NEW RESULTS ON α_s FROM THE LATTICE AND HADRONIC τ DECAYS

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OUTLINE

- *Context/tension between α_s from lattice, τ decay*
- *Recent updates of UKQCD/HPQCD lattice approach*
- *New results on the hadronic τ decay determination*
- *Future directions/issues*

CONTEXT ETC.

- HPQCD/UKQCD, PRL95 (2005) 052002: perturbative analysis of UV-sensitive lattice observables [dominant input to PDG08 assessment $\alpha_s(M_Z) = 0.1176(20)$]

$$[\alpha_s(M_Z)]_{latt} = 0.1170(12)$$

- ALEPH, OPAL [e.g., EPJC56 (2008) 305]: "(k,m) spectral weight" hadronic τ decay determination

$$[\alpha_s(M_Z)]_{\tau} = 0.1212(11)$$

- c.f. other recent determinations [Bethke+: 0908.1135]

Source	$\alpha_s(M_Z)$
Global EW fit	0.1193(28)
H1+ZEUS NLO inclusive jets	0.1198(32)
H1 high-Q^2 NLO jets	0.1182(45)
Non-singlet structure functions	0.1142(23)
NNLO+NLLA LEP event shapes	0.1224(39)
NNLO+NLLA JADE event shapes	0.1172(51)
$\Gamma[\Upsilon(1s) \to \gamma X]/\Gamma[\Upsilon(1s) \to X]$	0.1190(60)
Lattice PS $\bar{c}c$ correlator moments	0.1174(12)
$\sigma[e^+e^- \to hadrons]$ (2-10.6 GeV)	0.1190(110)
NNNLL ALEPH+OPAL thrust distributions	0.1172(21)

- expt'l determination errors large c.f. nominal lattice, τ

- Non-τ, non-HPQCD/UKQCD Bethke input weighted (naive) average: $\alpha_s(M_Z) = 0.1179(13)$

UPDATES OF HPQCD/UKQCD LATTICE

- Based on perturbative analyses of observables, O_k, measured on MILC (asqtad) $n_f = 2+1$ ensembles

- $O(\alpha_s^3)$ $D=0$ ($m_q = 0$) expansion

  $$[O_k]_{D=0} = D_k \alpha_T(Q_k) \left[1 + c_1^{(k)} \alpha_T(Q_k) + c_2^{(k)} \alpha_T^2(Q_k) + \cdots\right]$$

  with $Q_k = d_k/a$ the BLM scale for O_k

- D_k, $c_1^{(k)}$, $c_2^{(k)}$, d_k: Q. Mason et al. 3-loop lattice PT

- Original HPQCD/UKQCD analysis [PRL 95 (2005) 052002]: $a \sim 0.18$, 0.12, 0.09 fm ensembles

- HPQCD [PRD78 (2008) 114507], CSSM [PRD78 (2008) 114504] updates add new $a \sim 0.15$, 0.06 fm ensembles, one (am_ℓ, am_s) $a \sim 0.045$ fm ensemble (HPQCD only)(results dominated by finer ensembles)

- m_q-dependent NP contributions: linear m_q extrapolation/subtraction

- m_q-independent NP: estimate/subtract via LO $\langle aG^2 \rangle$ (+ fitted $D > 4$ for more long-distance-sensitive observables in 2008 HPQCD)

Some relevant details

- $D = 0$ to $O(\alpha_s^3)$ insufficient to account for observed scale dependence $\Rightarrow$ MUST fit additional HO term(s)

- 2008 HPQCD, CSSM: different $D = 0$ expansion parameter choices $\Rightarrow$ different (complementary) handling of residual HO perturbative uncertainties

- $m_q \rightarrow 0$ extrapolation very reliable:

  - many (am_ℓ, am_s) for $a \sim 0.12$ fm, very good linearity (plus good linearity for other a as well)

  - extrapolation very stable to added non-linear terms

- Re m_q-independent NP subtraction:
  - $\langle aG^2 \rangle = 0 \pm 0.012\ GeV^4$ (HPQCD), with independent fit for each O_k
  - $\langle aG^2 \rangle = 0.009 \pm 0.007\ GeV^4$ (CSSM), common input for all O_k
  - estimated $D = 4$ correction tiny for shortest-distance-sensitive observables (e.g., $log(W_{11})$, $log(W_{12})$)
  - After fitted m_q-independent NP subtractions, HPQCD observables with LARGE estimated $D = 4$ corrections yield α_s in good agreement with $log(W_{11})$ etc.

