Semileptonic $bc$ to $cc$ Baryon Decay and Heavy Quark Spin Symmetry

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- **PRD 76** (2007) 017502
- **PLB 663** (2008) 234 (QM)

**Motivation**: Separate heavy quark spin symmetries make it possible to describe the semileptonic decays

$$
\Xi^{(*)}_{bc} \rightarrow \Xi^{(*)}_{cc} l \bar{\nu}_l, \quad \Omega^{(*)}_{bc} \rightarrow \Omega^{(*)}_{cc} l \bar{\nu}_l
$$

in the limit $m_{b,c} \gg \Lambda_{QCD}$ and close to the zero recoil point.
\[ q^2 = m_{bc}^2 + m_{cc}^2 - 2m_{bc}m_{cc} \omega, \quad 1 \leq \omega \leq \frac{m_{bc}^2 + m_{cc}^2 - m_l^2}{2m_{bc}m_{cc}} \]

zero recoil

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<tr>
<th>SL FF's</th>
<th>S</th>
<th>J^P</th>
<th>I</th>
<th>( S_{hh}^{\pi} )</th>
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<tr>
<td>( \Xi_{cc} )</td>
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<td>( \frac{1}{2}^+ )</td>
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HQS constraints on SL FF’s and \( \Gamma \)’s of doubly heavy baryons.

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For instance, let us study $\Xi_{bc}^{(*)} \to \Xi_{cc}^{(*)}$ SL decays,

$$p_\mu = m_{\Xi_{bc}^{(*)}} v_\mu, \quad p'_\mu = m_{\Xi_{cc}^{(*)}} v'_\mu = m_{\Xi_{cc}^{(*)}} v_\mu + k_\mu$$

Near the zero-recoil point $\omega = 1$ ($\omega = v \cdot v'$) $k$ small residual momentum $\Rightarrow k \cdot v = \mathcal{O}(1/m_{\Xi_{cc}^{(*)}})$.

To represent the lowest-lying $S$-wave $bcq$ baryons we use wavefunctions comprising tensor products of Dirac matrices and spinors, namely:

$$\Xi'_{bc} = \left[ \frac{(1 + \psi)}{2} \gamma_5 \frac{(1 - \psi)}{2} \right]_{\alpha\beta} u_\gamma(v, r)$$

$$\Xi_{bc} = \left[ \frac{(1 + \psi)}{2} \gamma_\mu \frac{(1 - \psi)}{2} \right]_{\alpha\beta} \left[ \frac{1}{\sqrt{3}} (v^\mu + \gamma^\mu) \gamma_5 u(v, r) \right]_\gamma$$

$$\Xi^*_{bc} = \left[ \frac{(1 + \psi)}{2} \gamma_\mu \frac{(1 - \psi)}{2} \right]_{\alpha\beta} u^\mu_\gamma(v, r)$$

$\alpha, \beta, \gamma$ Dirac indices and $r$ baryon helicity label. These wavefunctions can be considered as matrix elements of the form $\langle 0 | b_\alpha \bar{c}^c_\beta q_\gamma | \Xi_{bc}^{(*)} \rangle$ where $\bar{c}^c = c^T C$ with $C$ the charge-conjugation matrix.
Under a Lorentz ($\Lambda$), and $b$ and $c$ quark spin ($S_b$ and $S_c$) transformations, a wavefunction $\Gamma_{\alpha\beta} u_\gamma$ transforms as:

$$\Gamma u \rightarrow S(\Lambda)\Gamma S^{-1}(\Lambda) S(\Lambda)u$$
$$\Gamma u \rightarrow S_b \Gamma S_c^\dagger u$$

**States** normalised using $\bar{u}u \text{Tr}(\Gamma\bar{\Gamma})$: mutually orthogonal and have a common normalisation ($\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$).

