# Derivation of a $1/r^2$ Potential Term for QQ-bar in a Potential Model

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**Based on**

- Isgur-Wise Function in a semi-relativistic potential model,
  Prog. Theor. Phys. 118 (2007) . 1087-1114 (hep-ph/0703158)
- **Structure of mass gap between two spin multiplets,**
  Phys. Lett. B659 (2007) 593 **(**hep-ph/0710.0325)
- **Radial Excitations of Heavy-Light System,**
  Eur. Phys. J. C31(2007) 701
- **New Heavy-Light Mesons $Q\bar{q}$ ,**
  Prog. Theor. Phys. 51 (2007) 1077 (hep-ph/0605019)
- **$0^+$ and $1^+$ States of $B$ and $B_s$ Mesons,**
  Phys. Lett. B606 (2005) 329 (hep-ph/0411034)
- **Spectroscopy of heavy mesons expanded in $1/m_Q$ ,**
  Phys. Rev. D56 (1997) 5646 (hep-ph/9702366)

# Contens

1. What is a reason why we succeed in predicting/reproducing mass levels
2. Definition of true vacuum
3. Various transformations to derive models
4. Make a list of models depending on if they satisfy criteria or not, i.e., starting from the dominant vacuum, including negative energy states, etc.
5. Derive $1/r^2$ term for $Q\bar{Q}$ state by using the Foldy-Tani-Wouthuysen transformation : with a positive coefficient contrary to Koma-Koma-Wittig results
6. Recent/Future experiments to confirm our model

# Where the point is

- $D_{s0}^{*}(2317)$ & $D_{s1}^{'}(2460)$ were found by BaBar and CLEO
- Conventional potential model fails to explain only these two (GIK)
  - Do we need new dynamics?
  - Do we need a new concept ?
  - Do we need a tetra-quark?
  - May need to explain only these two by some other mechanism (e.x., molecular state) : ***typical attitude these days***

- Numerical results of our model
  - Succeed in predicting/reproducing the experiments for heavy hadrons
- Reasons why our model succeeds while others do not
  - ***true vacuum in HQET ?!***

# True Vacuum in HQET

- true vacuum in heavy hadrons
  - Theory should be expanded around it
  - Transformation should be unitary
- Approximation is different from transformation
  - Intuitive approximation : does not change the vacuum -> in principle the same parameters as before; higher order interactions are the same as before
  - Transformation : change the vacuum -> change light quark mass values (reason why $m_u$=300 MeV in GIK while $m_u$=85 MeV in MMS); higher order interactions become different from the original
- Classify all models into those with approximation or transformation

# HQET in Field Theory

- HQET
  - Approximation first proposed by Eichten and Hill
    - be at rest = propagator in configuration space $\propto \delta^3\left(x - x'\right)\theta(t - t')$
    - propagator in momentum space $\propto 1/(p_0 + i\varepsilon) \rightarrow 1/(v\ \ p + i\varepsilon)$
- Georgi Transformation for the wave function
  - velocity-dependent Lagrangian in a Lorentz invariant form
  - $L = \bar{\psi}\left(i\not\partial - m\right)\psi = \bar{\psi}_v iv\ \partial\psi_v$
  - To obtain this Lagrangian, Georgi propopsed the transformation
  - 1) $\psi \rightarrow \exp(im\not{v}\ v\ \ x)\psi_v$ non-unitary
  - 2) $\psi \rightarrow \frac{1+\not{v}}{2}\exp(imv\ \ x)\psi_v$ unitary (improved) with projection op.

