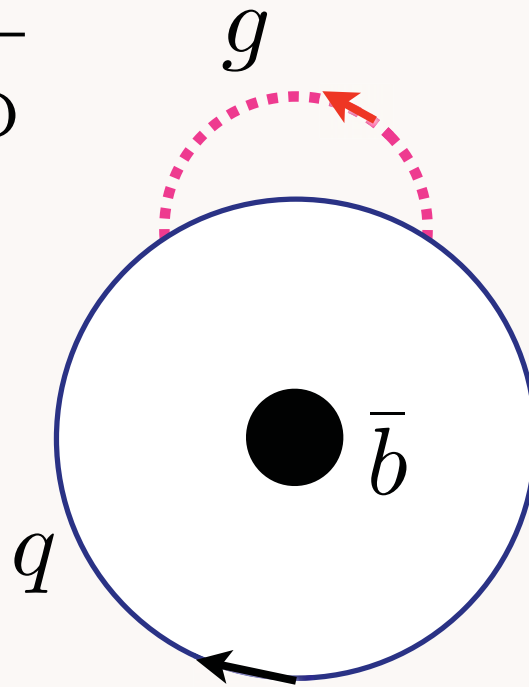


Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

$$k > \frac{1}{\Lambda_{\text{QCD}}}$$

$$\lambda < \Lambda_{\text{QCD}}$$



B-Meson

Shrock, sjb

*gluon and quark propagators cutoff in IR
because of color confinement*

Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain** $(x - y)^2 < \Lambda_{QCD}^{-2}$
- **All perturbative calculations regulated in IR**
- **High momentum calculations unaffected**
- **Bound-state Dyson-Schwinger Equation**
- **Analogous to Bethe's Lamb Shift Calculation**

*Quark and Gluon vacuum polarization insertions
decouple: IR fixed Point*

J. D. Bjorken,
SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

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IR Conformal Window for QCD?

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

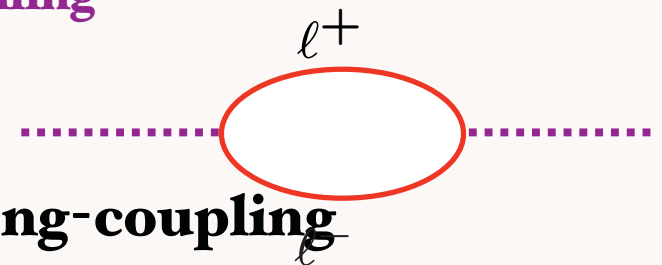
- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

Shrock, de Teramond, sjb

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

The Holographic Correspondence

- In the semi-classical approximation to QCD with massless quarks and no quantum loops the β function is zero and the approximate theory is scale and conformal invariant.
- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

- Semi-classical correspondence as a first approximation to QCD (strongly coupled at all scales).
- $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z .
- Different values of z correspond to different scales at which the hadron is examined: AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
- There is a maximum separation of quarks and a maximum value of z at the IR boundary
- Truncated AdS/CFT model: cut-off at $z_0 = 1/\Lambda_{\text{QCD}}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ usual Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Use mapping of conformal group $SO(4,2)$ to AdS_5
- Scale Transformations represented by wavefunction in 5th dimension
$$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$
- Match solutions at small z to conformal twist dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates “bag” boundary conditions

$$0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$.

- Action for massive scalar modes on AdS_{d+1} :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ -coordinates, $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\left[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \rightarrow z^\Delta$ as $z \rightarrow 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$

Let $\Phi(z) = z^{3/2}\phi(z)$

*AdS Schrodinger Equation for bound state
of two scalar constituents:*

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

**L: light-front orbital angular
momentum**

Derived from variation of Action in AdS_5

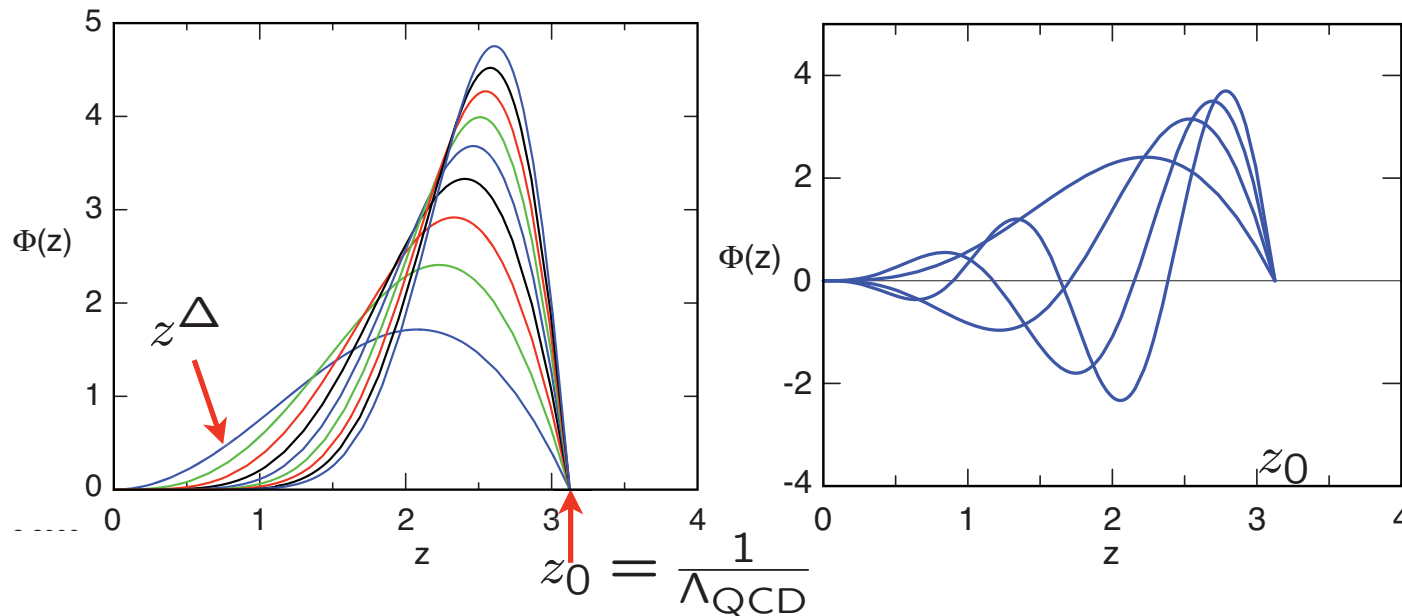
Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

*Match fall-off at small z to conformal twist-dimension,
at short distances*

twist

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$
- 4- d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



$S = 0$ Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

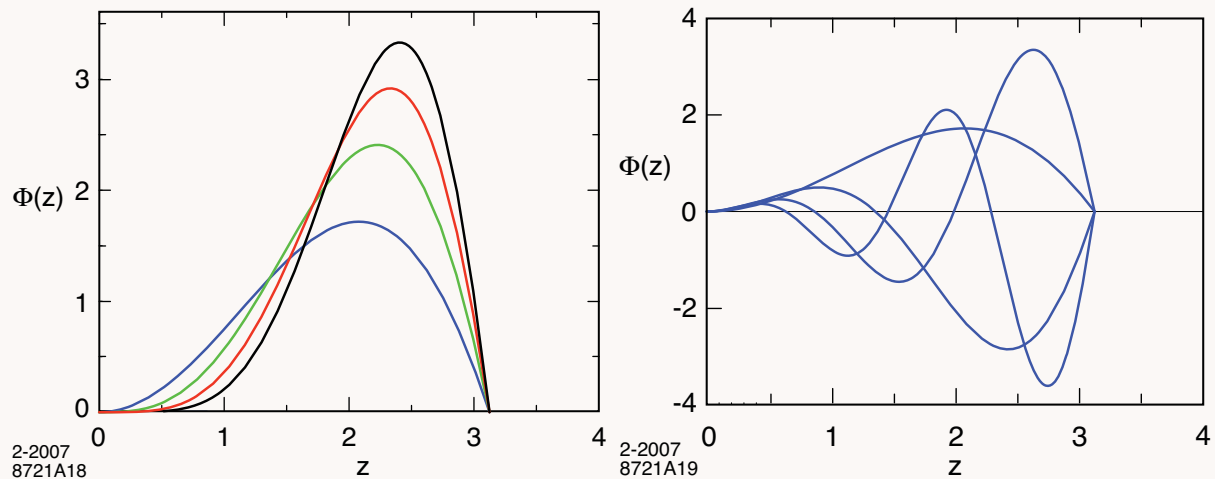


Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$.

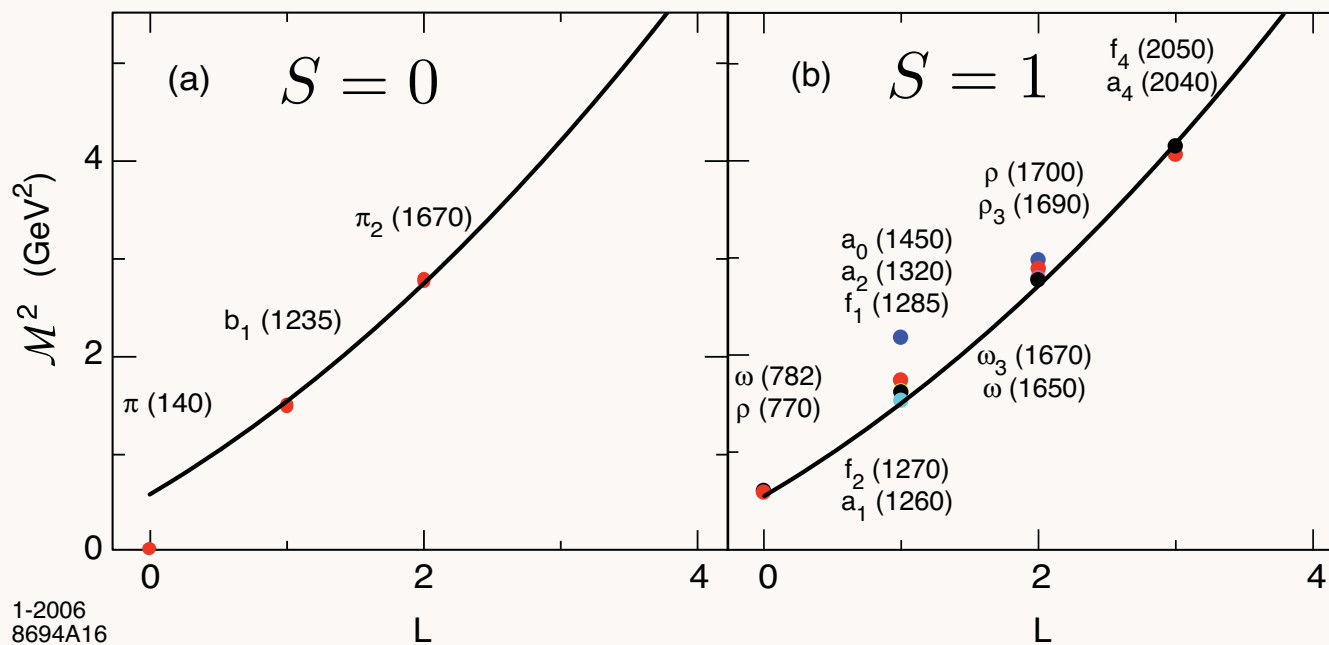


Fig: Light meson and vector meson orbital spectrum $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$

Higher Spin Bosonic Modes HW

- Each hadronic state of integer spin $S \leq 2$ is dual to a normalizable string mode

$$\Phi(x, z)_{\mu_1 \mu_2 \dots \mu_S} = \epsilon_{\mu_1 \mu_2 \dots \mu_S} e^{-iP \cdot x} \Phi_S(z).$$

with four-momentum P_μ and spin polarization indices along the 3+1 physical coordinates.