**We use similar wavefunctions for the $cc$ baryons.**
... construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements,

\[ \text{SL } \Xi^{(*)}_{bc} \rightarrow \Xi^{(*)}_{cc} \text{ decays } \leftrightarrow \text{ME weak current } J^\mu = \bar{c}\gamma^\mu(1-\gamma_5)b \]

The most general form for the ME respecting the HQSS is \((j^\mu = \gamma^\mu(1-\gamma_5))\):

\[
\langle \Xi^{(*)}_{cc}, v, k, M' | J^\mu(0) | \Xi^{(*)}_{bc}, v, M \rangle = \bar{u}_{cc}(v, k, M')\Omega \ u_{bc}(v, M) \text{Tr}[\Gamma_{cc} j^\mu \Gamma_{bc}]
\]

\[
\Gamma_{bc} \rightarrow S_b \Gamma_{bc} S_c^\dagger, \quad u_{bc} \rightarrow u_{bc}
\]

\[
\Gamma_{cc} \rightarrow S_c \Gamma_{cc} S_c^\dagger, \quad \bar{u}_{cc} \rightarrow \bar{u}_{cc}
\]

\[
\bar{c}\ j^\mu b: j^\mu \rightarrow S_c j^\mu S_b^\dagger
\]
... construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements,
\[ \text{SL } \Xi^{(f*)}_{bc} \rightarrow \Xi^{(*)}_{cc} \text{ decays} \leftrightarrow \text{ME weak current } J^{\mu} = \bar{c} \gamma^{\mu} (1 - \gamma_5) b \]

The most general form for the ME respecting the HQSS is \( (j^{\mu} = \gamma^{\mu}(1 - \gamma_5)) \):

\[
\langle \Xi^{(*)}_{cc}, v, k, M' | J^{\mu}(0) | \Xi^{(f*)}_{bc}, v, M \rangle = \bar{u}_{cc}(v, k, M') \Omega u_{bc}(v, M) \text{Tr}[S_c \Gamma_{cc} S_c^\dagger S_c j^{\mu} S_b^\dagger S_b \Gamma_{bc} S_c^\dagger] \\
\]

\[
\Gamma_{bc} \rightarrow S_b \Gamma_{bc} S_c^\dagger, \quad u_{bc} \rightarrow u_{bc} \\
\bar{\Gamma}_{cc} \rightarrow S_c \bar{\Gamma}_{cc} S_c^\dagger, \quad \bar{u}_{cc} \rightarrow \bar{u}_{cc} \\
\bar{c} j^{\mu} b : j^{\mu} \rightarrow S_c j^{\mu} S_b^\dagger 
\]
where $M$ and $M'$ are the helicities of the initial and final states

$$\Omega \propto \eta(v \cdot v')$$

is the most general Dirac matrix that can be written in terms of the vectors $k$ and $v$.

- terms with a factor of $\psi$ can be omitted because of the equations of motion ($\psi u = u$, $\psi \Gamma = \Gamma$, $\gamma_\mu u^\mu = 0$, $v_\mu u^\mu = 0$),
- terms with $k$ will always lead to contributions proportional to $v \cdot k = \mathcal{O}(1/m_{\Xi_{cc}^*})$. 
\[ \Xi_{bc} \rightarrow \Xi_{cc} \quad \frac{1}{\sqrt{2}} \eta \bar{u}_{cc} \left( 2 \gamma^\mu - \frac{4}{3} \gamma^\mu \gamma_5 \right) u_{bc} \]

\[ \Xi'_{bc} \rightarrow \Xi_{cc} \quad -\sqrt{\frac{2}{3}} \eta \bar{u}_{cc} \left( -\gamma^\mu \gamma_5 \right) u_{bc} \]

\[ \Xi_{bc} \rightarrow \Xi^*_{cc} \quad -\sqrt{\frac{2}{3}} \eta \bar{u}_{cc}^\mu u_{bc} \]

\[ \Xi'_{bc} \rightarrow \Xi^*_{cc} \quad -\sqrt{2} \eta \bar{u}_{cc}^\mu u_{bc} \]

\[ \Xi_{bc}^* \rightarrow \Xi_{cc} \quad -\sqrt{\frac{2}{3}} \eta \bar{u}_{cc}^\mu u_{bc} \]

\[ \Xi_{bc}^* \rightarrow \Xi^*_{cc} \quad -\sqrt{2} \eta \bar{u}_{cc}^\lambda (\gamma^\mu - \gamma^\mu \gamma_5) u_{bc} \lambda \]

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Remarks:

• If the $b$ and $c$ quarks become degenerate, then vector current conservation ensures that $\eta(1) = 1$.

