# Semileptonic $bc$ to $cc$ Baryon Decay and Heavy Quark Spin Symmetry

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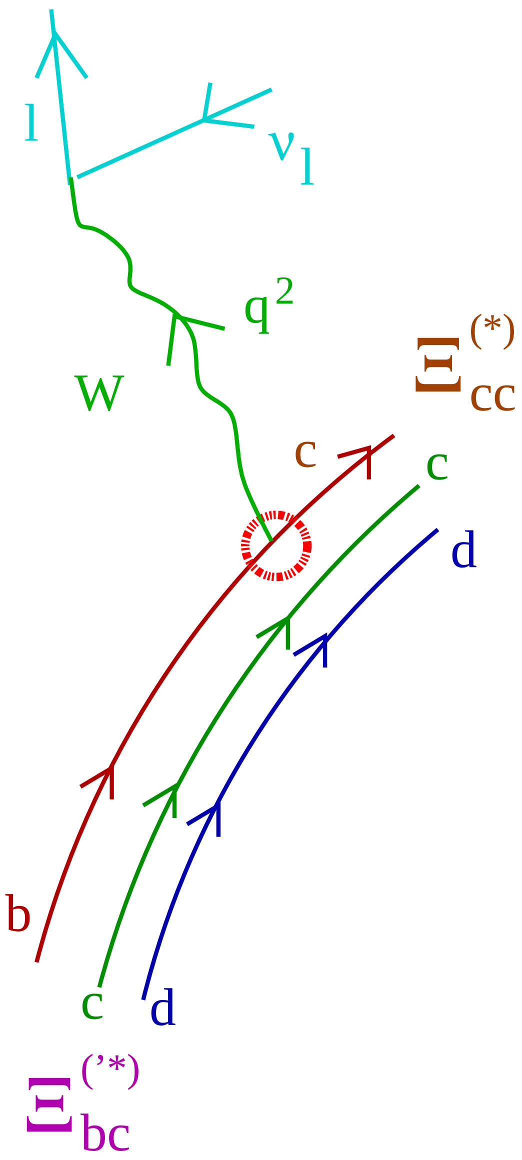
**Motivation :** Separate **heavy quark spin symmetries** make it possible to describe the **semileptonic decays**

$$\Xi_{bc}^{(\prime *)} \to \Xi_{cc}^{(*)}\, l\, \bar{\nu}_l, \qquad \Omega_{bc}^{(\prime *)} \to \Omega_{cc}^{(*)}\, l\, \bar{\nu}_l$$

