CTCs in Galileon Theories

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Lorentz and CPT Violation in Astrophysics and Cosmology

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Talk Outline

Part I:

Motivation for IR modifications of gravity which break time diffeomorphism invariance and a discussion of model-*independent* features

Part II:

Review of the Galileon model, description of a configuration with CTCs that apparently evolve from an otherwise reasonable configuration

The existence of this configuration is strong evidence that the Galileon model is at best a low energy effective theory, however *in contrast* with a previous study of the consistency of the model, it suggests the validity of this effective theory within the validity of the derivative expansion even in the presence of appreciable nonlinearities.

Part I: Motivation for the Galileon Theory

Perturbatively we know how to make sense of a nonabelian gauge theory:

(1) Expand the matter fields about their VEVs and the gauge field about zero
(2) Choose a symmetry group such that, when quotiented by the group action, the quadratic terms in the Lagrangian density are invertible
(3) Impose Gauss constraints for the corresponding charges and quotient by the symmetry, identify the inverse of the quadratic terms with the propagators

This procedure fails when applied to gravity, because the gauge group ISO(3,1) is *noncompact*

How is Gravity Different?

(Discussion from Arkani-Hamed, Cheng, Luty and Mukohyama, 2003)

Indefinite Killing metric of gauge algebra
$\rightarrow$ Fluctuation kinetic terms with both signs

This leads to an instability in which energy flows from the fluctuation of one vierbein component to another

The solution is to expand about a nonzero value of the vierbein, corresponding to a nondegenerate metric. The fluctuations then have kinetic terms with the same signs and the instability disappears.

Comparison with tachyonic instability

Wrong sign potential term
→ Low k modes are tachyonic and decay
Wrong sign kinetic term
→ High k modes are ghosts and decay

In the first case the speed of the decay is given by the highest k instable modes, and so in principle the decay may happen slowly. In the second, the faster the mode, the greater the instability and so the decay is instantaneous.

Therefore only the first may correspond to a phase transition/spontaneous symmetry breaking.

In the case of gravity it is essential from the beginning to expand about a nonzero vierbein/metric.

What initial metric? With what symmetry?

In a perturbative theory of gravity, the metric can be written as a background metric plus a perturbation.

For simplicity, one does not wish to include the backreaction of the uneven distribution of matter in the background metric.

The leaves two natural/popular choices:

(1) Minkowski space and so Poincaré symmetry

(2) FRW metric and so ISO(3) symmetry

What initial metric? Minkowski or FRW?

Depending on the cosmology to be studied, the spacetime may only be a finite deformation of one of these choices.

Example (1): In a hot big bang cosmology, or its UV completion such as a string gas model, in the phase in which general relativity is reliable time has a beginning. Thus the metric is *not* a finite perturbation of Minkowski space.

Example (2): A bouncing universe cosmology may be, at each time, a finite but unbounded perturbation of Minkowski space. However it is a bounded perturbation of an FRW solution.

Conclusion: FRW with ISO(3) symmetry is often better and sometimes the only reasonable choice for a background metric.

A new paradigm of symmetry breaking?

In such a formulation of gravity, time-diffeomorphism symmetry is broken.

Yesterday Yifu mentioned models with explicit and spontaneous Lorentz symmetry breaking, but as described above this breaking is neither.

For example, as there is no energy in the UV in which it is restored, the breaking may affect the renormalizability of the theory and so qualitatively differs from spontaneous breaking.

Gravity without temporal diffeomorphisms

In the usual case with 4d diffeomorphism invariance, generated by 4 generators, the counting of degrees of freedom is as follows:

Start with the 10 independent components of the metric. Impose the 4 Gauss constraints corresponding to the diffeomorphism generators, and then fix the corresponding 4 gauge choices

This leaves $10\text{-}4\text{-}4 = 2$ graviton polarizations

Now there are only 3 unbroken symmetries, but this does NOT give 4 polarizations. Indeed, the time diffeomorphism symmetry MUST be imposed to arrive at an invertible kinetic term and so a nonzero propagator.