• Savage and White (PLB 271 (1991) 410) found similar results: approach where the two heavy quarks bind into a colour antitriplet which appears as a pointlike colour source to the light degrees of freedom + “superflavor” formalism of Georgi and Wise. We find two differences to their results.

• Our approach, where we consider the spin transformations of each heavy quark explicitly, is straightforward and similar to that used to describe $B_c$ meson decays: Jenkins, Luke, Manohar and Savage, NPB 390 (1993) 463.
• Spin symmetry for both the $b$ and $c$ quarks enormously simplifies the description of all $\Xi_{bc}^{(*)} \rightarrow \Xi_{cc}^{(*)} l \bar{\nu}_l$ decays in the heavy quark limit and near the zero recoil point. All the weak transition matrix elements are given in terms of a single universal function. Lorentz covariance alone allows a large number of form factors (six form factors to describe $\Xi_{bc} \rightarrow \Xi_{cc}$, another six for $\Xi'_{bc} \rightarrow \Xi_{cc}$, eight each for $\Xi_{bc} \rightarrow \Xi_{cc}^{*}$, $\Xi'_{bc} \rightarrow \Xi_{cc}^{*}$ and $\Xi_{bc}^{*} \rightarrow \Xi_{cc}$, and even more for $\Xi_{bc}^{*} \rightarrow \Xi_{cc}^{*}$).
Test: QM [EPJA 32 (2007) 183 ]

\[
\eta (\mathbf{v} \cdot \mathbf{v}') = \int d^3 r_1 d^3 r_2 \exp \left[ -i \mathbf{k} \cdot \mathbf{r}_{12}/2 \right] [\Psi_{cc}^\Xi (r_1, r_2, r_{12})]^* \Psi_{bc}^\Xi (r_1, r_2, r_{12})
\]

\[ r_{12} \ll r_1, r_2 \rightarrow \Psi_{Qc}^\Xi (r_1, r_2, r_{12}) \approx \Phi_{Qc}(r_{12}) \phi(r_q) \]

\( \equiv \) Jacobi’s coordinates, \( Q, Q' = c, b \).

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\[
\left\langle \Xi_{cc}, r' \bar{p}' | \bar{c} \gamma^\mu (1 - \gamma_5) b(0) | \Xi_{bc}, r \bar{p} \right\rangle = \bar{u}_{r',c} (\bar{p}') \{ \gamma^\mu \left( F_1 (w) - \gamma_5 G_1 (w) \right) + \nu^\mu \left( F_2 (w) - \gamma_5 G_2 (w) \right) + \nu'^\mu \left( F_3 (w) - \gamma_5 G_3 (w) \right) \} u_{r,bc} (\bar{p})
\]
... $1/2 \rightarrow 3/2$ spin transitions

$$\langle \Xi_{cc}^*, r' \bar{p}' | \bar{c} \gamma^\mu (1 - \gamma_5) b(0) | \Xi_{bc}^{(r)}; r \bar{p} \rangle = \bar{u}^{\Xi_{cc}^*}_{\lambda r'}(\bar{p}') \Gamma^\lambda_\mu u^{\Xi_{bc}^{(r)}}_r(\bar{p})$$

$$\Gamma^\lambda_\mu = \left( \frac{C^V_3(\omega)}{m_{\Xi^{(r)}_{bc}}} (g^{\lambda \mu} \not{q} - q^{\lambda} \gamma^\mu) + \frac{C^V_4(\omega)}{m^2_{\Xi^{(r)}_{bc}}} (g^{\lambda \mu} q p' - q^{\lambda} p'^\mu) \right)\gamma_5$$

$$+ \left( \frac{C^V_5(\omega)}{m^2_{\Xi^{(r)}_{bc}}} (g^{\lambda \mu} q p - q^{\lambda} p^\mu) + C^V_6(\omega) g^{\lambda \mu} \right)\gamma_5$$

$$+ \left( \frac{C^A_3(\omega)}{m_{\Xi^{(r)}_{bc}}} (g^{\lambda \mu} \not{q} - q^{\lambda} \gamma^\mu) + \frac{C^A_4(\omega)}{m^2_{\Xi^{(r)}_{bc}}} (g^{\lambda \mu} q p' - q^{\lambda} p'^\mu) + C^A_5(\omega) g^{\lambda \mu} + \frac{C^A_6(\omega)}{m^2_{\Xi^{(r)}_{bc}}} q^{\lambda} q^\mu \right)$$

and $3/2 \rightarrow 1/2$ transitions...