- Wave equation for spin S -mode **W. S. I'Yi, Phys. Lett. B 448, 218 (1999)**

$$\left[z^2 \partial_z^2 - (d+1-2S)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_S(z) = 0,$$

- Solution

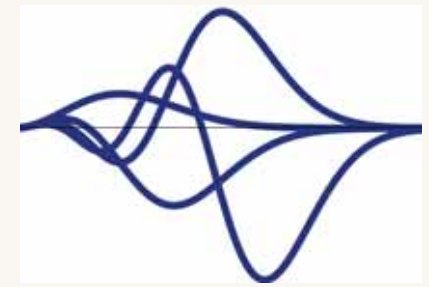
$$\tilde{\Phi}(z)_S = \left(\frac{z}{R} \right)^S \Phi(z)_S = C e^{-iP \cdot x} z^{\frac{d}{2}} J_{\Delta - \frac{d}{2}}(z\mathcal{M}) \epsilon(P)_{\mu_1 \mu_2 \dots \mu_S},$$

- We can identify the conformal dimension:

$$\Delta = \frac{1}{2} (d + \sqrt{(d-2S)^2 + 4\mu^2 R^2}).$$

- Normalization:

$$R^{d-2S-1} \int_0^{\Lambda_{\text{QCD}}^{-1}} \frac{dz}{z^{d-2S-1}} \Phi_S^2(z) = 1.$$



Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce a smooth cutoff which depends on the profile of a dilaton background field $\varphi(z) = \pm \kappa^2 z^2$

$$S = \int d^d x dz \sqrt{g} e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (d - 1 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with $(\mu R)^2 \geq -4$. See also [Metsaev (2002), Andreev (2006)]

+ sign: Fen Zuo

- LH holography requires 'plus dilaton' $\varphi = +\kappa^2 z^2$. Lowest possible state $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$ a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

Massless pion

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

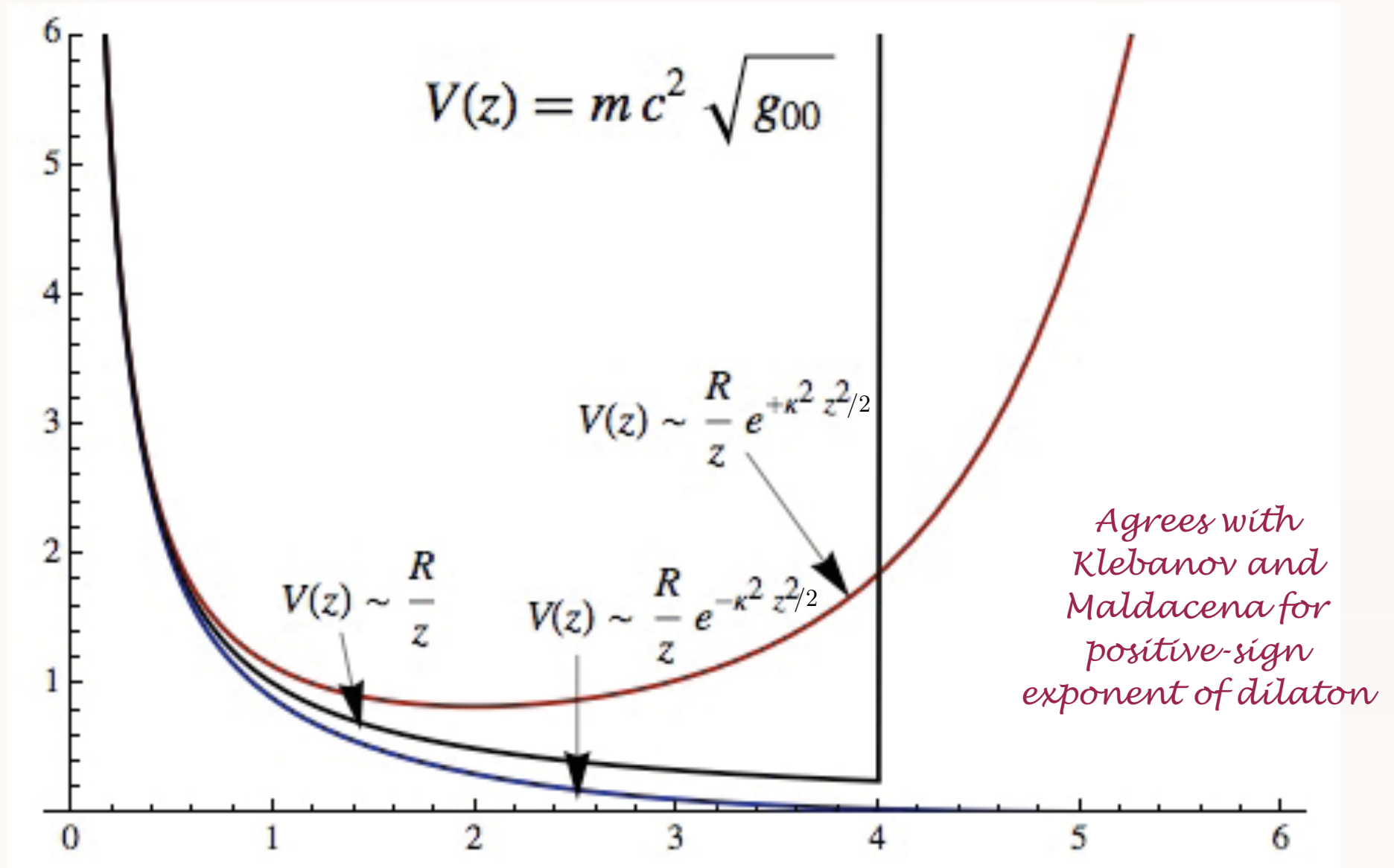
$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

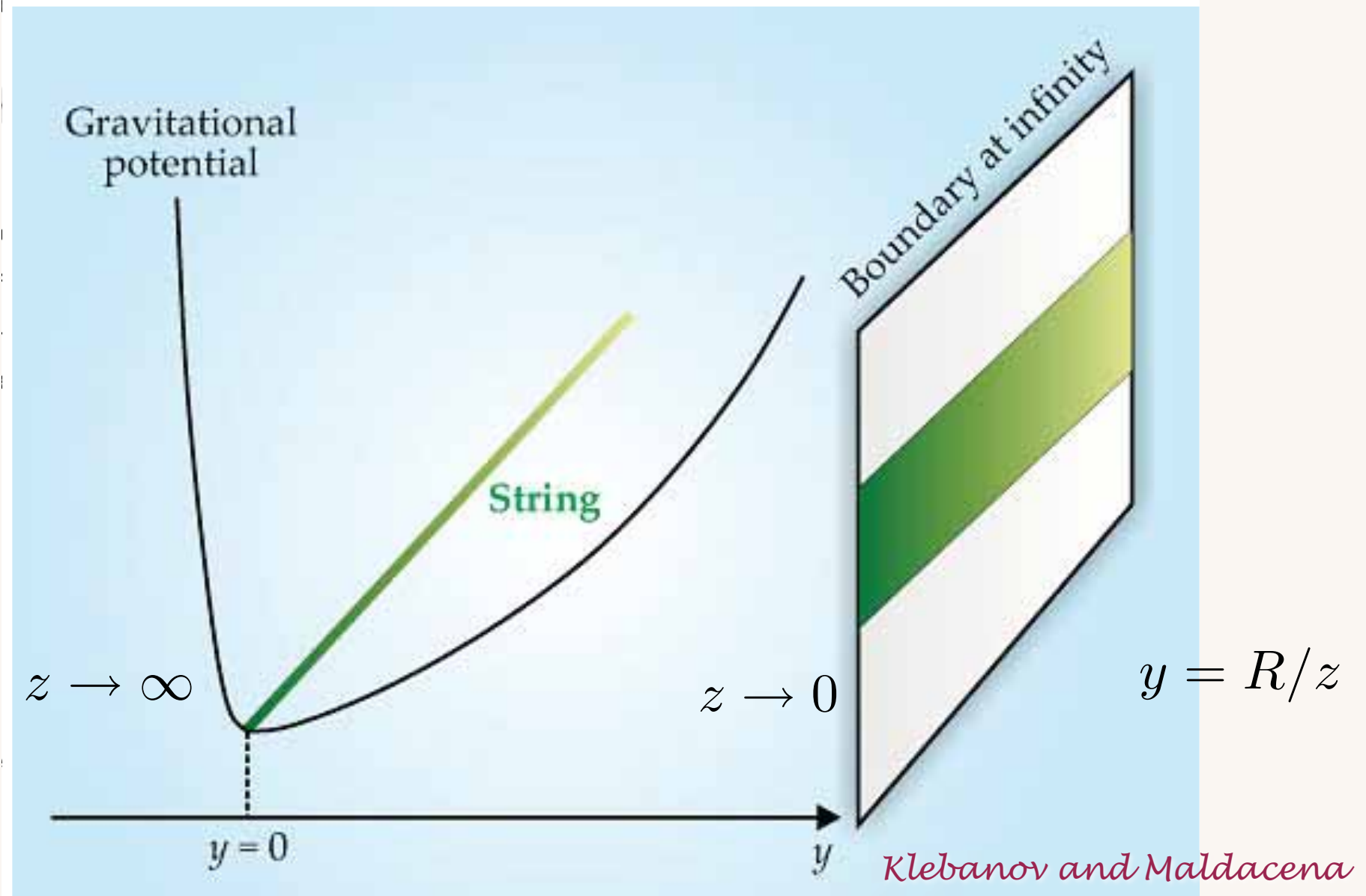
*Derived from variation of Action
Dilaton-Modified AdS₅*

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2)$$



$$ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2)$$



$$ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2$$

Quark separation
increases with L

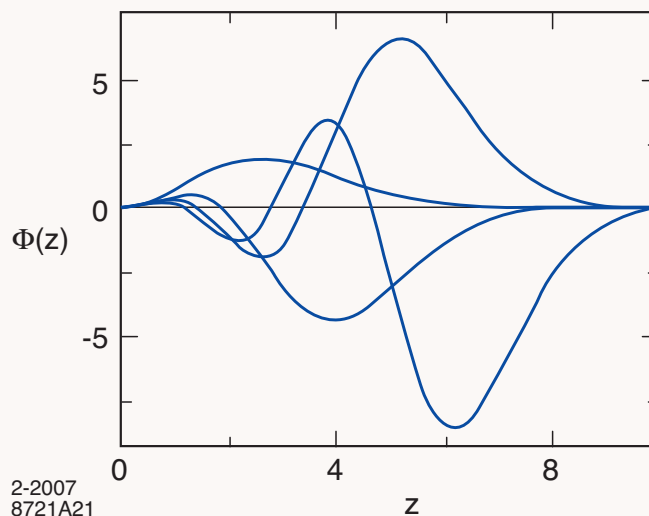
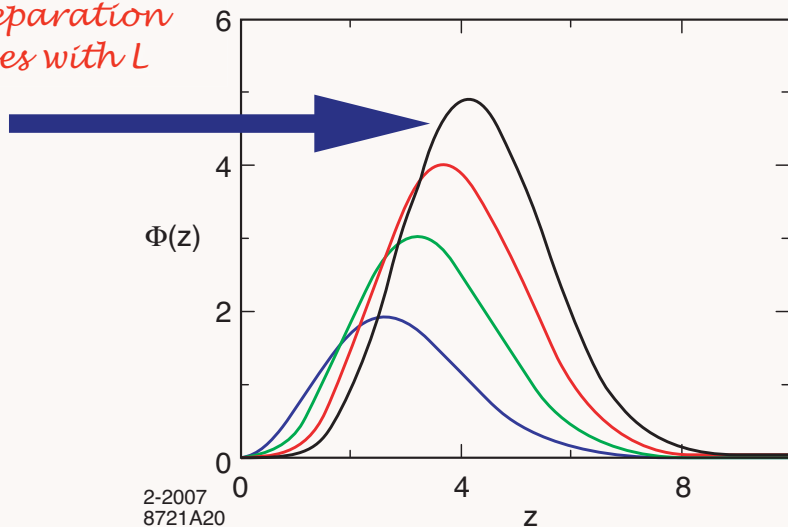
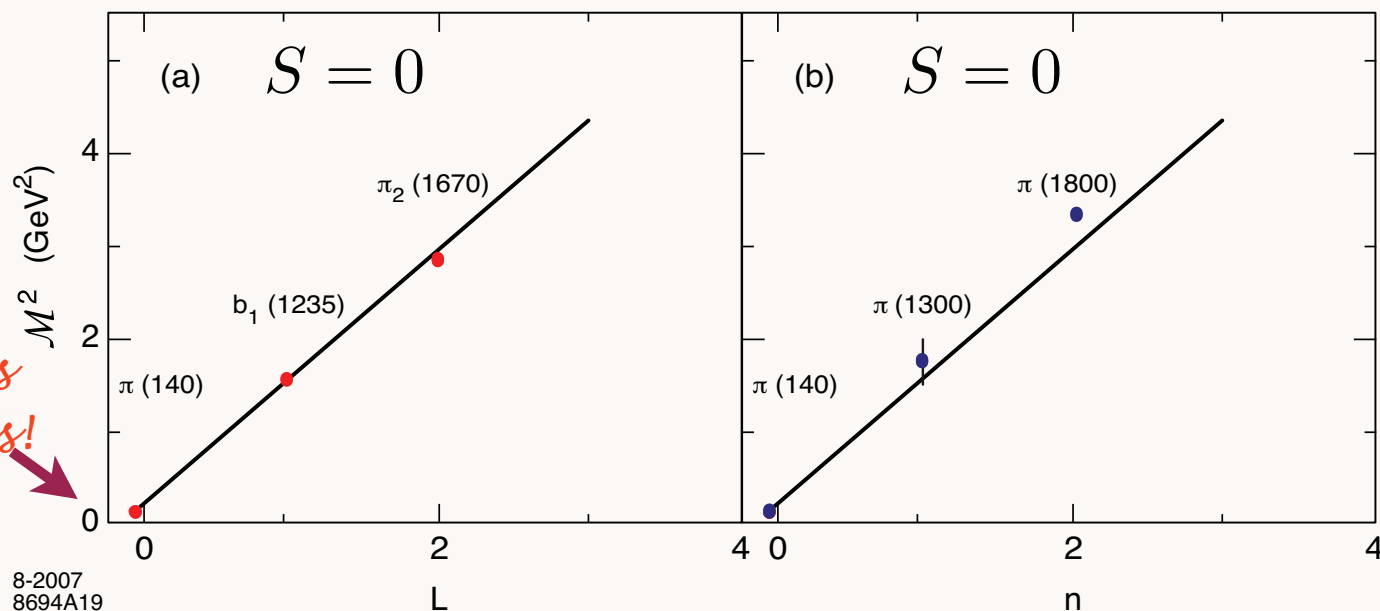