- COMPARISON OF HPQCD, CSSM RESULTS
  - Results for a selection of three least-NP and four most-NP observables
  - $\delta_{D=4} \equiv$ fractional change from scale dependence of "raw" observable to that of m_q-independent NP-subtracted version between $a \sim 0.12$ and ~ 0.06 fm ($\langle aG^2 \rangle = 0.009\ GeV^4$ as input)
  - common overall central scale $r_1 = 0.321$ fm as input
  - NOTE: re estimated NP $D = 4$ corrections
    * corrections far and away the largest for the 3 HPQCD "outliers"
    * despite *large* corrections, α_s agree with results from observables where NP corrections negligible

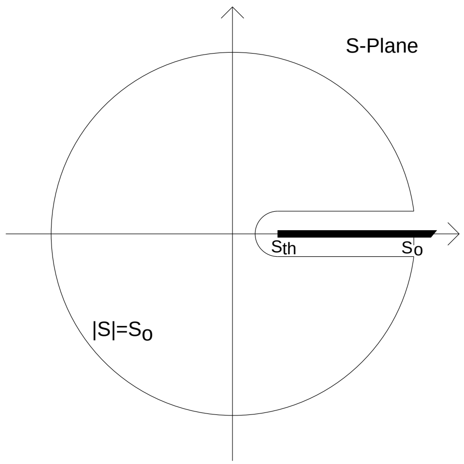
– $\delta_{D=4}$ and resulting $\alpha_s(M_Z)$ values

O_k	$\alpha_s(M_Z)$ (HPQCD)	$\alpha_s(M_Z)$ (CSSM)	$\delta_{D=4}$
$\log(W_{11})$	0.1185(8)	0.1190(11)	0.7%
$\log(W_{12})$	0.1185(8)	0.1191(11)	2.0%
$\log\left(\frac{W_{12}}{u_0^6}\right)$	0.1183(7)	0.1191(11)	5.2%
$log\left(\frac{W_{11}W_{22}}{W_{12}^2}\right)$	0.1185(9)	N/A	32%
$log\left(\frac{W_{23}}{u_0^{10}}\right)$	0.1176(9)	N/A	53%
$log\left(\frac{W_{14}}{W_{23}}\right)$	0.1171(11)	N/A	79%
$log\left(\frac{W_{11}W_{23}}{W_{12}W_{13}}\right)$	0.1174(9)	N/A	92%

THE HADRONIC τ DETERMINATION

- Based on FESRs for $\Pi^{(0+1)}_{T;ud}$, $T = V, A, V + A$

$$\int_0^{s_0} w(s)\, \rho^{(0+1)}_{T;ud}(s)\, ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\, \Pi^{(0+1)}_{T;ud}(s)\, ds$$



  - valid for any s_0, analytic $w(s)$
  - LHS: data; RHS: OPE (hence α_s) for $s_0 >> \Lambda^2_{QCD}$

- The spectral integrals
  - V, A, $I=1$ spectral function $\rho^{(J)=(0+1)}_{V/A;ud}(s)$ from experimental differential decay distributions $\frac{dR_{V/A;ud}}{ds}$, with $R_{V/A;ud} \equiv \frac{\Gamma[\tau \to \nu_\tau \, \mathrm{hadrons}_{V/A;ud}(\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]}$
  - $\Rightarrow$ experimental access to generic $(J)=(0+1)$; $w(s)$-weighted, $0 < s \leq s_0 \leq m_\tau^2$ spectral integrals

$$I^w_{spec;T}(s_0) = \int_0^{s_0} ds\, w(s) \rho^{(0+1)}_{T;ud}(s)$$

- The OPE side:
  - $D = 0$: fixed by α_s (known to 5 loops); strongly dominant for $s_0 \gtrsim 2$ GeV^2
  - $D = 2$: $\propto (m_d \pm m_u)^2$, hence negligible
  - $D = 4$: fixed by $\langle aG^2 \rangle$, $\langle m_\ell \bar{\ell}\ell \rangle$, $\langle m_s \bar{s}s \rangle$
  - $D = 6, 8, \cdots$:
    * not known phenomenologically, hence fitted to data (or guesstimated)
    * for $\sim 1\%$ $\alpha_s(M_Z)$ determination need integrated $D > 4$ to $\lesssim 0.5\%$ of $D = 0$

– More on fitting the $D > 4$ contributions

* $w(y) = \sum_{m=0} b_m y^m$, $y = s/s_0$ to distinguish contribs with different D (differing s_0 dependence)

* integrated $D = 2k + 2 \geq 2$ contribution $\Leftrightarrow b_k \neq 0$ (up to $O[\alpha_s^2(m_\tau^2)]$) $\Rightarrow$ contributions up to $D_{max} = 2N + 2$ for degree N $w(y)$

* integrated $D = 2k + 2$ contributions $\propto 1/s_0^k$

$$\frac{-1}{2\pi i} \oint_{|s|=s_0} ds\, w(y) \sum_{D>4} \frac{C_D}{Q^D} = \sum_{k\geq 2} (-1)^k \frac{b_k\, C_{2k+2}}{s_0^k}$$

Summary of recent τ-based determinations

- Differences in 6-loop $D = 0$ Adler function coeff, d_5; $D = 0$ series integral prescription; $D > 4$ treatment