$$\langle \Xi_{cc}, r' \bar{p}' | \bar{c} \gamma^\mu (1 - \gamma_5) b(0) | \Xi_{bc}^*; r \bar{p} \rangle = \bar{u}^{\Xi_{cc}}_{r \lambda}(\bar{p}') \hat{\Gamma}^\lambda_\mu u^{\Xi_{bc}^*}_\lambda (\bar{p})$$
\[ \Xi_{bc} \rightarrow \Xi_{cc} \quad \Xi'_{bc} \rightarrow \Xi_{cc} \]

\[ \frac{(F_1 + F_2 + F_3)}{\sqrt{2}} \quad \frac{3}{2\sqrt{2}} \quad G_1 \]

\[ -\frac{\sqrt{3}}{\sqrt{2}} \quad G_1 \]

\[ \Xi_{bb} \rightarrow \Xi_{bc} \quad \Xi_{bb} \rightarrow \Xi'_{bc} \]

\[ \Xi_{bb} \rightarrow \Xi^*_{bc} \quad \Xi^*_{bb} \rightarrow \Xi_{bc} \quad \Xi^*_{bb} \rightarrow \Xi'_{bc} \]
and $3/2 \rightarrow 3/2$ transitions, $\Xi_{bc}^* \rightarrow \Xi_{cc}^* \sim 15$ FF’s
HQSS constraints on semileptonic decay widths

\[ \Gamma = \frac{G_F^2 |V_{cb}|^2 m_{\Xi_{cc}}^3 (\ast)}{2\pi^4} \int_{1}^{\omega_{max}} d\omega \sqrt{\omega^2 - 1} \mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu} \]

hadron FF's

\[ \mathcal{L}^{\mu\nu} = \int \frac{d^3k_1}{2E_1} \frac{d^3k_2}{2E_2} \delta^{(4)}(q - k_1 - k_2) \left( k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu} k_1 \cdot k_2 + i\epsilon^{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right) \]

\[ = A(q^2) g^{\mu\nu} + B(q^2) \frac{q^\mu q^\nu}{q^2} \]

For the actual doubly heavy baryon masses \( \omega_{max} \approx 1.22 (1.08) \) for \( bc \to cc (bb \to bc) \) transitions. The different differential decay widths \( d\Gamma/d\omega \) show a maximum at around \( \omega \approx 1.05 (1.01) \) \( \Rightarrow \)

\[ \eta(\omega) \to \mathcal{H}_{\mu\nu} \]

and approximating

\[ m_{\Xi_{bb}} \approx m_{\Xi_{bb}^\ast} ; \ m_{\Xi_{bc}} \approx m_{\Xi_{bc}^\ast} \approx m_{\Xi_{bc}} ; \ m_{\Xi_{cc}} \approx m_{\Xi_{cc}^\ast} \]
predict that some ratios between different decay widths should be approximately 1...

$b_{c} \rightarrow c_{c}$

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<td>$\Xi$</td>
<td>$1.04^{+0.03}_{-0.01}$</td>
<td>$1.04_{-0.03}$</td>
<td>$0.79$</td>
<td>$0.68$</td>
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<tr>
<td>$\Omega$</td>
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<td>$0.82$</td>
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<tr>
<td>$\Xi$: $\Omega$</td>
<td>$0.82^{+0.06}_{-0.01}$</td>
<td>$0.84^{+0.13}_{-0.01}$</td>
<td>$1.22$</td>
<td>$1.17$</td>
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<tr>
<td>$\Xi$: $\Omega$</td>
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<td>$0.97^{+0.10}_{-0.01}$</td>
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\[ bb \rightarrow bc \]

