TECHNISCHE UNIVERSITÄT DARMSTADT

Bethe-Salpeter studies of mesons beyond rainbow-ladder

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Richard Williams and Christian Fischer

C. S. Fischer and RW, Phys. Rev. D **78** 2008 074006, [arXiv:0808.3372]
C. S. Fischer and RW, Phys. Rev. Lett. **103** 2009 122001, [arXiv:0905.2291]
C. S. Fischer and RW, *unpublished*

24. September 2009

Motivation

Desire: description of hadronic bound-states in QCD

- Non-perturbative
- Formulated in the continuum
- Poincaré covariant
- Applicable to chiral/heavy quarks

Framework that satisfies all of these requirements:

- Bethe-Salpeter/Faddeev equations (mesons/baryons)
- Input Green's functions from Dyson-Schwinger equations.

Description is

- *ab initio*, modulo necessary *truncations*. Work in Landau gauge.
- complementary to lattice QCD, effective theories, quark models etc.

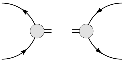
- Mesons: described by Bethe-Salpeter equation
- Baryons: quark-diquark model/Faddeev equation.

[G. Eichmann, PhD thesis (KFU Graz) [arXiv:0909.0703]]

Most common truncation: Rainbow-Ladder – *Time to do better?*

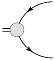
[P. Maris, P. C. Tandy, PRC **60** (1999) 055214]

Introduction: Bound-state amplitudes



$$\overline{(2\pi)^4\left(P^2+M^2\right)}$$

- Meson bound-states: poles in two-body propagator.
- Depend on two momenta: total P and relative q.
- Amplitude defined on-shell $P^2=-M^2$ (Euclidean Space).



$$=\Gamma^{(\mu)}(q,P)=\sum_i \tau_i(q,P)T_i^{(\mu)}(q,P)$$

Relativistic covariants

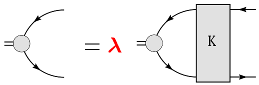
[C. H. Llewellyn-Smith, Annals Phys. **53** (1969) 521.]

		Component	0^{-+}	0^{++}
T_1	:	$\mathbb{1}$	○	○
T_2	:	$-i\not{P}$	○	•
T_3	:	$-i\not{p}$	•	○
T_4	:	$[\not{p},\not{P}]$	○	○

$J^{PC}=0^{-+}$, 1^{++}, 1^{+-}: prefixed by γ_5.

		Component	1^{--}	1^{++}	1^{+-}
T_1^μ	:	$i\gamma_T^\mu$	○	○	
T_2^μ	:	$\gamma_T^\mu\not{P}$	○	•	
T_3^μ	:	$-\gamma_T^\mu\not{p}+p_T^\mu\mathbb{1}$	•	○	
T_4^μ	:	$i\gamma_T^\mu[\not{P},\not{p}]+2ip_T^\mu\not{P}$	○	○	
T_5^μ	:	$p_T^\mu\mathbb{1}$	○		○
T_6^μ	:	$ip_T^\mu\not{P}$	•		○
T_7^μ	:	$-ip_T^\mu\not{p}$	○		•
T_8^μ	:	$p_T^\mu[\not{P},\not{p}]$	○		○

Introduction: Bound-state amplitudes



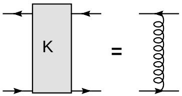
Eigenvalue problem $\lambda(P^2) = 1$ corresponds to discrete bound-states.

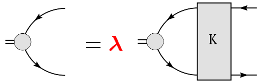
Solve homogeneous Bethe-Salpeter Equation:

- Obtain Bound-state mass
- Electroweak decay constants
- Electromagnetic form factors
  - pion charge radius
  - magnetic moments
- transition form factors (hadronic decays)
- structure functions

Need:

- quark propagator from Dyson-Schwinger equation
- Normalisation (provided by an auxiliary condition)
- Dynamical information – Bethe-Salpeter kernel K





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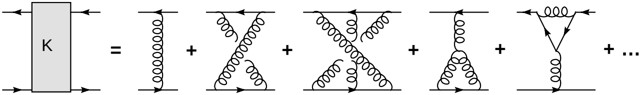
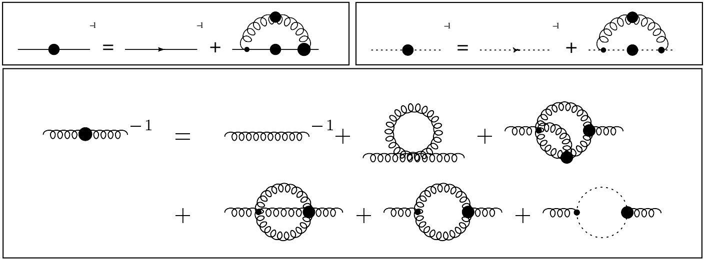


