Vacuum Cherenkov radiation in Lorentz violating quantum electrodynamics and experimental limits on the scale of Lorentz violation

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Lorentz symmetry is a basic ingredient of the Standard Model of particle physics.

The parameters of the Lorentz violating extension of the Standard Model have been measured with great precision, particularly in the power-counting renormalizable sector

It turns out that Lorentz symmetry is a very precise symmetry of Nature, at least in low-energy domain.

Several (dimensionless) parameters have bounds

$$10^{-13} \; - \; 10^{-32}$$

V.A. Kostelecký and N. Russell, *Data tables for Lorentz and CTP violation*, arXiv:0801.0287

However, several authors have argued that at high energies Lorentz symmetry and possibly CPT could be broken.

If so, its violation may be associated with a specific energy scale

For example, if we consider CPT-invariant QED in the power-counting renormalizable sector,

$$\mathcal{L}_{\rm LV} = -\frac{1}{4}(k_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} + \frac{1}{2}\bar{\psi}\left(ic^{\mu\nu}\gamma_\mu \overleftrightarrow{D}_\nu + id^{\mu\nu}\gamma_5\gamma_\mu \overleftrightarrow{D}_\nu - H_{\mu\nu}\sigma^{\mu\nu}\right)\psi$$

the present bounds read

$c_{XX}+c_{YY}-2c_{ZZ}$	$(-106\pm147)\times10^{-16}$
λ^{ZZ}	$(13.3\pm9.8)\times10^{-16}$
$c_{(YZ)}$	$(2.1\pm4.6)\times10^{-16}$
$c_{(XZ)}$	$(-1.6\pm6.3)\times10^{-16}$
$c_{(XY)}$	$(7.6\pm3.5)\times10^{-16}$
$c_{XX}-c_{YY}$	$(1.15\pm0.64)\times10^{-15}$

$\lvert d_{XX}\rvert$	$<2\times10^{-14}$
$\lvert d_{YY}\rvert, \lvert d_{ZZ}\rvert$	$<3\times10^{-15}$
$\lvert d_{(XY)}\rvert$	$<2\times10^{-15}$
$\lvert d_{(XZ)}\rvert$	$<2\times10^{-14}$
$\lvert d_{(YZ)}\rvert$	$<7\times10^{-15}$
$\lvert d_{TX}\rvert$	$<5\times10^{-14}$
$\lvert d_{TY}\rvert$	$<5\times10^{-15}$
$\lvert d_{TZ}\rvert$	$<8\times10^{-17}$

$(\tilde{\kappa}_{e+})^{XY}$	10^{-32}
$(\tilde{\kappa}_{e+})^{XZ}$	10^{-32}
$(\tilde{\kappa}_{e+})^{YZ}$	10^{-32}
$(\tilde{\kappa}_{e+})^{XX}-(\tilde{\kappa}_{e+})^{YY}$	10^{-32}
$(\tilde{\kappa}_{e+})^{ZZ}$	10^{-32}
$(\tilde{\kappa}_{o-})^{XY}$	10^{-32}
$(\tilde{\kappa}_{o-})^{XZ}$	10^{-32}
$(\tilde{\kappa}_{o-})^{YZ}$	10^{-32}
$(\tilde{\kappa}_{o-})^{XX}-(\tilde{\kappa}_{o-})^{YY}$	10^{-32}
$(\tilde{\kappa}_{o-})^{ZZ}$	10^{-32}
$(\tilde{\kappa}_{e-})^{XY}$	10^{-16}
$(\tilde{\kappa}_{e-})^{XZ}$	10^{-16}
$(\tilde{\kappa}_{e-})^{YZ}$	10^{-16}
$(\tilde{\kappa}_{e-})^{XX}-(\tilde{\kappa}_{e-})^{YY}$	10^{-17}
$(\tilde{\kappa}_{e-})^{ZZ}$	10^{-17}
$(\tilde{\kappa}_{o+})^{XY}$	10^{-13}
$(\tilde{\kappa}_{o+})^{XZ}$	10^{-13}
$(\tilde{\kappa}_{o+})^{YZ}$	10^{-13}
$\tilde{\kappa}_{\rm tr}$	10^{-14}

V.A. Kostelecký and N. Russell, *Data tables for Lorentz and CTP violation*, arXiv:0801.0287

Recent analysis of gamma-ray bursts suggests a bound on the

scale $\bar{M}$ of Lorentz violation in the dispersion relation $c(E) \sim c\left(1 - \frac{E}{\bar{M}}\right)$

and found $\bar{M} \geqslant 1.3 \cdot 10^{18}\,\mathrm{GeV}$

The Fermi LAT and Fermi GBM Collaborations, Fermi observations of High-energy gamma-ray emission from GRB 080916C, Science 323 (1009) 1688 and DOI: 10.1126/science.1169101.