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<td>1.00( ^+0.03 )</td>
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\[ \Gamma(\bar{B}_{bb} \rightarrow B_{bc} l \bar{\nu}_l) \]

\[ \frac{3}{2} \Gamma(\bar{B}_{bb} \rightarrow B_{bc} l \bar{\nu}_l) \]
Diquark Picture and link to $B_c$ Meson Decays

$$\eta(\mathbf{v} \cdot \mathbf{v}') = \int d^3r_1 d^3r_2 \exp[-i\mathbf{k} \cdot \mathbf{r}_{12}/2][\Psi_{cc}^\Xi(r_1, r_2, r_{12})]^* \Psi_{bc}^\Xi(r_1, r_2, r_{12})$$

$\iff$ Jacobi’s coordinates, $Q, Q' = c, b$.

$$r_{12} \ll r_1, r_2 \rightarrow \Psi_{Qc}^\Xi(r_1, r_2, r_{12}) \approx \Phi_{Qc}(r_{12}) \varphi(r_q) \Phi_{Qc}(\mathbf{r}_{12} \cdot \mathbf{r}_q)$$

RELATIVE MOTION OF $q$ AND A POINTLIKE $Qc$ DIQUARK
\[ \eta(v \cdot v') = \int d^3r_{12} \exp[-i\vec{k} \cdot \vec{r}_{12}/2][\Phi_{cc}(r_{12})]^* \Phi_{bc}(r_{12}) \int d^3r \phi^*(r)\phi(r) \]

where \( \vec{r} = \vec{r}_{ccq} \) and in the \( d^3r \) integral we have replaced \( \phi(r_{bcq}) \) by \( \phi(r) \) since \( \vec{r}_{bcq} = \vec{r}_{ccq} + O(\vec{r}_{12}) \). This approximation leads to uncertainties of \( O(r_{12}^2) \) after integration,

\[ \eta(v \cdot v') = \int d^3r_{12} \exp[-i\vec{k} \cdot r_{12}/2][\Phi_{cc}(r_{12})]^* \Phi_{bc}(r_{12}) \]

which has an identical form to the expression of the form factor \( \Delta \), unique form factor which describes the \( B_c \) to \( \eta_c \) and \( J/\psi \) semileptonic decays, in terms of wavefunctions of the \( \bar{b}c \) and \( \bar{c}c \) bound states (Jenkins et al., NPB 390 (1993) 463).
This does not mean that $\eta$ and $\Delta$ are identical because the $QQ$ and $Q\bar{Q}$ potentials used to compute the diquark and meson wavefunctions are not the same. For example a $\lambda_i\lambda_j$ colour dependence ($\lambda_i$ are the usual Gell-Mann matrices) would lead to $V_{QQ} = V_{Q\bar{Q}}/2$. [approx wfnt overlaps (solid lines) vs IW funcs (dashed lines)]
The $\omega^2$ slope of the $\Delta$ form factor is indeed smaller than that of $\eta$, but the ratio is around 1 to 3 rather than 1 to 6, so there are significant corrections to the Coulomb wavefunction description.

Conclusions

• Separate HQSS make it possible to describe all SL

$$\Xi^{(\ast)}_{bc} \rightarrow \Xi^{(\ast)}_{cc} l \bar{\nu}_l, \quad \Omega^{(\ast)}_{bc} \rightarrow \Omega^{(\ast)}_{cc} l \bar{\nu}_l$$

decays using a single form factor. Similarly for $bb \rightarrow bc$ decays.

• We have discussed the resemblance of the $bc$ baryon decays to those of $B_c$ mesons to $\eta_c$ and $J/\psi$ mesons
• Lattice QCD simulations work best near the zero-recoil point and thus are well-suited to check the validity of the results.

• QM calculations consistent with HQSS?
  – Results by Hernández et al. (FF’s and decay width ratios), Ebert et al. and Faessler et al. (decay width ratios), compare well, within expectations with HQSS
  – We detect problems either in the model or in the calculation performed by Guo et al.

• HQSS and triply heavy baryon SL decays?