- Duality violation typically assumed negligible

Source	d_5	$D > 4$ self-consistency	PT scheme	$\alpha_s(M_Z^2)$
BCK08	275	No	$\frac{1}{2}$(FO+CI)	0.1202(19)
ALEPH08	383	No	CI	0.1211(11)
BJ08	283	No	FO	0.1185(14)
	283	No	model	0.1179(8)
MY08	275	Yes	CI	0.1187(16)
N09	0	partly	$\frac{1}{2}$(FO+CI)	0.1192(10)
M09	400	No	$\frac{1}{2}$(RC+CI)	0.1213(11)
CF09	283	No	modified CI	0.1186(13)

THE ALEPH, OPAL (AND RELATED) ANALYSES

- $w_{(00)}(y) = 1 - 3y^2 + 2y^3 \Rightarrow$ OPE up to $D = 6, 8$
- $\Gamma[\tau \to hadrons_{ud}\,\nu_\tau]$ alone $(\leftrightarrow\ I^{w_{(00)}}_{spec;V+A}(m_\tau^2))$ insufficient to fix α_s, C_6, C_8
- ALEPH, OPAL approach
  - add $s_0 = m_\tau^2$, $(km) = (10), (11), (12), (13)$ "spectral weight" FESRs $[w(y) \to y^m\,(1-y)^k\,w_{(00)}(y)]$
  - neglect (in ppl present) $D = 10, \cdots, 16$ contribs
  - α_s, $\langle aG^2\rangle$, C_6, C_8 fitted to 5 integral set

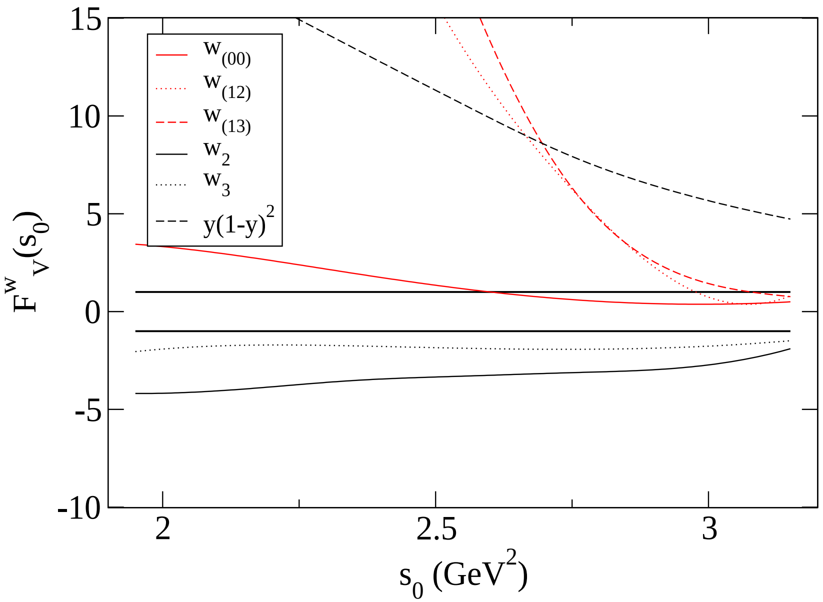
- NOTE: ALEPH C_6, C_8 is input to most other analyses

- Potential problem: single s_0 $(= m_\tau^2)$ $\Rightarrow$ $D > 8$ (if non-negligible) distort $D = 0, 4, 6, 8$ fit parameters

- Test for possible symptoms (systematic s_0-dependence problems) using "fit qualities"

$$F_T^w(s_0) \quad \equiv \left[I^w_{spec;T}(s_0) - I^w_{OPE;T}(s_0) \right] / \delta I^w_{spec;T}(s_0)$$

- FIGURE: $F_V^w(s_0)$ for ALEPH data, OPE fit, and 3 $w_{(k,m)}$ used in ALEPH/OPAL fit, PLUS 3 other degree 3 $w(y)$ (to provide independent $C_{6,8}$ tests)



- OPE-spectral mismatch $\Rightarrow$ either a problem with assumption that $D > 8$ negligible, or OPE breakdown (either way a problem for extracted α_s)

A MODIFIED ANALYSIS

- V, A and V+A, $w_N(y) \equiv 1 - \frac{N}{N-1} y + \frac{1}{N-1} y^N$ FESRs

  [KM,T. Yavin, PRD78 (2008) 094020 (arXiv:0807.0650)]

- single unsuppressed $D = 2N + 2 > 4$ contrib ($N \geq 2$), $(-1)^N C_{2N+2} / \left[(N-1) s_0^N \right]$

- $1/s_0^{N+1}$ scaling c.f. $D = 0 \Rightarrow$ joint α_s, C_{2N+2} fit

- $1/(N-1)$ $D = 2N + 2$ suppression, no $D = 0$ suppression $\Rightarrow$ MUCH better α_s emphasis than $w_{(k,m)}$ set