    Separation of $\psi_{v+}$ and $\psi_{v-}$ in the Lagrangian

# Georgi Transformation

- $\psi \to \frac{1+\not v}{2}\exp(imv\ x)\psi_v$ improved expression
  - $\exp(imv\ x)\psi$ momentum shift operator (unitary)
  - $i\not\partial \to i\not\partial + m\not v \to i\not\partial + m$
  - because projection operator is multiplied
  - $\frac{1+\not v}{2}\not v = \frac{1+\not v}{2}$ i.e., $\not v \to 1$ and also $\not\partial \to v\ \partial$
- Projection operator is inserted by hand (not derived)
- Actually it is derived by taking the transformation
  - $\boxed{\psi \to \exp(imv\ x)\psi_v}$ proposal (without projection operator)
  - $L = \bar\psi\left(i\not\partial - m\right)\psi = \bar\psi_v\left(i\not\partial + m\not v - m\right)\psi_v$
  - $\left(m\not v - m\right)\psi_v = -m(1-\not v)\psi_v = 0$ lowest order in 1/***m***, or $\frac{1+\not v}{2}\psi_v = \psi_v$
  - using this, we have in the lowest order
  - $\boxed{L = \bar\psi\left(i\not\partial - m\right)\psi = \bar\psi_v iv\ \partial\psi_v}$ the same form as the original

# HQET in Potential Model

- HQET in our model
  - lowest approximation adopted by MM (Matsuki and Morii)
    - Dominant wave function = positive energy state = $Q_+$
    - Heavy quark free energy $Q^\dagger\left(ip\,\alpha + m\beta\right)Q \to Q^\dagger{}_{FWT}\sqrt{m^2+p^2}\,\beta Q_{FWT}$
- Foldy-Tani-Wouthuysen Transformation for the wave function
  - dominant positive energy state (diagonalize kinetic term)

    $E(1-\beta)\psi_{FTW} = 0$
  - To obtain this kinetic term for heavy quark, FWT gives
  - $\psi \to \exp(W\,\hat{p}\,\gamma)\psi_{FTW}$ $\quad\tan W = p/(m+E)$ : our choice
  - Expanding the Hamiltonian, energy, and wave function in 1/***m***, we obtain $m(1-\beta)\psi_{FTW} = 0$ or $\frac{1+\beta}{2}\psi_{FTW} = \psi_{FTW}$

    and DsJ mass values lower than the thresholds (effective light quark mass : $m_u$+b=85 MeV after chiral symmetry is broken and before heavy quark symmetry is not broken)

# Lowest FTW Transformation

- Consider the following transformation instead of FTW
  - first order in 1/***m*** (unitary)

$$\psi \rightarrow \exp(W\ \hat{p}\ \gamma)\psi_{LFTW} \quad \begin{pmatrix} 1 & -\dfrac{\sigma\ p}{2m} \\ \dfrac{\sigma\ p}{2m} & 1 \end{pmatrix} \psi_{LFTW}, \quad \tan W = p/(2m)$$

- which gives

$$U_{LFWT}(-p)\left(\alpha\cdot p+\beta m\right)U_{LFWT}^{-1}(p) = \beta\left(m+\frac{p^2}{2m}\right)-\frac{p^2}{m^2}\alpha\cdot p+O(x^3)$$

- Up to 1/***m***, these terms give the same interactions as those of FTW, hence we could start from this transformation for the heavy hadrons (including at least one heavy quark) if calculating only up to 1/***m***.

# Bloch Transformation

- Possible to separate positive and negative energy state completely in a potential model = Bloch transformation
  - TM and K. Seo or Appendix of MMS (PTP, 51 1077 (2007))
- which gives complete projection on positive energy eigenstate
  - (too complicated to show here)

$$H_{eff}(P_+P_+\psi_j) = E_j(P_+P_+\psi_j)$$

- Solvable up to 1/***m***, but it includes $V^2$ terms, i.e., square of the coupling constant $\alpha_s^2$
  - $V = -\frac{4\alpha_s}{3}\frac{1}{r}$ one gluon exchange term

