Relativistic BCS-BEC Crossover in Quark Matter

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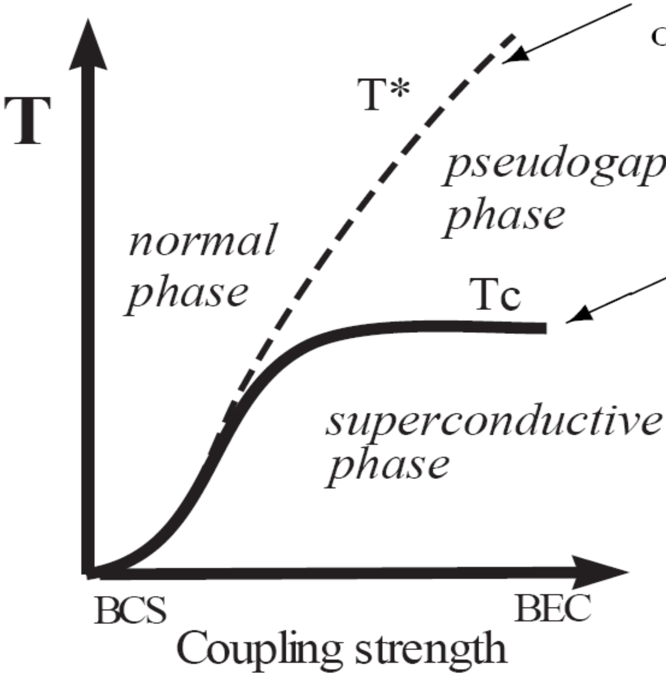
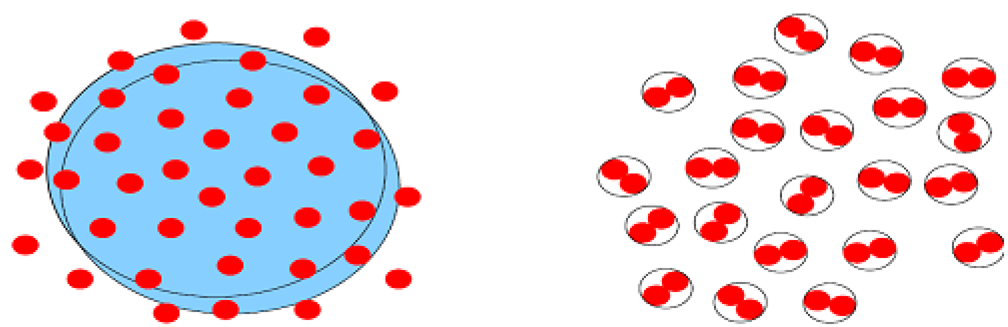
3) Fluctuations
4) Nuclear Matter and Quark Matter
5) Conclusions

Introduction: Pairing



Tc in the BEC region is independent of the coupling between fermions, since the coupling only affects the internal structure of the bosons.

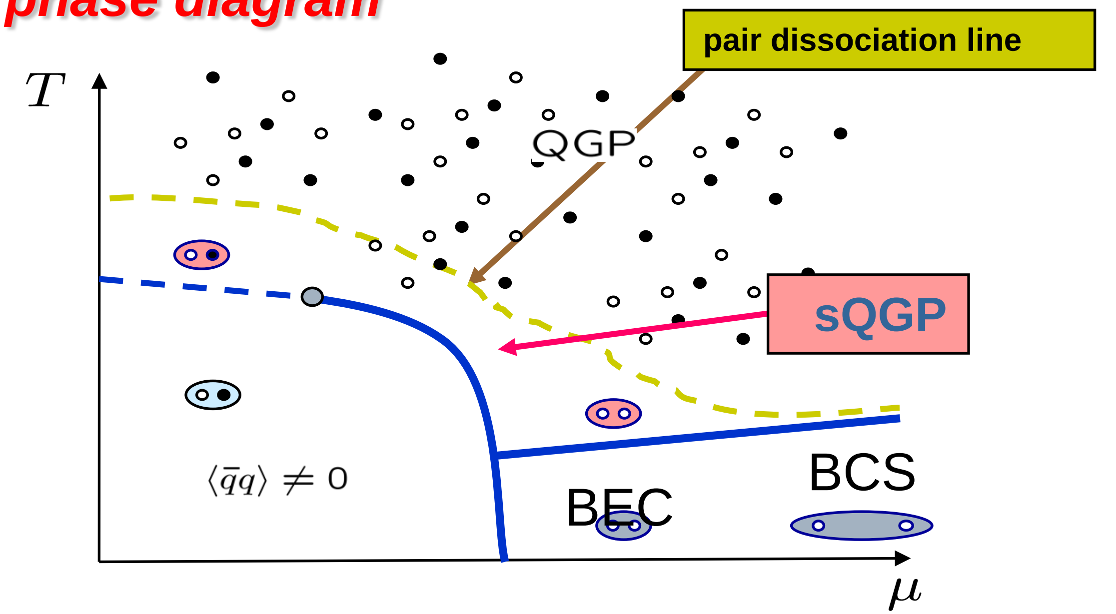


$T^* = T_c$ $\qquad$ $T^* >> T_c$

in BCS, T_c is determined by thermal excitation of fermions,

in BEC, T_c is controlled by thermal excitation of collective modes

QCD phase diagram



strongly coupled quark matter with both quarks and bosons

there may exist BCS-BEC crossovers in quark matter !

new phenomena in BCS-BEC crossover of QCD:

relativistic systems,

anti-fermion contribution,

rich inner structure (color, flavor),

medium dependent mass, ……

Introduction: History of BCS-BEC Theory

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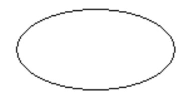
*) non-relativistic mean field theory at T=0 (Leggett, 1980)

*) non-relativistic theory at $T \neq 0$ (Nozieres and Schmitt-Rink, 1985)

extension to relativistic system (Nishida and Abuki (2006,2007)

$$\Omega_{fl} = \int \frac{d^4 q}{(2\pi)^4} \ln\left[\frac{1}{G} - \chi(q)\right], \qquad \chi =$$

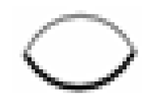


$$\sim G_0 G_0$$

*) non-relativistic G_0G scheme (Chen, Levin et al., 1998, 2000, 2005)

asymmetric pair susceptibility

$$\chi =$$



$$\sim G_0 G$$

extension to relativistic G_0G scheme (He, Jin, PZ, 2006, 2007)