Breaking temporal diffeomorphism symmetry

To have a nonzero propagator, one must still impose all 4 constraints and quotient by all 4 diffeomorphism generators.

Diff symmetry may be broken by introducing an operator which transforms nontrivially under time reparametrizations: A universal clock

Without loss of generality this universal clock may be taken to be a scalar field π (which as Alex stressed yesterday is likely not fundamental), and so there are now 3 degrees of freedom, in other words we are necessarily considering a scalar/tensor theory of gravity

Note that there exists a unitary gauge $\pi = t$ in which the scalar is eaten by the graviton, and so this is a theory of a graviton with 3 polarizations

IR modification of gravity?

At small distances, away from the big bang, FRW looks like Minkowski space. Therefore we must demand that further in the UV this scalar-tensor theory is consistent with the usual measurements of general relativity. The scalar must decouple.

This decoupling may happen already at the level of linear perturbations, or else due to the nonlinear backreaction of the spacetime (Vainshtein effect) as in the Pauli-Fierz theory.

In the case of the Galileon theory to be discussed below, the Vainshtein effect has not been demonstrated, it has only been shown that there is some similar phenomenology. If it fails, then the Galileon is NOT an IR modification of gravity.

UV Complete or Effective theory?

Einstein gravity cannot be UV complete, as it is nonrenormalizable.

As Damiano said this morning, Lorentz-invariant modifications are known which destroy unitarity but lead to a renormalizable theory (Stelle, 1977). These terms have projections which are not Lorentz-invariant but do not violate unitarity and are likely to preserve renormalizability.

Therefore this scalar-tensor theory may be UV complete or just a low energy effective theory.

Clearly it is important to determine which is the case, and in the latter to determine the range of validity of the effective theory

Summary of Criteria for Consistency

We have argued that ANY IR modification of gravity which violates time diffeomorphisms must:

(1) Contain a graviton and a scalar field π whose VEV acts as a universal clock.

(2) At higher energies than the breaking scale, the scalar must decouple either linearly or via the Vainshtein mechanism.

(3) Break time-diffeomorphism invariance not spontaneously, but already in the initial conditions.

Additional Criterion for Consistency

In addition locality and consistency of the quantum theory demand that once local constraints are imposed on an initial Cauchy surface, all future constraints are automatically solved and so:

(4) There must be local criteria which, when imposed on an Cauchy initial surface, guarantee that closed timelike curves (CTCs) will not form.

Superluminal propagation in itself does not imply an inconsistency, but in many cases implies that there are configurations in which one may create CTCs.

Candidate IR modifications of gravity

Some examples of candidate models of time-diffeomorphism breaking in gravity:

1) k-essence and braiding models: You heard about these from Alex.

2) Ghost condensate models: This is a general effective field theory of such a scalar. The scalar theory on its own is not Lorentz invariant, as demanded by Hubble friction when placed in an FRW background decoupled to gravity, and as demanded by the spatial diffeomorphism invariance of gravity when they are coupled.

3) Galileon model: Lorentz-invariant model of a scalar with a Galilean symmetry $\pi \to \pi + x \cdot a + b$

These are all models of a cosmological fluid with bizarre hydrodynamical properties.

Part II: The Galileon model

Introduced by Nicolis, Rattazzi and Trincherini in 2008

It is defined to be the most general Lorentz-invariant, local model of a scalar field π whose classical equation of motion:

(1) Possesses Galilean symmetry $\pi \to \pi + x \cdot a + b$
(2) Is a second order PDE for π

This second requirement is necessary to avoid the presence of ghosts in arbitrary configurations.
The Galilean-invariance guarantees that only Galilean-invariant terms will be generated by RG flow.
However coupling to gravity cannot be achieved while preserving both conditions (1) and (2)
(Demonstrated by Deffayet, Esposito-Farese and Vikman, 2009)

