
However, new experimental results appeared: Auger 2007

Lessons from new results

- The observation of GZK cut-off by HiRes and Auger put strong constraints on previous models for the super-GZK events.
- There are still uncertainties on the re-construction of the energy, so final conclusion still may change.
- Detailed features are important: how large of the GZK events, shape, and direction.
Lorentz Violation & Super-GZK events

- The earlier reports on super-GZK events triggered attention on Lorentz-Violation (LV or LIV).
  
  S.R. Coleman and S.L. Glashow, PRD 59 (1999) 116008

- The new results of observation of GZK cut-off put strong constraints on Lorentz-Violation parameters,
  see, e.g.,

  X.J. Bi, Z. Cao, Y. Li, Q. Yuan, PRD 79 (2009) 083015.
Lorentz Violation as a mechanism for Super-GZK events *Coleman&Glashow*

- **Starting from a free field Lagrangian,**

\[ \mathcal{L} = \partial_\mu \Psi^* Z \partial^\mu \Psi - \Psi^* M^2 \Psi. \]

and adding a LV term

\[ \mathcal{L} \rightarrow \mathcal{L} + \partial_i \Psi \epsilon \partial^i \Psi, \]

- **Modified dispersion relation with LV effect**

\[ p^2 = E^2 - \vec{p}^2 = m^2 + \epsilon \vec{p}^2. \]

\[ E_a^2 = \vec{p}_a c_a^2 + m_a^2 c_a^4, \quad c_a = \sqrt{1 + \epsilon c}, \quad m_a = m/(1+\epsilon) \]

where \( c_a \) is the maximal attainable velocity for the \( a \)th particle.
Lorentz violation & enhancement of threshold energy

- Take the nucleon-photon to Delta process as example

\[ P + \gamma (\text{CMB}) \rightarrow \Delta (1232) \quad \omega + E_p \geq E_\Delta \]

- With LV effect

\[ \omega + E_p \geq \sqrt{(|\vec{P}_p| - \omega)^2 c_\Delta^2 + m_\Delta^2 c_\Delta^4} \]

\[ E_p = \frac{\omega \sqrt{1 - 1/2 \frac{K}{\omega^2} \left(1 - \frac{c_\Delta}{c_p}\right)}}{1 - \frac{c_\Delta}{c_p}} - \omega \approx -\frac{K}{4\omega} - \frac{K^2}{32\omega^3} \left(1 - \frac{c_\Delta}{c_p}\right) + \ldots \]

\[ 1 - \frac{c_\Delta}{c_p} = -\frac{2\omega (E_p - E_{\text{thre}})}{E_{\text{thre}}^2} \]

simply assume \( c_\Delta = 1 \)

Constraints on LV parameters


A rough estimate

\[ \epsilon_p \sim 10^{-24} \]

- X.J.Bi, Z.Cao, Y.Li, Q.Yuan, PRD 79 (2009) 083015.

An analysis with shape

\[ \epsilon_p \sim 10^{-23} \]


**Figure 4.** Comparison of the latest Auger data with calculated spectra for various values of \( \delta_p \), taking \( \delta_p = 0 \) (see text). From top to bottom, the curves give the predicted spectra for \( \delta_p = 1 \times 10^{-22}, 6 \times 10^{-23}, 4.5 \times 10^{-23}, 3 \times 10^{-23}, 2 \times 10^{-23}, 1 \times 10^{-23}, 3 \times 10^{-23} \) and 0 (no Lorentz violation) [44].
LV from cosmological VHE photon emissions

Z.Xiao and B.-Q.Ma, PRD 80 (09) 116005, arXiv:0909.4927
Modified photon dispersion relation from LV

\[ \nu(E) = c_0 \left( 1 - \frac{E}{M_P c^2} - \frac{E^2}{M_P^2 c^4} \right) \]

\[ \sqrt{\frac{\hbar c}{G}} \simeq 1.22 \times 10^{19} \text{ GeV}/c^2 \]

For reviews, see, e.g.,
Kostelecky & Mewes’09, PRD
Mattingly’05, Living Rev. Rel.
Amelino-Camelia & Smonlin’09, PRD
Gammy-ray Bursts (GRBs)

- the most energetic astrophysical process except the Big Bang
- 2 types [Piran’05, Rev. Mod. Phys.]
  - long GRBs: duration > 2 s; collapses of massive rapidly rotating stars
  - short GRBs: duration < 2 s; coalescence of two neutron stars or a neutron star and a black hole
- use GRBs to test LV [Amelino-Camelia et al.’98, Nature]
Time lag by LV effect