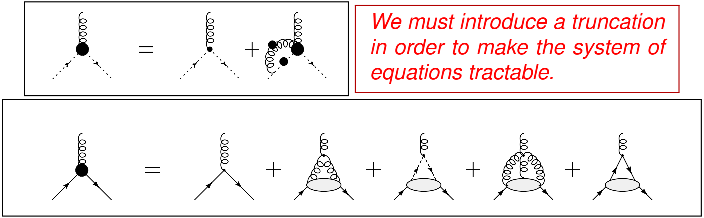
Introduction: Dyson-Schwinger equations

infinite tower of coupled integral equations relating the Greens functions of QCD

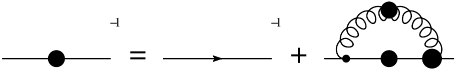
- Propagators, Vertices and higher n-point Greens functions



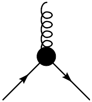
We must introduce a truncation in order to make the system of equations tractable.



Truncation: Minimum requirements

$-i\not{p}\, A(p^2) + B(p^2) =$ 

 $= \left(\delta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \frac{Z(q^2)}{q^2}$

 $= \sum_{i=1}^{12} \lambda_i(k,p;q) L_i^\mu(k,p;q)$

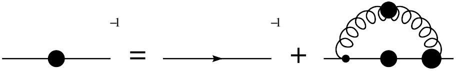
- Quark propagator essential for the Bethe-Salpeter equation
- Specification of:
  - Gluon propagator and the Quark-gluon vertex
- sufficient to close the system and provide input for BSE.

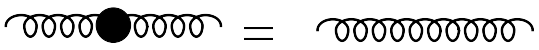
That means provision of:

- One scalar dressing for the gluon
- Twelve scalar dressings for the quark, dependent upon the incoming and outgoing quark momenta.

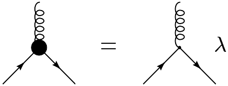
Simplest truncation: the Rainbow

$-i\not{p}\,A(p^2)+B(p^2) =$





$\frac{Z(q^2)}{q^2}$



$\lambda_1\left((k-p)^2\right)$

Simplifications:

- Construct a reduced quark-gluon vertex:
  - Consider only the tree-level component, $\Gamma(k,p)^\mu = \lambda_1(k,p)\gamma^\mu$
  - $\lambda_1(k,p) \to \lambda_1(k-p)$ (important for BS equation)
- Provide a gluon dressing by
  - either solve Dyson-Schwinger equation for gluon
  - provide Ansatz that provides dynamical chiral symmetry breaking (IR strength, matching of perturbation theory etc.)

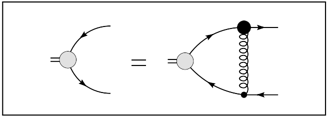
Axial-vector Ward–Takahashi identity

$$P_\mu \Gamma^a_{5\mu}(k;P) = S^{-1}(k_+)\frac{1}{2}\lambda^a_f i\gamma_5 + \frac{1}{2}\lambda^a_f i\gamma_5 S^{-1}(k_-) - M_\zeta i\Gamma^a_5(k;P) - i\Gamma^a_5(k;P)M_\zeta$$

Satisfaction of this gives rise to

- Gell-Mann–Oakes-Renner
- Massless pion in the chiral limit

$$m^2_\pi f_\pi \simeq 2m_q \langle \bar{q}q \rangle$$



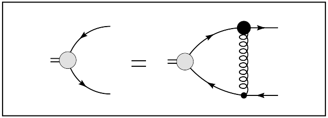
Symmetry preserving truncation

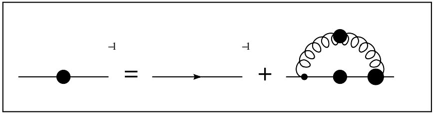
- Replace kernel by single gluon-exchange
- Quark-gluon vertex minimally dressed:

Require consistency between quark DSE and BS kernel

- Gluon dressing function $Z(q^2)$ must match
- Vertex dressing $\lambda_1(q)$ must match

Philosophy of Rainbow-Ladder





- Gluon dressing function $Z(q^2)$
- Quark-gluon vertex $\Gamma^{\mu} = \lambda_1 \left(q^2\right) \gamma^{\mu}$

Phenomenological Model

- Provide model interaction $\alpha_{\mathit{eff}} = \alpha Z(q^2)\lambda_1(q^2)$

→ Dynamical chiral symmetry breaking

→ Chiral condensate

→ Meson observables

Entirely an Ansatz.