Since in local QFT odd powers of the energy mean that CPT is violated also, we can take this bound to be a bound on the scale Λ_{CPT} of CPT violation.

However, if you report the bound above to the case of a quadratic dependence $c(E) = c\left(1 \pm \frac{E^2}{\bar{M}^2}\right)$

you just find $\bar{M} \geqslant 5 \cdot 10^{9}\,\mathrm{GeV}$

L. Shao, Z. Xiao, B. -Q. Ma, Astropart. Phys. **33**, 312-315 (2010). [arXiv:0911.2276 [hep-ph]]

Yesterday's talks by Wu XueFeng and Ma BoQiang

On the other hand, Lorentz violation does not necessarily imply CPT violation, so we may assume that there exist two scales, one scale Λ_L for the Lorentz violation, and one scale Λ_{CPT} for the CPT violation, with

$$\Lambda_{\mathrm{CPT}} \geqslant \Lambda_L$$

For the same reason, we may assume that there exists an energy range

$$\Lambda_L \leqslant E \leqslant \Lambda_{\mathrm{CPT}}$$

that is well described by a Lorentz violating, but CPT invariant quantum field theory.

I will try to convince you that

$$\Lambda_L \sim 10^{14}\mathrm{GeV}$$

which will force us to think about quantum gravity anew.

The best existing bounds on operators of higher dimensions come from the photon sector:

$$\frac{1}{\Lambda_L^2} F\partial^2 F \qquad\qquad \frac{1}{\Lambda_L^4} F\partial^4 F$$

$$\lesssim 10^{-29}\mathrm{GeV}^{-2} \qquad\qquad \lesssim 10^{-23}\mathrm{GeV}^{-4}$$

V.A. Kostelecký and N. Russell, *Data tables for Lorentz and CTP violation*, arXiv:0801.0287
V.A. Kostelecký and M. Mewes, Electrodynamics with Lorentz-violating operators of arbitrary dimension, Phys. Rev. D 80 (2009) 015020 and arXiv:0905.0031 [hep-ph].

They are compatible with our claim $\Lambda_L \sim 10^{14}\text{-}10^{15}\mathrm{GeV}$

Much stronger bounds exist on CPT violating terms, for example

$$\frac{1}{\Lambda_{CPT}} F\partial F \qquad\qquad \lesssim \quad 10^{-33}\mathrm{GeV}$$

It has been claimed that ultrahigh-energy cosmic rays push Λ_L above the Planck scale also

O. Gagnon and G. Moore, Limits on Lorentz violation from highest energy cosmic rays, Phys. Rev. D 70 (2004) 065002 and arXiv:hep-ph/0404196.

I claim that this piece of evidence is too weak and actually the scale of Lorentz violation with preserved CPT can be as small as $\Lambda_L \sim 10^{14}\text{-}10^{15}\mathrm{GeV}$

Why is it interesting to consider quantum field theories where Lorentz symmetry Is explicitly broken?

The set of power-counting renormalizable theories is considerably “small”

Relaxing some assumptions can enlarge it, but often it enlarges it too much

Without locality in principle every theory can be made finite

Without unitarity even gravity can be renormalized

Relaxing Lorentz invariance appears to be interesting in its own right

Here we are interested in the renormalization of Lorentz violating theories obtained improving the behavior of propagators with the help of higher space derivatives and study under which conditions no higher time derivatives are turned onto be consistent with unitarity.

For this you need to break Lorentz symmetry explicitly.

You can do it , but only in flat space, because with gravity you would break a gauge symmetry explicitly!

