Low energy $D^{*+}\bar{D}_1^0$ scattering and $Z^+(4430)$

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Details of this work: PRD 80, 034503, (2009)

Outline

- Motivation
- Theoretical framework
- One- and two-particle correlation matrix
- Simulation results
  - Parameters
  - Dispersion relations
  - Scattering phase & scattering length
  - Possible bound state?
- Conclusions

Motivations

S.K. Choi et al., PRL 100,142001 (2008)

- ✓A peak observed in $\pi\psi'$ invariant mass spectrum in B decays to $K\pi\psi'$
- ✓Quantum numbers:
  - ✓$Q=+1,\ J^P=0^-,1^-,2^-$
- ✓Breit-Wigner fit results in:
  - ✓M=4433 MeV
  - ✓Γ=45 MeV
- ✓Mass close to the threshold of D^* and D_1

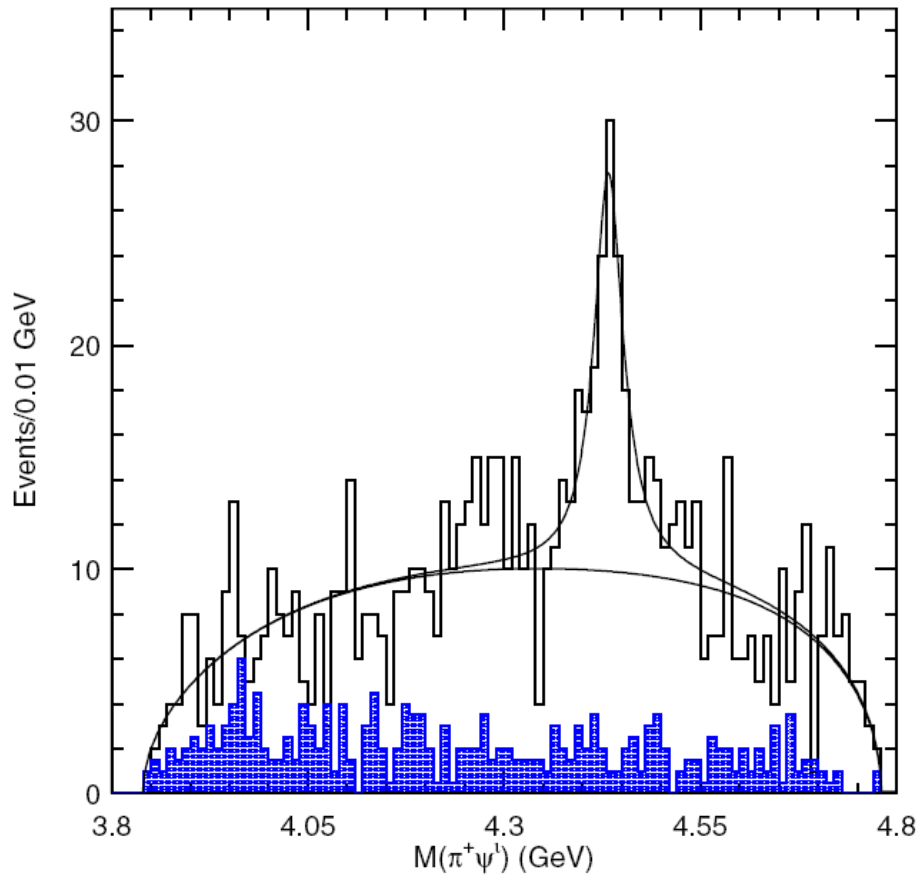


FIG. 2 (color online). The $M(\pi^+\psi')$ distribution for events in the $M_{bc}-\Delta E$ signal region and with the K^* veto applied. The shaded histogram show the scaled results from the ΔE sideband. The solid curves show the results of the fit described in the text.