Towards *ab initio*

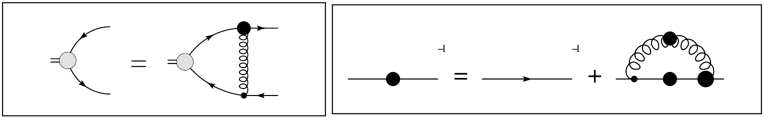
- Solve Gluon DSE
- Solve Quark-Gluon DSE

→ Discard tensor structure

→ restrict momentum dependence.

Rainbow-ladder but
Guided by calculations.

Philosophy of Rainbow-Ladder



Phenomenological models successful

Interaction features enhancement in infrared

$\longrightarrow$ Dynamical chiral symmetry breaking

$\longrightarrow$ Reasonable description of

- light-meson masses
- leptonic decay constants
- hadronic decays
- electromagnetic form factors

Result? Majority of properties are governed by chiral symmetry breaking, not the details of the interaction.
Limitations? Intrinsic to rainbow-ladder: restriction to vector-vector coupling limits important differences in different meson channels.
Interaction has no quark mass dependence (unless we model it)

Beyond Rainbow-Ladder

Any scheme must

- Respect the axial-vector Ward-Takahashi identity

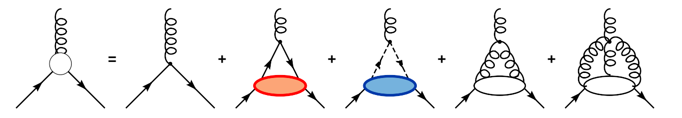
$\longrightarrow$ Intimate relation between BS kernel and quark DS kernel

[H. J. Munczek, Phys. Rev. D **52** (1995) 4736.], [A. Bender, C. D. Roberts, L. von Smekal, Phys. Rev. Lett. B **380** (1996) 7]

Symmetry preserving truncation: obtained by cutting internal quark-lines in the quark Dyson-Schwinger equation

Look at the quark-gluon vertex

Dyson-Schwinger equation involves several unknown Green's functions



- Perform dressed skeleton expansion to perform truncation
- IR power counting, $1/N_c$ expansion yields dominant contributions.

Beyond Rainbow-Ladder: what has been done?

Any scheme must

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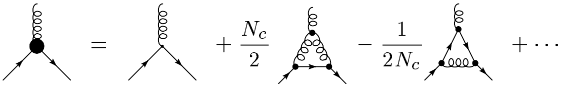
Symmetry preserving truncation may be obtained by cutting internal quark-lines in the quark Dyson-Schwinger equation

Semi-perturbative expansion (one-loop)



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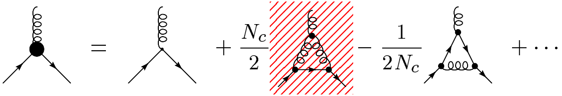

Trivialization of the gluon momentum dependence (reduces # integrals):

$$D_{\mu\nu}(q) \propto \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{(4)}(q) \qquad \sim \textit{(Munczek–Nemirovsky)}$$

[A. Bender, C. D. Roberts, L. von Smekal, Phys. Rev. Lett. B **380** (1996) 7]
[P. Watson, W. Cassing and P. C. Tandy, Few Body Syst. **35** (2004) 129]
[H. H. Matevosyan, A. W. Thomas and P. C. Tandy, Phys. Rev. C **75** (2007) 045201]

Beyond Rainbow-Ladder: what must be done?

To investigate dominant contributions we need:

- a gluon with non-trivial momentum dependence
- appropriate internal vertex dressings

We must bite the bullet and tackle two-loop integrals over non-perturbative propagators and vertex dressings. *Challenging on a technical level*.