The Galileon action and equations of motion

The Lagrangian density is an arbitrary linear combination of 5 terms which satisfy the two criteria on the previous transparency:

$$\mathcal{L} = \sum_{i=1}^{5} c_i \mathcal{L}_i$$

$$\begin{aligned} \mathcal{L}_1 &= \pi \\ \mathcal{L}_2 &= -\frac{1}{2}\partial\pi \cdot \partial\pi \\ \mathcal{L}_3 &= -\frac{1}{2}(\partial \cdot \partial\pi)\partial\pi \cdot \partial\pi \end{aligned}$$

The expressions for $\mathcal{L}_4$ and $\mathcal{L}_5$ are not illuminating, but are respectively contractions of 2 and 3 factors of $\partial\partial\pi$ with one of $\partial\pi \cdot \partial\pi$.

The terms $\mathcal{L}_2$ and $\mathcal{L}_3$ alone reproduce a UV truncation of the 5-dimensional DGP model

Superluminal propagation in the Galilean theory

One year after creating the model, Nicolis, Rattazzi and Trincherini demonstrated that superluminal propagation is a generic feature of nontrivial Galilean solutions.

Decompose the Galileon field $\pi = \pi_0 + \delta\pi$. Expanding the Lagrangian to second order they find

$$\delta\mathcal{L} = -\frac{1}{2} G^{\mu\nu} \partial_\mu \delta\pi \partial_\nu \delta\pi$$

G is the inverse effective metric describing π propagation.

Superluminal propagation in the Galilean theory II

Consider only the terms $\mathcal{L}_2$ and $\mathcal{L}_3$ in the Lagrangian, then the inverse metric is

$$G^{\mu\nu} = (1 + 2c_3\partial\partial\pi_0)\eta^{\mu\nu} - c_3\partial^\mu\partial^\nu\pi_0$$

If for simplicity π is harmonic and time-independent then (rescaling away c_3)

$$G^{\mu\nu} = \eta^{\mu\nu} - \partial^\mu\partial^\nu\pi_0$$

The spatial Hessian is traceless and so generically has a negative eigenvalue. In the corresponding direction the inverse metric is increased and so the metric component is decreased. There will be superluminal propagation in that direction.
Superluminality is generic when the nonlinear terms (π dependence of the metric) are considered.

Chronology Protection?

If the Galilean theory is only a low energy effective theory, and if the UV theory is causal, then Hawking's chronology protection implies that the low energy effective theory breaks down whenever causality is violated by the Galilean theory.

Invoking chronology protection whenever there is superluminal propagation, they conclude that the nonlinear terms in the Lagrangian are not reliable. Removing the nonlinearities, the theory becomes free and so there are higher derivative terms in the equations of motion and ghosts.

Therefore the Galileon model does not possess a UV completion which is free of superluminal propagation.

What then?

There are now two possibilities:

1. The Galileon model is UV complete

2. The Galileon model is a low energy effective description of a UV theory with superluminal propagation, for example it may be that the microscopic theory is not Lorentz-invariant

In this last part of the talk, we will argue that the first possibility is likely to be inconsistent by constructing an otherwise healthy configuration with CTCs

A new class of Galileon solutions

In the absence of the tadpole $\mathcal{L}_1$, for an arbitrary function f

$$\pi_0 = f(x+t)$$

is a solution to the full nonlinear Galileon equations of motion, corresponding to an arbitrary left-moving wave-form.

Absorbing c_3 into π_0 and setting $c_4 = c_5 = 0$ and $h = f^{''}$ for simplicity the effective (x,t) metric for Galileon fluctuations is

$$G_{\mu\nu} = \begin{pmatrix} 1+h & -h \\ -h & -1+h \end{pmatrix}$$

The causal structure of these solutions

The metric on the previous transparency has two nullvectors in the (x, t) plane:

$$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1-h \\ -h-1 \end{pmatrix}$$

Conclusions:

1) Right-moving modes are unaffected by the left-moving background

2) $h > 0$: Left-moving modes are superluminal

3) $h > 1$: Left-moving modes travel backwards in time, however precisely in this case $|\mathcal{L}_3| > |\mathcal{L}_2|$ are so the derivative expansion cannot be trusted IF the Galileon model is considered to be a low energy effective theory.