The approach is based of a modified criterion of power counting, dubbed weighted power counting (2007)

D.A. and M. Halat, *Renormalization of Lorentz violating theories*, Phys. Rev. D 76 (2007) 125011 and arxiv:0707.2480 [hep-th]

Consider the free theory

$$\mathcal{L} = \frac{1}{2}(\hat{\partial}\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{\tau_{n-1}}{2}(\overline{\partial}\varphi)^2 \cdots - \frac{\tau_0}{2\Lambda_L^{2n-2}}(\overline{\partial}^n\varphi)^2$$

Its propagator is

$$\frac{1}{E^2 - m^2 - \tau_{n-1}\bar{p}^2 - \tau_{n-2}\frac{(\bar{p}^2)^2}{\Lambda_L^2} \cdots - \tau_0\frac{(\bar{p}^2)^n}{\Lambda_L^{2n-2}}}$$

and the dispersion relation reads

$$E = \sqrt{m^2 + \tau_{n-1}\bar{p}^2 + \tau_{n-2}\frac{(\bar{p}^2)^2}{\Lambda_L^2} \cdots + \tau_0\frac{(\bar{p}^2)^n}{\Lambda_L^{2n-2}}}$$

The improved ultraviolet behavior allows us to renormalize otherwise non-renormalizable vertices. They can be classified using a weighted power counting

D.A. and M. Halat, *Renormalization of Lorentz violating theories*, Phys. Rev. D 76 (2007) 125011 and arxiv:0707.2480 [hep-th]

High-energy-Lorentz-violating Standard Model (2008)

Consider the vertex

$$\mathcal{L}_{LH} = \frac{\bar{g}^2}{4\Lambda_L}(LH)^2$$

that gives Majorana masses to left-handed neutrinos after symmetry breaking,

and the four fermion interactions

$$\mathcal{L}_{4f} \sim \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi$$

that can describe proton decay.

Such vertices are renormalizable by weighted power counting

Matching the vertex $(LH)^2$ with estimates of the electron neutrino Majorana

mass the scale of Lorentz violation has roughly the value

$$\Lambda_L \sim 10^{14}\mathrm{GeV}$$

D.A., *Weighted power counting, neutrino masses and Lorentz violating extensions of the Standard Model*, Phys. Rev. D 79 (2009) 025017 and arXiv:0808.3475 [hep-ph]

The (simplified) model reads

$$\mathcal{L}' = \mathcal{L}'_Q + \mathcal{L}_{\mathrm{kin}f} + \mathcal{L}'_H + \mathcal{L}_Y - \frac{\bar{g}^2}{4\Lambda_L}(LH)^2 - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g\bar{D}\bar{F}\left(\bar{\chi}_I \bar{\gamma}\chi_I\right) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3$$

$$-\frac{1}{\Lambda_L^2} g\bar{g}\bar{\psi}\psi\bar{F}H - \frac{1}{\Lambda_L^2}\left(\bar{g}^3\bar{\psi}\psi H^3 + \bar{g}^2\bar{\psi}\bar{D}\psi H^2 + \bar{g}\bar{\psi}\bar{D}^2\psi H\right) - \frac{1}{\Lambda_L^4}\left(g\bar{D}^2\bar{F} + g^2\bar{F}^2\right) H^\dagger H$$

where
$$\mathcal{L}'_Q = \frac{1}{4}\sum_G \left(2F^G_{\hat{\mu}\hat{\nu}}F^G_{\hat{\mu}\hat{\nu}} - F^G_{\bar{\mu}\bar{\nu}}\tau'^G(\bar{\Upsilon})F^G_{\bar{\mu}\bar{\nu}}\right),$$

$$\mathcal{L}'_H = \mathcal{L}_H - \frac{\lambda_4^{(3)}\bar{g}^2}{4\Lambda_L^2}|H|^2|\bar{D}_{\bar{\mu}}H|^2 - \frac{\lambda_4^{(2)}\bar{g}^2}{4\Lambda_L^2}|H^\dagger \bar{D}_{\bar{\mu}}H|^2 - \frac{\bar{g}^2}{4\Lambda_L^2}\left[\lambda_4^{(1)}(H^\dagger\bar{D}_{\bar{\mu}}H)^2 + \mathrm{h.c.}\right] - \frac{\lambda_6\bar{g}^4}{36\Lambda_L^2}|H|^6$$

$$\mathcal{L}_H = |\hat{D}_{\hat{\mu}}H|^2 - \frac{a_0}{\Lambda_L^4}|\bar{D}^2\bar{D}_{\bar{\mu}}H|^2 - \frac{a_1}{\Lambda_L^2}|\bar{D}^2 H|^2 - a_2|\bar{D}_{\bar{\mu}}H|^2 - \mu_H^2|H|^2 - \frac{\lambda_4\bar{g}^2}{4}|H|^4,$$