# Free Fermion Field Transformation

- Consider the free fermion field transformation

$$\psi \to \begin{pmatrix} \sqrt{\dfrac{E+m}{2E}} & 0 \\ \dfrac{\sigma \;\, p}{\sqrt{2E(E+m)}} & 0 \end{pmatrix} \psi_{FFF} \qquad \begin{pmatrix} 1 & 0 \\ \dfrac{\sigma \;\, p}{2m} & 0 \end{pmatrix} \psi_{FFF} = \begin{pmatrix} 1 & -\dfrac{\sigma \;\, p}{2m} \\ \dfrac{\sigma \;\, p}{2m} & 1 \end{pmatrix} \frac{1+\beta}{2} \psi_{FFF}$$

- non-unitary
- Adopted by many people, e.g., Morishita, Kawaguchi and Morii (PRD37, 159 (1988)), Zeng, Van Orden and Roberts (PRD52, 5229 (1995)), etc.
- They did not derive dominance of a positive energy state
- 1) the former paper mixed different order of interactions in 1/***m***, and 2) the latter paper adopted constituent light quark mass values

# Various Models for $D_{(s)J}$

| Method | Authors | Successful or Not | True V.C. | Perturbation |
|---|---|---|---|---|
| Conventional P. M. | Godfrey, Isgur, Kokoski | 2.48, 2.55 GeV | No | Yes |
| Rel. Potential Model | Morishita, Kawaguchi, Morii | 2.525, 2.593 | No | Yes |
| Rel. Potential Model | Matsuki, Morii, Sudoh | 2.325, 2.467 | Yes | Yes |
| Rel. Potential Model (Bloch) | Matsuki, Seo | 2.297, 2.544 | Yes | Yes |
| BS Model | Zeng, Orden, Roberts | 2.38, 2.51 | No | No |
| Another R. Potential Modle | Faustov, Galkin, Ebert | 2.463, 2.535 | No | Yes |
| Tetra-Quark | Cheng, Hou,Terasaki | Yes (Qualitative) | N/A | N/A |
| DK Molecule | Close et al. | Yes (Qualitative) | N/A | N/A |
| Coupled Channel Method | Beveren, Rupp | Yes (Wide Range) | N/A | N/A |
| Effective Lagrangian | Bardeen et al. | Phenomenology | Yes | Yes |

N/A=Not Applicable
Threshold values:
D+K=2.367, D*+K=2.505 GeV

- Potential Model
  - Bloch Method (Matsuki, Seo) : too accurate
  - Bardeen et al. : perturbation in effective field theory but no calculation

# Application to QQ-bar

- Apply FTW transformation to QQ-bar (m1=m2)
  - Obtain 1/r$^2$ potential term with positive sign
  - No problem to solve it at the origin

$$H = \left(\alpha_1 \cdot p + \beta_1 m_1\right) + \left(-\alpha_2 \cdot p + \beta_2 m_2\right) + H_{int}$$

$$H_{int} = \beta_1 \beta_2 S + \left\{1 - \frac{1}{2}\left[\alpha_1 \cdot \alpha_2 + \left(\alpha_1 \cdot n\right)\left(\alpha_2 \cdot n\right)\right]\right\} V$$

$$\left(U_{total} H U_{total}^{-1}\right) \otimes \left(U_{total} \psi\right) = E U_{total} \psi, \quad U_{total} = U_c U_{FWT\,2} U_{FWT\,1}$$