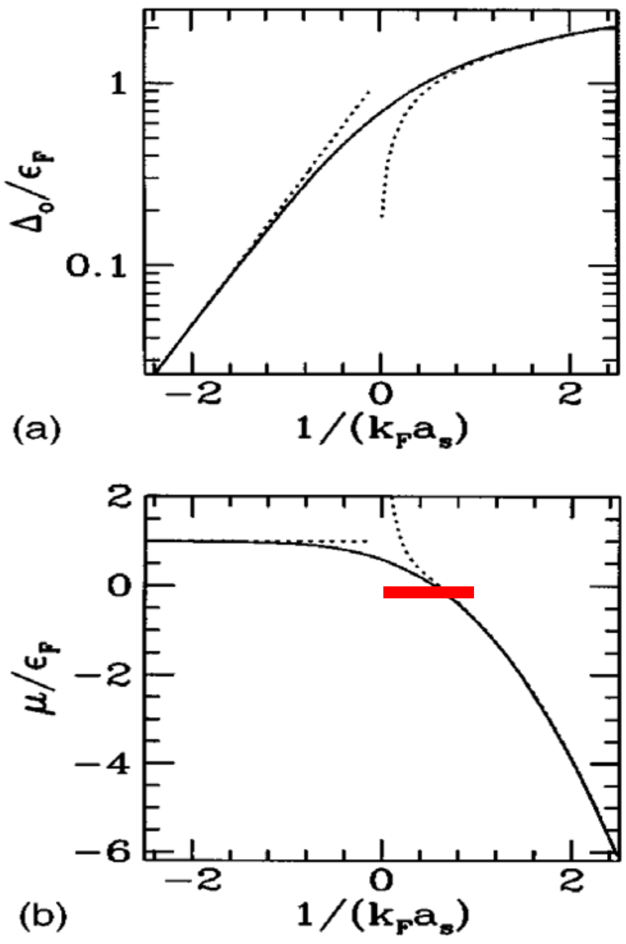
*) bose-fermion model (Friedderg, Lee, 1989, 1990)
extension to relativistic systems (Deng, Wang, 2007)

*) Kitazawa, Rischke, Shovkovy, 2007, NJL+phase diagram
Brauner, 2008, collective excitations ……

Non-relativistic Mean Field at T=0

A.J.Leggett, 1980

universality behavior



BCS limit

$$\eta = \frac{1}{k_F a_s} \to -\infty, \qquad \tilde{\Delta} = \frac{8}{e^2} e^{2\eta/\pi}, \qquad \hat{\mu} = 1$$

BEC limit

$$\eta \to \infty, \qquad \tilde{\Delta} = \sqrt{\frac{16\eta}{3\pi}}, \qquad \hat{\mu} = -\eta^2$$

$$\mu = -\frac{\varepsilon_b}{2}, \qquad \varepsilon_b = \frac{1}{m a_s^2}$$

$$n(p) = \frac{1}{e^{(\varepsilon-\mu)/T} - 1} \qquad \Rightarrow \qquad \mu \le 0$$

BCS-BEC crossover

$\eta < 0 \quad \to \quad \eta > 0,$

small $\Delta \quad \to \quad$ large $\Delta,$

$\mu > 0 \quad \to \quad \mu < 0$

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Lianyi He, PZ, 2007

NJL-type model at moderate density

$$L = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + \frac{g}{4}\left(\bar{\psi} i\gamma_5 C\bar{\psi}^T\right)\left(\psi^T iC\gamma_5\psi\right)$$

order parameter

$$\Delta = \frac{g}{2}\left\langle \psi^T iC\gamma_5\psi\right\rangle$$

mean field thermodynamic potential

$$E_k^\pm = \sqrt{\left(\xi_k^\pm\right)^2 + \Delta^2}$$

$$\Omega = \frac{\Delta^2}{g} - \int \frac{d^3\vec{k}}{(2\pi)^3}\left[E_k^+ + E_k^- - \xi_k^+ - \xi_k^-\right] \qquad \xi_k^\pm = \sqrt{k^2+m^2} \pm \mu$$

fermion and anti-fermion contributions

gap equation and number equation:

$$\begin{cases} -\dfrac{\pi}{2}\eta = \displaystyle\int_0^z dx x^2 \left[\left(\frac{1}{E_x^-} - \frac{1}{\varepsilon_x - 2\zeta^{-2}}\right) + \left(\frac{1}{E_x^+} - \frac{1}{\varepsilon_x + 2\zeta^{-2}}\right)\right] \\ \dfrac{2}{3} = \displaystyle\int_0^z dx x^2 \left[\left(1 - \frac{\xi_x^-}{E_x^-}\right) - \left(1 - \frac{\xi_x^+}{E_x^+}\right)\right] \end{cases}$$

$$\eta = \frac{1}{k_F a_s}, \qquad \zeta = \frac{k_F}{m}$$

broken universality

extra density dependence

Relativistic Mean Field BCS-BEC

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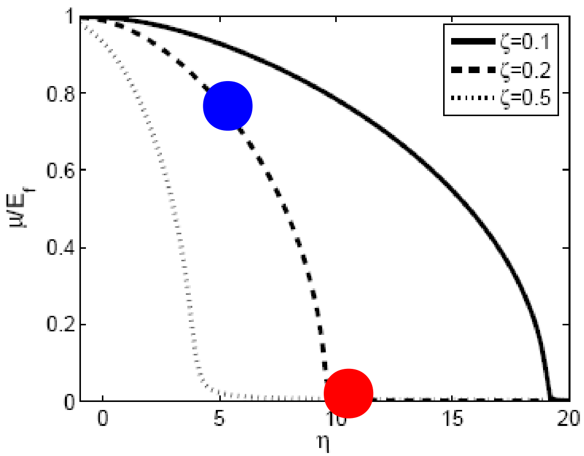
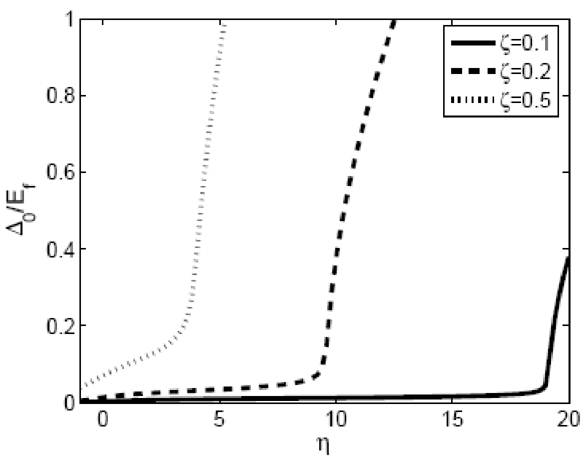
Lianyi He, PZ, 2007