First study: *compare to existing literature*

Investigate the new truncation with a model gluon that provide DχSB:

- no trivialization of the gluon momentum

$\longrightarrow$ study qualitative impact of vertex corrections

To perform a quantitative investigation: replace the gluon Ansatz with something more appropriate, and specify the internal vertex dressings.

$$\alpha\lambda_1(q^2)Z(q^2) = \frac{\pi \boldsymbol{D}}{\boldsymbol{\omega}^2}\; q^4\; e^{-q^2/\boldsymbol{\omega}^2}$$

- Enhanced at intermediate momentum
- Damped in UV (no renormalisation)

Beyond Rainbow-Ladder: what must be done?

To investigate dominant contributions we need:

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We must bite the bullet and tackle two-loop integrals over non-perturbative propagators and vertex dressings. *Challenging on a technical level*.

What do we gain?

- Truncation does not restrict what mesons we can study:

	π	σ	ρ	a_1	b_1
(J^{PC})	0^{-+}	0^{++}	1^{--}	1^{++}	1^{+-}

- Quark-gluon vertex:
  - all twelve components – dynamically generate scalar parts
  - full momentum dependence

  $\longrightarrow$ imparts quark-mass dependence on interaction.
- Different interactions:
  - vector $\times$ vector
  - vector $\times$ scalar
  - different degrees of attraction/repulsion in different meson channels.

To investigate dominant contributions we need:

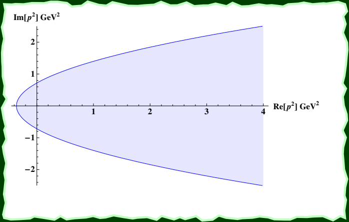
- a gluon with non-trivial momentum dependence
- appropriate internal vertex dressings

We must bite the bullet and tackle two-loop integrals over non-perturbative propagators and vertex dressings. *Challenging on a technical level*.

Technical challenges?

Auxiliary normalisation condition: (*Leon–Cutkosky*)

- momentum depdendent kernel complicates evaluation

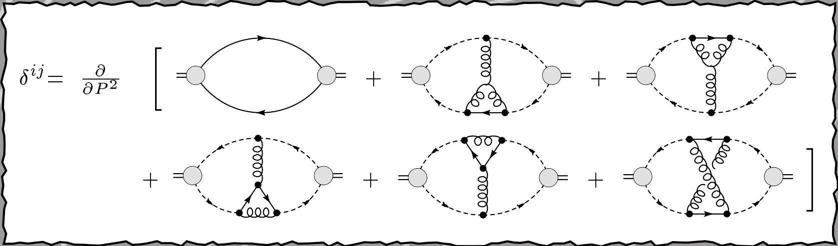


Bound state in Eucl. space, $P^0 = iM$ quark and vertex momenta $(q \pm P/2)^2$:

- must be evaluated for complex momenta
- region bounded by parabola dependent upon meson mass.

Normalisation: Leon–Cutkosky

$$\delta^{ij} = \frac{\partial}{\partial P^2} tr \int_k \left[\left(\overline{\Gamma}^i_\pi S \Gamma^j_\pi S \right) + \int_q \left([\overline{\chi}^i_\pi]_{sr} K_{tu;rs} [\chi^j_\pi]_{ut} \right) \right]$$



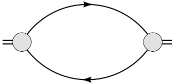
lines	quark propagator.
dashed lines	quark propagator fixed wrt. derivative.
wiggles	gluon propagator.

[R. E. Cutkosky and M. Leon, Phys. Rev. **135** (1964) B1445.]

[P. Maris and P. C. Tandy, Nucl. Phys. Proc. Suppl. **161** (2006) 136.]

Normalisation: Nakanishi

$$\left(\frac{d\ln(\lambda)}{dP^2}\right)^{-1} = tr\int_k \bar{\Gamma} S \Gamma S$$

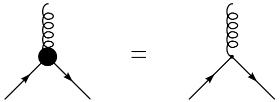
$$\left(\frac{d\ln(\lambda)}{dP^2}\right)^{-1} = \Bigg[$$  $$\Bigg]$$

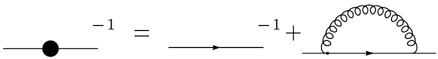
Where λ is the eigenvalue obtained via $\Gamma = \lambda K \Gamma$.