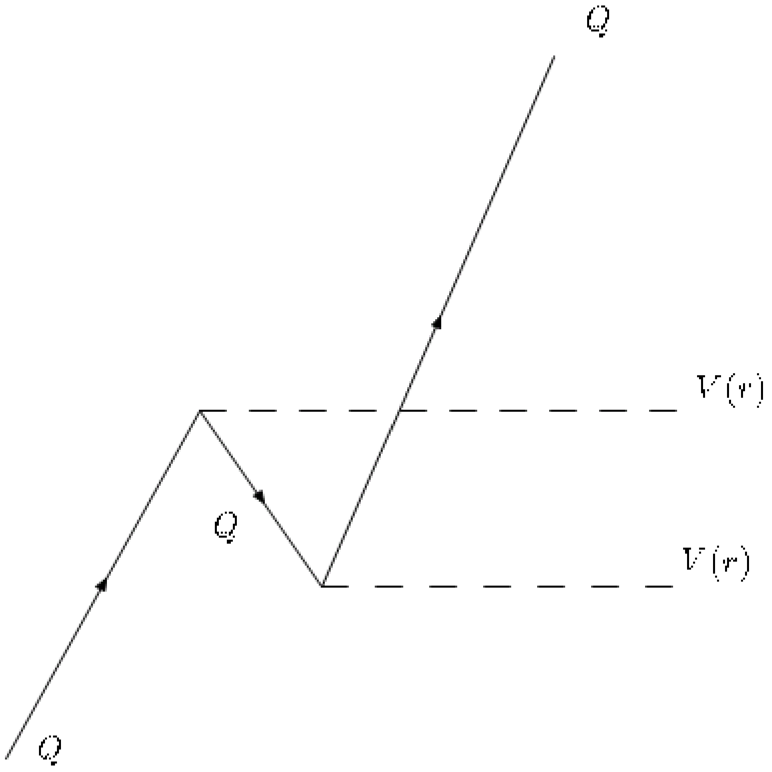
1/r$^2$ term

- Non-trivial equation is given by

$$\frac{p^2}{m} \psi_{0+1}^{+,+} + \left[ S(r) + V(r) + \frac{V^2}{4m} \left(\sigma_1 \ \sigma_2\right)^2 \otimes \right] \psi_{0+1}^{+,+} = E_{0+1} \psi_{0+1}^{+,+}$$

# $V(r)^2$ Term

- $V^2$ term may be drawn in a figure as follows



# Mass difference of Xc and $\eta_c$

- Linear+Coulomb vs. linear + modified Coulomb
    - Morishita, Oka, Kaburagi, Munakata and T. Kitazoe, Z. Phys. C 19 (1983) 167.

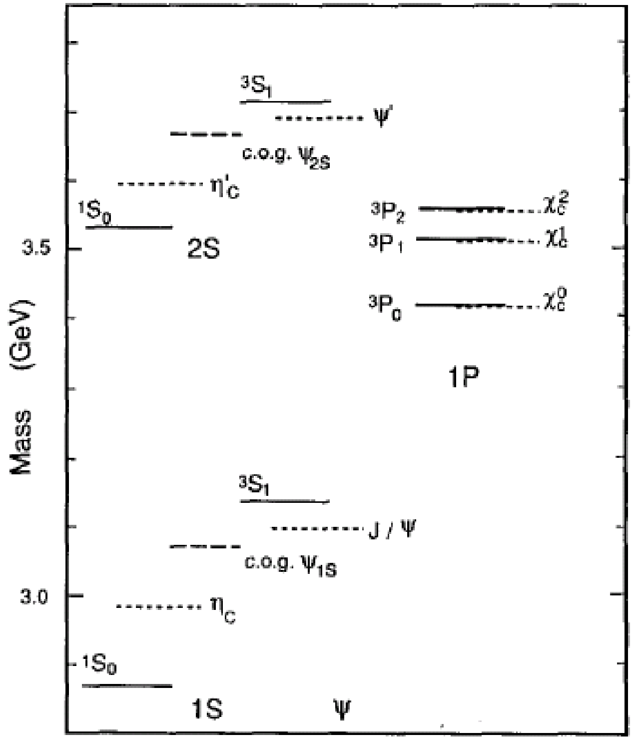
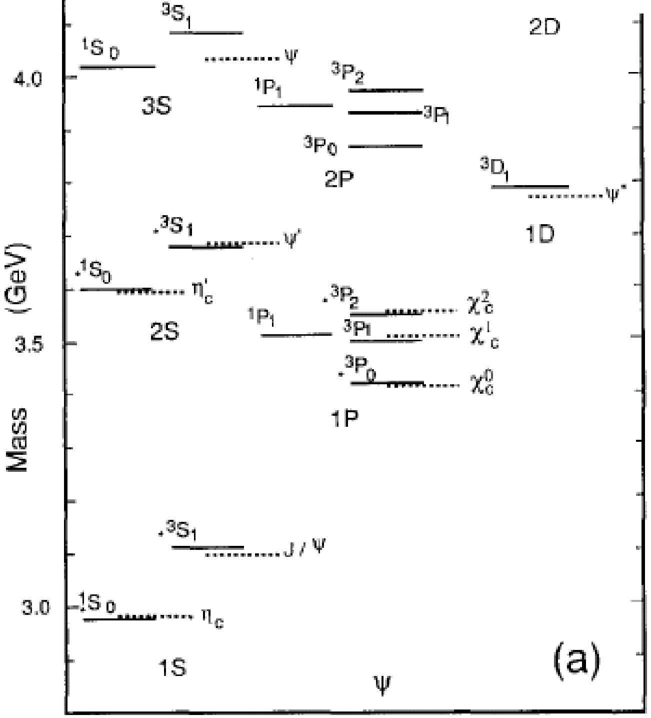