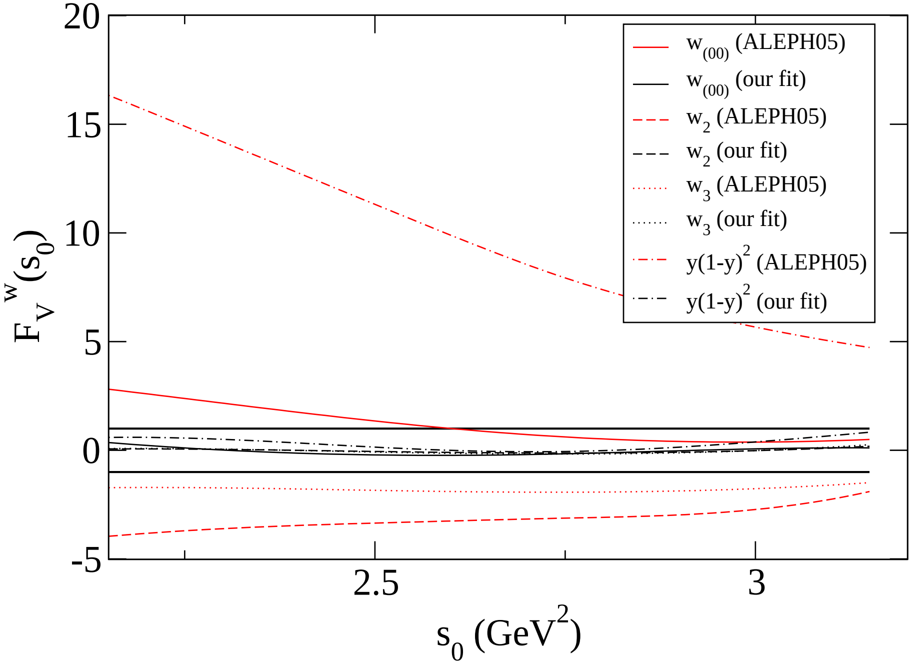
RESULTS

- Results for $\alpha_s(m_\tau^2)$ using the CIPT $D = 0$ prescription

$w(y)$	ALEPH V+A	OPAL V+A
w_2	0.320(5)(12)	0.322(7)(12)
w_3	0.320(5)(12)	0.322(7)(12)
w_4	0.320(5)(12)	0.322(7)(12)
w_5	0.320(5)(12)	0.322(7)(12)
w_6	0.320(5)(12)	0.322(8)(12)

$w(y)$	ALEPH V	ALEPH A	ALEPH V+A
w_2	0.321(7)(12)	0.319(6)(12)	0.320(5)(12)
w_3	0.321(7)(12)	0.319(6)(12)	0.320(5)(12)
w_4	0.321(7)(12)	0.319(6)(12)	0.320(5)(12)
w_5	0.321(7)(12)	0.319(6)(12)	0.320(5)(12)
w_6	0.321(7)(12)	0.319(6)(12)	0.320(5)(12)

- Much improved $F_V^w(s_0)$ for $w = w_N$ c.f. $w = w_{(k,m)}$



- CIPT $w_2, \cdots, w_6$ fit values consistent to ± 0.0001

- Averaging ALEPH and OPAL based results with non-normalization component of error $\Rightarrow$

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.3209(46)_{exp}(118)_{th}$$

- standard self-consistent combination of 4-loop running, 3-loop matching at flavor thresholds $\Rightarrow$

$$\alpha_s^{(n_f=5)}(M_Z) = 0.1187(3)_{evol}(6)_{exp}(15)_{th}$$

CONCLUSIONS/SUMMARY

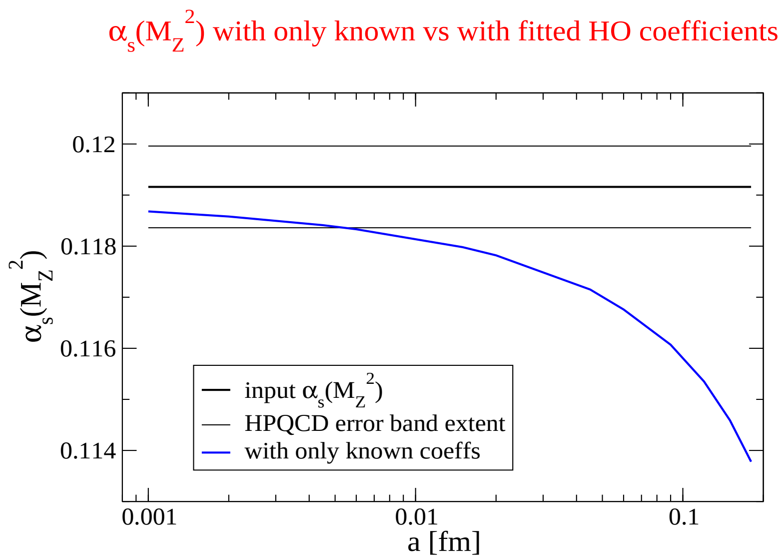
- Lattice ($log\,(W_{11})$ to be specific) and τ determinations now in excellent agreement

$$[\alpha_s(M_Z)]_{latt} = 0.1185(8),\ 0.1190(11)$$
$$[\alpha_s(M_Z)]_\tau = 0.1187(16)$$