$\mu - m$ ***plays the role of non-relativistic chemical potential***

$\mu - m = 0:$ ***BCS-NBEC crossover***

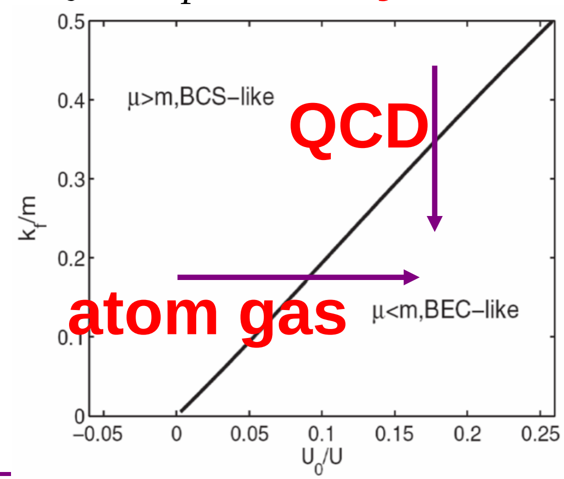
$\mu = 0:$ ***fermion and anti-fermion degenerate, NBEC-RBEC crossover***





in non-relativistic case, there is only one variable $\eta = 1/k_F \alpha_s$***, changing the density can not induce a BCS-BEC crossover.***

however, in relativistic case, the extra density dependence $\xi = k_F / m$ ***may induce a BCS-BEC.***



Fluctuations: G_0G Scheme

Lianyi He, PZ, 2007

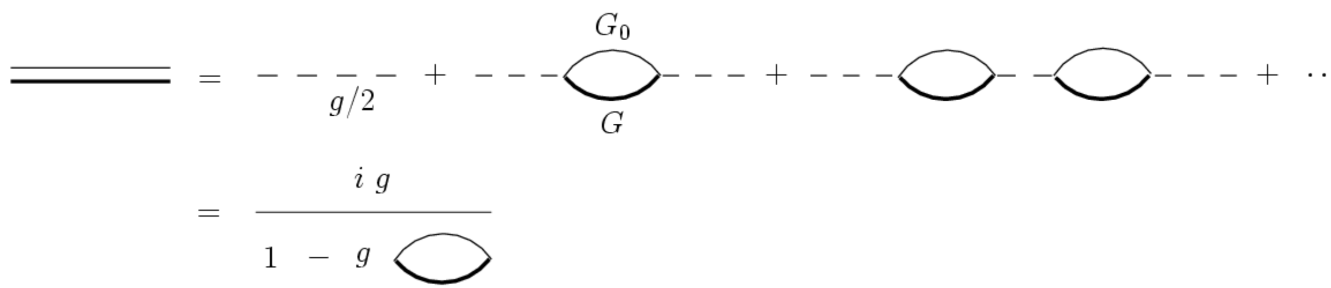
bare fermion propagator

$$G_0^{-1}(k,\mu) = (k_0+\mu)\gamma_0 - \vec{\gamma}\cdot\vec{k} - m$$

mean field fermion propagator

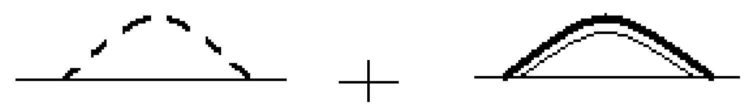
$$G^{-1}(k,\mu) = G_0^{-1}(k,\mu) - \Sigma_{mf}(k)$$

pair propagator



pair feedback to the fermion self-energy

$$\Sigma(k) = \Sigma_{mf}(k) + \Sigma_{fl}(k) =$$



fermions and pairs are coupled to each other

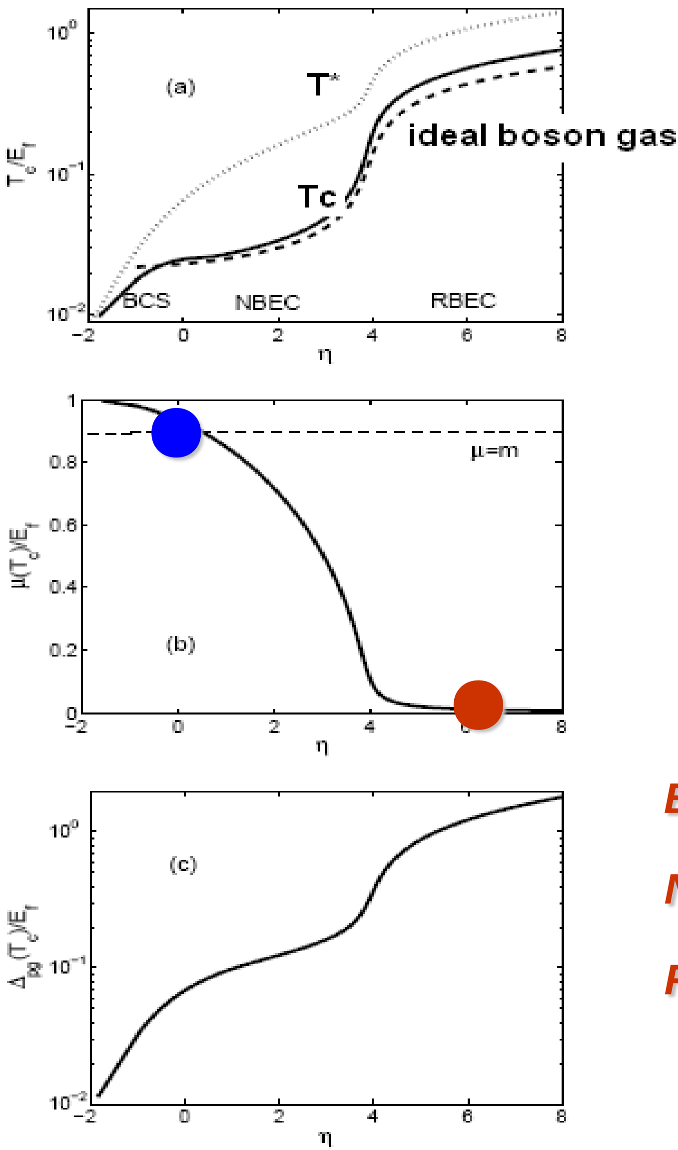
approximation

$$\Sigma_{fl}(k) \simeq -\Delta_{pg}^2 G_0(-k,\mu)$$

the pseudogap is related to the uncondensed pairs,
in G_0G scheme the pseudogap does not change the symmetry structure

