Low energy scattering and $Z^+(4430)$

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Details of this work: PRD 80, 034503, (2009)
Outline

- Motivation
- Theoretical framework
- One- and two-particle correlation matrix
- Simulation results
  - Parameters
  - Dispersion relations
  - Scattering phase & scattering length
  - Possible bound state?
- Conclusions
Motivations

✓ A peak observed in $\pi\psi'$ invariant mass spectrum in $B$ decays to $K\pi\psi'$
✓ Quantum numbers:
  ✓ $Q=+1, J^P=0^+, 1^-, 2^-$
✓ Breit-Wigner fit results in:
  ✓ $M=4433$ MeV
  ✓ $\Gamma=45$ MeV
✓ Mass close to the threshold of $D^*$ and $D_1$

S.K. Choi et al., PRL 100, 142001 (2008)

FIG. 2 (color online). The $M(\pi^+\psi')$ distribution for events in the $M_{bc} - \Delta E$ signal region and with the $K^*$ veto applied. The shaded histogram show the scaled results from the $\Delta E$ sideband. The solid curves show the results of the fit described in the text.
Motivations

- Close to $D^*D_1$ threshold
  - Shallow bound state of two $D$ mesons (S.L. Zhu et al, PRD77, 034003)
  - Tetraquark resonance above threshold (X.-H Liu et al, PRD77, 094005)
  - Threshold enhancement (J.L. Rosner, PRD76, 114002)
- To clarify the issue, scattering info near the threshold of $D^*D_1$ is needed!
  - Scattering length $a_0$
  - Effective range $r_0$
- We use quenched lattice QCD to study the problem
Theoretical framework

- In lattice QCD, one could infer hadron-hadron scattering information by measuring the energy eigenvalues of the two (interacting!) hadrons in a finite box.

- **Luescher’s formula** relates the two-particle energy eigenvalues in a box to the scattering phase shifts in the continuum.
Theoretical framework

- A box of size $L$, periodic in all three spatial directions:
  \[ \vec{k} = \left( \frac{2\pi}{L} \right) \vec{n}, \quad \vec{n} \in \mathbb{Z}^3 \]

- Two interacting hadrons

\[
E_{1\cdot2}(\vec{k}) = \sqrt{m_1^2 + \vec{k}^2} + \sqrt{m_2^2 + \vec{k}^2}, \quad q^2 = \frac{\vec{k}^2 L^2}{(2\pi)^2}
\]

\[
\tan \delta(q) = \frac{\pi^{3/2} q}{Z_{00}(1; q^2)}
\]
Theoretical framework

- We use asymmetric volumes: \( L \times (\eta_2 L) \times (\eta_3 L) \)
- We take \( \eta_2 = 1, \eta_3 > 1 \)
- The symmetry for the box is \( D_4 \)

\[
\tan \delta(q) = \frac{\pi^{3/2} q \eta_2 \eta_3}{Z_{00}(1; q^2; \eta_2, \eta_3)}
\]
Correlation matrices

- Single particle operators

\[ Q_i(x) = [\bar{d} \gamma^i c](x), \quad P_i(x) = [\bar{c} \gamma^i \gamma^5 u](x), \]

- Angular momentum decomposition

\[ 0 = A_1, \quad 1 = E \oplus A_2, \quad 2 = A_1 \oplus B_1 \oplus B_2 \oplus E. \]
Correlation matrices

- Two particle operators

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>Two-particle operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^-$</td>
<td>$O^{(A_1)(1)}(t)$</td>
</tr>
<tr>
<td>$1^-$</td>
<td>$O^{(A_2)}(t)$, $O^{(E)(1)}_1(t)$, $O^{(E)(1)}_2(t)$</td>
</tr>
<tr>
<td>$2^-$</td>
<td>$O^{(A_1)(2)}(t)$, $O^{(B_1)}(t)$, $O^{(B_2)}(t)$, $O^{(E)(2)}_1(t)$, $O^{(E)(2)}_2(t)$</td>
</tr>
</tbody>
</table>

$O^{(A_1)(1)}(t) = \sum_{R \in G} \left[ Q_1(t + 1, -R \circ k) P_1(t, R \circ k) + Q_2(t + 1, -R \circ k) P_2(t, R \circ k) + Q_3(t + 1, -R \circ k) P_3(t, R \circ k) \right]$,
Correlation matrices