- Significant improvement to lattice errors difficult

- Some improvement in τ decay analysis probable

- The lattice analysis case:
  - some improvement, further self-consistency checks from additional $a \sim 0.045$ fm MILC ensembles, BUT a small enough to avoid fitting additional $D = 0$ coefficients impractical [Figure]



- errors dominated by overall scale-setting and residual HO $D = 0$ perturbative issues hence difficult to significantly improve

- The τ decay analysis case:

  Significant improvement requires better understanding of $D = 0$ truncation uncertainty and residual duality violation (if any)

  - Theory error currently dominant ($\sim$ 2.5 times expt'l)
  - $D = 0$ truncation dominant theory error source (for $|FOPT - CIPT| \oplus O(a^5)$ estimate $\sim$ 0.010 of 0.012 total) $\Rightarrow$ main bottleneck for future improvements

– Beneke-Jamin-like exploration (taking into account divergent nature of $D = 0$ series) crucial to reducing truncation uncertainty

– interesting possibilities in this regard in recent Caprini-Fischer work, but needs to be coupled to simultaneous fitting of $D > 4$ OPE coefficients

– Work on further constraining models of duality violation (see, e.g., recent Cata, Goltermann, Peris papers), estimates of impact on α_s extraction known to be feasible, and in preliminary stages of investigation (KM, Goltermann et al.)

SUPPLEMENTARY τ MATERIAL

- More on consistency of V+A fit results
- More on the independence of the $w_2, \cdots, w_6$ FESRs
- Some observations on the Beneke-Jamin calculation

More on the consistency of the V+A fit results

V+A fit results for $\alpha_s(m_\tau)$

$w(y)$	CIPT full fit	$s_0 = m_\tau^2$ CIPT $D > 4 \to 0$	FOPT full fit
w_2	0.320	0.310	0.320
w_3	0.320	0.316	0.315
w_4	0.320	0.319	0.313
w_5	0.320	0.321	0.312
w_6	0.320	0.322	0.312

More on the independence of the $w_2, \cdots, w_6$ FESRs

Fitted ALEPH-based V+A $\alpha_s(m_\tau^2)$ from pseudo-FESRs employing one w_N for the spectral integrals (row label) and another for the OPE integrals (column heading)

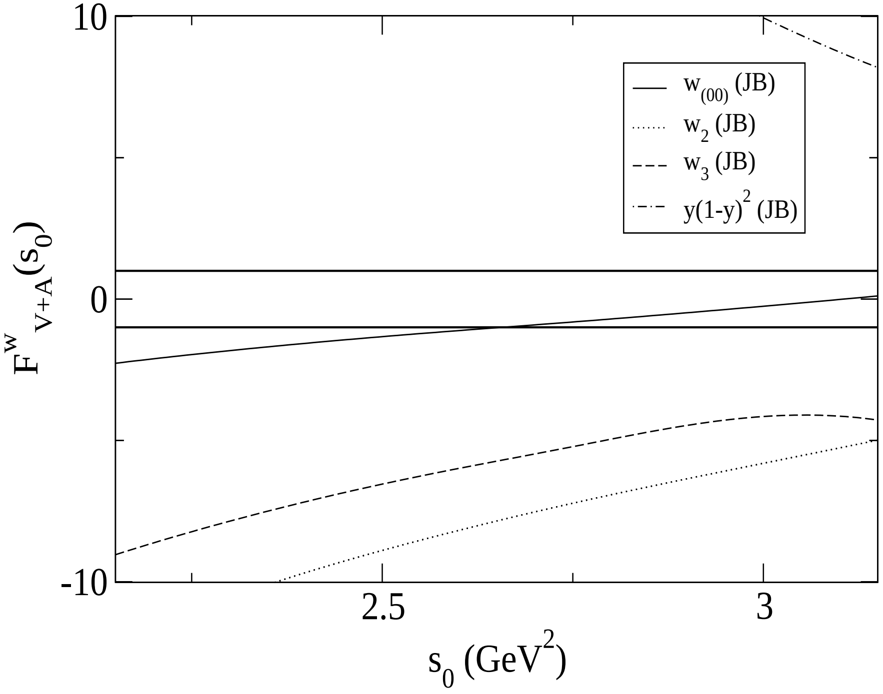
	w_2	w_3	w_4	w_5	w_6
w_2	0.320	0.175	—	—	—
w_3	0.435	0.320	0.249	0.194	0.149
w_4	0.499	0.384	0.320	0.277	0.243
w_5	0.541	0.423	0.361	0.320	0.291
w_6	—	0.450	0.388	0.349	0.320