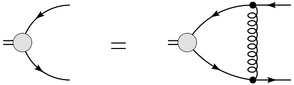
- considerably simpler
- valid for all truncations
- first time applied beyond rainbow-ladder

[N. Nakanishi, Phys. Rev. **138**, B1182 (1965)]

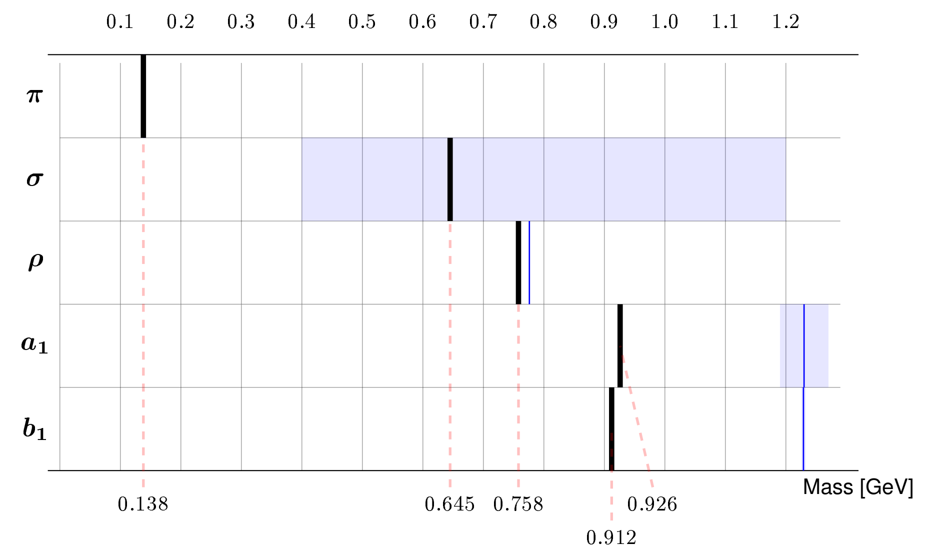
Calculate bound-state masses with bare vertex:







Results: Rainbow-Ladder benchmark



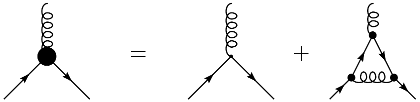
Results: Abelian corrections

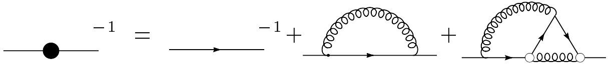
Consider sub-leading Abelian corrections to vertex

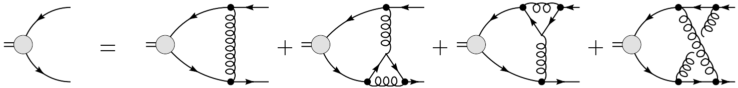
[A. Bender, C. D. Roberts, L. von Smekal, Phys. Rev. Lett. B **380** (1996) 7]

[A. Bender, W. Detmold, C. D. Roberts and A. W. Thomas, Phys. Rev. C **65** (2002) 065203]

[P. Watson, W. Cassing and P. C. Tandy, Few Body Syst. **35** (2004) 129]



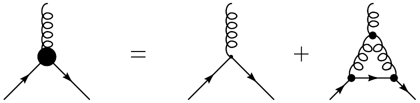


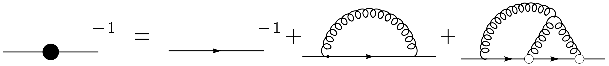


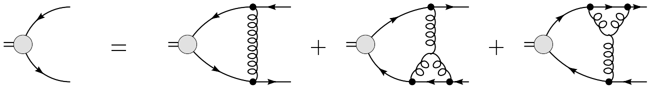
Results: Abelian corrections (unpublished)