$$\mathcal{L}_{\mathrm{kin}f} = \sum_{a,b=1}^{3}\sum_{I=1}^{5}\bar{\chi}_I^a\, i\left(\delta^{ab}\hat{\not{D}} - \frac{b_0^{Iab}}{\Lambda_L^2}\bar{\not{D}}^3 + b_1^{Iab}\bar{\not{D}}\right)\chi_I^b,$$

$$\mathcal{L}_Y = -\bar{g}\Omega_i H^i + \mathrm{h.c.}, \qquad \Omega_i = \sum_{a,b=1}^{3} Y_1^{ab}\bar{L}^{ai}\ell_R^b + Y_2^{ab}\bar{u}_R^a Q_L^{bj}\varepsilon^{ji} + Y_3^{ab}\bar{Q}_L^{ai}d_R^b,$$

At low energies we have the Colladay-Kostelecky Standard-Model Extension

It can be shown that the gauge anomalies vanish, since they coincide with those of the Standard Model

Scalarless high-energy-Lorentz-violating Standard Model (2009)

$$\mathcal{L}_{\mathrm{noH}} = \mathcal{L}_F + \mathcal{L}_{\mathrm{kinf}} - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g\bar{D}\bar{F}\left(\bar{\chi}_I \bar{\gamma} \chi_I\right) + \frac{Y_f}{\Lambda_L^2}\bar{\chi}\chi\bar{\chi}\chi - \frac{g}{\Lambda_L^2}\bar{F}^3$$

$$\mathcal{L}_F = \frac{1}{4}\sum_G \left(2F_{0i}^G F_{0i}^G - F_{ij}^G \tau^G(\bar{\Upsilon}) F_{ij}^G\right)$$

$$\mathcal{L}_{\mathrm{kinf}} = \sum_{a,b=1}^{3}\sum_{I=1}^{5} \bar{\chi}_I^a\, i\left(\delta^{ab}\gamma^0 D_0 - \frac{b_0^{Iab}}{\Lambda_L^2}\bar{\not{D}}^3 + b_1^{Iab}\bar{\not{D}}\right)\chi_I^b$$

The model contains four fermion interactions at the fundamental level. It is possible to describe the known low-energy physics in the Nambu—Jona-Lasinio spirit, which gives masses to fermions and gauge bosons dynamically.

The Higgs field is a composite field and arises as a low-energy effect. Actually, in general more than one doublet appears at low energies.

D.A., *Standard Model Without Elementary Scalars And High Energy Lorentz Violation,* Eur. Phys. J. C 65 (2010) 523 and arXiv:0904.1849 [hep-ph]

D.A. and E. Ciuffoli, *Low-energy phenomenology of scalarless Standard Model extensions with high-energy Lorentz violation*, arXiv:1101.2014 [hep-ph]

High-energy Lorentz violating QED

$$\mathcal{L} = \frac{1}{2}F_{0i}^2 - \frac{1}{4}F_{ij}\left(\tau_2 - \tau_1\frac{\bar{\partial}^2}{\Lambda_L^2} + \tau_0\frac{(-\bar{\partial}^2)^2}{\Lambda_L^4}\right)F_{ij}$$
$$+\bar{\psi}\left(i\gamma^0 D_0 + \frac{ib_0}{\Lambda_L^2}\bar{\not{D}}^3 + ib_1\bar{\not{D}} - m - \frac{b'}{\Lambda_L}\bar{\not{D}}^2\right)\psi$$
$$+\frac{e}{\Lambda_L}\bar{\psi}\left(b''\sigma_{ij}F_{ij} + \frac{b_0'}{\Lambda_L}\gamma_i\partial_j F_{ij}\right)\psi + ie\frac{b_0''}{\Lambda_L^2}F_{ij}\left(\bar{\psi}\gamma_i\frac{\overleftrightarrow{\bar{D}}_j}{2}\psi\right)$$

Gauge symmetry is unmodified

D.A. and M. Taiuti, *Renormalization of high-energy Lorentz violating QED*, Phys. Rev. D 81 (2010) 085042 and arXiv:0912.0113 [hep-ph]

Scale of Lorentz violation

(with preserved CPT)

D.A. and M. Taiuti, *Vacuum Cherenkov radiation in quantum electrodynamics with high-energy Lorentz violation*, arXiv:1101.2019 [hep-ph]

When Lorentz symmetry is violated several otherwise forbidden phenomena are allowed to take place. Examples are the Cherenkov radiation in vacuo and the pair production from a single photon.