Some observations on the Beneke-Jamin calculation

- As for the spectral weight analysis, control of $D > 4$ contributions essential for precision α_s (independent of choice of FOPT or CIPT for $D = 0$ contributions)

- Can test BJ input assumptions for $C_{6,8}$ for consistency with output FOPT fit α_s using $F^w_{V+A}(s_0)$ for various degree ≤ 3 $w(y)$ (FIGURE)

- Find problems for combination of assumed $D = 6, 8$ and FOPT fitted α_s



10
0
-10
2.5
3
s_0 (GeV^2)
$F^w_{V+A}(s_0)$
$w_{(00)}$ (JB)
w_2 (JB)
w_3 (JB)
$y(1-y)^2$ (JB)

- Exercise to test implications of (minimal, 5-parameter) BJ model for the resummed $D = 0$ series

  - Features of the minimal model:

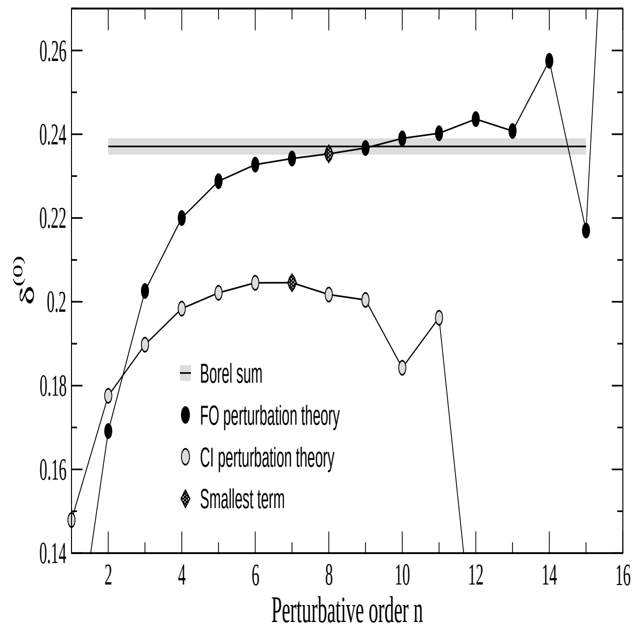
    - $*$ good approximation to full model sum using FOPT for a range of $w(y)$ (FIGURES)

    - $*$ CIPT approximation inferior to FOPT *most strongly so for* $w_{(0,0)}$ (FIGURES)

    - $*$ $\Rightarrow$ expect consistency of various FOPT fits, reduced consistency for CIPT fits

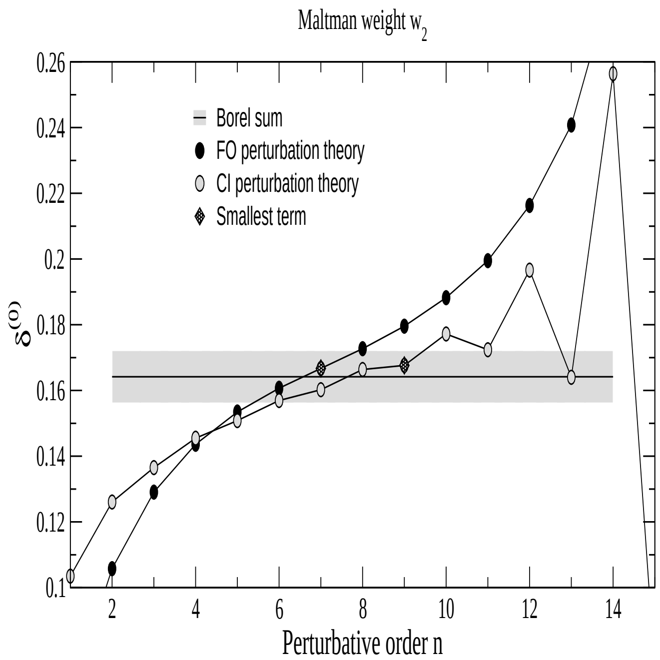
  - FIGURE: FOPT, CIPT vs. Borel sum for BJ model