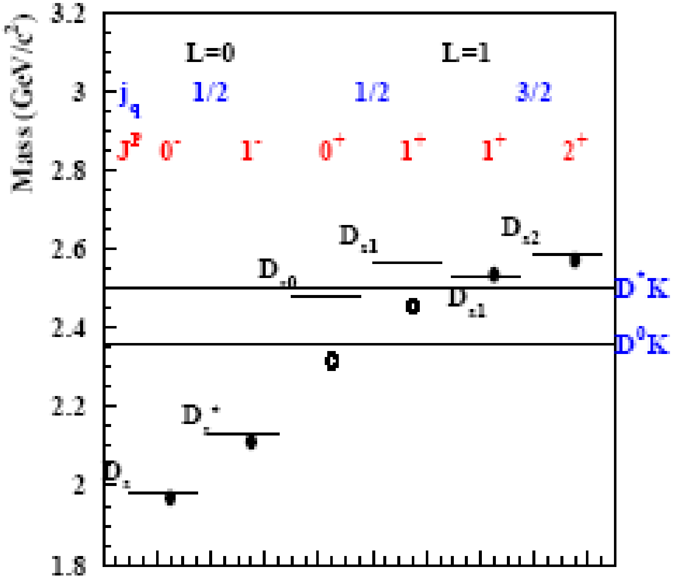
Fig. 2.9. Ψ spectra in the linear plus Coulomb potential model

$$V(r) = -\frac{4}{3}\frac{\alpha_s^0}{r}\left\{1 - \exp\left[-\left(\frac{r}{R_c}\right)^\sigma\right]\right\}$$

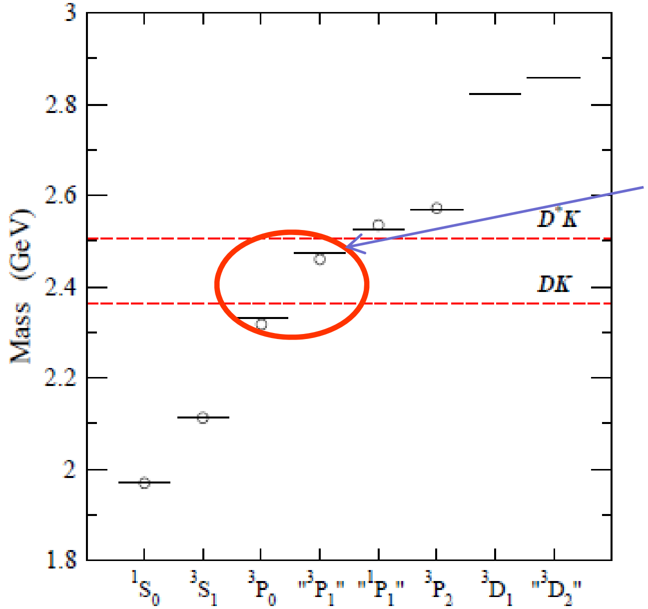


# Mass Spectra of $D_{sJ}$

- Successful prediction/reproduction of $D_s$ mass spectra using our semi-relativistic potential model
  - Lowering $0^+$ and $1^+$ of $D_{s0}^*(2317)$ and $D_1'(2427)$ compared with other potential models