The non-observation of such phenomena can be used to put bounds on the parameters of the Lorentz violation.

Ultrahigh-energy cosmic rays are very energetic particles never observed directly, but only through the showers they produce when they interact with the earth atmosphere.

Their nature is still unclear. Most people assume they are protons, but they could also be heavy nuclei or even neutral particles. Their highest claimed energy is

$$3 \cdot 10^{11} \mathrm{GeV}$$

The naivest estimate tell us that they are incompatible with Lorentz violation unless

$$\Lambda_L \quad \gtrsim \quad 10^{22\text{-}23} \mathrm{GeV}$$

However, this result is based on the loose assumption that all dimensionless parameters we know nothing about are equal to one. Given that already in the Standard Model parameters, for examples masses, differ from one another by several orders of magnitude, this assumption should be reconsidered.

The Lorentz violating Standard Model (LVSM) I have proposed offers a framework where the set of new parametes is large enough to describe the phenomena allowed by Lorentz violation, yet enough restricted to ensure a certain degree of predictivity.

In the LVSM not all parameters are on the same footing. Some of them could even be zero without affecting the internal consistency. Others must absolutely be nonzero, otherwise the model is not renormalizable. We call these parameters crucial. They are the coefficients of the quadratic terms of largest dimensions, e.g.

$$-\frac{\tau_0}{4\Lambda_L^4}F_{ij}(-\bar{\partial}^2)^2F_{ij}, \qquad \frac{ib_0}{\Lambda_L^2}\bar{\psi}\bar{\not{D}}^3\psi,$$

Every term of higher dimension

$$\frac{1}{\Lambda_{iL}^{d_i-4}}\mathcal{O}^i$$

can be used to define a scale of Lorentz violation Λ_{iL} , normalizing its dimensionless coefficient to one. So the question is: which is the scale of Lorentz violation?

A possible answer is: the scale of Lorentz violation is the smallest Λ_{iL}

namely the scale at which Lorentz violation may first manifest itself.

Among the two crucial terms

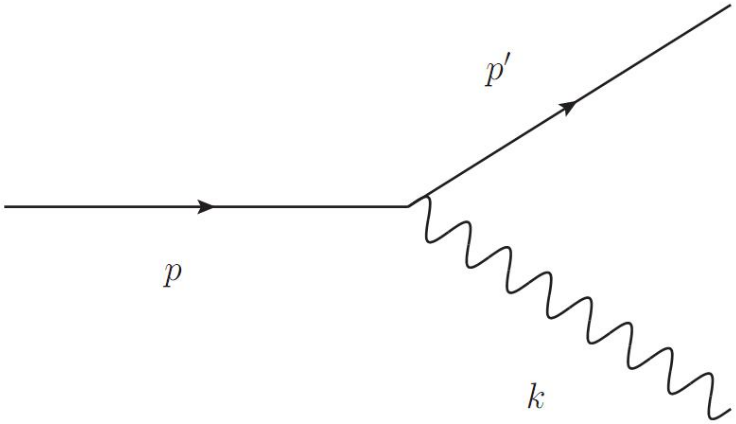
$$-\frac{\tau_0}{4\Lambda_L^4}F_{ij}(-\bar{\partial}^2)^2F_{ij}, \qquad \frac{ib_0}{\Lambda_L^2}\bar{\psi}\bar{\not{D}}^3\psi,$$

we may expect that it is the photon term, which is much better tested so far.

Thus, we normalize τ_0 to one. Then, however, we have no reason to assume that b_0 is also one. Our question is: how small should it be to explain data?

“Reasonably small” or “unreasonably small” ?