Motivations

- Close to D^*D_1 threshold
  - Shallow bound state of two D mesons(S.L. Zhu et al, PRD77,034003)
  - Tetraquark resonance above threshold(X.-H Liu et al, PRD77, 094005)
  - Threshold enhancement (J.L. Rosner, PRD76,114002)
- To clarify the issue, scattering info near the threshold of D^*D_1 is needed!
  - Scattering length $\boldsymbol{a_0}$
  - Effective range $\boldsymbol{r_0}$
- We use quenched lattice QCD to study the problem

Theoretical framework

- In lattice QCD, one could infer hadron-hadron scattering information by measuring the energy eigenvalues of the two (interacting !) hadrons in a finite box
- Luescher's formula relates the two-particle energy eigenvalues in a box to the scattering phase shifts in the continuum

Theoretical framework

- A box of size $\boldsymbol{L}$, periodic in all three spatial directions: $\vec{k} = (2\pi / L)\vec{n} \ , \ \vec{n} \in Z^3$

- Two interacting hadrons

$$E_{1\bullet 2}(\bar{k}) = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2} \ , \ q^2 = \bar{k}^2 L^2 / (2\pi)^2$$

$$\tan \delta(q) = \frac{\pi^{3/2} q}{Z_{00}(1; q^2)}$$

Theoretical framework

- We use asymmetric volumes: $L \times (\eta_2 L) \times (\eta_3 L)$
- We take $\eta_2=1$, $\eta_3>1$
- The symmetry for the box is D_4

$$\tan\delta(q) = \frac{\pi^{3/2} q \eta_2 \eta_3}{Z_{00}(1; q^2; \eta_2, \eta_3)}$$

Correlation matrices

- Single particle operators

$\overline{D}_1^0$

$$Q_i(x) = [\bar{d}\gamma^i c](x), \qquad P_i(x) = [\bar{c}\gamma^i\gamma^5 u](x),$$

D^{*+}

- Angular momentum decomposition

$$\mathbf{0} = A_1, \qquad \mathbf{1} = E \oplus A_2, \qquad \mathbf{2} = A_1 \oplus B_1 \oplus B_2 \oplus E.$$

Correlation matrices

- Two particle operators

TABLE I. The two-particle operators defined in Eq. (12) and their corresponding angular momentum quantum number J in the continuum.

J^P	Two-particle operators
$\mathbf{0}^-$	$O^{(A_1)(1)}(t)$
$\mathbf{1}^-$	$O^{(A_2)}(t),\ O_1^{(E)(1)}(t),\ O_2^{(E)(1)}(t)$
$\mathbf{2}^-$	$O^{(A_1)(2)}(t),\ O^{(B1)}(t),\ O^{(B_2)}(t),\ O_1^{(E)(2)}(t),\ O_2^{(E)(2)}(t)$

$$
\begin{aligned}
O^{(A_1)(1)}(t) = \sum_{R\in G} [&Q_1(t+1, -R\circ\mathbf{k})P_1(t, R\circ\mathbf{k}) \\
&+ Q_2(t+1, -R\circ\mathbf{k})P_2(t, R\circ\mathbf{k}) \\
&+ Q_3(t+1, -R\circ\mathbf{k})P_3(t, R\circ\mathbf{k})],
\end{aligned}
$$

Correlation matrices

- Only the correlation matrix in A_1 channel shows signal

$$C_{mn}^{(A_1)(1)}(t) = \langle O_m^{(A_1)(1)\dagger}(t) O_n^{(A_1)(1)}(0) \rangle,$$

- We have computed 5 lowest non-zero momentum modes together with the zero momentum mode.

Simulation results

- Tadpole improved clover action on anisotropic lattices

TABLE II. Simulation parameters in this study. All lattices have the same aspect ratio $\xi = 5$.

	$\beta = 2.5$	$\beta = 2.8$	$\beta = 3.2$
N_{conf}	700	500	200
u_s^4	0.4236	0.4630	0.50679
ν_c	0.732	0.79	0.89
ν_{ud}	0.9305	0.96	1.0
$a_s(fm)$	0.2037	0.1432	0.0946
Lattice	$8 \times 8 \times 12 \times 40$	$12 \times 12 \times 20 \times 64$	$16 \times 16 \times 24 \times 80$
κ_{max}^c	0.0577	0.0598	0.0595
$\kappa_{\mathrm{max}}^{ud}$	0.0613	0.0611	0.0606