Fluctuations: BCS-NBEC-RBEC

Lianyi He, PZ, 2007



T_c : critical temperature

T^* : pair dissociation temperature

$T < T_c$: $\Delta \neq 0,\ \Delta_{pg} \neq 0,$

condensed phase

$T_c < T < T^*$: $\Delta = 0,\ \Delta_{pg} \neq 0,$

normal phase with both fermions and pairs

$T > T^*$: $\Delta = 0,\ \Delta_{pg} = 0$

normal phase with only fermions

BCS: $\eta < 0, \quad \mu > m$ **no pairs**

NBEC: $0 < \eta < m/k_F, \quad 0 < \mu < m$ **heavy pairs, no anti-pairs**

RBEC: $\eta > m/k_F, \quad \mu \sim 0$ **light pairs, almost the same number of pairs and anti-pairs**

BCS-BEC in Asymmetric Nuclear Matter

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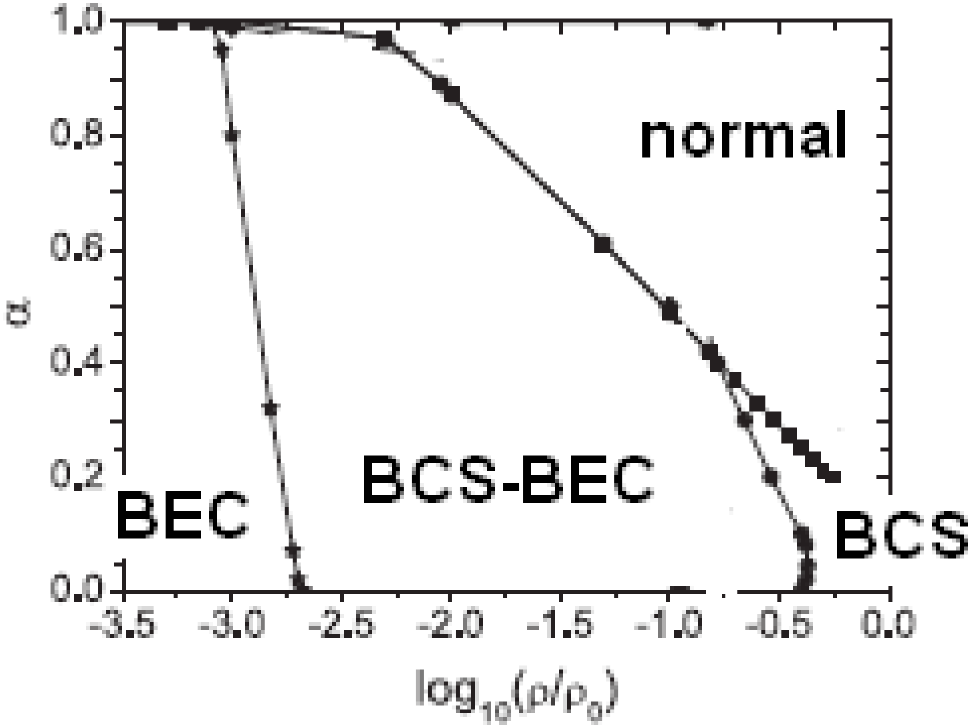
Shijun Mao, Xuguang Huang, PZ, 2009

asymmetric nuclear matter with both np and nn and pp pairings

density-dependent contact interaction (Garrido et al, 1999)

$$V(\mathbf{x},\mathbf{x}') = v\left(1-\eta\left\{\frac{\rho[(\mathbf{x}+\mathbf{x}')/2)]}{\rho_0}\right\}^{\gamma}\right)\delta(\mathbf{x}-\mathbf{x}'),$$

and density-dependent nucleon mass (Berger, Girod, Gogny, 1991)



by calculating the three coupled gap equations, there exists only np pairing BEC state at low density and no nn and pp pairing BEC states.

BCS-BEC in Color Superconductivity

Lianyi He, PZ, 2007

order parameters of spontaneous chiral and color symmetry breaking

$$\sigma = \langle \bar{\psi}\psi \rangle \qquad \Delta = \Delta^3 = \langle \bar{\psi}^C_{i\alpha}\varepsilon^{ij}\varepsilon^{\alpha\beta 3} i\gamma_5 \psi_{j\beta} \rangle$$

color breaking from SU(3) to SU(2)

quarks at mean field and mesons and diquarks at RPA

quark propagator in 12D Nambu-Gorkov space

$$\Psi = \begin{pmatrix} \psi_{u1} \\ \psi^C_{d2} \\ \psi_{d2} \\ \psi^C_{u1} \\ \psi_{d1} \\ \psi^C_{u2} \\ \psi_{u2} \\ \psi^C_{d1} \\ \psi_{u3} \\ \psi^C_{u3} \\ \psi_{d3} \\ \psi^C_{d3} \end{pmatrix}$$

$$\bar{\Psi} = \left(\bar{\psi}_{u1} \quad \bar{\psi}^C_{d2} \quad \bar{\psi}_{d2} \quad \bar{\psi}^C_{u1} \quad \bar{\psi}_{d1} \quad \bar{\psi}^C_{u2} \quad \bar{\psi}_{u2} \quad \bar{\psi}^C_{d1} \quad \bar{\psi}_{u3} \quad \bar{\psi}^C_{u3} \quad \bar{\psi}_{d3} \quad \bar{\psi}^C_{d3} \right)$$