General kinematics



$E(p^2)$ = energy of incoming proton

$\omega(k^2)$ = frequency of emitted photon

Assumptions:

$$E \geqslant 0, \qquad \omega \geqslant 0, \qquad \frac{\mathrm{d}E}{\mathrm{d}p} > 0, \qquad \frac{\mathrm{d}\omega}{\mathrm{d}k} > 0, \qquad \frac{\mathrm{d}^2E}{\mathrm{d}p^2} > 0, \qquad \frac{\mathrm{d}^2\omega}{\mathrm{d}k^2} \geqslant 0$$

These assumptions are obeyed by all common relativistic and non-relativistic dispersion relations

We find the following condition for the emission of Cherenkov radiation:

$$\left.\frac{\mathrm{d}\omega}{\mathrm{d}k}\right|_0 < \frac{\mathrm{d}E}{\mathrm{d}p}$$

and the *k*-range

$$0 \leqslant k \leqslant k_{\mathrm{max}}(p)$$

where $k_{\mathrm{max}}(p)$ is obtained from the forward emission.

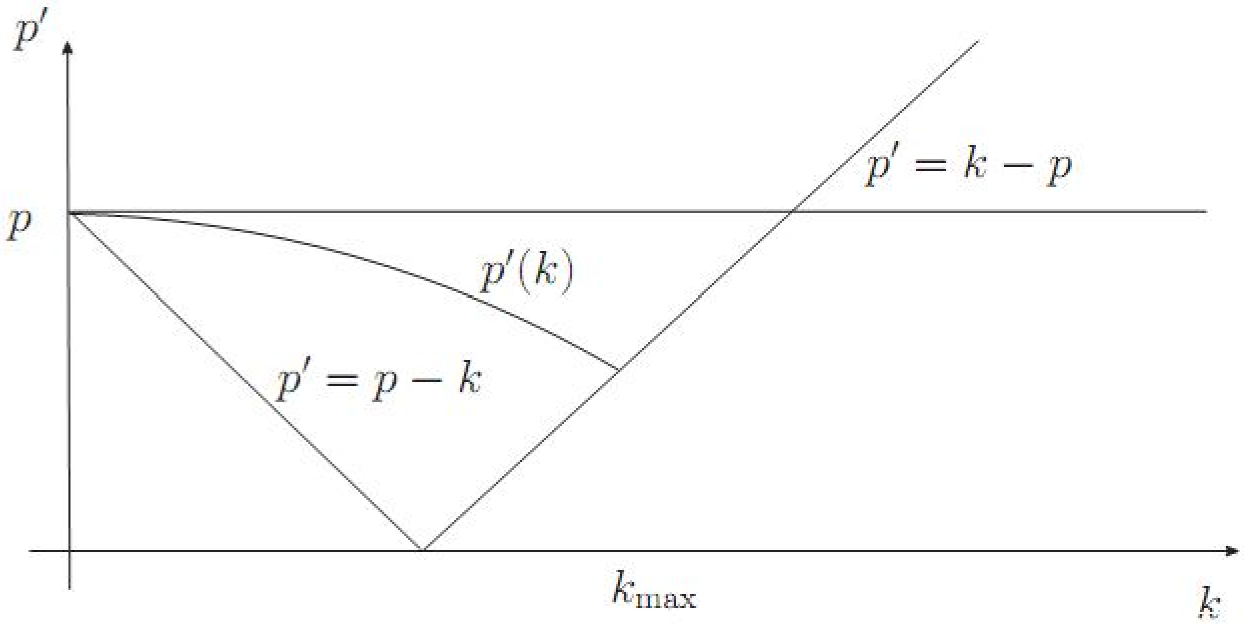
The threshold is proved differentiating the solution $p'(k)$ to the conditions

$$E(p) = \omega(k) + E(p'), \qquad p' = \sqrt{p^2 + k^2 - 2pku}.$$

of energy- and momentum-conservation: the solution must be motonically decreasing and concave

$$\frac{\mathrm{d}p'}{\mathrm{d}k} = -\frac{\mathrm{d}\omega}{\mathrm{d}k}\left(\left.\frac{\mathrm{d}E}{\mathrm{d}p}\right|_{p'}\right)^{-1} < 0, \qquad \frac{\mathrm{d}^2p'}{\mathrm{d}k^2} = -\left[\frac{\mathrm{d}^2\omega}{\mathrm{d}k^2} + \left.\frac{\mathrm{d}^2E}{\mathrm{d}p^2}\right|_{p'}\left(\frac{\mathrm{d}p'}{\mathrm{d}k}\right)^2\right]\left(\left.\frac{\mathrm{d}E}{\mathrm{d}p}\right|_{p'}\right)^{-1} < 0$$

which gives the picture:



$$\left.\frac{\mathrm{d}\omega}{\mathrm{d}k}\right|_0 < \frac{\mathrm{d}E}{\mathrm{d}p}$$

