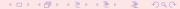
# Comparing AdS/CFT inspired dipole model to HERA $F_2$ data

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- Brief introduction to AdS/CFT dipole model
- 2 Fitting AdS/CFT inspired dipole model to HERA  $F_2$  data
- Predictions
  - Inclusion of charm and prediction for  $F_2^c$
  - prediction for F<sub>L</sub>
  - prediction for total photoproduction cross section
- Summary

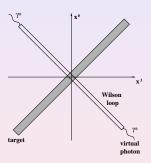
#### Reference:

Yuri Kovchegov, ZL, Amir Rezaeian, arxiv:0906.4197



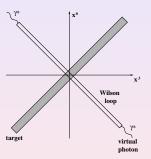
• Total deeply inelastic cross-section at large s

$$\sigma_{tot}^{\gamma^{\star}A}(Q^2, x_{Bj}) \propto \int \frac{d^2\mathbf{r}dz}{2\pi} \Psi(r, z; Q^2) N(\mathbf{r}, Y), \qquad Y = In\left(\frac{1}{x_{Bj}}\right)$$



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• N related to the expectation value of Wilson loop  $\langle W \rangle$  :

$$\begin{array}{rcl} \mathcal{N}(\mathbf{r},Y) & = & 1 - \mathcal{S}(\mathbf{r},Y) \\ \mathcal{S}(\mathbf{r},\mathbf{b},Y) & = & \frac{1}{\mathcal{N}_c} \mathrm{Re} \left\langle W\left(\mathbf{b} + \frac{1}{2}\mathbf{r},\mathbf{b} - \frac{1}{2}\mathbf{r},Y\right) \right\rangle \end{array}$$

$$N(r,s) = 1 - S(r,s) = 1 - \exp\left[-rac{\lambda_{YM}}{\sqrt{2}\pi s}\left(rac{c_0^2 r^2}{
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- $\rho$  is the function of r,s and  $c_0$



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$$x = \frac{Q^2}{s + Q^2} \Rightarrow s = \frac{Q^2(1 - x)}{x}$$

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$$\Rightarrow N(r,x) = 1 - \exp\left[-\frac{\lambda_{YM} x r}{\mathcal{M}_0^2 (1-x)\pi\sqrt{2}} \left(\frac{1}{\rho_m^3} + \frac{2}{\rho_m} - 2\mathcal{M}_0 \sqrt{\frac{1-x}{x}}\right)\right]$$

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$$\rho_m = \begin{cases} (\frac{1}{3m})^{1/4} \sqrt{2\cos(\frac{\theta}{3})} & : m \le \frac{4}{27} \\ \sqrt{\frac{1}{3m\Delta} + \Delta} & : m > \frac{4}{27} \end{cases}$$

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$$m = \frac{\mathcal{M}_0^4 (1-x)^2}{x^2}$$

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$$\begin{split} |\Psi_{T}^{(f)}(r,z;Q^{2})|^{2} &= \frac{\alpha_{EM} N_{c}}{2 \pi^{2}} \sum_{f} e_{f}^{2} \left\{ a_{f}^{2} \left[ K_{1}(r \, a_{f}) \right]^{2} \left[ z^{2} + (1-z)^{2} \right] \right. \\ &+ \left. m_{f}^{2} \left[ K_{0}(r \, a_{f}) \right]^{2} \right\} \\ |\Psi_{L}^{(f)}(r,z;Q^{2})|^{2} &= \frac{\alpha_{EM} N_{c}}{2 \pi^{2}} \sum_{f} e_{f}^{2} \left\{ 4 \, Q^{2} z^{2} (1-z)^{2} \left[ K_{0}(r \, a_{f}) \right]^{2} \right\} \end{split}$$

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# AdS/CFT-based fit to $F_2$ data

$$\sigma_{q\bar{q}}(r,x) = \frac{\sigma_0}{\sigma_0} \left( 1 - \exp\left[ -\frac{\frac{\lambda_{\text{YM}} x r}{\mathcal{M}_0^2 (1-x) \pi \sqrt{2}} \left( \frac{1}{\rho_m^3} + \frac{2}{\rho_m} - 2 \mathcal{M}_0 \sqrt{\frac{1-x}{x}} \right) \right] \right)$$

$\lambda_{YM}$	$M_0/10^{-3}$	$\sigma_0$ [mb]	$\chi^2/{\rm d.o.f.}$
5	9.85	31.164	110.70/78 = 1.42
20	6.36	22.65	141.12/78 = 1.81
5	10.114	30.97	44.24/60 = 0.74
10	8.16	26.08	49.22/60 = 0.82
20	6.54	22.47	55.195/60 = 0.92
	5 20 5 10	5 9.85 20 6.36 5 10.114 10 8.16	5     9.85     31.164       20     6.36     22.65       5     10.114     30.97       10     8.16     26.08

Table: Parameters of the AdS/CFT dipole model determined from a fit to  $F_2$  data reported by ZEUS in two Bjorken x bins. The value of quark mass  $m_f = 140$  MeV (f = u, d, s) is taken for all fits. The data for the first two rows and the rest are within  $Q^2/\text{GeV}^2 \in [0.045, 6.5]$  and  $Q^2/\text{GeV}^2 \in [0.045, 2.5]$  respectively.