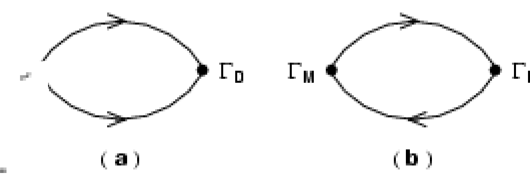
$$S = \begin{pmatrix} S_A & & & & & \\ & S_B & & & & \\ & & S_C & & & \\ & & & S_D & & \\ & & & & S_E & \\ & & & & & S_F \end{pmatrix}$$

$$S_I = \begin{pmatrix} G_I^+ & \Xi_I^- \\ \Xi_I^+ & G_I^- \end{pmatrix} \qquad I = A, B, C, D, E, F$$

$$E_k = \sqrt{\vec{k}^2 + M_q^2} \qquad M_q = m_0 - 2G_S\sigma$$

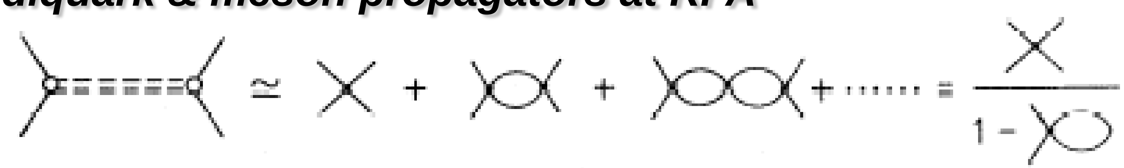
$$E_\Delta^\pm = \sqrt{(E_k \pm \mu)^2 + (2G_D\Delta)^2}$$

diquark & meson polarizations



Π_D $\qquad$ Π_M

diquark & meson propagators at RPA



BCS-BEC with Color Neutrality

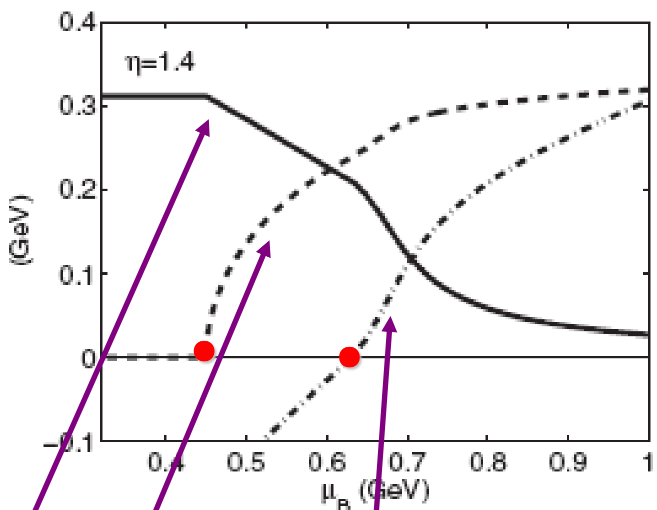
Lianyi He, PZ, 2007

gap equations for chiral and diquark condensates at T=0

$$\begin{cases} m - m_0 = 8G_s m \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_p} \left[\frac{E_k - \mu_B/3}{E_\Delta^-} + \frac{E_k + \mu_B/3}{E_\Delta^+} + \Theta\left(E_k - \mu_B/3\right) \right] \\ \Delta = 8G_d \Delta \int \frac{d^3k}{(2\pi)^3} \left[\frac{1}{E_\Delta^-} + \frac{1}{E_\Delta^+} \right] \end{cases}$$

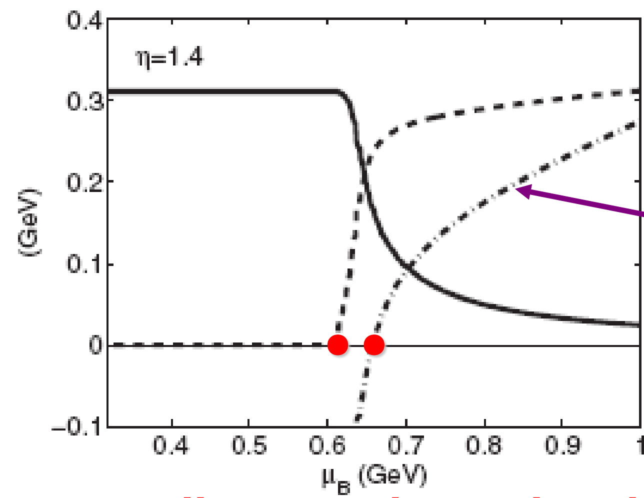
to guarantee color neutrality, we introduce color chemical potential:

$$\mu_r = \mu_g = \mu_B/3 + \mu_8/3, \qquad \mu_b = \mu_B/3 - 2\mu_8/3$$



m Δ $\mu_B/3 - m$ $\eta = G_d/G_s$

there exists a BCS-BEC crossover

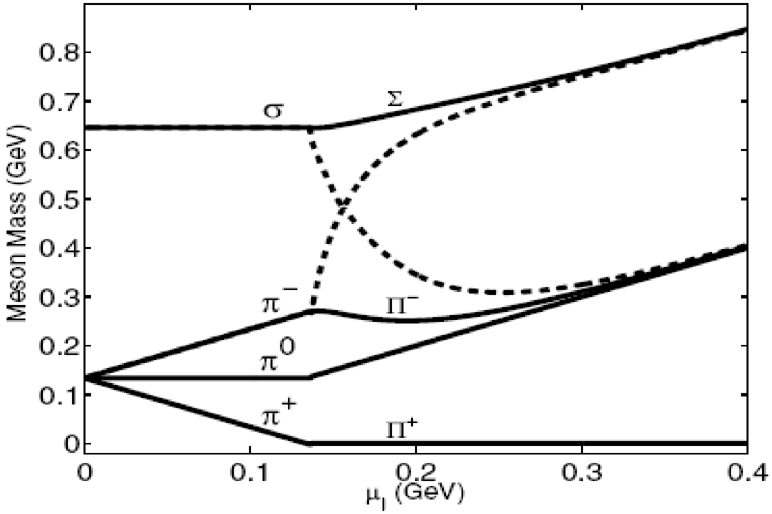


$\mu_r - m$

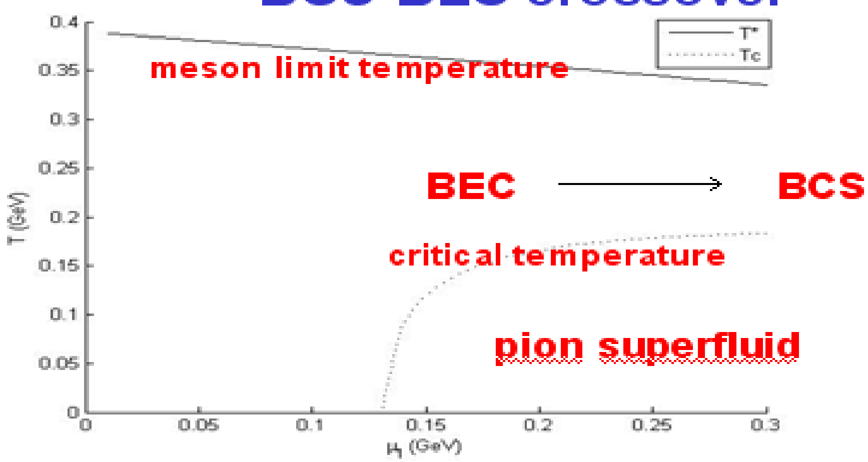
color neutrality speeds up the chiral restoration and reduces the BEC region

BCS-BEC in Pion Superfluidity

meson mass, Goldstone mode



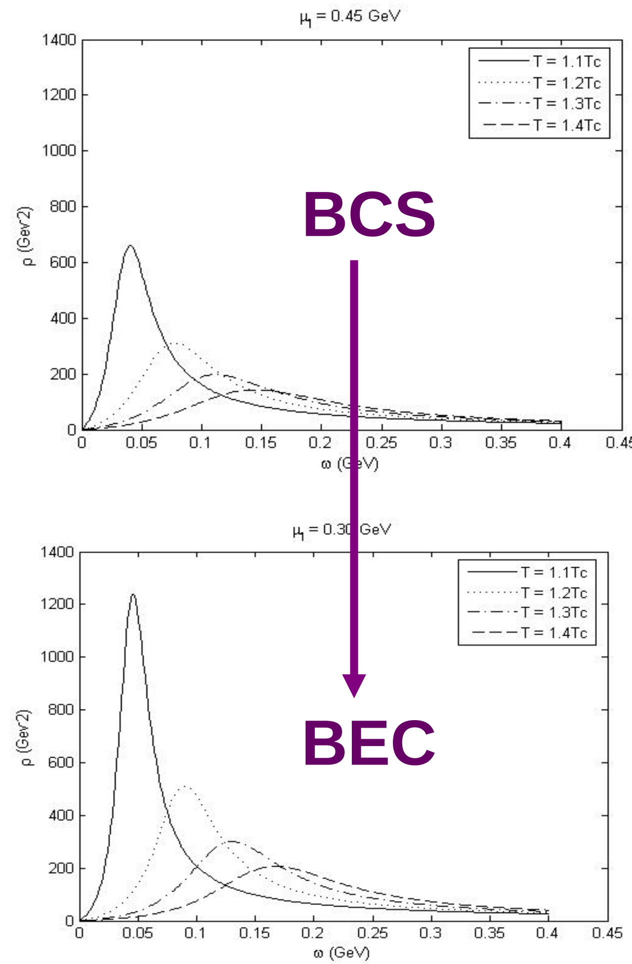
BCS-BEC crossover



Gaofeng Sun, Lianyi He, PZ, 2007

meson spectra function

$$\rho(\omega,\vec{k}) = -2\,\mathrm{Im}\,D(\omega,\vec{k})$$



Conclusions

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* BCS-BEC crossover is a general phenomena from cold atom gas to quark matter.

* BCS-BEC crossover is closely related to the QCD key problems: vacuum, color symmetry, chiral symmetry, isospin symmetry ……

* BCS-BEC crossover in color superconductivity and pion superfluidity is not induced by simply increasing the coupling constant of the attractive interaction, but by changing the corresponding charge number.

* there are potential applications in heavy ion collisions (at CSR/Lanzhou, FAIR/GSI, NICA/JINR and RHIC/BNL) and compact stars.

thanks for your patience

backups

vector meson coupling and magnetic instability

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vector-meson coupling

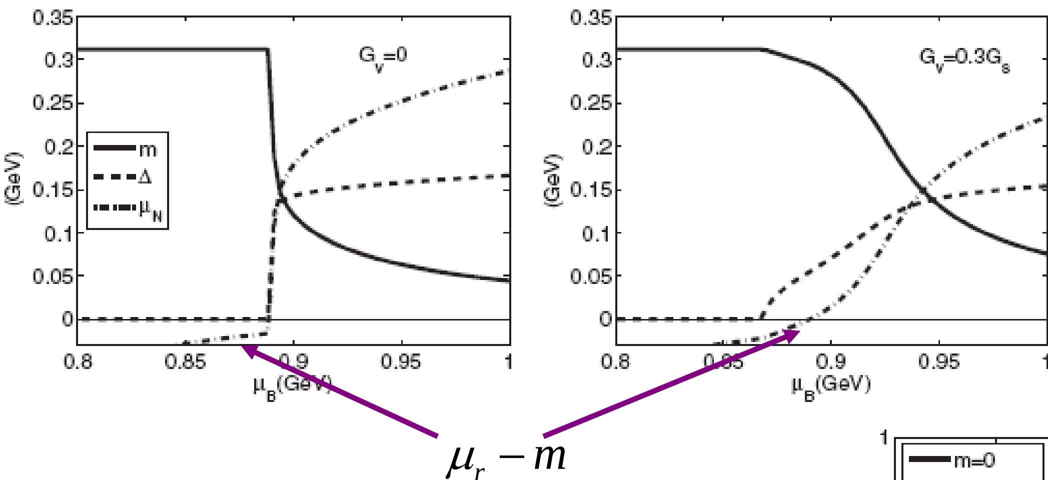
$$L_V = -G_V\left[\left(\bar{\psi}\gamma_\mu\psi\right)^2 + \left(\bar{\psi}\gamma_\mu\gamma_5\tau\psi\right)^2\right]$$