Typical scenario

$$E(p^2) = \sqrt{m^2 + p^2\left(1 + \frac{b_0 p^2}{\Lambda_L^2}\right)^2}, \qquad \omega(k^2) = \sqrt{k^2 + \tau_0 \frac{(k^2)^3}{\Lambda_L^4}}.$$

$$\bar{\psi}\left(i\gamma^0 D_0 + \frac{ib_0}{\Lambda_L^2}\bar{\not{D}}^3 + ib_1\bar{\not{D}} - m - \frac{b'}{\Lambda_L}\bar{\not{D}}^2\right)\psi$$

$$\tau_2 = 1, \qquad \tau_1 = 0, \qquad b_1 = 1, \qquad b' = 0, \qquad b_0 > 0$$

The condition for emission is

$$\xi^2 \equiv \frac{m^2\Lambda_L^2}{6b_0 p^4} < \left(1 + \frac{b_0 p^2}{\Lambda_L^2}\right)^2\left(1 + \frac{3b_0 p^2}{2\Lambda_L^2}\right) \qquad \xi < 1 \qquad E > E_{\rm lim} \sim \frac{m^{1/2}\Lambda_L^{1/2}}{6^{1/4} b_0^{1/4}}$$

The values compatible with the observation of ultrahigh-energy protons are

Λ_L	10^{14}GeV	10^{15}GeV	10^{16}GeV	10^{17}GeV	10^{19}GeV	$2.4 \cdot 10^{23}$GeV
b_0	$1.8 \cdot 10^{-19}$	$1.8 \cdot 10^{-17}$	$1.8 \cdot 10^{-15}$	$1.8 \cdot 10^{-13}$	$1.8 \cdot 10^{-9}$	1

If ultrahigh-energy primaries are iron atoms we have

Λ_L	10^{14}GeV	10^{15}GeV	10^{16}GeV	10^{17}GeV	10^{19}GeV	$4.2 \cdot 10^{21}$GeV
b_0	$5.6 \cdot 10^{-16}$	$5.6 \cdot 10^{-14}$	$5.6 \cdot 10^{-12}$	$5.6 \cdot 10^{-10}$	$5.6 \cdot 10^{-6}$	1

In all cases of interest we have studied the energy loss and verified that it is so fast that in practice it can be assumed to occur instantaneously any time it is allowed by kinematics (the particle looses almost all of its energy in a fraction of a second).

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\int_0^{\omega_{\max}} \omega \frac{\mathrm{d}\Gamma}{\mathrm{d}\omega} \mathrm{d}\omega \qquad \mathrm{d}\Gamma = \frac{1}{2E}\overline{|\mathcal{M}|^2}(2\pi)\delta(E-\omega-E')(2\pi)^3\delta^3(\mathbf{p}-\mathbf{k}-\mathbf{p}')\frac{\mathrm{d}^3\mathbf{k}}{2\omega(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}'}{2E'(2\pi)^3}$$

We find formulas

$$\left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{\xi^2\ll 1} = -\frac{11\alpha p^4 b_0}{12\Lambda_L^2} \qquad \left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{1-\xi^2\ll 1} = -\frac{\alpha p^4\left(1-\xi^2\right)^3}{4\Lambda_L^2} b_0$$

Typical radiation times we find are

$$t'_f \sim 7\cdot 10^{-12}\mathrm{sec} \quad t''_f \sim 8\cdot 10^{-10}\mathrm{sec} \quad \text{If } \Lambda_L = 10^{14}\mathrm{GeV} \text{ and } b_0 = 1.8\cdot 10^{-19}$$

$$t'_f \sim 10^{-29}\mathrm{sec} \quad t''_f \sim 2\cdot 10^{-14}\mathrm{sec} \quad \text{if } b_0 \sim 1 \text{ and } \Lambda_L = 10^{14}\mathrm{GeV}$$

Compositeness

Quite surprisingly, compositeness favors small numbers. For example, assume that a

Composite particle is made of constituents that have dispersion relations

$$E_i = |\mathbf{p}_i| \sqrt{1 + \left(\frac{\eta_i^2 \mathbf{p}_i^2}{\Lambda_L^2} \right)^{n-1}}$$