Simulation results

- D^* and D_1 mass:
  - fixing charm quark mass parameter

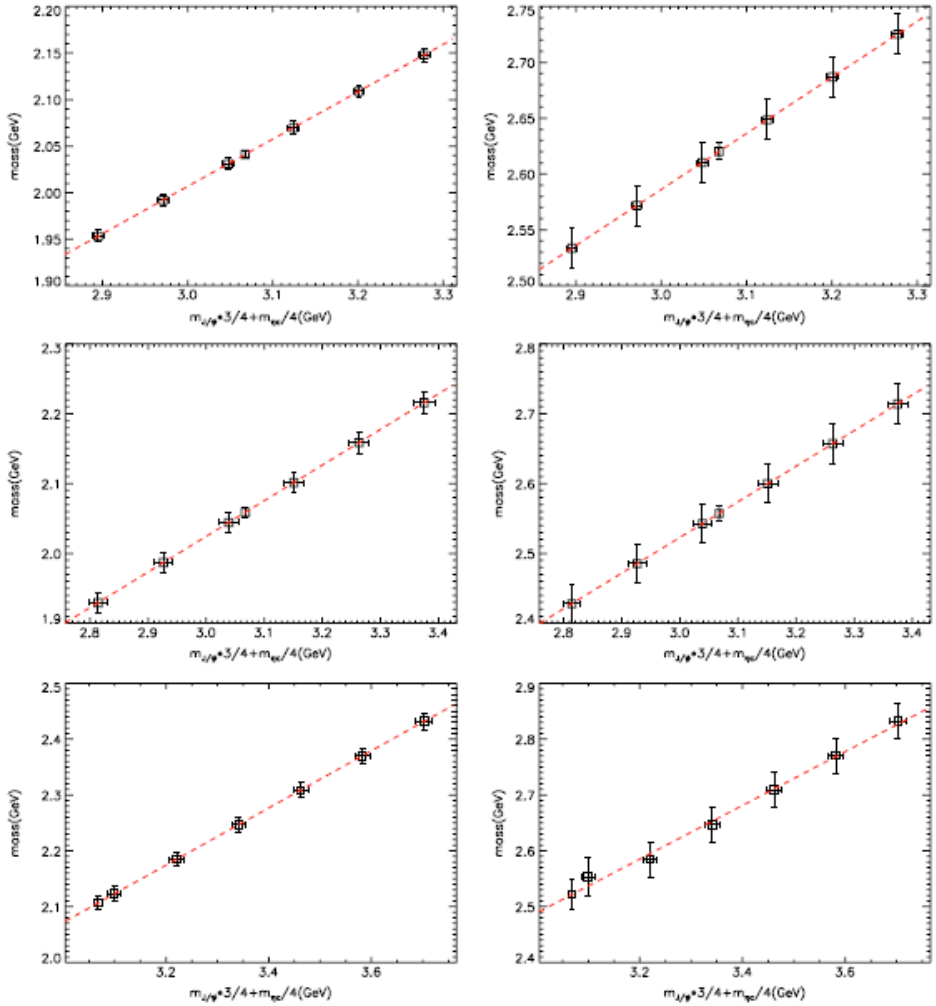
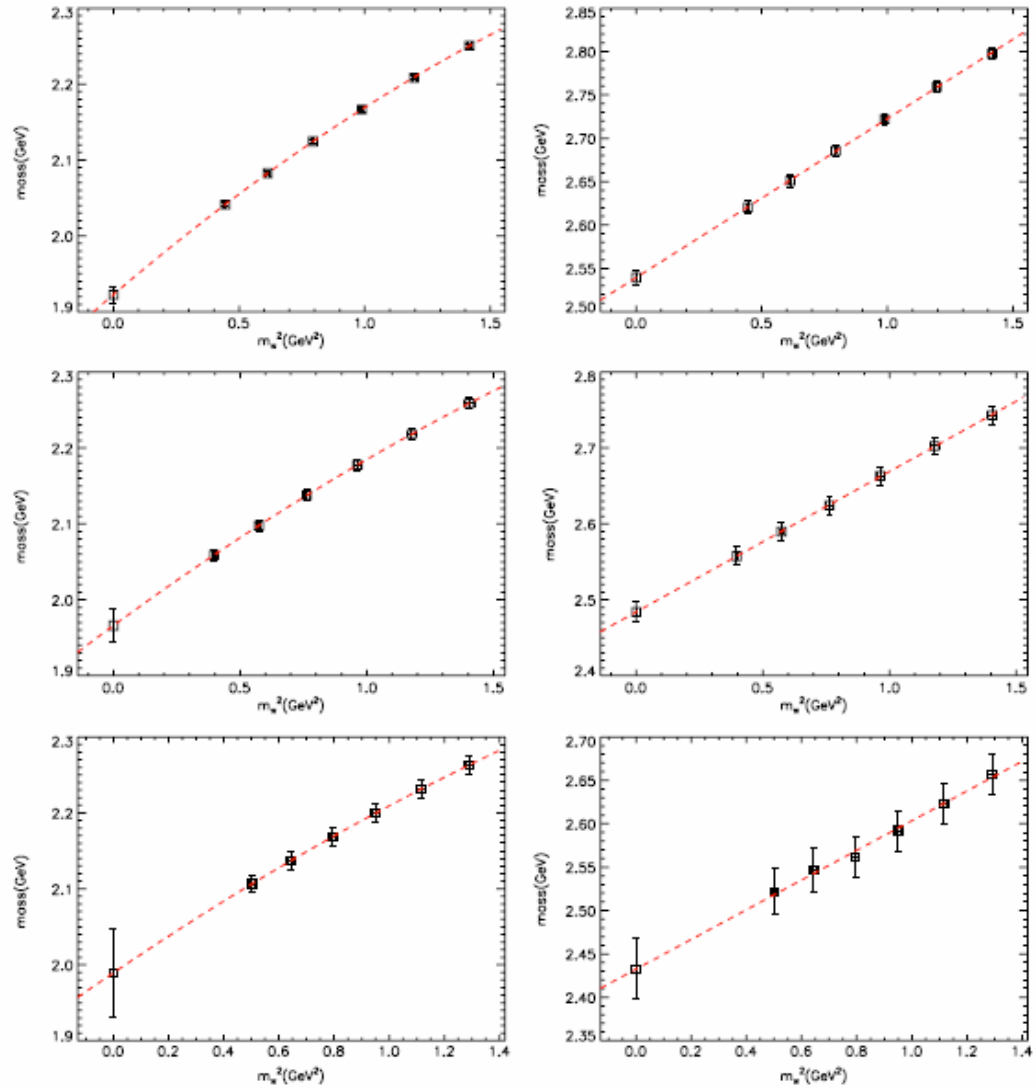