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

*Soft Wall
Model*

**Pion mass
automatically
zero!**

$$m_q = 0$$



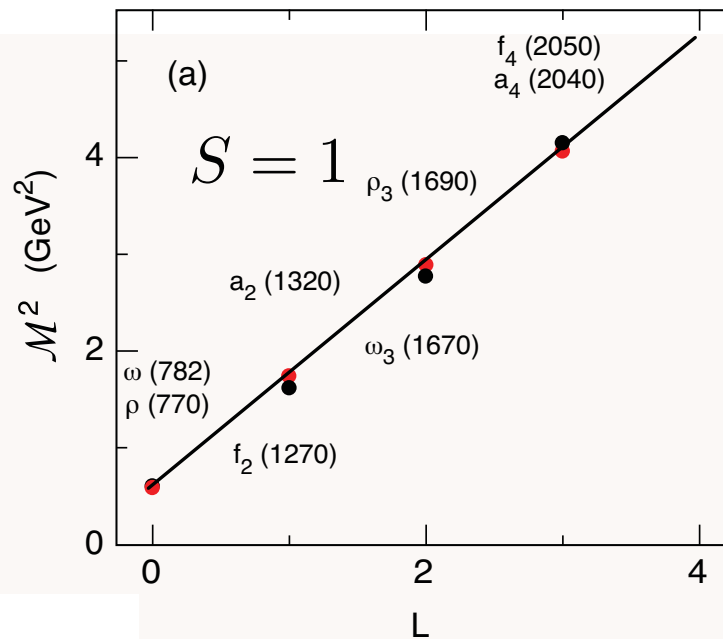
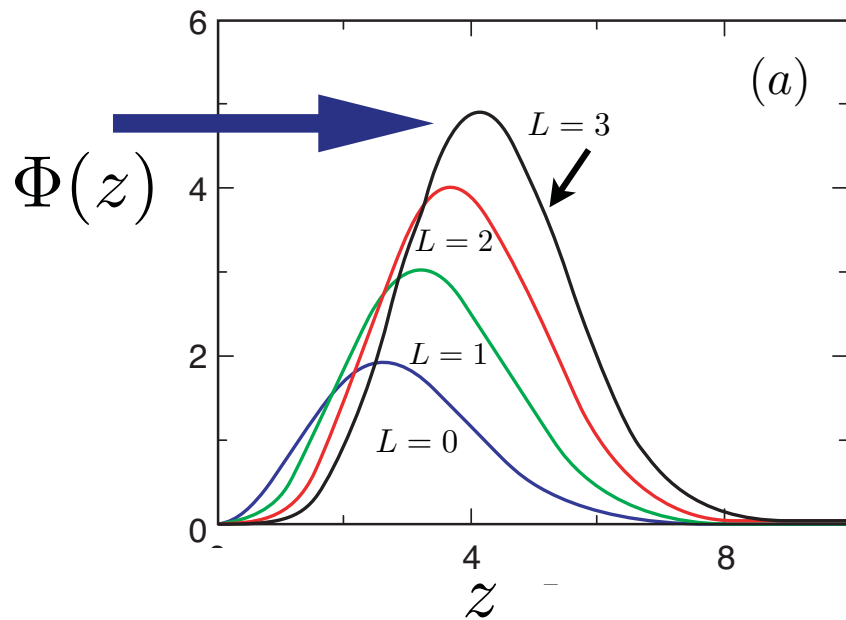
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

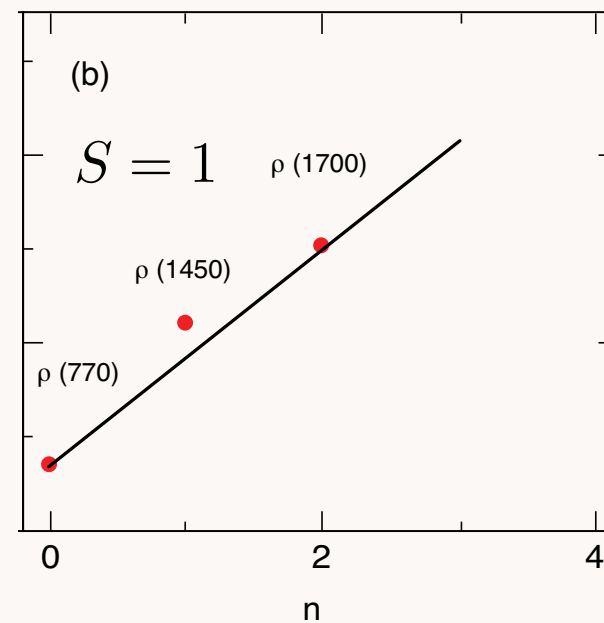
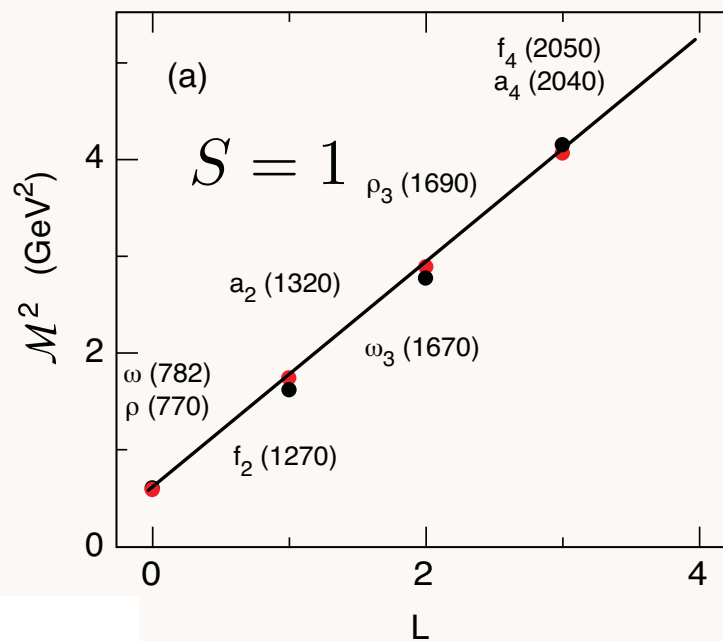
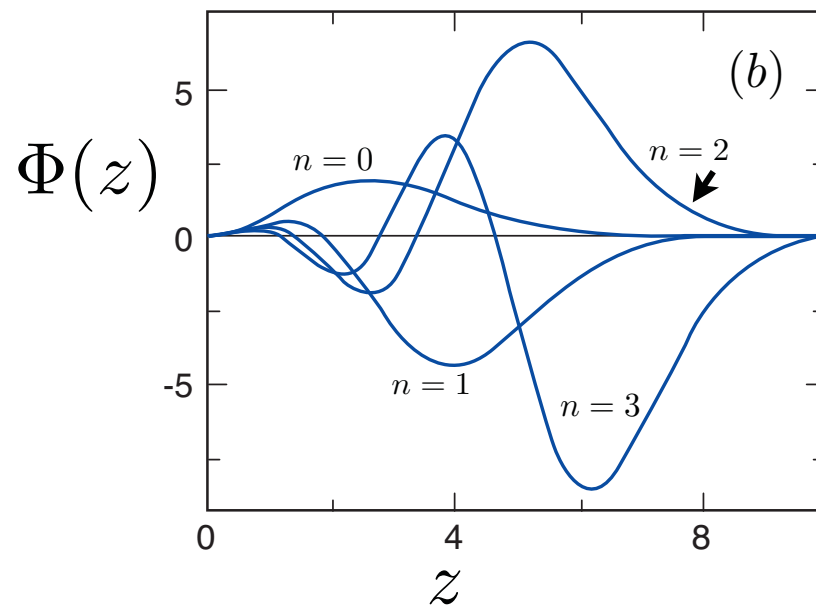
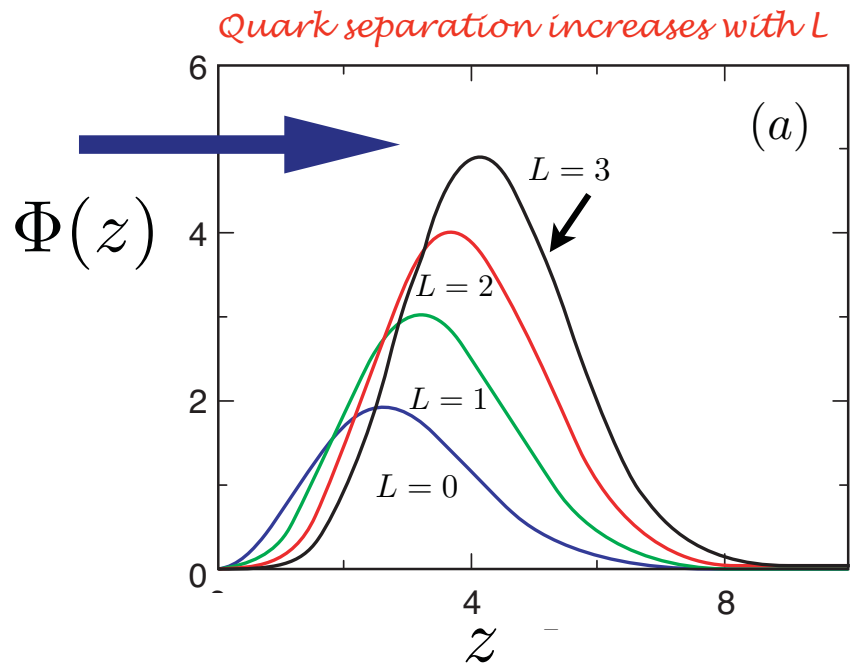
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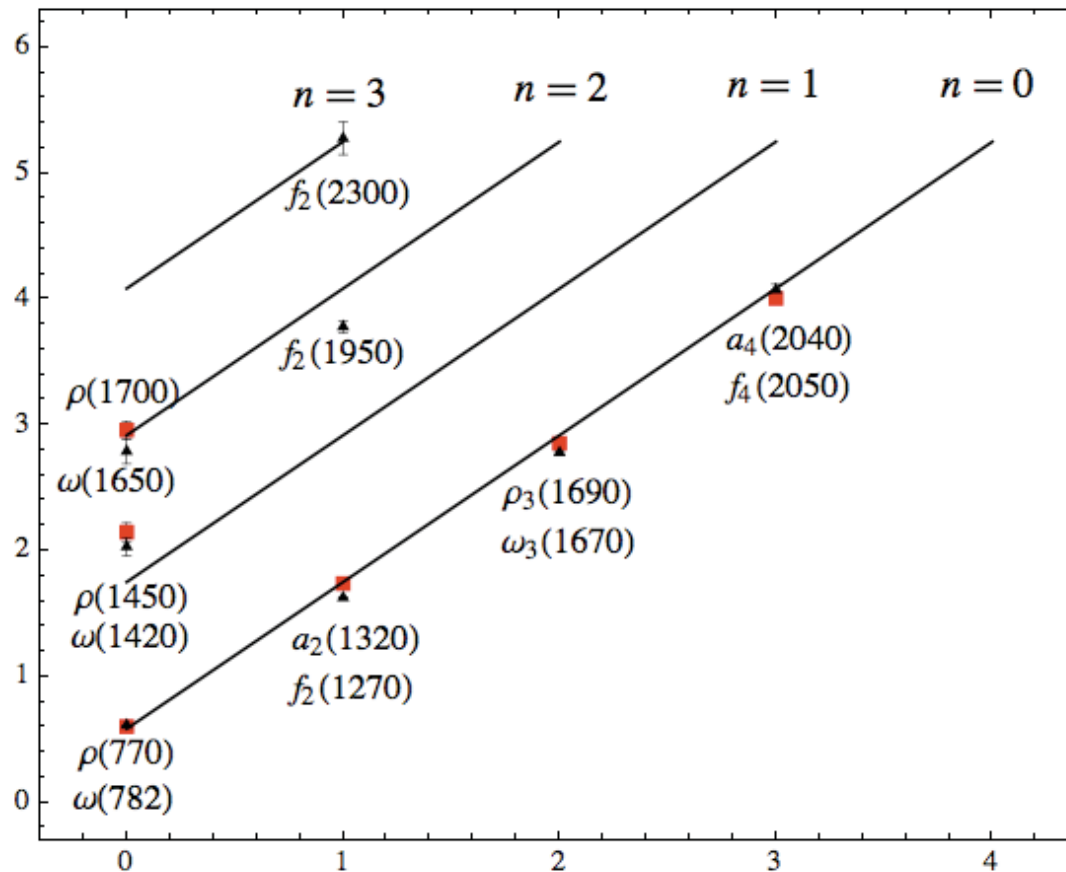
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Quark separation increases with L