$w_{(0,0)}$



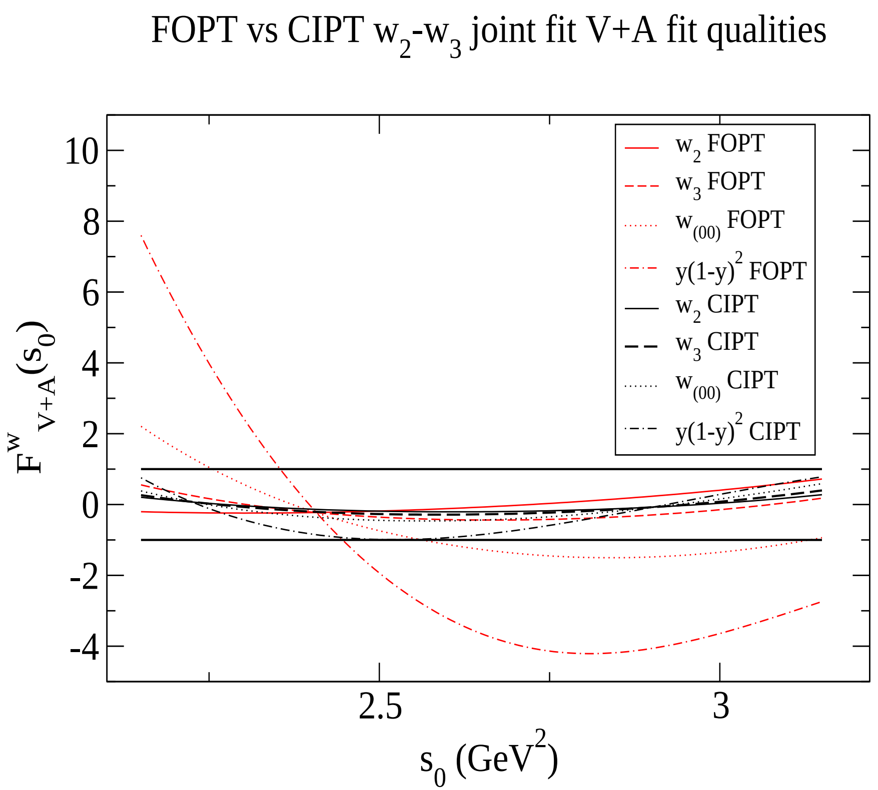
Borel sum
FO perturbation theory
CI perturbation theory
Smallest term
δ(o)
Perturbative order n
0.14
0.16
0.18
0.2
0.22
0.24
0.26
2
4
6
8
10
12
14
16

w_2



Maltman weight w2
Borel sum
FO perturbation theory
CI perturbation theory
Smallest term
δ(o)
Perturbative order n
0.1
0.12
0.14
0.16
0.18
0.2
0.22
0.24
0.26
2
4
6
8
10
12
14

- Test expectations with combined FOPT, CIPT w_2-w_3 fit
  - combined fit yields α_s, C_6, C_8, hence OPE integrals fixed for any degree ≤ 3 $w(y)$
  - test agreement of CIPT, FOPT OPE with corresponding spectral integrals for $w_{(0,0)}$, $y(1-y)^2$
- find good (not good) CIPT (FOPT) consistency (contrary to model expectations) (FIGURE)
- suggests alternate non-minimal modelling possible using such observations as constraints


FOPT vs CIPT w2-w3 joint fit V+A fit qualities
10
8
6
4
2
0
-2
-4
2.5
3
F^w_V+A(s0)
s0 (GeV^2)
w2 FOPT
w3 FOPT
w(00) FOPT
y(1-y)^2 FOPT
w2 CIPT
w3 CIPT
w(00) CIPT
y(1-y)^2 CIPT

SUPPLEMENTARY PAGES ON LATTICE ANALYSIS

- Original 2005 HPQCD/UKQCD, 2008 HPQCD:
  - r_1, $\frac{r_1}{a}$, $\langle aG^2 \rangle$: independent fit w/ priors *for each* O_k
  - r_1, $\frac{r_1}{a}$: small (measured) prior widths $\Rightarrow$ possible unphysical observable-dependence effects small
  - Relation of expansion parameter, α_V, to $\alpha_s^{\overline{MS}}$ unknown beyond 4^{th} order
  - O_k with potentially sizeable m_q-independent NP subtractions included in analysis
  - (2008 update): better agreement of $\langle aG^2 \rangle$ from different O_k when $D > 4$ forms included, with fitted coefficients, for more NP observables [HPQCD private communications]

- 2008 CSSM re-analysis:
  - measured r_1, $\frac{r_1}{a}$, charmonium sum-rule $\langle aG^2\rangle$ (with errors): common, external input for all O_k
  - LO $D = 4$ $\langle aG^2\rangle$ estimate of m_q-independent NP contribution/subtraction
  - Relation of expansion parameter to $\alpha_s^{\overline{MS}}$ exactly specified
  - focus on O_k where estimated $D = 4$ NP $\langle aG^2\rangle$ subtraction small, hence $D > 4$ presumably even smaller

More on the two $D = 0$ expansion parameters choices

- $D = 0$ expansion parameter α_T, β function β^T to 4-loops from $\beta^{\overline{MS}} \Rightarrow \beta^T_{4,5,\cdots}$ incompletely known