FIG. 1 (color online). Heavy quark mass interpolation for m_{D^*} and m_{D_1}, from top to bottom: $\beta = 2.5$, 2.8, and 3.2.

Simulation results

- D^* and D_1 mass:
  - Chiral extrap.



Simulation results

- D^* and D_1 mass:
  - Continuum extrap.:

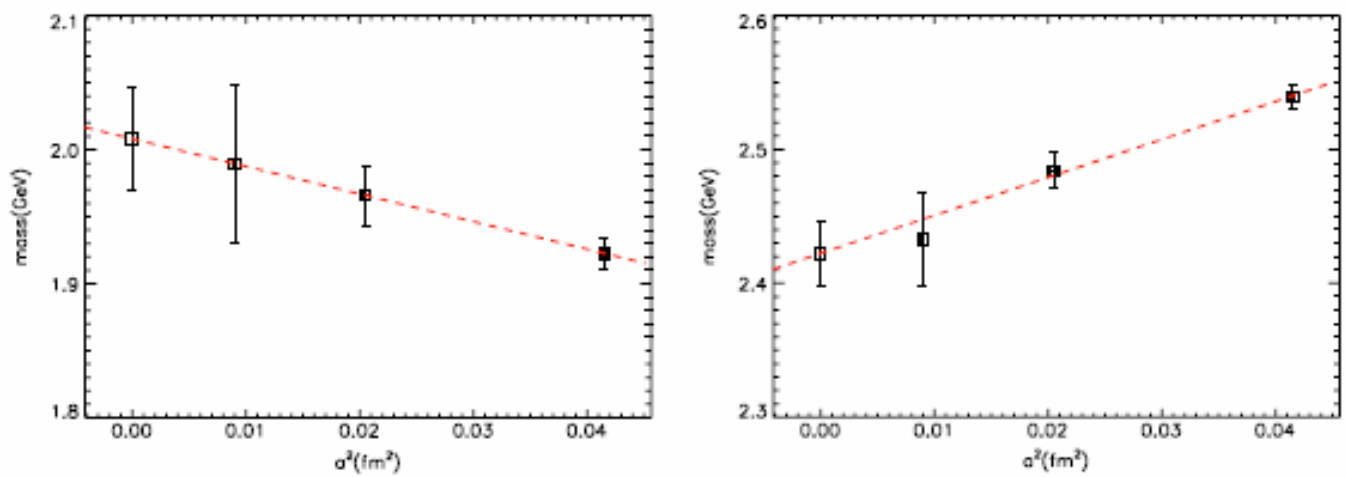


FIG. 3 (color online). Continuum limit extrapolations of m_{D^*} and m_{D_1}.