$\mathcal{M}^2$ 1^{--} 2^{++} 3^{--} 4^{++} J^{PC}  L

Parent and daughter Regge trajectories for the $I = 1$ ρ -meson family (red)
and the $I = 0$ ω -meson family (black) for $\kappa = 0.54$ GeV

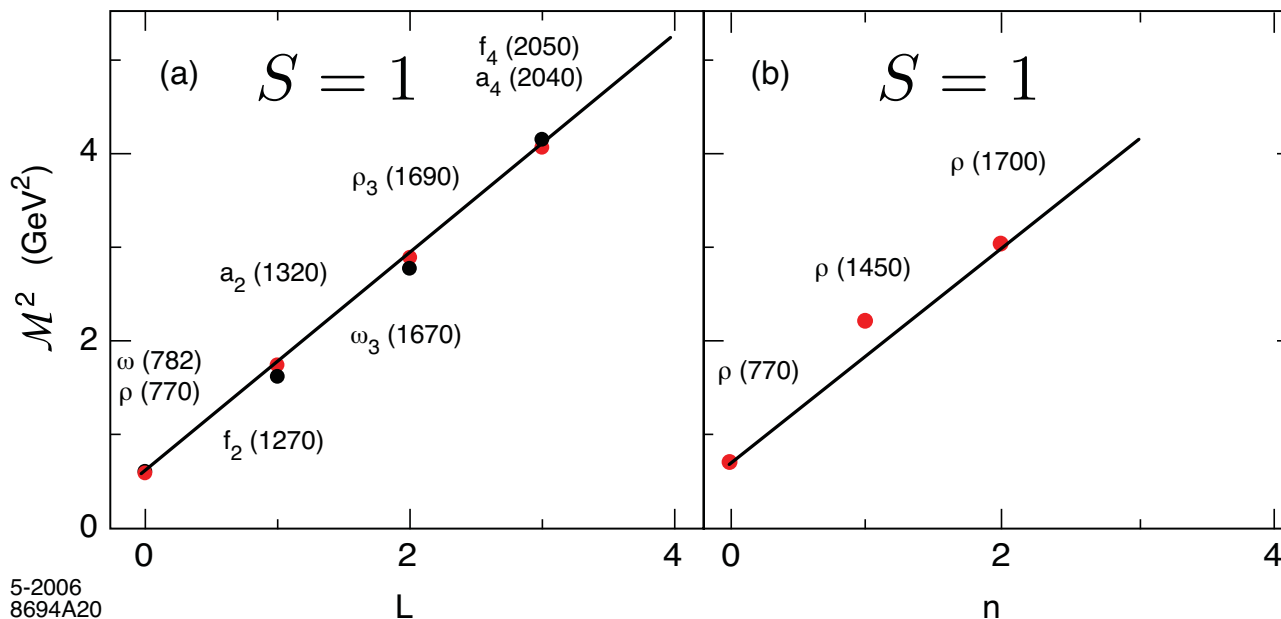
- Effective LF Schrödinger wave equation

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L + S - 1) \right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)$$

with eigenvalues $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$.

Same slope in n and L

- Compare with Nambu string result (rotating flux tube): $M_n^2(L) = 2\pi\sigma(n + L + 1/2)$.



Vector mesons orbital (a) and radial (b) spectrum for $\kappa = 0.54$ GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Fazio, Jugeau and Nicotri (2007).

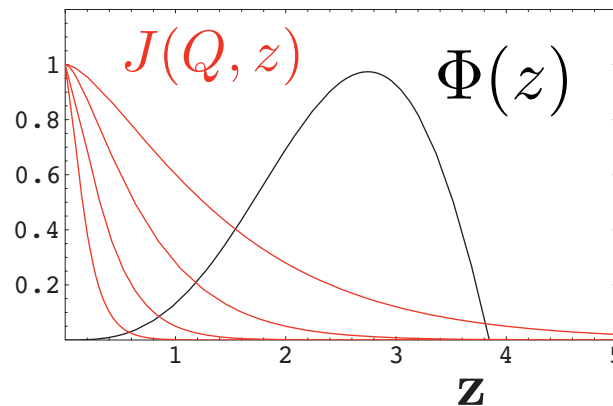
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQ K_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High Q^2
from
small $z \sim 1/Q$



Polchinski, Strassler
de Teramond, sjb

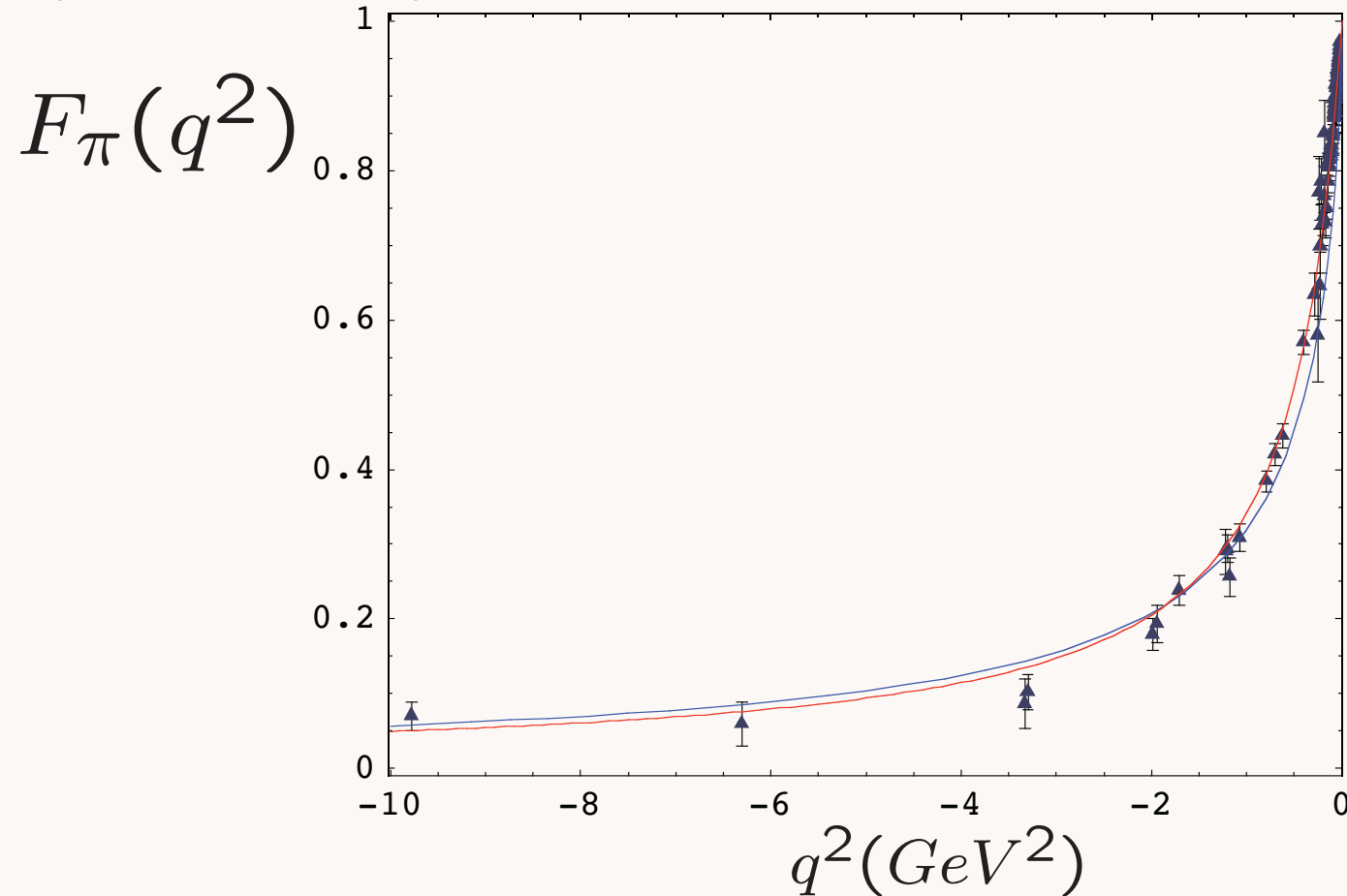
Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z , Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

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- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