Results: non-Abelian corrections

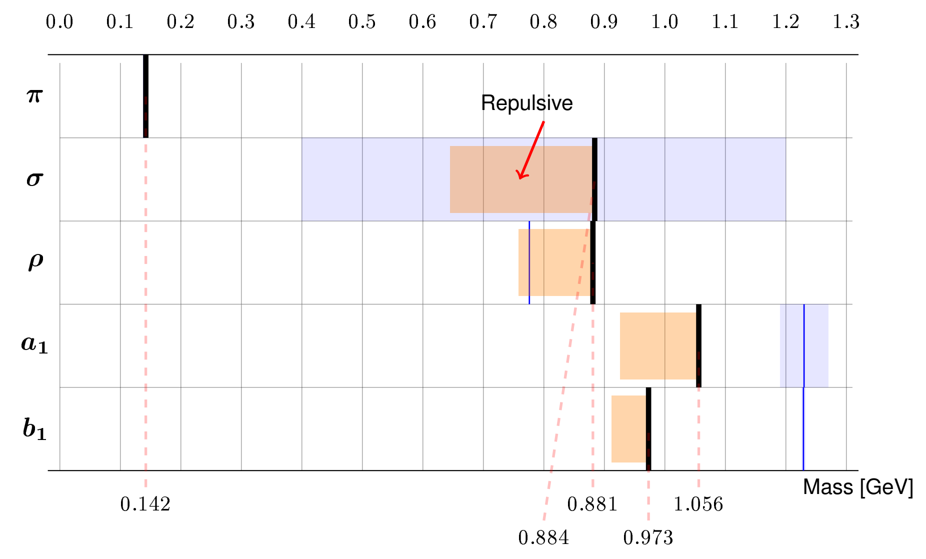
Consider dominant non-Abelian corrections to vertex





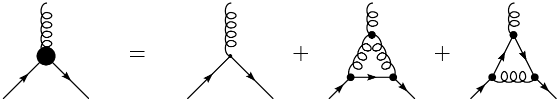


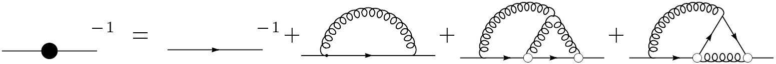
Results: non-Abelian corrections

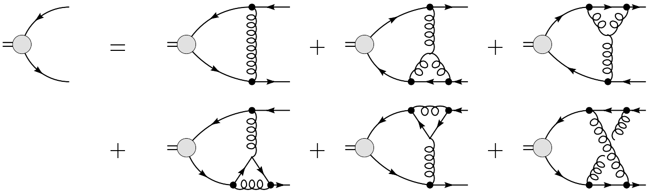


Results: non-Abelian and Abelian corrections

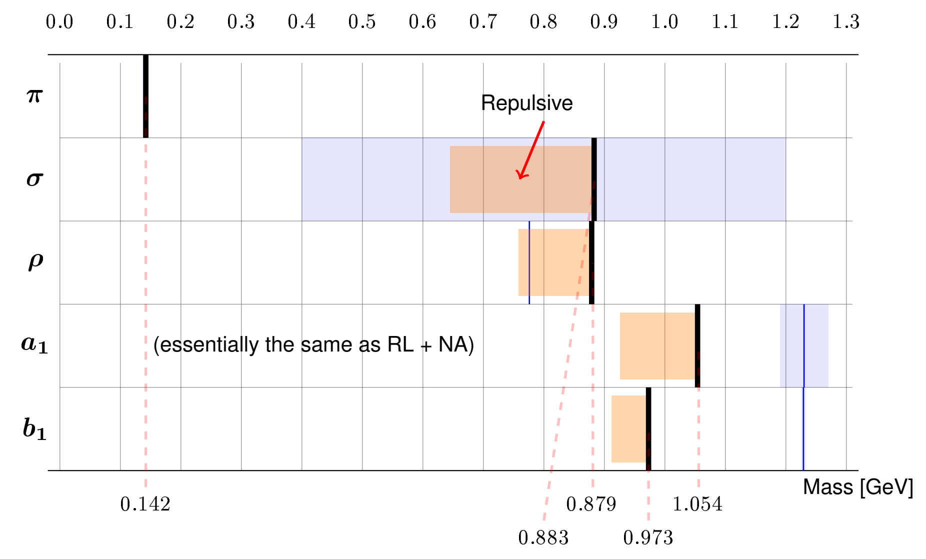
Put them all-together!