vector condensate

$$\rho_V = 2G_V\left\langle\bar{\psi}\gamma_0\psi\right\rangle$$

gap equation

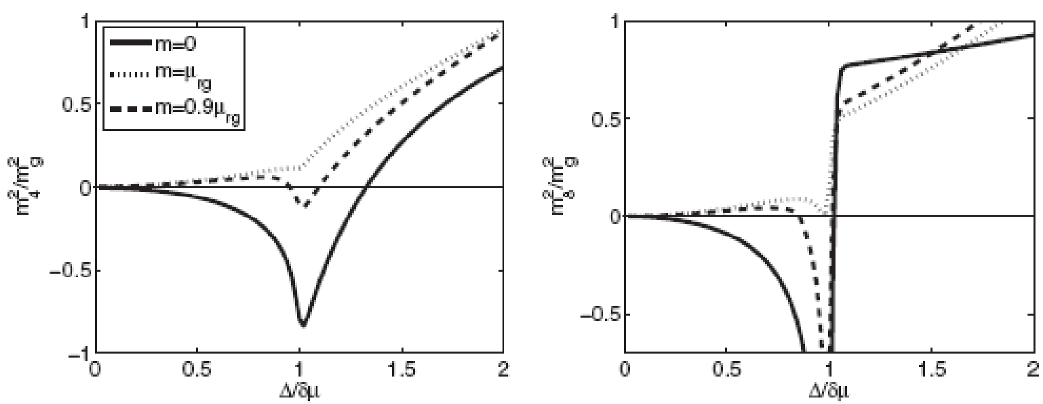
$$\rho_V = 8G_V\int\frac{d^3k}{(2\pi)^3}\left[\frac{E_k+\mu_B/3}{E_\Delta^+} - \frac{E_k-\mu_B/3}{E_\Delta^-} + \Theta\left(-E_k+\mu_B/3\right)\right]$$



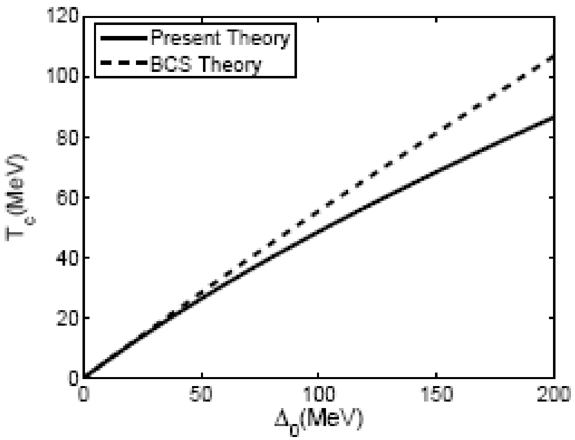
$\eta = 1$

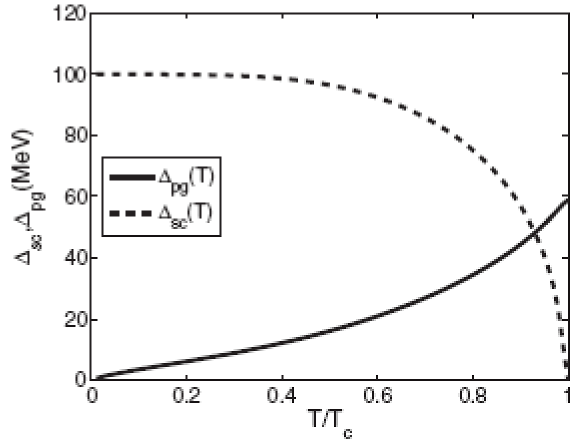
vector meson coupling slows down the chiral symmetry restoration and enlarges the BEC region.

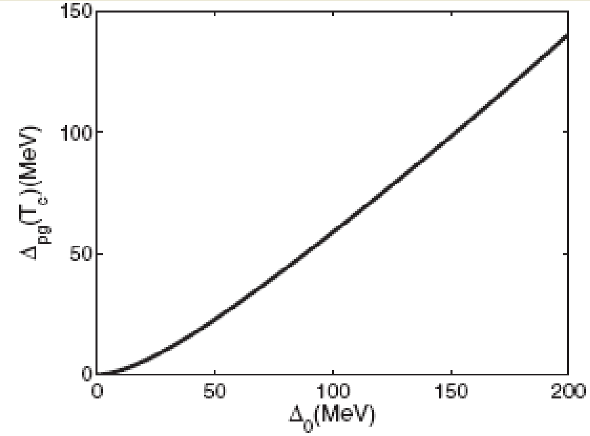
Meissner masses of some gluons are negative for the BCS Gapless CSC, but the magnetic instability is cured in BEC region.



beyond mean field







$\Delta_0 = \Delta(T=0)$ ***is determined by the coupling and chemical potential***

$$\Delta_0 = 100 - 200 \text{ MeV} \leftrightarrow \mu_q = 300 - 500 \text{ MeV}$$

- ***going beyond mean field reduces the critical temperature of color superconductivity***
- ***pairing effect is important around the critical temperature and dominates the symmetry restored phase***

pion superfluid

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NJL with isospin symmetry breaking

$$L_{NJL}=\bar\psi\left(i\gamma^\mu\partial_\mu-m_0+\mu\gamma_0\right)\psi+G\left(\left(\bar\psi\psi\right)^2+\left(\bar\psi i\tau_i\gamma_5\psi\right)^2\right)$$

quark chemical potentials

$$\mu=\begin{pmatrix}\mu_u & 0\\ 0 & \mu_d\end{pmatrix}=\begin{pmatrix}\mu_B/3+\mu_I/2 & 0\\ 0 & \mu_B/3-\mu_I/2\end{pmatrix}$$

chiral and pion condensates with finite pair momentum

$$\sigma=\langle\bar\psi\psi\rangle=\sigma_u+\sigma_d,\quad \sigma_u=\langle\bar u u\rangle,\quad \sigma_d=\langle\bar d d\rangle$$

$$\pi_+=\sqrt{2}\langle\bar u i\gamma_5 d\rangle=\frac{\pi}{\sqrt{2}}e^{2i\vec q\cdot\vec x},\quad \pi_-=\sqrt{2}\langle\bar d i\gamma_5 u\rangle=\frac{\pi}{\sqrt{2}}e^{-2i\vec q\cdot\vec x}$$

quark propagator in MF

$$S^{-1}(p,\vec q)=\begin{pmatrix}\gamma^\mu p_\mu-\vec\gamma\cdot\vec q+\mu_u\gamma_0-m & 2iG\pi\gamma_5\\ 2iG\pi\gamma_5 & \gamma^\mu k_\mu+\vec\gamma\cdot\vec q+\mu_d\gamma_0-m\end{pmatrix}\qquad m=m_0-2G\sigma$$