*Soft Wall
Model*

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

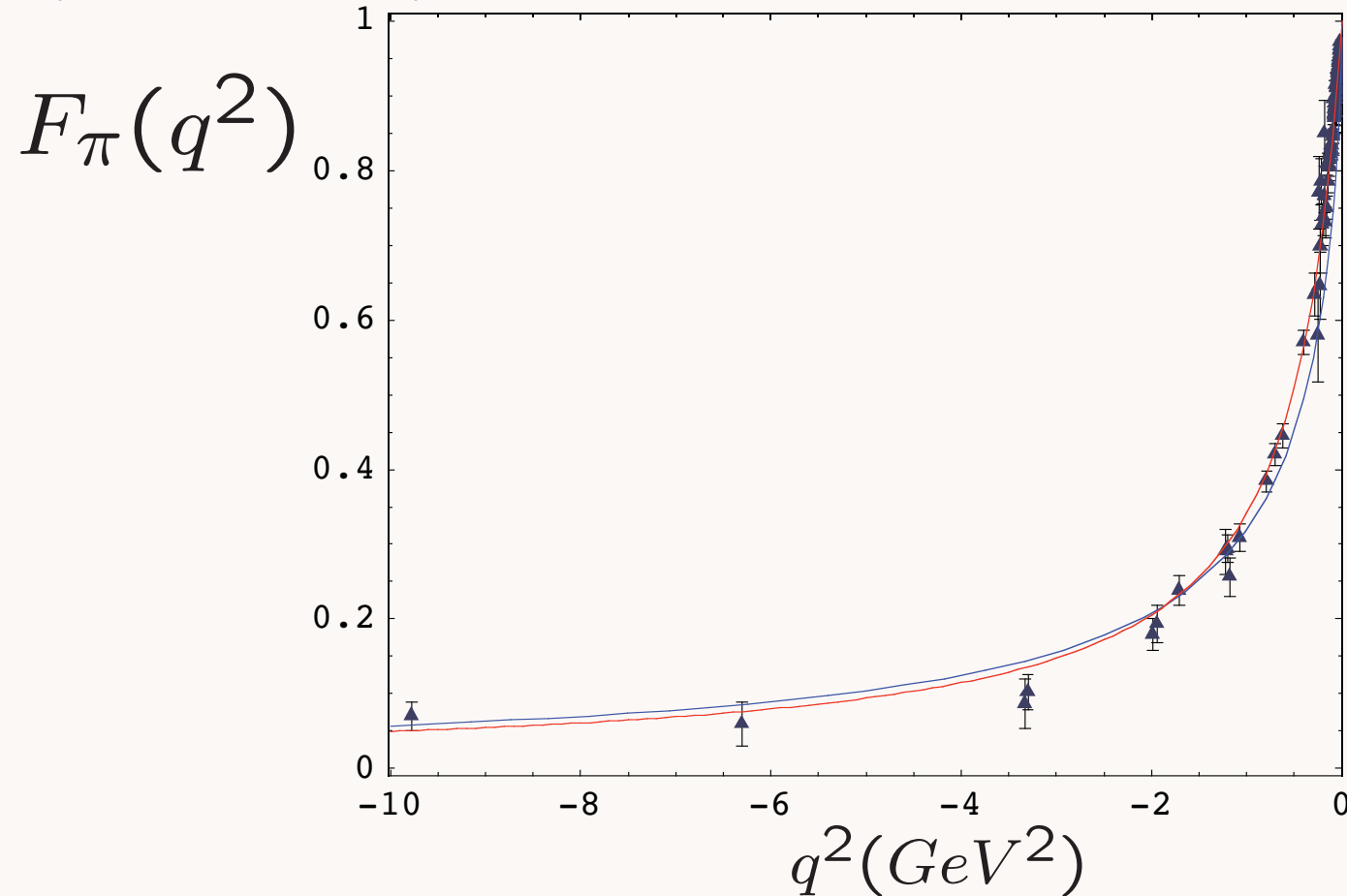
$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



Data Compilation
Baldini, Kloe and Volmer

- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

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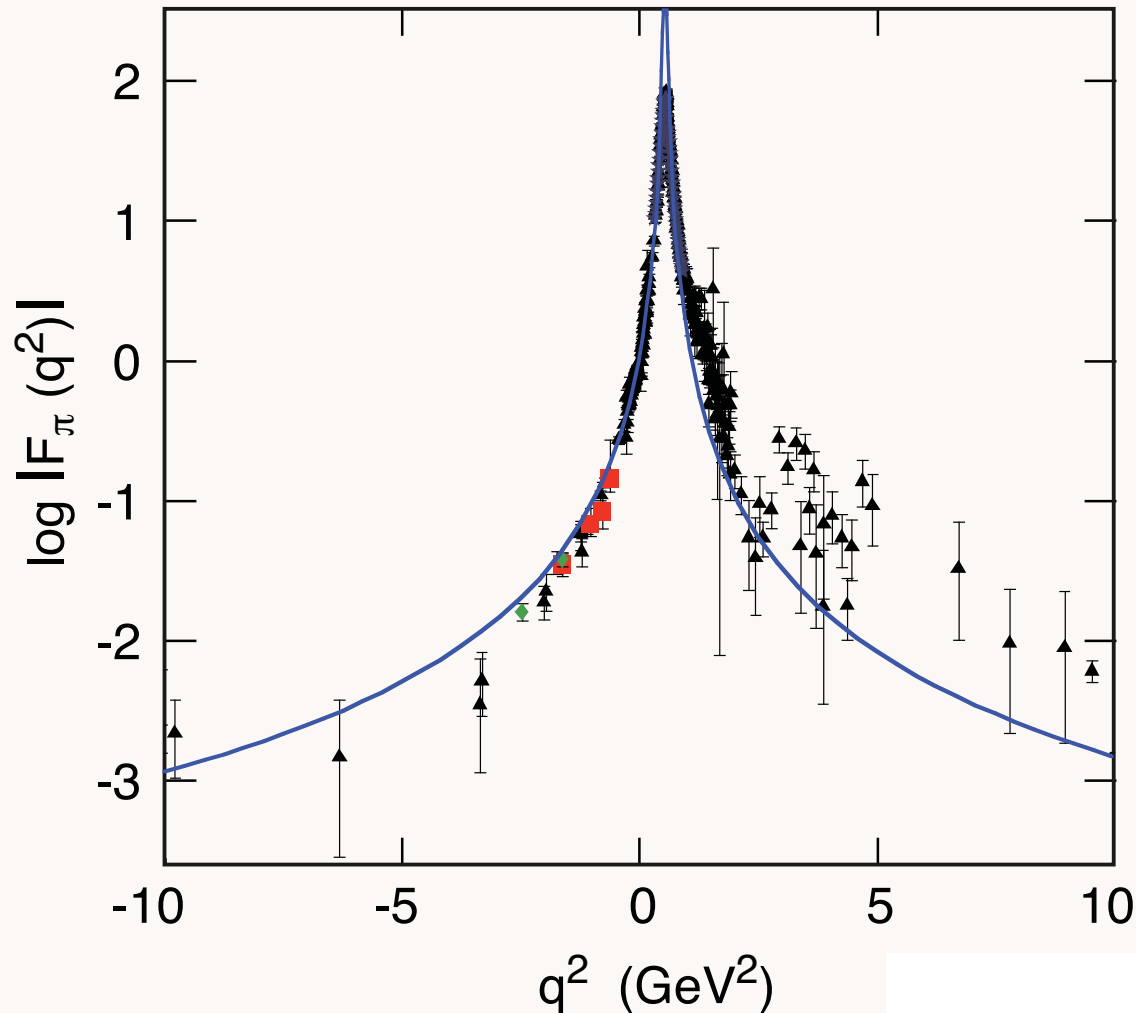
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- Analytical continuation to time-like region $q^2 \rightarrow -q^2$

$$M_\rho = 2\kappa = 750 \text{ MeV}$$

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375 \text{ GeV}$ in the SW model.

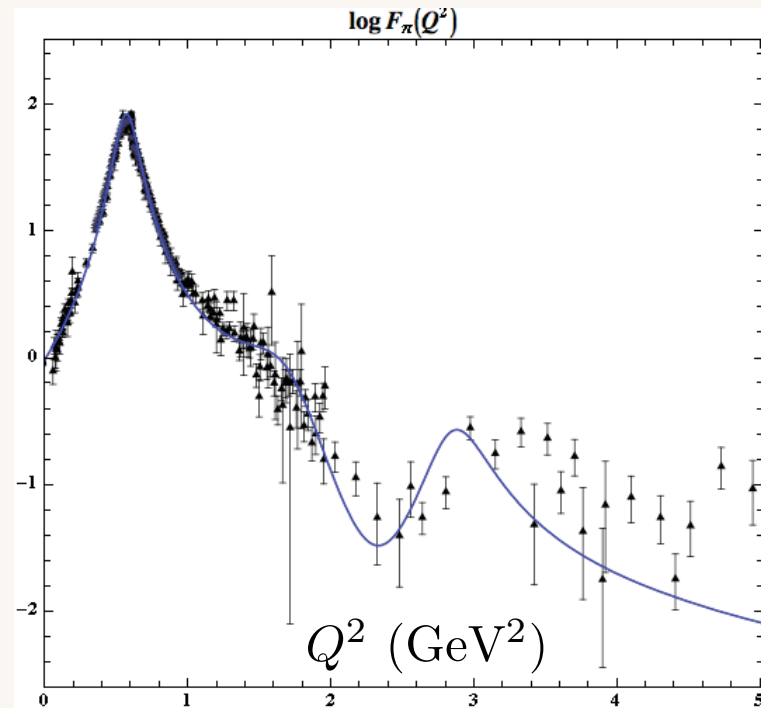
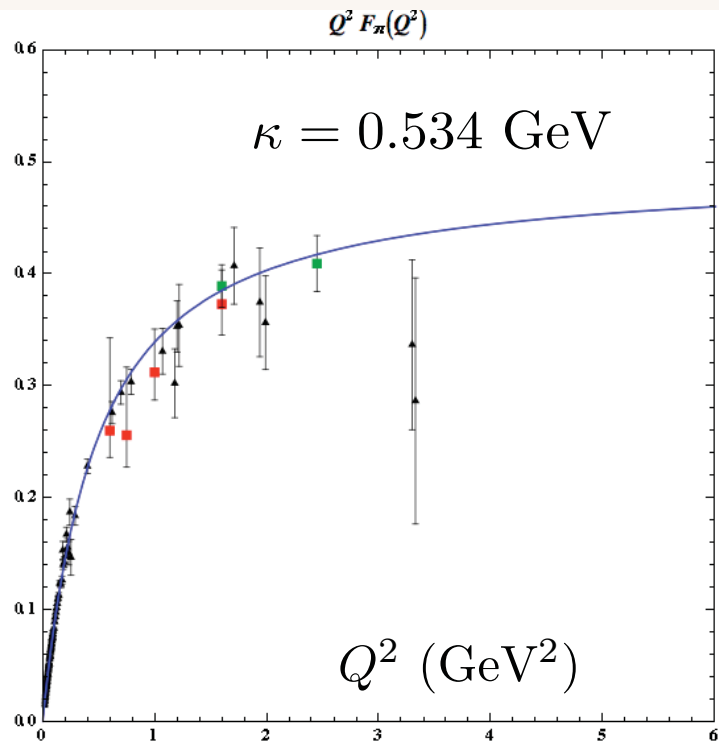
- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

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Spacelike and timelike pion form factor