- expansion universe [Jacob & Piran’08, JCAP]

\[
\Delta t_{LV} = \frac{1 + n}{2H_0} \left( \frac{E_h^n - E_1^n}{M_{QG}^n c^{2n}} \right) \int_0^z \frac{(1 + z')^n dz'}{h(z')}
\]

\[M_{QG,L} = |\xi|^{-1} M_P \quad \text{and} \quad M_{QG,Q} = |\zeta|^{-1/2} M_P\]

\[h(z) = \sqrt{\Omega_\Lambda + \Omega_M (1 + z)^3}\]

\[H_0 \approx 71 \text{ km s}^{-1} \text{ Mpc}^{-1}\]

\[\Omega_\Lambda \approx 0.73 \quad \Omega_M \approx 0.27\]
Fermi instruments

"Typical" Prompt GRB Spectrum

\[ E^2 N_E (\text{erg cm}^{-2} \text{s}^{-1}) \]

- GBM
- LAT

\[ \sim 300 \text{ GeV} \]

trigger photons \( \sim 0.1 \text{ MeV} \)

Pulsar (Kaaret 99|Ellis 06)
GRB (Biller 98)
AGN (Boggs 04)
GRB (Albert 08)
AGN (Albert 08)

\[ \text{Planck mass} \]

\[ \text{min } M_{\text{QG}} \]

(\text{GeV/c}^2)
Lag determinations

GRB080916C -- Abdo et al.’09, Science
Time lags are affected both artificially and instrumentally.

GRB090510
Abdo et al.'09, Nature
Time-lag by GRB
Four Fermi observations

the arrival of the highest energy photon to GBM trigger

<table>
<thead>
<tr>
<th>GRBs</th>
<th>$z$</th>
<th>$E$ (GeV)</th>
<th>$\Delta t_{\text{obs}}$ (s)</th>
<th>$M_{\text{QG,L}}$ (GeV/c$^2$)</th>
<th>$M_{\text{QG,Q}}$ (GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>080916C [19]</td>
<td>4.35 [21]</td>
<td>13.22</td>
<td>16.54</td>
<td>$1.5 \times 10^{18}$</td>
<td>$9.7 \times 10^9$</td>
</tr>
<tr>
<td>090510 [20]</td>
<td>0.903 [22]</td>
<td>31</td>
<td>0.829</td>
<td>$1.7 \times 10^{19}$</td>
<td>$3.4 \times 10^{10}$</td>
</tr>
<tr>
<td>090902B [23]</td>
<td>1.822 [24]</td>
<td>33.4</td>
<td>82</td>
<td>$3.7 \times 10^{17}$</td>
<td>$5.9 \times 10^9$</td>
</tr>
<tr>
<td>090926A [25]</td>
<td>2.1062 [26]</td>
<td>19.6</td>
<td>26</td>
<td>$7.8 \times 10^{17}$</td>
<td>$6.8 \times 10^9$</td>
</tr>
</tbody>
</table>

$\Delta t_{\text{obs}} = \Delta t_{\text{LV}}$

$M_{\text{QG,L}} \sim (4.9 \pm 8.1) \times 10^{18}$ GeV

$M_{\text{QG,Q}} \sim (1.4 \pm 1.3) \times 10^{10}$ GeV
Formulas in our analysis of LV parameter

linear and quadratic energy dependence

\[ v(E) = c_0 \left( 1 - \frac{E}{M_{\text{QG}} c^2} \right) \]

\[ v(E) = c_0 \left( 1 - \frac{E^2}{M_{\text{QG}}^2 c^4} \right) \]
Separation of astrophysical time lags from LV delay

- imperfect knowledge of radiation mechanism of GRBs
- a survey of GRBs at different redshifts
  - the time lag induced by LV accumulates with propagation distance
  - the intrinsic source induced time lag is likely to be a distance independent quantity
- A robust survey [Ellis et al.’06 & 08, Astropart. Phys.]
\[
\Delta t_{LV} = \frac{1 + n}{2H_0} \left( \frac{E_h^n - E_1^n}{M_{QG}^{n} c^{2n}} \right) \int_0^z (1 + z')^n dz' \frac{h(z')}{h(z)}
\]

\[
\Delta t_{\text{obs}} = \Delta t_{LV} + \Delta t_{\text{in}}(1 + z)
\]

**Linear fits**

\[
M_{QG,L} = (2.2 \pm 0.2) \times 10^{17} \text{ GeV}/c^2 \text{ and } M_{QG,Q} = (5.4 \pm 0.2) \times 10^9 \text{ GeV}/c^2
\]