# GBW dipole model fit to the same data bins

 Golec-Biernat-Wüsthoff (GBW) dipole model (Phys. Rev. D59 (1999) 014017; D60 (1999) 114023):

$$\sigma_{q\bar{q}}^{\mathsf{GBW}}(r,x) = \sigma_0 \left(1 - e^{-r^2\left(\frac{x_0}{x}\right)^{\lambda}/4}\right)$$

GBW dipole model	$x_0/10^{-4}$	λ	$\sigma_0$ [mb]	$\chi^2/{\rm d.o.f.}$
$x \in [6.2 \times 10^{-7}, 10^{-4}]$	2.225	0.299	22.77	63.09/78 = 0.81
$x \in [6.2 \times 10^{-7}, 6 \times 10^{-5}]$	2.371	0.368	21.13	39.35/60 = 0.66

Table: Parameters of the GBW color dipole model determined from a fit to  $F_2$  data from ZEUS in two Bjorken x bins. The data for the first two rows and the rest are within  $Q^2/\text{GeV}^2 \in [0.045, 6.5]$  and  $Q^2/\text{GeV}^2 \in [0.045, 2.5]$ 

# AdS/CFT-based fit to $F_2$ data

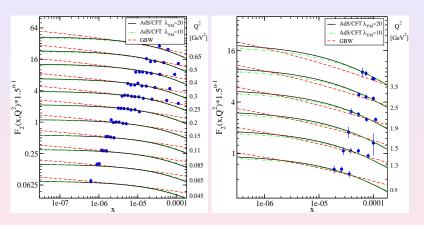
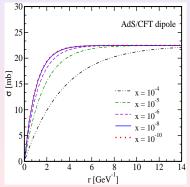


Figure: Results of our AdS/CFT-based fit to the proton structure function  $F_2$ .

# AdS/CFT-based dipole cross-section

$$\sigma_{q\bar{q}}(r,x) = \frac{\sigma_0}{\sigma_0} \left( 1 - \exp\left[ -\frac{\frac{\lambda_{YM} \times r}{\mathcal{M}_0^2 (1-x)\pi\sqrt{2}} \left( \frac{1}{\rho_m^3} + \frac{2}{\rho_m} - 2\mathcal{M}_0 \sqrt{\frac{1-x}{x}} \right) \right] \right)$$



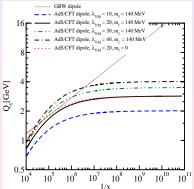
The AdS/CFT dipole cross-section obtained from the fit for  $\lambda_{YM}=20$  and  $m_q=140$  MeV at various fixed Bjorken-x as a function of r

$$N(r_s = \sqrt{2}/Q_s, x) = \mathcal{N}_0 \equiv 1 - e^{-1/2} \approx 0.4$$

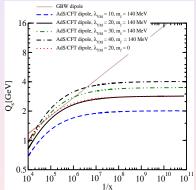
$$egin{aligned} N(\emph{r}_s = \sqrt{2}/\emph{Q}_s, \emph{x}) &= \mathcal{N}_0 \equiv 1 - e^{-1/2} pprox 0.4 \ \emph{Q}_s^{\mathsf{GBW}}(\emph{x}) &= \left(rac{\emph{x}_0}{\emph{x}}
ight)^{\lambda/2} \end{aligned}$$

$$\begin{split} & \textit{N}(\textit{r}_{\textit{s}} = \sqrt{2}/\textit{Q}_{\textit{s}}, \textit{x}) = \mathcal{N}_{0} \equiv 1 - e^{-1/2} \approx 0.4 \\ & \textit{Q}_{\textit{s}}^{\text{GBW}}(\textit{x}) = \left(\frac{x_{0}}{\textit{x}}\right)^{\lambda/2} \qquad \textit{Q}_{\textit{s}}^{\text{AdS}}(\textit{x}) = \frac{2\,\lambda_{\textit{YM}}\,\textit{x}}{\mathcal{M}_{0}^{2}\,(1-\textit{x})\,\pi}\,\left(\frac{1}{\rho_{\textit{m}}^{3}} + \frac{2}{\rho_{\textit{m}}} - 2\mathcal{M}_{0}\sqrt{\frac{1-\textit{x}}{\textit{x}}}\right) \end{split}$$

$$\begin{split} N(r_s &= \sqrt{2}/Q_s, x) = \mathcal{N}_0 \equiv 1 - e^{-1/2} \approx 0.4 \\ Q_s^{\text{GBW}}(x) &= \left(\frac{x_0}{x}\right)^{\lambda/2} \qquad Q_s^{\text{AdS}}(x) = \frac{2 \, \lambda_{\text{YM}} \, x}{\mathcal{M}_0^2 \, (1-x) \, \pi} \, \left(\frac{1}{\rho_m^3} + \frac{2}{\rho_m} - 2 \mathcal{M}_0 \sqrt{\frac{1-x}{x}}\right) \end{split}$$