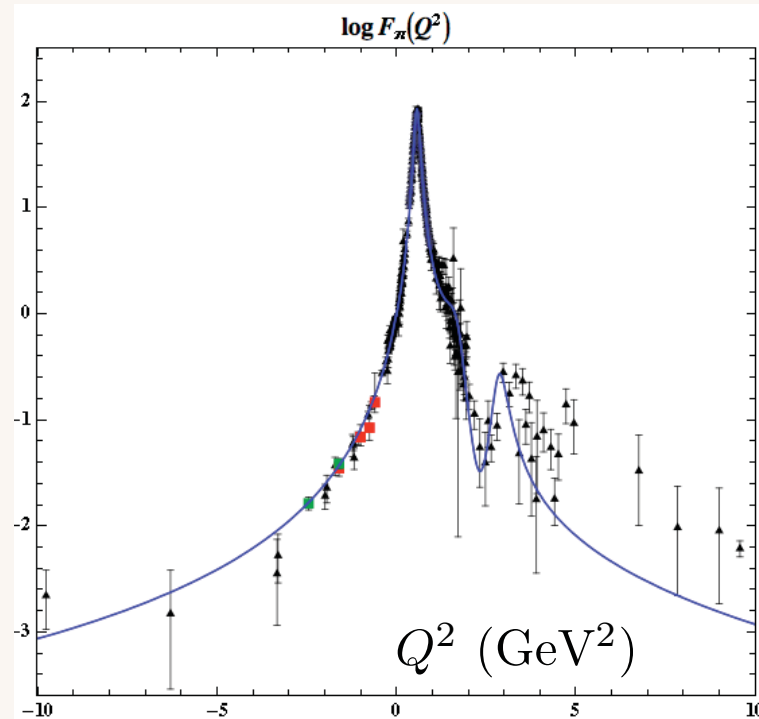


GdT and SJB
preliminary

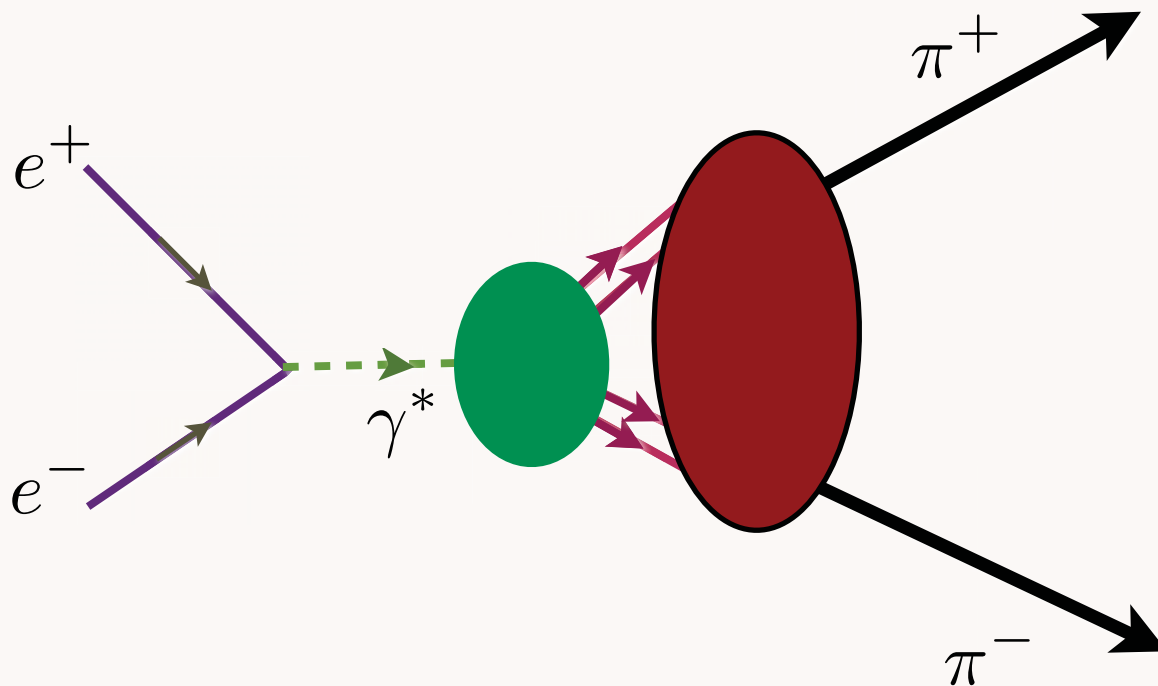
$$|\pi\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle$$

$$\Gamma_\rho = 120 \text{ MeV}, \Gamma'_\rho = 300 \text{ MeV}$$

$$P_{q\bar{q}q\bar{q}} = 15\%$$



Dressed soft-wall current bring in higher Fock states and more vector meson poles



Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ , Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \dots (1 + z)\Gamma(1 + z)$.
- Form factor expressed as $N - 1$ product of poles

$$\begin{aligned} F(Q^2) &= \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \\ F(Q^2) &= \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\ &\dots \\ F(Q^2) &= \frac{(N - 1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \dots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N. \end{aligned}$$

- For large Q^2 :

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

$$\vec{q}_\perp^2 = Q^2 = -q^2$$

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

- Fourier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

Soper

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

- Electromagnetic form-factor in AdS space:

$$F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2,$$

where $J(Q^2, z) = zQK_1(zQ)$.

- Use integral representation for $J(Q^2, z)$

$$J(Q^2, z) = \int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Write the AdS electromagnetic form-factor as

$$F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with electromagnetic form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}$$

with $\zeta = z$, $0 \leq \zeta \leq \Lambda_{\text{QCD}}$

$LF(3+1)$ AdS_5

$\psi(x, \vec{b}_\perp)$



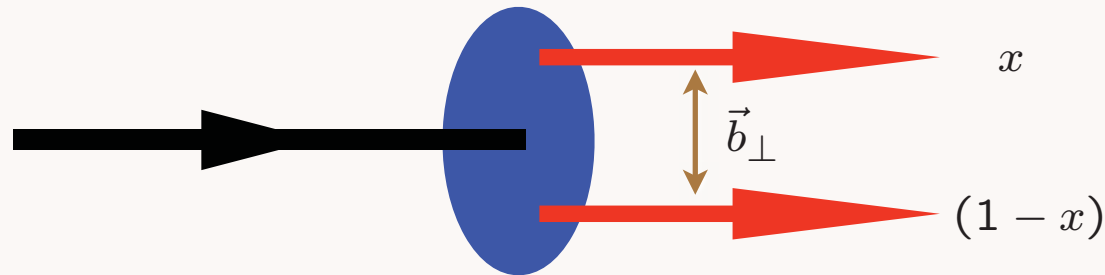
$\phi(z)$

$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$



z

$\psi(x, \vec{b}_\perp)$



$$\psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta)$$

*Light-Front Holography: Unique mapping derived from equality of
LF and AdS formula for current matrix elements*

Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

$$A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2,$$

Abidin & Carlson

where $H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ)$

- Use integral representation for $H(Q^2, z)$

$$H(Q^2, z) = 2 \int_0^1 x dx J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right)$$

- Write the AdS gravitational form-factor as

$$A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0 \left(zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2$$

- Compare with gravitational form-factor in light-front QCD for arbitrary Q

$$\left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4},$$

Identical to LF Holography obtained from electromagnetic current

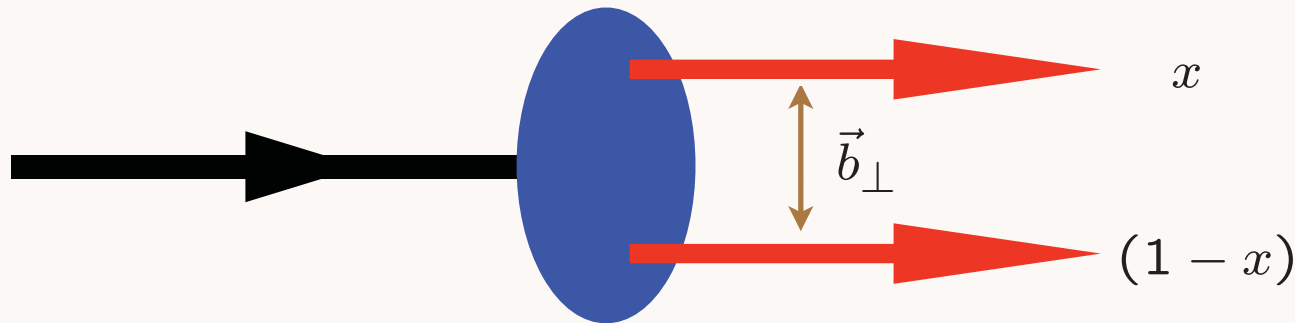
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

G. de Teramond, sjb

*soft wall
confining potential:*

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Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

$$\begin{aligned}\mathcal{M}^2 &= \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \\ &= \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}_{\vec{b}_\perp}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions}.\end{aligned}$$

**Change
variables**

$$(\zeta, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)} \vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}$$

$$\begin{aligned}\mathcal{M}^2 &= \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \\ &\quad + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \\ &= \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta)\end{aligned}$$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

Coupled Fock states

$$\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell+1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

Azimuthal Basis ζ, ϕ

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

Semiclassical first approximation to QCD

Confining AdS/QCD potential

Prediction from AdS/CFT: Meson LFWF

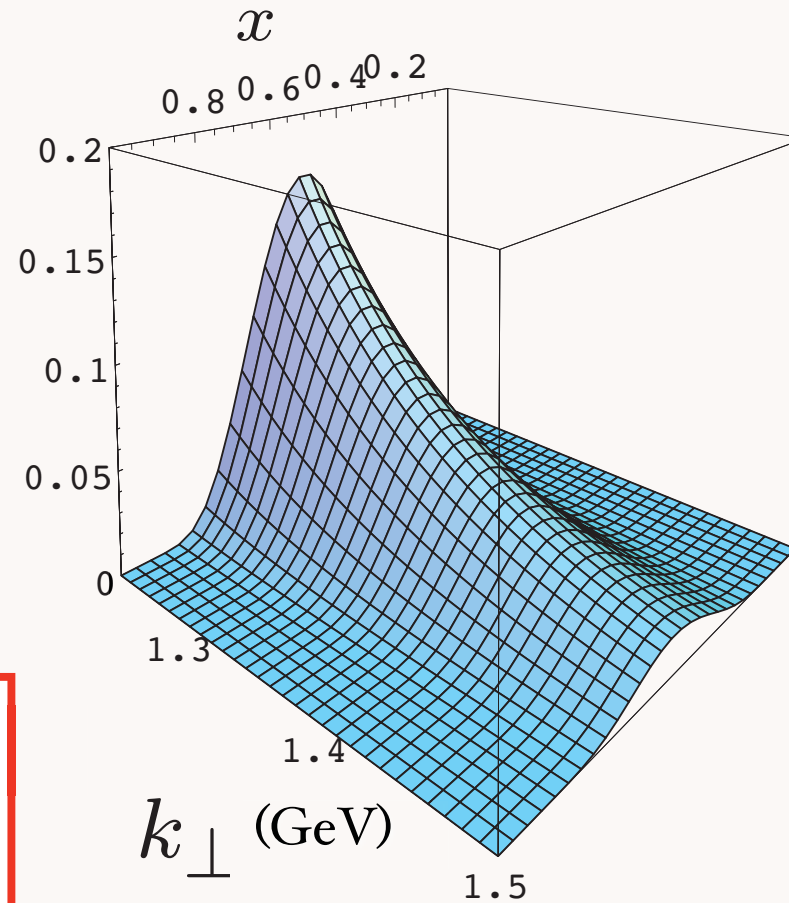
de Teramond, sjb

**“Soft Wall”
model**

$\kappa = 0.375 \text{ GeV}$
massless quarks

Note coupling

$$k_{\perp}^2, x$$



$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

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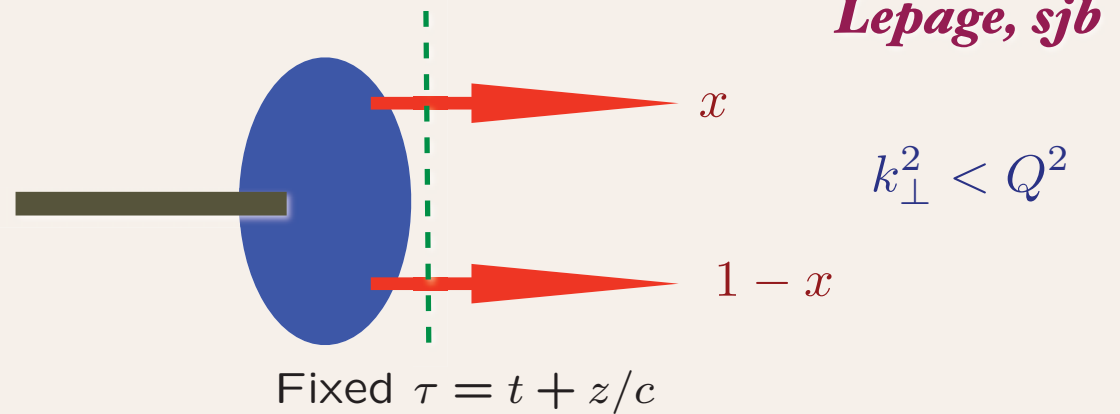
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Hadron Distribution Amplitudes