- Only the correlation matrix in $A_I$ channel shows signal

$$C_{mn}^{(A_1)(1)}(t) = \langle O_m^{(A_1)(1)^\dagger}(t) O_n^{(A_1)(1)}(0) \rangle,$$

- We have computed 5 lowest non-zero momentum modes together with the zero momentum mode.
Simulation results

- Tadpole improved clover action on anisotropic lattices

| TABLE II. Simulation parameters in this study. All lattices have the same aspect ratio $\xi = 5.$ |
|-----------------|-----------------|-----------------|
|                 | $\beta = 2.5$   | $\beta = 2.8$   | $\beta = 3.2$   |
| $N_{\text{conf}}$ | 700             | 500             | 200             |
| $u_s^4$         | 0.4236          | 0.4630          | 0.50679         |
| $\nu_c$         | 0.732           | 0.79            | 0.89            |
| $\nu_{ud}$      | 0.9305          | 0.96            | 1.0             |
| $a_s (fm)$      | 0.2037          | 0.1432          | 0.0946          |
| Lattice         | $8 \times 8 \times 12 \times 40$ | $12 \times 12 \times 20 \times 64$ | $16 \times 16 \times 24 \times 80$ |
| $\kappa^c_{\text{max}}$ | 0.0577          | 0.0598          | 0.0595          |
| $\kappa^{ud}_{\text{max}}$ | 0.0613          | 0.0611          | 0.0606          |
Simulation results

- $D^*$ and $D_1$
  - mass:
    - fixing charm quark mass parameter

FIG. 1 (color online). Heavy quark mass interpolation for $m_{D^*}$ and $m_{D_1}$, from top to bottom: $\beta = 2.5$, 2.8, and 3.2.
Simulation results

- \( D^* \) and \( D_1 \) mass:
  - Chiral extrapolation.
Simulation results

- $D^*$ and $D_1$ mass:
  - Continuum extrap.:
Simulation results

Single particle dispersion

FIG. 5. Dispersion relations for various mesons obtained from single meson energies. From top to bottom: $D^*$, $D_1$, $\eta_c$, $J/\psi$; from left to right: $\beta = 2.5, 2.8, 3.2$. 
Simulation results

Two-particle energy
Simulation results

- Scattering length $a_0$
- & effective range $r_0$

$$\frac{k}{\tan \delta(k)} = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots,$$

**FIG. 7.** The quantity $k/\tan \delta(k)$ versus $q^2$ in the $A_1^{(1)}$ channel. From top to bottom: $\beta = 2.5, 2.8,$ and $3.2$. 
Simulation results

- After all the extrapolations

\[ a_0 = 2.53 \pm 0.47 \text{ fm}, \quad r_0 = 0.70 \pm 0.10 \text{ fm.} \]
Simulation results

- Possible bound state?
  - Bound state: $q^2 \to -\infty, \cot \sigma(q^2) \to (-1)$

**TABLE III.** Results for the lowest $q^2$ and the corresponding values for $\cot \sigma(q)$ as given by Eq. (8) for different values of $\beta$ in the simulation. Corresponding errors for the quantities are also given in the parenthesis.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$q^2$</th>
<th>$\cot \sigma(q^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>$-0.026(0.003)$</td>
<td>5.23(0.65)</td>
</tr>
<tr>
<td>2.8</td>
<td>$-0.064(0.005)$</td>
<td>0.16(0.18)</td>
</tr>
<tr>
<td>3.2</td>
<td>$-0.053(0.016)$</td>
<td>0.92(0.93)</td>
</tr>
</tbody>
</table>
Simulation results

- Possible bound state?
  - For shallow bound state, \( a_0 \rightarrow -\infty \)
  - But our scattering lengths are all positive
  - On the verge of developing a bound state
  - Using the square well potential model to estimate, the potential well is \( V_0=70(10) \text{ MeV}, R=0.7 \text{fm} \).

- Our results is not in favor of a shallow resonance
Conclusions

- Low energy $D^{*+}D_{1}^{0}$ scattering is studied in the channel of $J^{P} = 0^{-}$ using quenched lattice QCD on anisotropic lattices.
- Scattering length and effective range obtained.
- The interaction between the two $D$ mesons is attractive in nature.
- But it is unlikely that they can form a shallow bound state below the threshold.
- More studies are welcome.