**prediction by conventional potential model** (Godfrey & Kokski, PRD43, 1679 (1991))

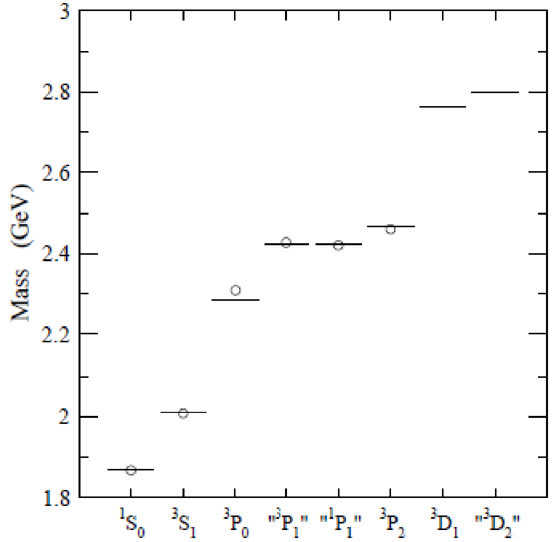


Below threshold

**prediction by our semi-relativisitic potential model** (Prog. Theor. Phys.117 (2007) 1077)

# Other Mass Spectra of Our Model

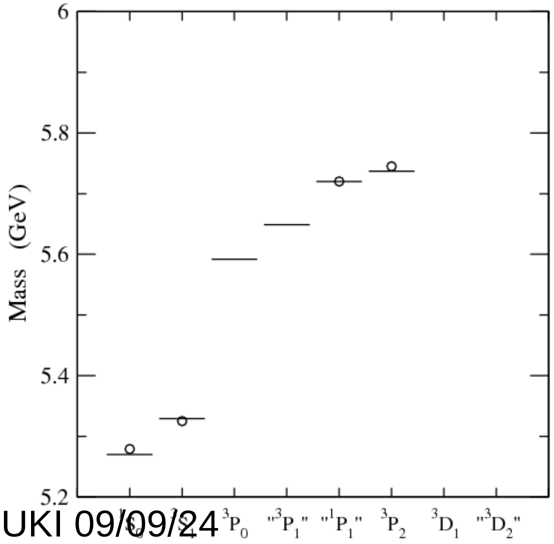
**D**



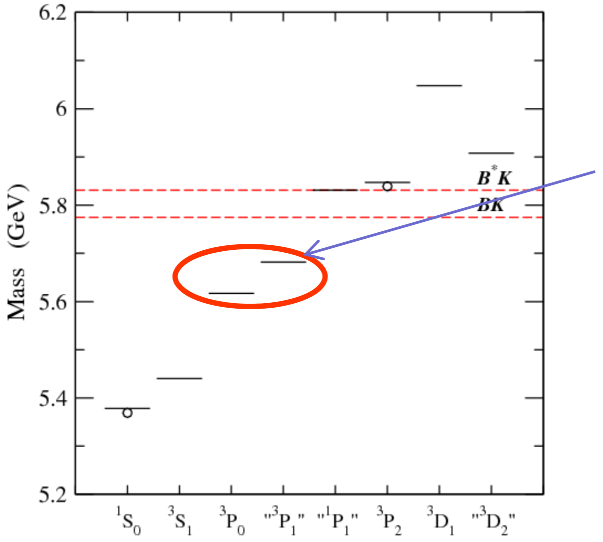
Successful reproduction of the following spectra

- ●$D_0^*(2308)$ and $D_1'(2427)$ by Belle
- ●$D_{s0}(2860)$ and $D_s^*(2715)$ by BaBar & Belle (n=2; $0^+$ and $1^-$ states of $D_s$)
- ●$B_1(5720)$ and $B_2^*(5745)$ by D0 (1+ and 2+ states of B)
- ●$B_{s2}^*(5839)$ by D0 ($2^+$ state of $B_s$)