Simulation results

- Single particle dispersion

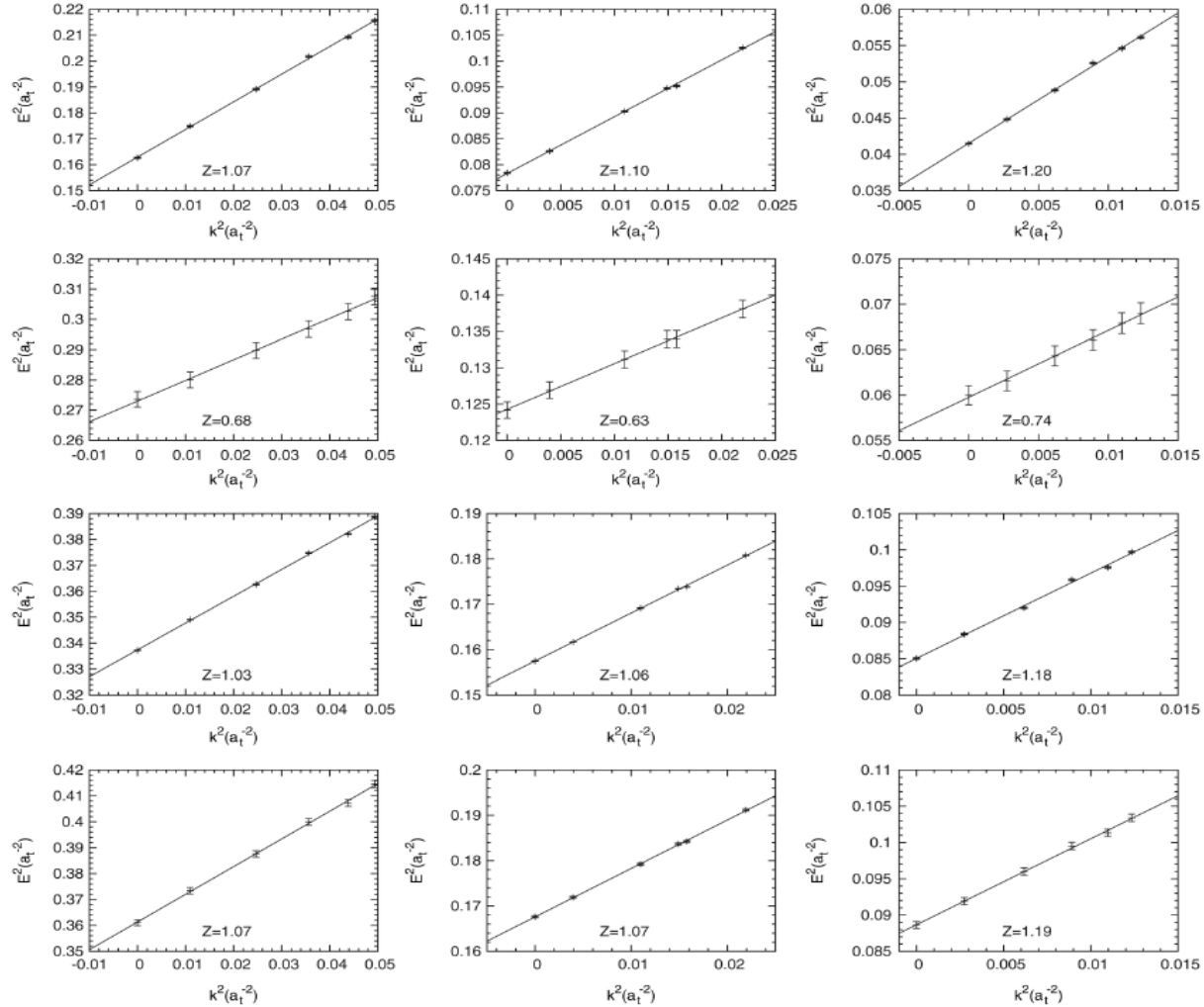
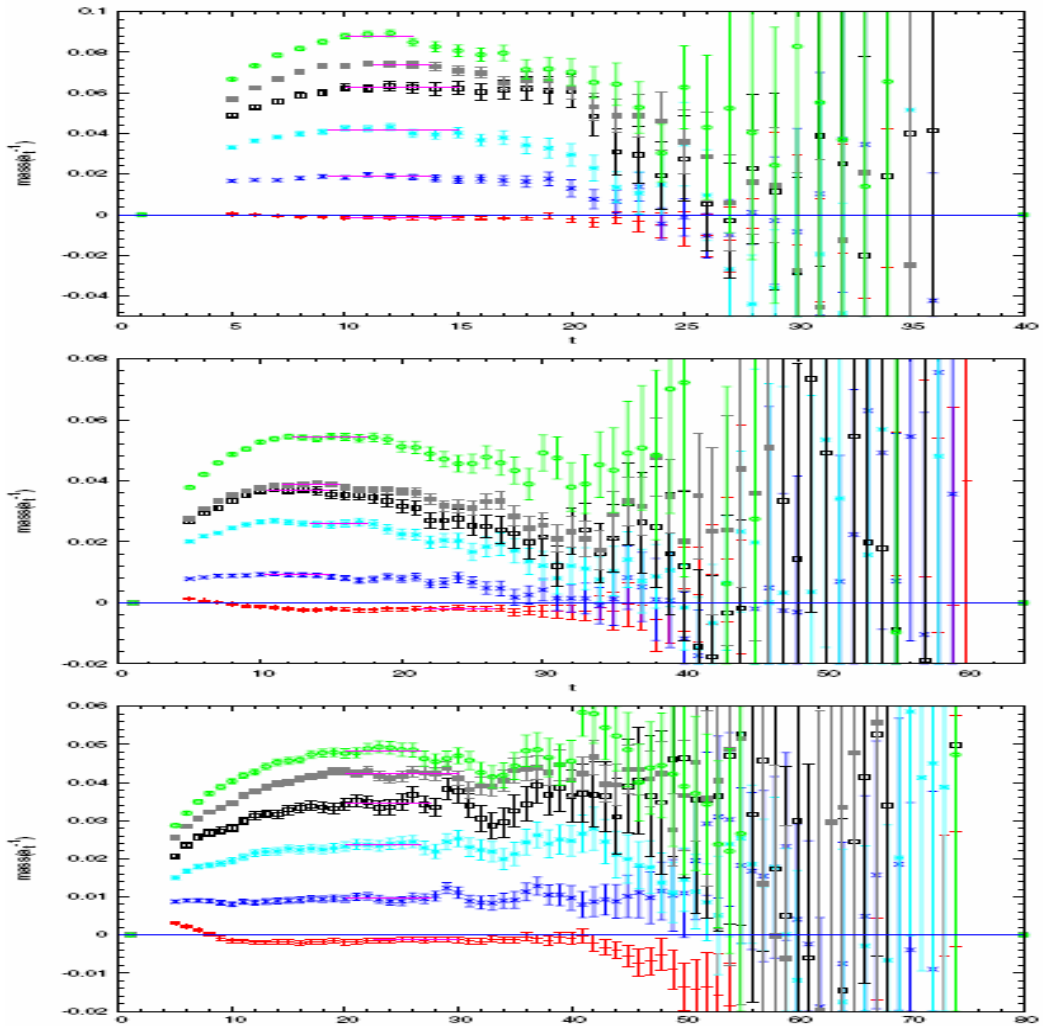


FIG. 5. Dispersion relations for various mesons obtained from single meson energies. From top to bottom: D^*, $\bar{D}_1$, η_c, J/ψ; from left to right: $\beta = 2.5, 2.8, 3.2$.