- Expand α_T in $\alpha_0 = \alpha_T(Q_k^{max})$, $t_k = log[(Q_k/Q_k^{max})^2]$

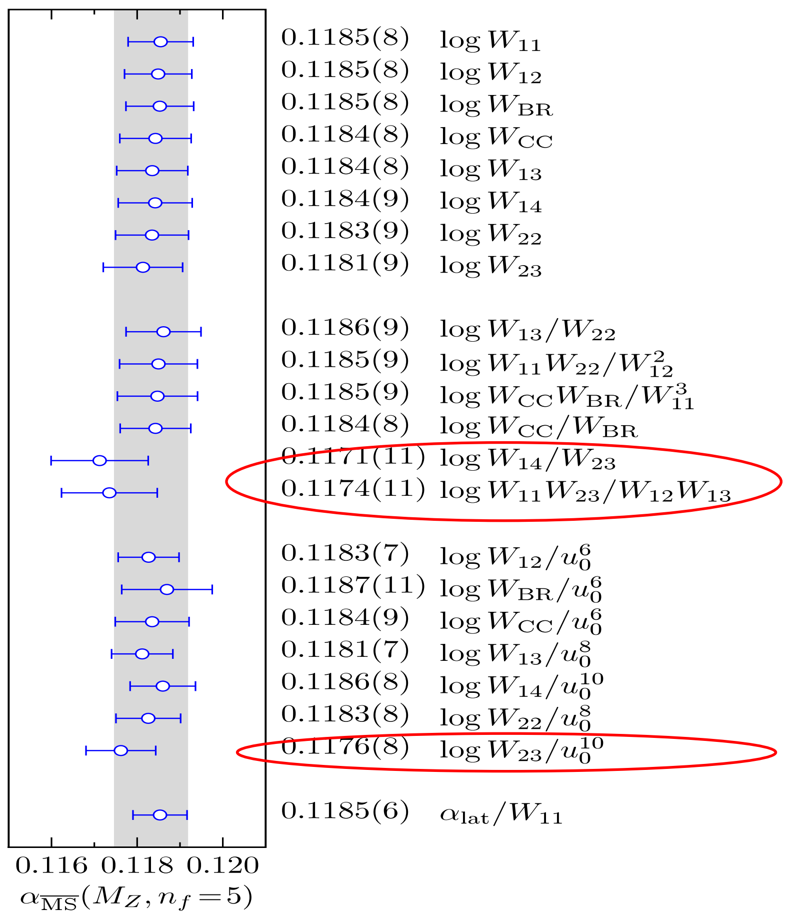
$$
\begin{aligned}
\frac{O_k}{D_k} = \quad & \cdots + \alpha_0^4 \left(c_3^{(k)} + \cdots \right) + \alpha_0^5 \left(c_4^{(k)} - 2.87 c_3^{(k)} t_k + \cdots \right) \\
& + \alpha_0^6 \left(c_5^{(k)} - 0.0033 \beta_4^T t_k - 3.58 c_4^{(k)} t_k \right. \\
& \left. + [5.13 t_k^2 - 1.62 t_k] c_3^{(k)} + \cdots \right) + \alpha_0^7 \left(c_6^{(k)} \right. \\
& - 0.0010 \beta_5^T t_k + [0.0094 t_k^2 - 0.0065 c_1^{(k)} t_k] \beta_4^T \\
& - 4.30 c_5^{(k)} t_k + [7.69 t_k^2 - 2.03 t_k] c_4^{(k)} \\
& \left. + [-7.35 t_k^3 + 6.39 t_k^2 - 4.38 t_k] c_3^{(k)} + \cdots \right) + \cdots
\end{aligned}
$$

- Incompletely known $\beta^T_{4,5,\cdots}$ distorts fit parameters

- HPQCD approach
  - $\alpha_T \to \alpha_V$ defined such that $\beta_4^V = \beta_5^V = \cdots \equiv 0$
  - $\Rightarrow$ no distortion of fit parameters
  - expansion for α_V in terms of $\alpha_s^{\overline{MS}}$ in principle well-defined
  - (however) expansion coefficients beyond 4^{th} order depend on $\beta_{4,5,\ldots}^{\overline{MS}}$, hence not known
  - impact of HO (after fitting $c_{3,4,\ldots}^{(k)}$) localized to conversion/running to $\alpha_s(M_Z)$

- CSSM approach
  - α_T defined as 3-order-truncated expansion of α_V^p
  - $\Rightarrow$ conversion to $\alpha_s^{\overline{MS}}$ exact but $\beta^T_{4,5\ldots}$ depend on $\beta^{\overline{MS}}_{4,5,\ldots}$, hence incompletely known
  - Fit parameter distortions reducible by hand:
    * focus on highest intrinsic scale O_k
    * restrict t_k (subset of finest lattices)
    * stability c.f. expanding subset as test

FULL HPQCD RESULTS



Large NP subtractions for 3 outliers

Ave (all):
0.1183(8)

Ave small NP:
0.1185(8)