**Quadratic fits**

\[
M_{QG,L} = (2.2 \pm 0.9) \times 10^{17} \text{ GeV}/c^2 \text{ and } M_{QG,Q} = (5.3 \pm 0.8) \times 10^9 \text{ GeV}/c^2
\]
Active galactic nuclei (AGNs)

- AGN is a compact region at the centre of a galaxy which has a much higher than normal luminosity over some or all of the electromagnetic spectrum [wikipedia]

- AGNs vs GRBs [Ellis et al.’09, PLB]
  - distance & time structure
  - energy of flares; rare & unpredictable
  - different types & distinct intrinsic time lags?
A brief review on LV AGNs

- **Markarian 421** – no time lag > 280 s between energy bands < 1 TeV and > 2 TeV [Biller et al.’99, PRL]
  \[ M_{QG,L} > 4.9 \times 10^{16} \text{ GeV}/c^2 \text{ and } M_{QG,Q} > 1.5 \times 10^{10} \text{ GeV}/c^2 \]

- **Markarian 501** – 4 min lag for \( \Delta E \sim 2 \text{ TeV} \) [Albert et al.’08, PLB]
  \[ M_{QG,L} \sim 1.2 \times 10^{17} \text{ GeV}/c^2 \]

- **PKS 2155-304** – ~20 s lag for \( \Delta E \sim 1.0 \text{ TeV} \) & \( \Delta E^2 \sim 2.0 \text{ TeV}^2 \) [Aharonian et al.’08, PRL]
  \[ M_{QG,L} \sim 2.6 \times 10^{18} \text{ GeV}/c^2 \quad M_{QG,Q} \sim 9.1 \times 10^{10} \text{ GeV}/c^2 \]

\[ \Delta t_{in} = 0 \]
Discussions

- GRBs vs AGNs
  - AGNs data are inadequate to carry out a robust analysis
  - a complementary probe: different observational method and distinct origins
  - Fermi LAT: GRB 090323, GRB 090328, & GRB 091003

- a set of VHE celestial events for LV hints or quantum gravity mass boundaries?
- disentangle astrophysical origins from LV effects?
Discussions

- negative intercepts

\[ \approx -20 \text{ s for linear dependence} \]
\[ -6 \text{ s} \sim -10 \text{ s for quadratic dependence} \]
A brief discussion:

- QG -> LV -> modified dispersion relation -> time lag -> high energy & long distance -> astrophysics -> GRBs & AGNs etc.
- We (re)analyse 4 Fermi LAT GRBs and review 3 AGNs -> surprising consistency
- A robust survey on GRBs of different redshifts to separate source effects -> how about AGNs?

\[ \sim 2 \times 10^{17} \text{ GeV/c}^2 \]
\[ \sim 5 \times 10^9 \text{ GeV/c}^2 \]
A new theory of Lorentz violation

- a replacement of the common derivative operators by covariant co-derivative ones

\[ \partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta, \quad D^\alpha \rightarrow M^{\alpha\beta} D_\beta, \]

- The effective minimal Standard Model

\[ \mathcal{L}_{SM} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_{HG} + \mathcal{L}_{HF}, \]
\[ \mathcal{L}_G = -\frac{1}{4} F^{a\alpha\beta} F^a_{\alpha\beta}, \]
\[ \mathcal{L}_F = i \bar{\psi} \gamma^\alpha D_\alpha \psi, \]
\[ \mathcal{L}_{HG} = (D_\alpha \phi)^\dagger D_\alpha \phi + V(\phi), \]

- A new standard model with supplement terms

\[ \mathcal{L}_{SM} = \mathcal{L}_{SM} + \mathcal{L}_{LV}, \]
\[ \mathcal{L}_{LV} = \mathcal{L}_{GV} + \mathcal{L}_{FV} + \mathcal{L}_{HFV} \]


physical independence of mathematical background manifolds
Fig. 1. $\delta x_{CE}$ azimuthal distribution vs angles of the GRAAL data of the years 1998-2005 on a plane ($x$-$y$ plane or $\theta = \pi/2$). $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$. 

Anisotropy of light speed


Fig. 2. $\delta x_{CE}$ azimuthal distribution vs angles of the GRAAL data of the year 2008 on a plane ($x$-$y$ plane or $\theta = \pi/2$). $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$. 
Summary

- New theory for Lorentz violation can be introduced.
- Lorentz violation can provide an alternative explanation for neutrino oscillation.
- Photon time-lag effect from Lorentz violation can be observed from GRBs, AGN, and Pulsars.
- Possible evidence for light speed anisotropy.
- Lorentz violation is being an active frontier both theoretically and experimentally.