thermodynamic potential and gap equations:

$$\Omega=G(\sigma^2+\pi^2)-\frac{T}{V}\mathrm{Tr}\,\mathrm{Ln}\,S^{-1}$$

$$\frac{\partial\Omega}{\partial\sigma_u}=0,\ \frac{\partial^2\Omega}{\partial\sigma_u^2}\ge 0,\qquad \frac{\partial\Omega}{\partial\sigma_d}=0,\ \frac{\partial^2\Omega}{\partial\sigma_d^2}\ge 0,\qquad \frac{\partial\Omega}{\partial\pi}=0,\ \frac{\partial^2\Omega}{\partial\pi^2}\ge 0,\qquad \frac{\partial\Omega}{\partial q}=0,\ \frac{\partial^2\Omega}{\partial q^2}\ge 0$$

mesons in RPA

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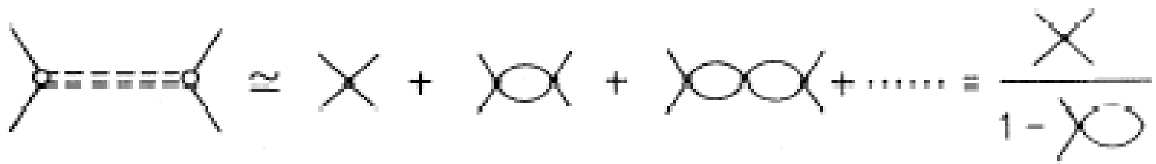
meson propagator D at RPA



considering all possible channels in the bubble summation

meson polarization functions

$$\Pi_{mn}(k) = i\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_m^* S(p+k)\Gamma_n S(p)\right) \qquad \Gamma_m = \begin{cases} 1, & m=\sigma \\ i\tau_+\gamma_5, & m=\pi_+ \\ i\tau_-\gamma_5, & m=\pi_- \\ i\tau_3\gamma_5, & m=\pi_0 \end{cases}$$

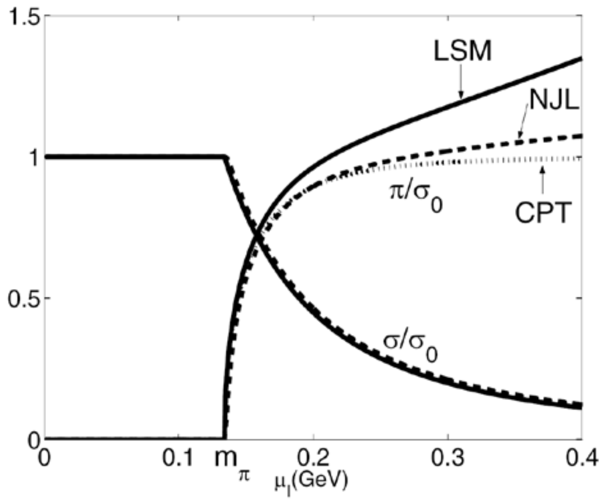
pole of the propagator determines meson masses M_m

$$\det\begin{pmatrix} 1-2G\Pi_{\sigma\sigma}(k) & -2G\Pi_{\sigma\pi_+}(k) & -2G\Pi_{\sigma\pi_-}(k) & -2G\Pi_{\sigma\pi_0}(k) \\ -2G\Pi_{\pi_+\sigma}(k) & 1-2G\Pi_{\pi_+\pi_+}(k) & -2G\Pi_{\pi_+\pi_-}(k) & -2G\Pi_{\pi_+\pi_0}(k) \\ -2G\Pi_{\pi_-\sigma}(k) & -2G\Pi_{\pi_-\pi_+}(k) & 1-2G\Pi_{\pi_-\pi_-}(k) & -2G\Pi_{\pi_-\pi_0}(k) \\ -2G\Pi_{\pi_0\sigma}(k) & -2G\Pi_{\pi_0\pi_+}(k) & -2G\Pi_{\pi_0\pi_-}(k) & 1-2G\Pi_{\pi_0\pi_0}(k) \end{pmatrix}_{k_0=M_m,\vec{k}=0} = 0$$

mixing among normal σ, π_+, π_- in pion superfluid phase,

the new eigen modes $\bar{\sigma}, \bar{\pi}_+, \bar{\pi}_-$ are linear combinations of σ, π_+, π_-

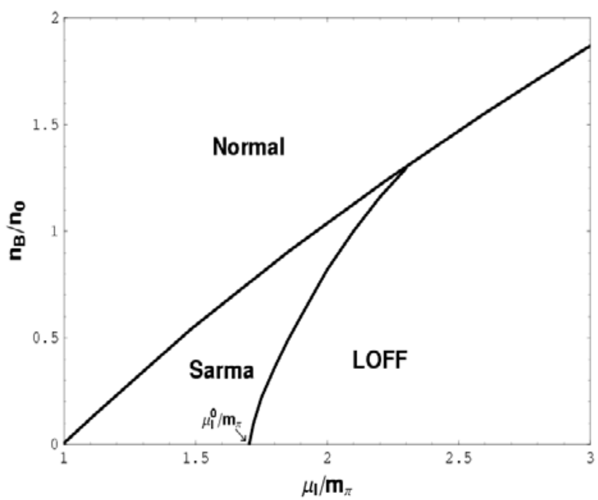
phase diagram of pion superfluid



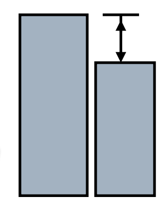
chiral and pion condensates at $T=\mu_B=\vec{q}=0$ in NJL, Linear Sigma Model and Chiral Perturbation Theory, there is no remarkable difference around the critical point.

analytic result:
critical isospin chemical potential for pion superfluidity is exactly the pion mass in the vacuum:

$$\mu_I^c = m_\pi$$



pion superfluidity phase diagram in $\mu_I-\mu_B$ plane at T=0



μ_I***: average Fermi surface***

$\mu_B(n_B)$***: Fermi surface mismatch***

homogeneous (Sarma, $\vec{q}=0$) and inhomogeneous pion superfluid (LOFF, $\vec{q}\neq 0$)

magnetic instability of Sarma state at high average Fermi surface leads to the LOFF state