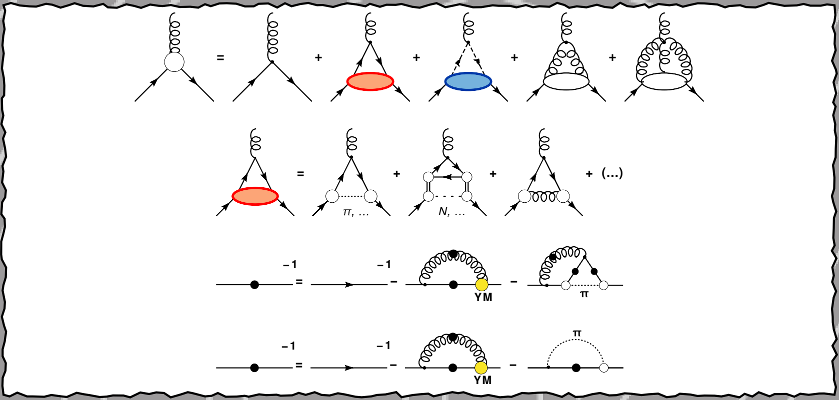




Results: non-Abelian and Abelian corrections (unpublished)



Contributions from the pion cloud



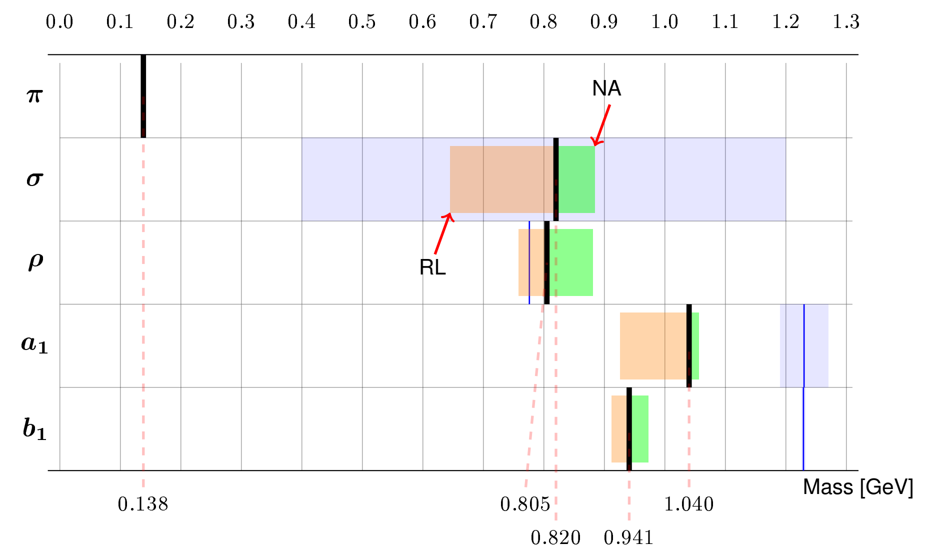
In addition to non-resonant diagrams

- resonant contributions from, *e.g.* pion exchange

⟶ *approximate and include*

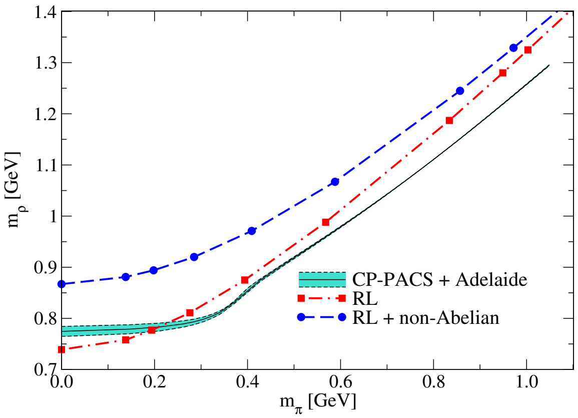
(Ignore contribution from sub-leading Abelian diagrams)

Results: non-Abelian and pion-cloud effects



Results: Mass dependence of Vector Meson

- Implicit mass-dependence of vertex corrections
- inc. pion cloud: shifts towards corrected lattice results.



[C. R. Allton, W. Armour, D. B. Leinweber, A. W. Thomas and R. D. Young, Phys. Lett. B **628** (2005) 125.]

Conclusions

Summary

Reached an era where rainbow-ladder is no longer the only feasible truncation

- first study **no trivialization of momenta**:
  - leading non-Abelian corrections
  - sub-leading Abelian corrections
  - *also* hadronic resonance contributions
- full calculation:
  - solve the full system of equations without overt simplifications
- Not just a toy-model
  - foundation on which to build future *ab initio* bound-state studies
  - $\longrightarrow$ just change the inputs to something realistic!

Outlook or *work in progress*

- incorporate solutions from gluon DSE.
- pion/electromagnetic form factors, decays
- do *exotics* and *diquarks* exist beyond rainbow-ladder – Bound/Unbound?
- excited states, tensor mesons, (and more quarks!).