Can you use these solutions to make CTCs?

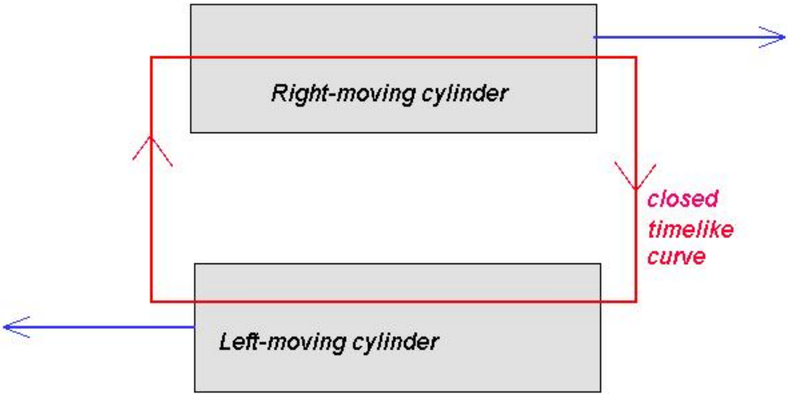
Yes! The key observations are:

(1) The right-moving modes still move at the speed of light in the presence of any left-moving solution

(2) $x \leftrightarrow -x$ produces a right-moving solution in which left-moving modes continue to move at the speed of light.

Consider a left-moving and a right-moving cylinder extended in the x direction, centered at $(y = \pm a, z = 0)$ respectively. In each cylinder, $h > 1$, outside $h = 0$. Then a closed timelike curve exists which moves left through the first cylinder and right through the second while remaining at an approximately constant time.

A CTC in a background with two cylinders



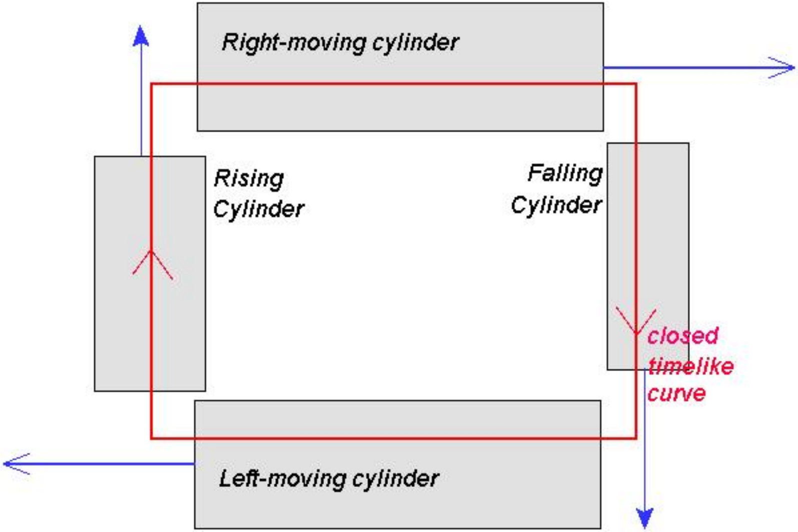
Are the derivatives out of control?

The trouble is that the derivatives of π become large as the curve moves between the two cylinders.

This can be resolved if one also considers an up-moving and down-moving cylinder extended in the y direction, centered at $(x = \pm a, z = 0)$ with again $\partial_y^2 f > 1$ inside. The configuration is symmetric under 90 degree rotations.

A square which threads all of the cylinders, always moving in the same direction, is a potential CTC. Recall, this is work in progress and several checks must be performed. However, evolving each cylinder backwards in time naively it becomes diffuse, and so the initial conditions which lead to such a configuration seem to be difficult to eliminate using any local selection criterion.

A CTC in a background with four cylinders



This strongly suggests that the Galileon theory cannot be quantized, and at best may be a low energy effective description of unknown microscopic physics.