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

Lepage, sjb

Efremov, Radyushkin

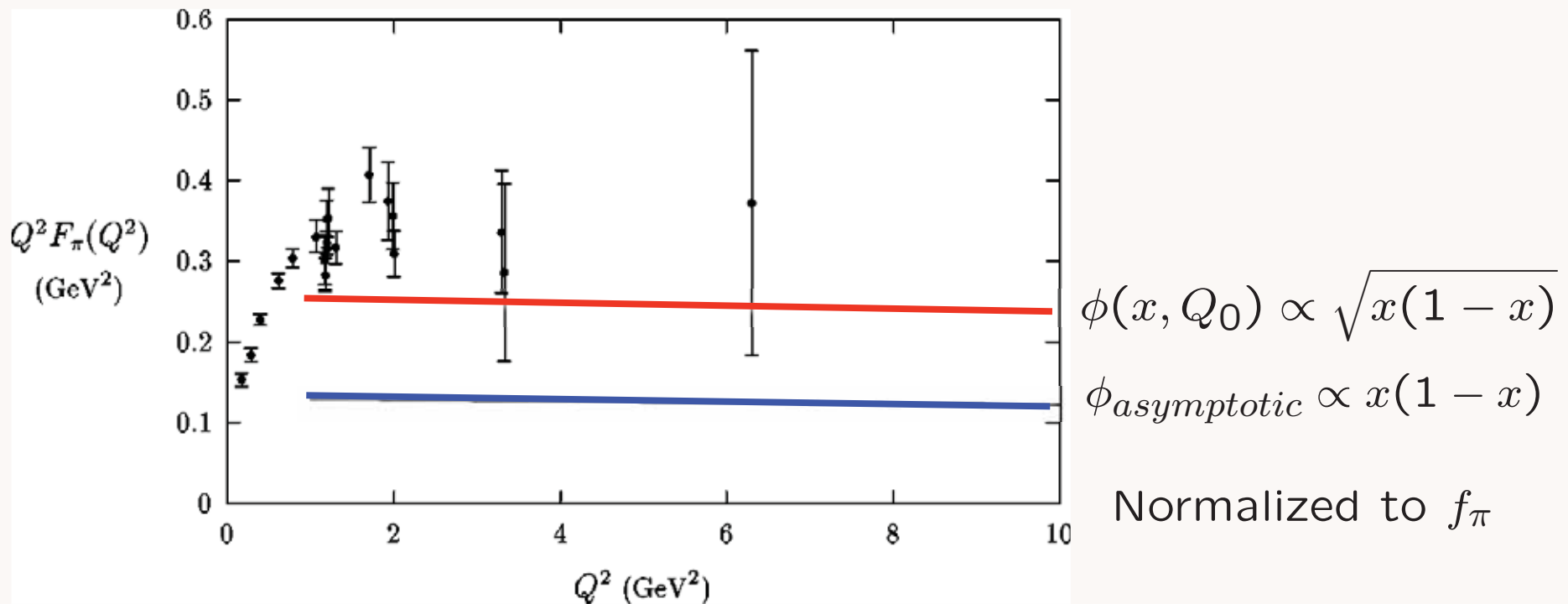
Sachrajda, Frishman Lepage, sjb

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \, \psi_{q\bar{q}}(x, \vec{k}_{\perp})$$

$$F_{\pi}(Q^2) = \int_0^1 dx \phi_{\pi}(x) \int_0^1 dy \phi_{\pi}(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2}$$



AdS/CFT:

Increases PQCD leading twist prediction for $F_{\pi}(Q^2)$ by factor 16/9

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Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20$$

$$\phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25$$

$$\phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

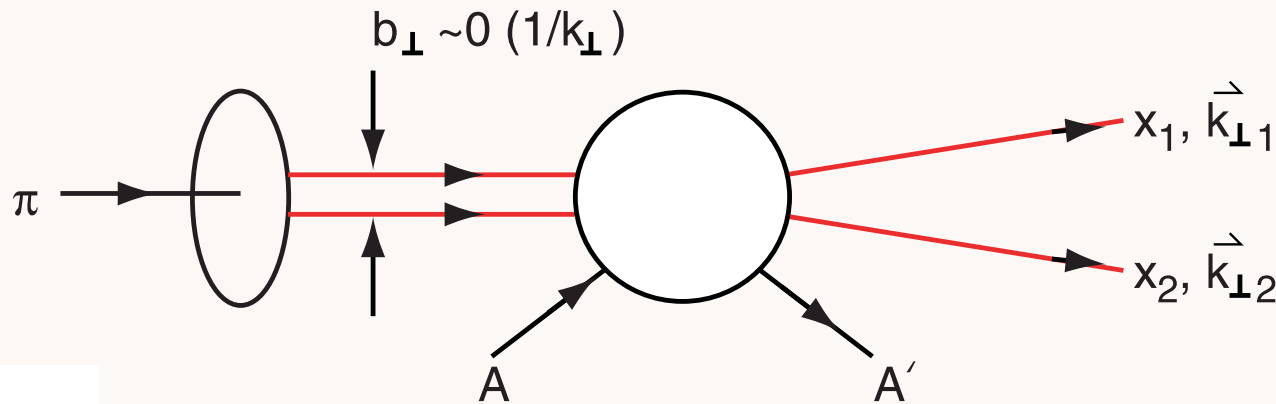
Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

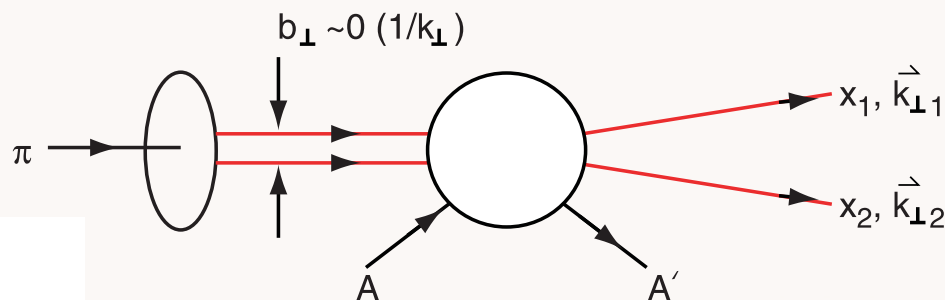


$$M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp})$$

Measure Light-Front Wavefunction of Pion

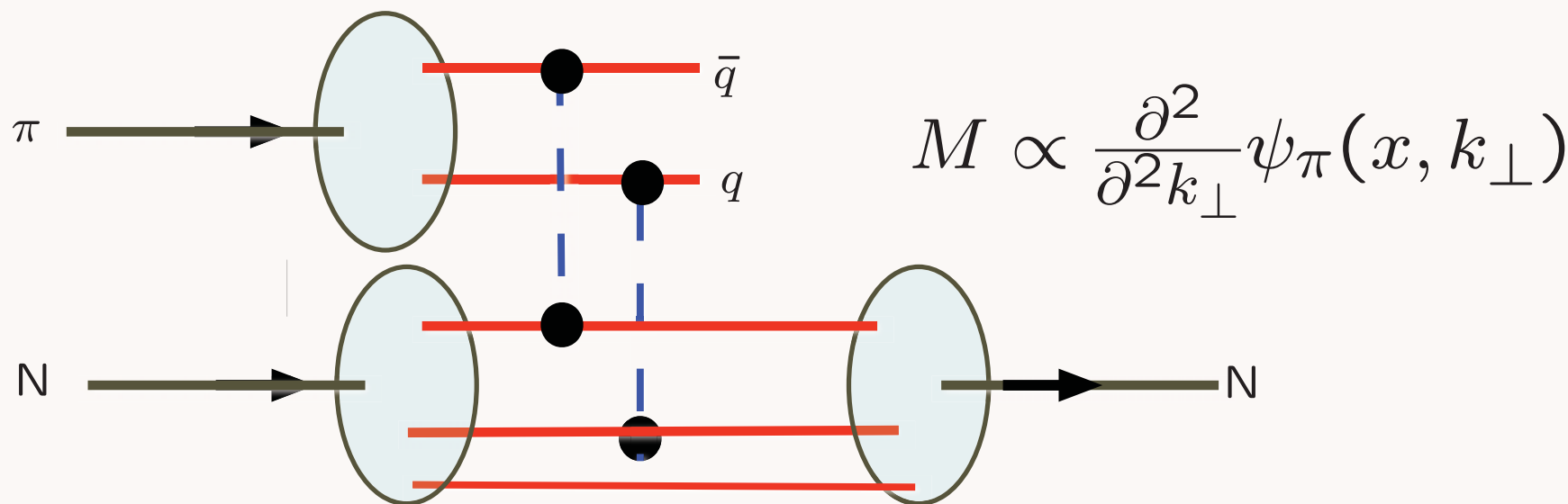
Minimal momentum transfer to nucleus
Nucleus left Intact!