The velocities are $$\mathbf{v}_i = \frac{\mathrm{d}E_i}{\mathrm{d}\mathbf{p}_i} = \frac{\mathbf{p}_i}{E_i} \left(1 + n \left(\frac{\eta_i^2 p_i^2}{\Lambda_L^2} \right)^{n-1} \right)$$

In the simplest situation, all constituents have the same velocity. Setting $\mathbf{v}_i = \mathbf{v}$

we get

$$v^2 (1 + x_i) = (1 + n x_i)^2 \qquad x_i = \left(\frac{\eta_i^2 p_i^2}{\Lambda_L^2} \right)^{n-1}$$

Then the energy $E = \sum_i E_i$ and momentum $\mathbf{P} = \sum_i \mathbf{p}_i$ of the composite particle are related by

$$E = |\mathbf{P}| \sqrt{1 + \left(\frac{\eta^2 \mathbf{P}^2}{\Lambda_L^2} \right)^{n-1}}$$

with

$$\frac{1}{\eta} = \sum_i \frac{1}{\eta_i} \qquad \eta_i^2 = |b_{0i}|$$

Recall that the dispersion relation of the constituents is

$$E_i = |\mathbf{p}_i| \sqrt{1 + \left(\frac{\eta_i^2 \mathbf{p}_i^2}{\Lambda_L^2} \right)^{n-1}}$$

The result can be immediately generalized to dispersion relations of the form

$$E_i = |\mathbf{p}_i| f(x_i), \qquad x_i = \left(\frac{\eta_i^2 \mathbf{p}_i^2}{\Lambda_L^2} \right)^{n-1}$$

From quarks to protons

$$|b_{0p}| = \left(\frac{2}{|b_{0u}|^{1/2}} + \frac{1}{|b_{0d}|^{1/2}} \right)^{-2}$$

From nucleans to iron atoms

$$b_{0\mathrm{iron}} = \left(\frac{82}{|b_{0u}|^{1/2}} + \frac{86}{|b_{0d}|^{1/2}} \right)^{-2}$$

If ultrahigh-energy cosmic rays are iron atoms they can be explained with

Λ_L	$10^{14}\mathrm{GeV}$	$10^{15}\mathrm{GeV}$	$10^{16}\mathrm{GeV}$	$10^{17}\mathrm{GeV}$	$10^{19}\mathrm{GeV}$	$4.9 \cdot 10^{19}\mathrm{GeV}$
b_{0d}	$4.1 \cdot 10^{-12}$	$4.1 \cdot 10^{-10}$	$4.1 \cdot 10^{-8}$	$4.1 \cdot 10^{-6}$	$.04$	1

In summary, patterns like

$$\tau_0 = 1, \qquad b_{0u} \sim 10^{-6}, \qquad b_{0d} \sim 4 \cdot 10^{-12}, \qquad \Lambda_L \sim 10^{14}\mathrm{GeV},$$
$$\tau_0 = 1, \qquad b_{0u} \sim 10^{-3}, \qquad b_{0d} \sim 4 \cdot 10^{-6}, \qquad \Lambda_L \sim 10^{17}\mathrm{GeV},$$

are compatible with a scale of Lorentz violation well below the Planck scale.

Conclusions

A weighted power-counting criterion can be used to renormalize Lorentz violating theories that contain higher space derivatives, but no higher time derivatives. The theories are unitary, polynomial and causal

Renormalizable theories can contain two scalar-two fermion vertices and four fermion vertices

We can construct an extended Standard Model that gives masses to left neutrinos without introducing right neutrinos or other extra fields. We can also describe proton decay

The scalarless variant of the model is able to reproduce all known low-energy physics without the ambiguities of usual Nambu—Jona-Lasinio non-renormalizable approaches.

It predicts relations among the parameters of the Standard Model and could possibly be disproved by experimental measurements.

The scale of Lorentz violation (with preserved CPT) could be smaller than the Planck scale. From considerations about neutrino masses and bounds on proton decay I have suggested that it could even be as small as 10^{14}GeV. This values is compatible with all present data but some estimates suggested by considerations on ultrahigh-energy cosmic rays.

However, we have shown that in the realm of our models there are reasonable scenarios that are compatible with a scale of Lorentz violation smaller than the Planck scale and the observation of ultrahigh-energy cosmic rays.

This would force us to reconsider quantum gravity from scratch.