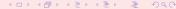


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x-independence of  $Q_s^{AdS}(x)$  at small x: saturation of saturation Conjectured in arxiv:0707.0811,

D. Kharzeev et.al.



# s-dependent dipole fit

$$\sigma_{q\bar{q}}(r,s) = \sigma_0 \left( 1 - \exp\left[ -\frac{\lambda_{YM}}{s\pi\sqrt{2}} \left( \frac{c_0^2 r^2}{\rho^3} + \frac{2}{\rho} - 2\sqrt{s} \right) \right] \right)$$

$\lambda_{YM}$	<i>c</i> <sub>0</sub>	$\sigma_0$ [mb]	$\chi^2/{\rm d.o.f.}$
5	0.00583	40.55	62.61/60 = 1.04
10	0.00440	36.30	77.17/60 = 1.29
20	0.00324	33.58	92.11/60 = 1.53

Table: Parameters of the s-dependent AdS/CFT dipole model determined from a fit to  $F_2$  data from ZEUS. The data are within  $x \in [6.2 \times 10^{-7}, 6 \times 10^{-5}]$  and  $Q^2/\text{GeV}^2 \in [0.045, 2.5]$ .

- In  $\mathcal{N}=4$  SYM:  $c_0=\frac{\Gamma^2(\frac{1}{4})}{(2\pi)^{3/2}}=0.834.$
- our fitting result is two orders of magnitude smaller: difference between QCD and  $\mathcal{N}=4$  SYM.

- Brief introduction to AdS/CFT dipole model
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- 4 Summary

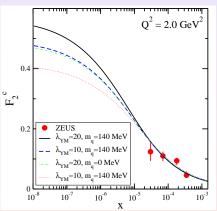
# Inclusion of charm and prediction for $F_2^c$

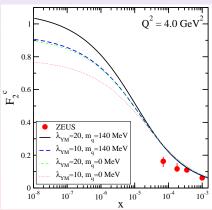
$$F_2 = F_2^u + F_2^d + F_2^s + F_2^c$$

$m_c[{\sf GeV}]$	$\lambda_{YM}$	$M_0/10^{-3}$	$\sigma_0$ [mb]	$\chi^2/{\sf d.o.f.}$
-	10	8.16	26.08	49.22/60 = 0.82
	20	6.54	22.47	55.195/60 = 0.92
1.4	10	7.66	24.72	61.66/60 = 1.03
1.4	20	6.16	21.31	70.99/60 = 1.18

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# Inclusion of charm and prediction for $F_2^c$





# prediction for $F_L$

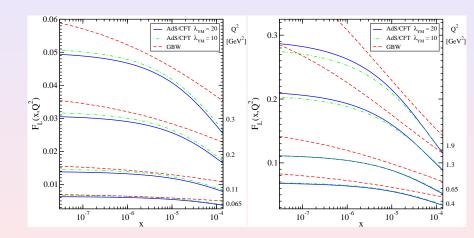


Figure:  $F_L$  predicted from our fit

# photoproduction

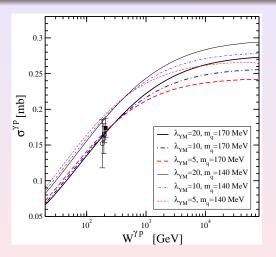


Figure: Photoproduction cross section at large  $\sqrt{s}$  predicted from our fit compared with ZEUS (square) and H1 (circle) data.

• HERA data for inclusive structure function  $F_2(x, Q^2)$  at small Bjorken-x and  $Q^2$  can be reasonably well described by an AdS/CFT-inspired dipole model with three parameters.

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- The saturation scale in AdS/CFT-inspired dipole model varies in the range of  $1 \div 3$  GeV becoming independent of energy (Bjorken-x) at very small x ( $10^{-8}$ ), leads to the prediction of x-independent of the  $F_2$  and  $F_L$  structure function at very small x.

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- Predictions for  $F_2$ ,  $F_2^c$ ,  $F_L$  and photoproduction in the kinematical regions of future experiments.
- AdS/CFT-based model of non-perturbative physics at large coupling could be viewed as complimentary to the perturbative description of data based on saturation/Color Glass Condensate physics.

Brief introduction to AdS/CFT dipole model Fitting AdS/CFT inspired dipole model to HERA  $F_2$  data Predictions Summary

Backup slides