E791 FNAL Diffractive DiJet

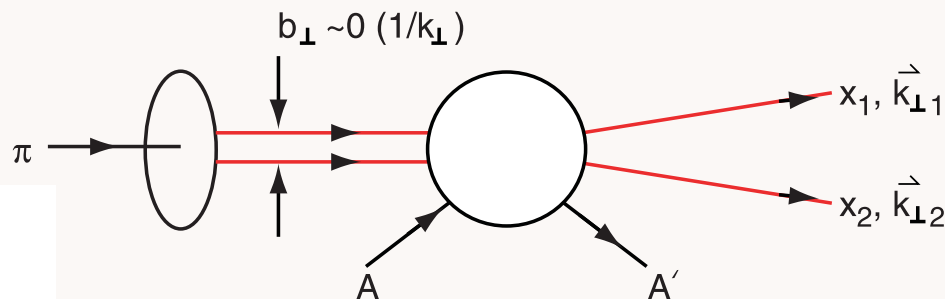


Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



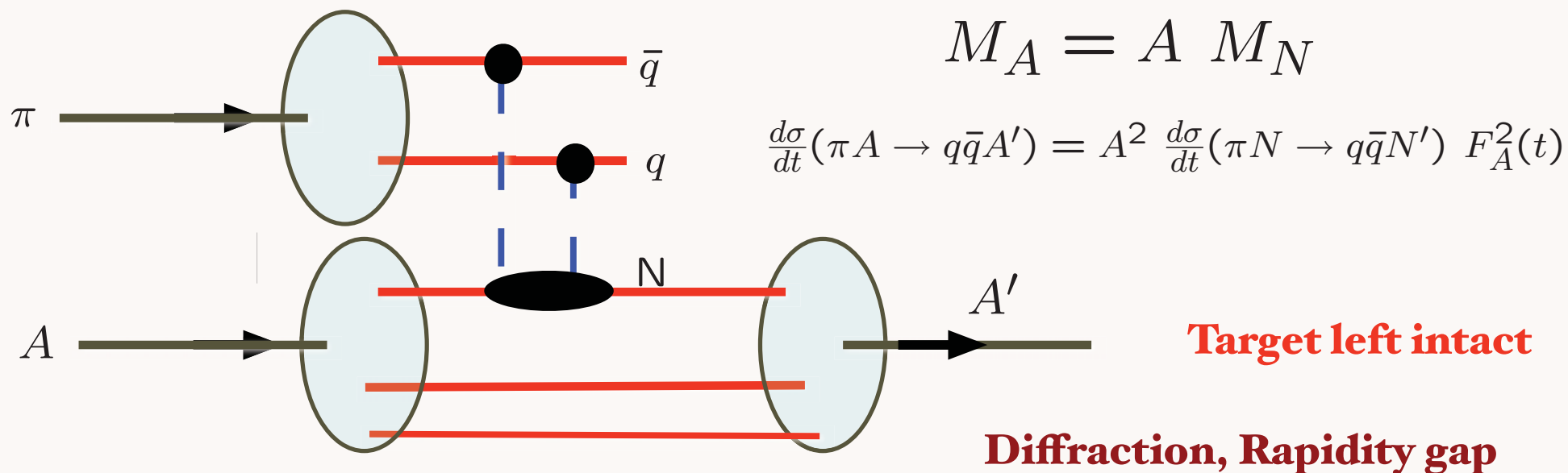
Key Ingredients in E791 Experiment



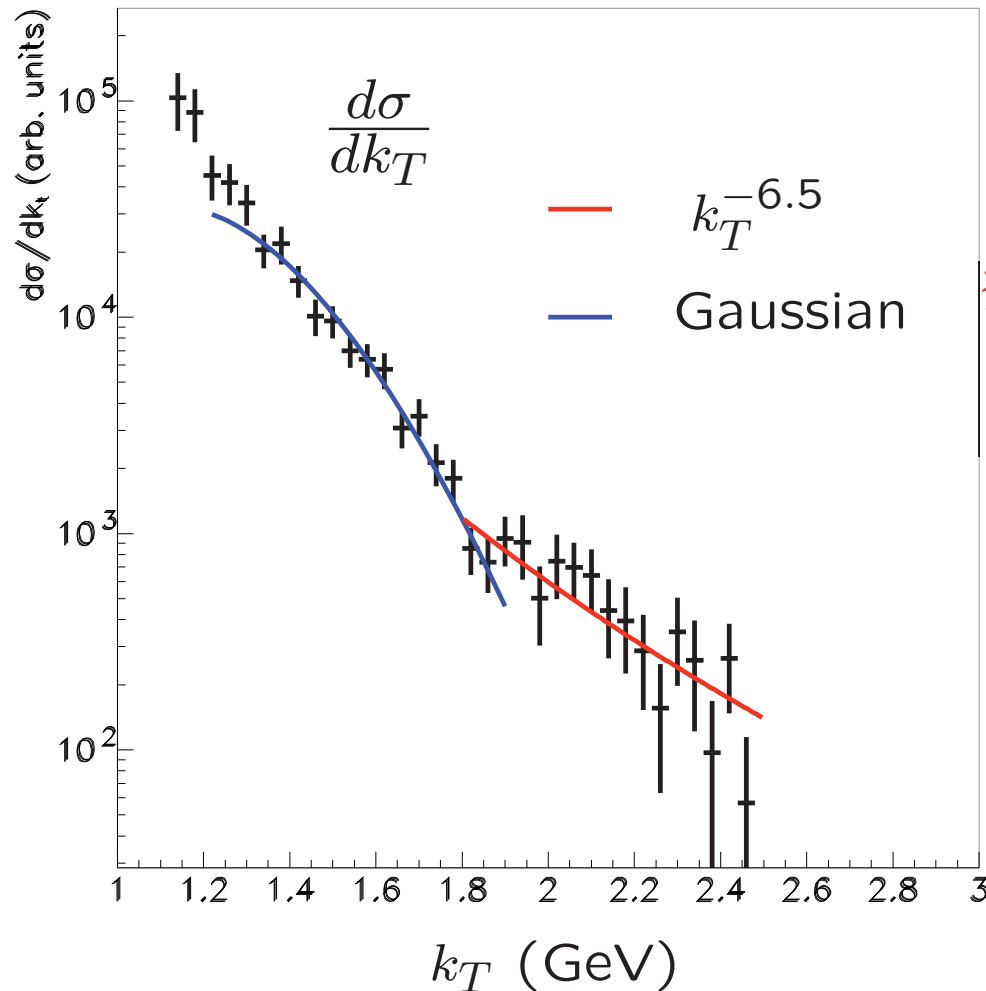
Brodsky Mueller
Frankfurt Miller Strikman

*Small color-dipole moment pion not absorbed;
interacts with each nucleon coherently*

QCD COLOR Transparency



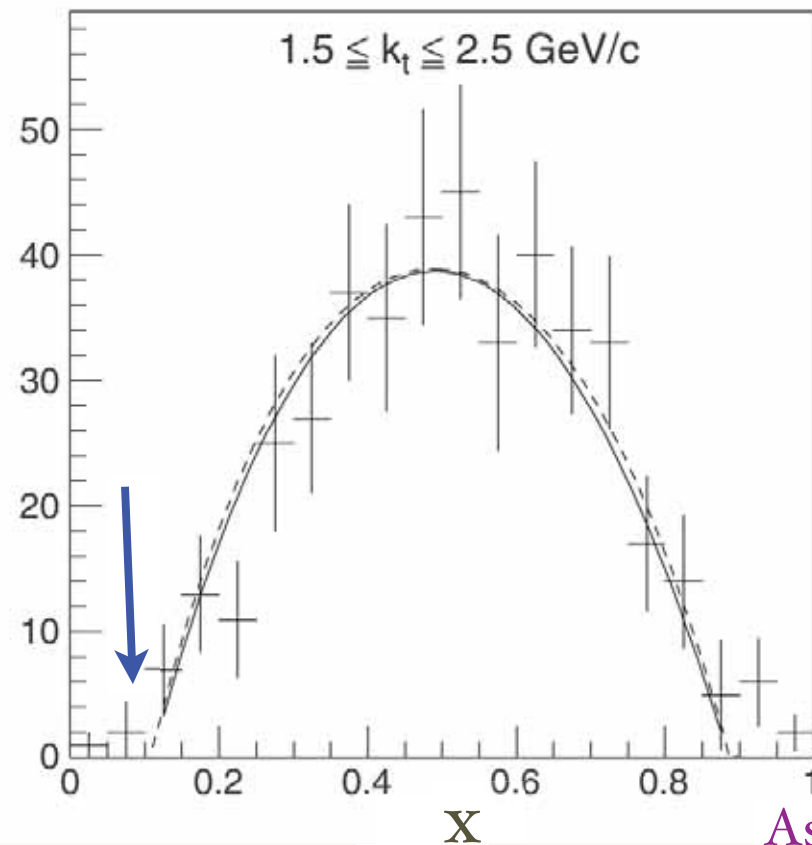
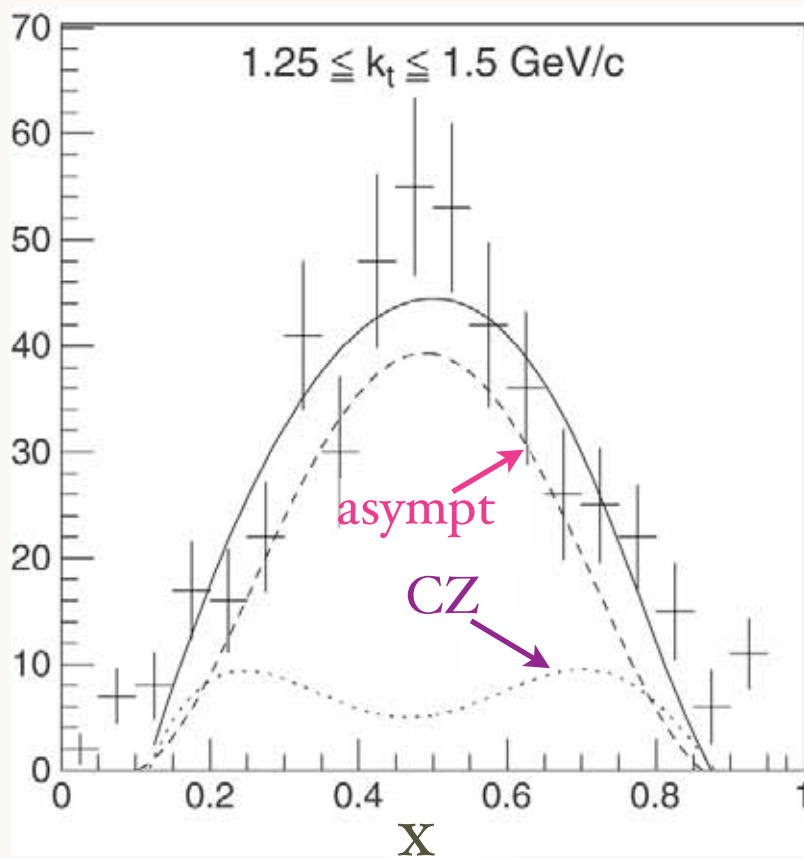
E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with $k_T^{-6.5}$ PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT H₀ LFWF



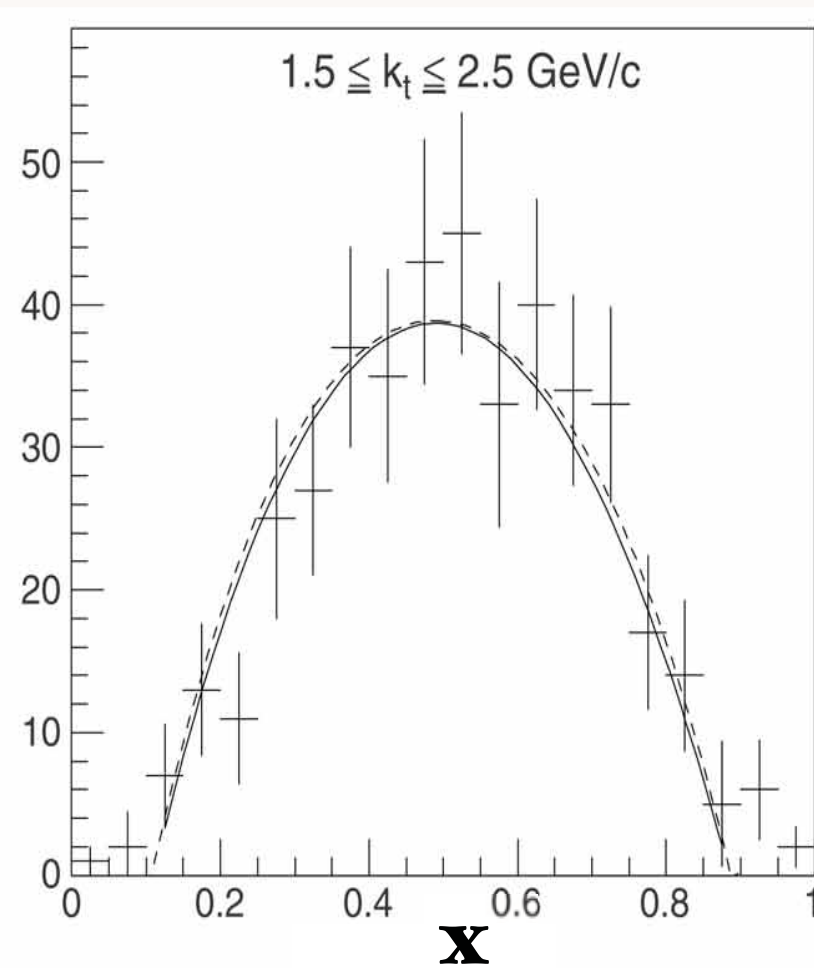
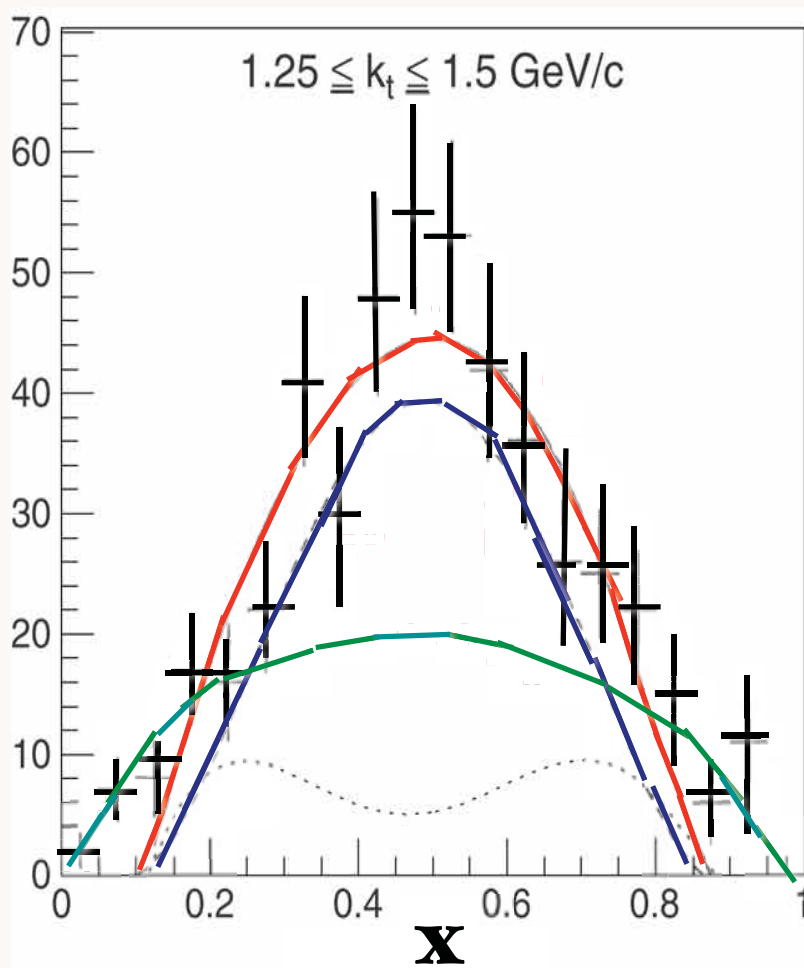
Ashery E791

Narrowing of x distribution at higher jet transverse momentum

x : distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components: Nonperturbative (AdS/CFT) and Perturbative (ERBL)
Evolution to asymptotic distribution

$$\phi(x) \propto \sqrt{x(1-x)}$$



Ashery
E791

Possibly two components:

Perturbative (ERBL) + Nonperturbative (AdS/CFT)

$$\phi(x) = A_{\text{pert}}(k_{\perp}^2)x(1-x) + B_{\text{nonpert}}(k_{\perp}^2)\sqrt{x(1-x)}$$

Narrowing of x distribution at high jet transverse momentum