Simulation results

- Two-particle energy



Simulation results

- Scattering length a_0
- & effective range r_0

$$\frac{k}{\tan\delta(k)} = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots,$$

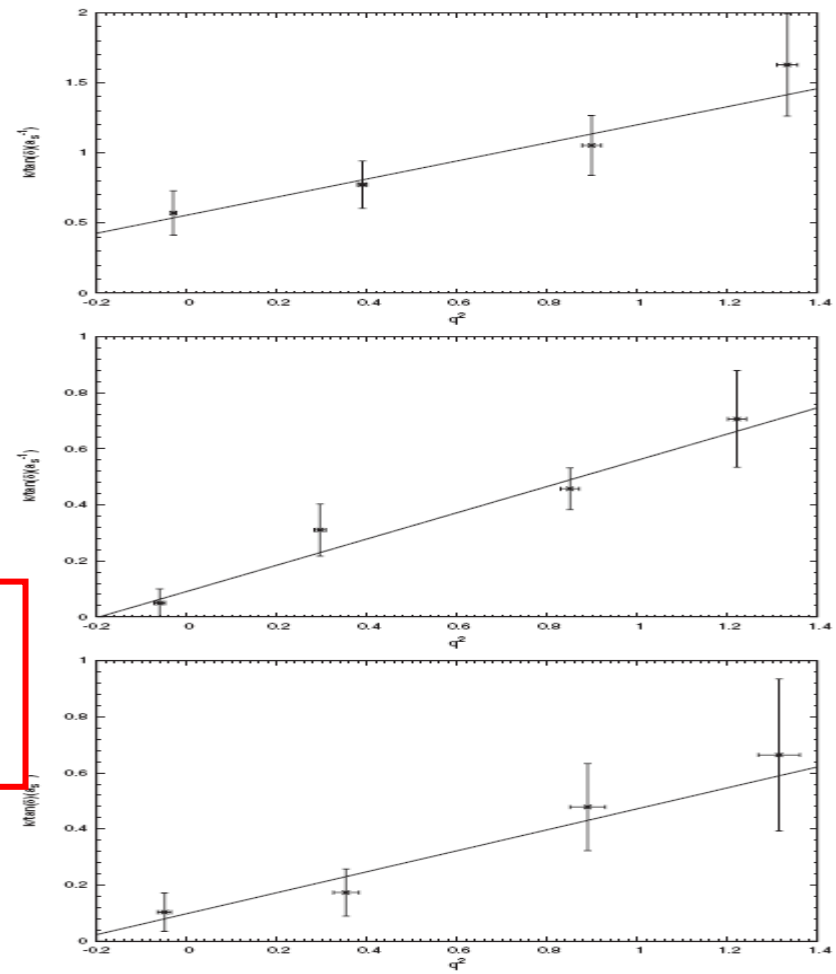
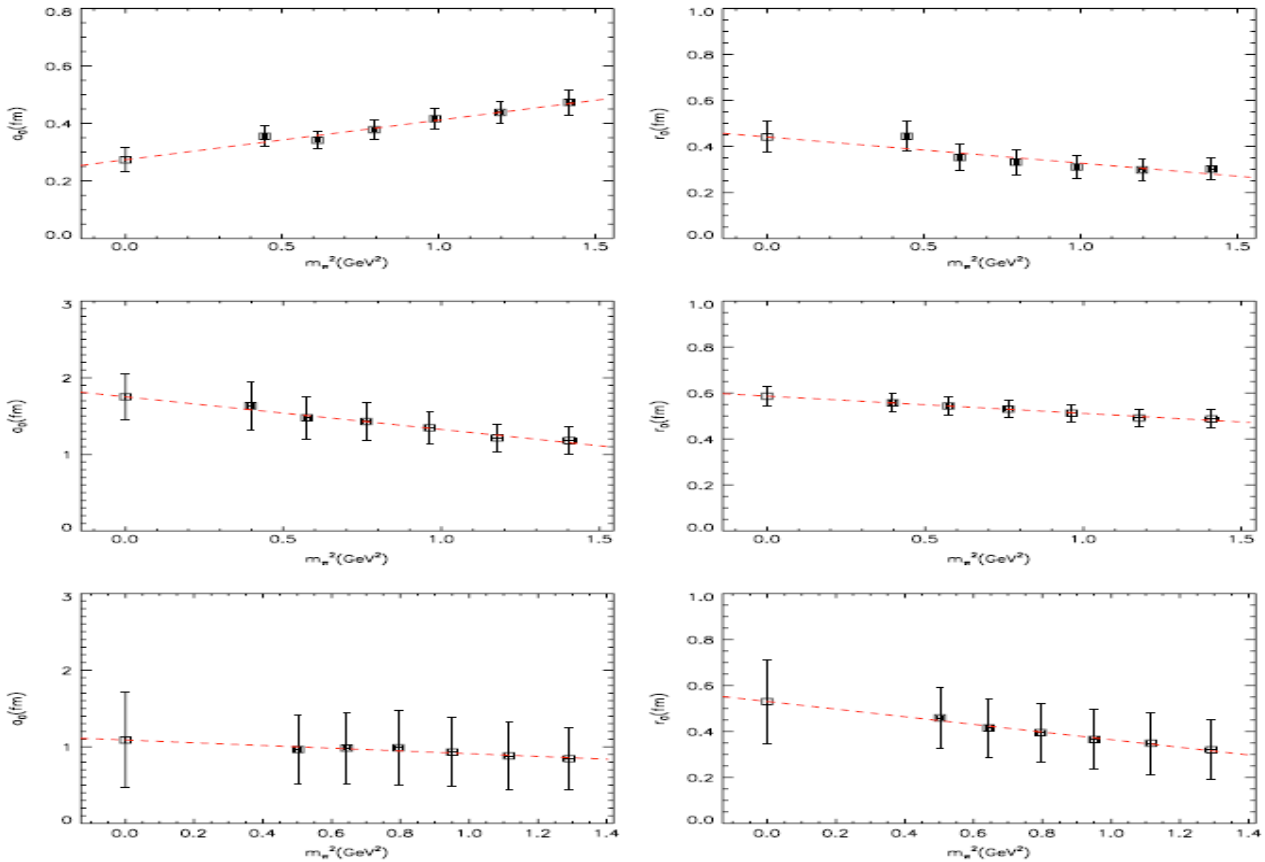