**B**



**$B_s$**



Below threshold

# Our Numerical Values

| $J^P$ | $D(0^-)$ | $D(1^-)$ | $D(0^+)$ | $D(1^+)$ |
|---|---|---|---|---|
| observed | 1867 | 2008 | 2308 | 2427 |
| predicted | 1869 | 2011 | 2283 | 2421 |

These two values determine whether ours or molecular model succeeds or not

Below ***BK*/*B*K*** threshold

| $J^P$ | $D_s(0^-)$ | $D_s(1^-)$ | $D_s(0^+)$ | $D_s(1^+)$ | $D_s(1^+)$ | $D_s(2^+)$ |
|---|---|---|---|---|---|---|
| observed | 1969 | 2112 | 2317 | 2460 | 2535 | 2572 |
| predicted | 1967 | 2120 | 2325 | 2467 | 2525 | 2568 |

Recent CDF data

| $J^P$ | $B(0^-)$ | $B(1^-)$ | $B(0^+)$ | $B(1^+)$ | $B(1^+)$ | $B(2^+)$ |
|---|---|---|---|---|---|---|
| observed | 5279 | 5325 | – | – | 5720 | 5745 |
| predicted | 5270 | 5329 | 5621 | 5663 | 5720 | 5737 |

| $J^P$ | $B_s(0^-)$ | $B_s(1^-)$ | $B_s(0^+)$ | $B_s(1^+)$ | $B_s(1^+)$ | $B_s(2^+)$ |
|---|---|---|---|---|---|---|
| observed | 5369 | – | – | – | 5829 | 5839 |
| predicted | 5378 | 5440 | 5617 | 5682 | 5831 | 5847 |

# Radial Excitations of Ds

n=2 radial excitation : Eur. Phys. J. A 31, 701 (2007)

| $J^P$ | $D_s(0^-)$ | $D_s(1^-)$ | $D_s(0^+)$ | $D_s(1^+)$ | $D_s(1^+)$ | $D_s(2^+)$ |
|---|---|---|---|---|---|---|
| observed | – | 2715 | 2856(?) | 3040 | – | – |
| predicted | 2563 | 2755 | 2837 | 3082 | 3094 | 3157 |

could be n=1,
higher state $3^+$

nearly two peaks

Very recent
BaBar data
by Palano et. al

- Predicted in 2007 “1+” with 3082 MeV as a radial excitation of Ds1(2460) (T. Matsuki, T. Morii, and K. Sudoh, Eur. Phys. J.A 31, 701 (2007))

# Valuse of 0+ and 1+ of Bs

| $J^P$ | $B_s(0^-)$ | $B_s(1^-)$ | $B_s(0^+)$ | $B_s(1^+)$ | $B_s(1^+)$ | $B_s(2^+)$ |
|---|---|---|---|---|---|---|
| observed | 5369 | – | – | – | 5829 | 5839 |
| predicted | 5378 | 5440 | 5617 | 5682 | 5831 | 5847 |

B+K  = 5773 MeV
B*+K= 5819 MeV

Recent CDF data

Below ***BK/B*K*** threshold

- These two values determine whether  ours or molecular model is preferable or not
- Mass gap between predictions and threshold values
  - Ours :  ~ 140 MeV
  - Molecular (B+K/B*+K) assumption :  ~ 40 MeV
- Wait for experiments of LHCb

# Summary

- Clarify the difference of our model HQET and other potential models together with field theoretical HQET
  - Principle to construct a model = use transformation to derive dominant positive energy state in HQET
- Recent data DsJ(3040) by BaBar is very close to a radial excitation of Ds(1+) predicted as 3082 MeV in 2007 by our model
- Succeed in deriving $1/r^2$ potential term for $Q\bar{Q}$
  - Positive coefficient contrary to that by Koma-Koma-Wittig
  - Safe to solve the Schroedinger eq. near the origin