FIG. 7. The quantity $k/\tan\delta(k)$ versus q^2 in the $A_1^{(1)}$ channel. From top to bottom: $\beta = 2.5$, 2.8, and 3.2.

Simulation results

- After all the extrapolations

$$a_0 = 2.53 \pm 0.47 \text{ fm}, \qquad r_0 = 0.70 \pm 0.10 \text{ fm}.$$



Simulation results

- Possible bound state?
  - Bound state: $\boxed{q^2 \to -\infty, \cot\sigma(q^2) \to (-1)}$

TABLE III. Results for the lowest q^2 and the corresponding values for $\cot\sigma(q)$ as given by Eq. (8) for different values of β in the simulation. Corresponding errors for the quantities are also given in the parenthesis.

β	q^2	$\cot\sigma(q^2)$
2.5	−0.026(0.003)	5.23(0.65)
2.8	−0.064(0.005)	0.16(0.18)
3.2	−0.053(0.016)	0.92(0.93)

Simulation results

- Possible bound state?
  - For shallow bound state, $\boxed{a_0 \rightarrow -\infty}$
  - But our scattering lengths are all positive
  - On the verge of developing a bound state
  - Using the square well potential model to estimate, the potential well is V_0=70(10) MeV, R=0.7fm.
- Our results is not in favor of a shallow resonance

Conclusions

- Low energy $\boxed{D^{*+}\overline{D}_1^0}$ scattering is studied in the channel of $J^P = 0^-$ using quenched lattice QCD on anisotropic lattices
- Scattering length and effective range obtained
- The interaction between the two *D* mesons is attractive in nature
- But it is unlikely that they can form a shallow bound state below the threshold
- More studies are welcome