in the limit $m_{b,c} \gg \Lambda_{QCD}$ and close to the zero recoil point

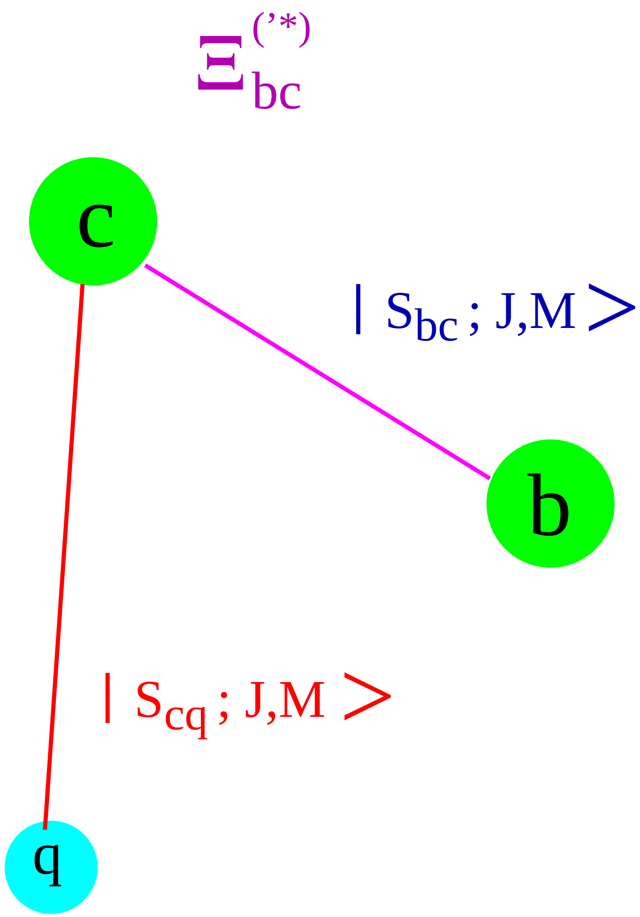



$$\mathbf{q^2} = m_{bc}^2 + m_{cc}^2 - 2 m_{bc} m_{cc} \omega, \qquad \underbrace{\boxed{1}}_{\text{zero recoil}} \leq \omega \leq \frac{m_{bc}^2 + m_{cc}^2 - m_l^2}{2 m_{bc} m_{cc}}$$

| | $S$ | $J^P$ | $I$ | $\mathbf{S^\pi_{hh}}$ | |
|---|---|---|---|---|---|
| $\mathbf{\Xi_{cc}}$ | 0 | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $ccl$ |
| $\mathbf{\Xi^*_{cc}}$ | 0 | $\frac{3}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $ccl$ |
| $\mathbf{\Omega_{cc}}$ | $-1$ | $\frac{1}{2}^+$ | 0 | $\mathbf{1}^+$ | $ccs$ |
| $\mathbf{\Omega^*_{cc}}$ | $-1$ | $\frac{3}{2}^+$ | 0 | $\mathbf{1}^+$ | $ccs$ |
| $\mathbf{\Xi_{bb}}$ | 0 | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $bbl$ |
| $\mathbf{\Xi^*_{bb}}$ | 0 | $\frac{3}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $bbl$ |
| $\mathbf{\Omega_{bb}}$ | $-1$ | $\frac{1}{2}^+$ | 0 | $\mathbf{1}^+$ | $bbs$ |
| $\mathbf{\Omega^*_{bb}}$ | $-1$ | $\frac{3}{2}^+$ | 0 | $\mathbf{1}^+$ | $bbs$ |

| | $S$ | $J^P$ | $I$ | $\mathbf{S^\pi_{hh'}}$ | |
|---|---|---|---|---|---|
| $\mathbf{\Xi'_{bc}}$ | 0 | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $\mathbf{0}^+$ | $bcl$ |
| $\mathbf{\Xi_{bc}}$ | 0 | $\frac{1}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $bcl$ |
| $\mathbf{\Xi^*_{bc}}$ | 0 | $\frac{3}{2}^+$ | $\frac{1}{2}$ | $\mathbf{1}^+$ | $bcl$ |
| $\mathbf{\Omega'_{bc}}$ | $-1$ | $\frac{1}{2}^+$ | 0 | $\mathbf{0}^+$ | $bcs$ |
| $\mathbf{\Omega_{bc}}$ | $-1$ | $\frac{1}{2}^+$ | 0 | $\mathbf{1}^+$ | $bcs$ |
| $\mathbf{\Omega^*_{bc}}$ | $-1$ | $\frac{3}{2}^+$ | 0 | $\mathbf{1}^+$ | $bcs$ |

**HQS constraints on SL FF's and Γ's of doubly heavy baryons.**




For instance, let us study $\mathbf{\Xi_{bc}^{(\prime *)}} \to \mathbf{\Xi_{cc}^{(*)}}$ SL decays,

$$p_\mu = m_{\Xi_{bc}^{(\prime *)}} \mathbf{v}_\mu, \quad p'_\mu = m_{\Xi_{cc}^{(*)}} v'_\mu = m_{\Xi_{cc}^{(*)}} \mathbf{v}_\mu + \mathbf{k}_\mu$$

Near the zero-recoil point $\omega = 1$ ( $\omega = \mathbf{v} \cdot \mathbf{v}'$) **k small residual momentum** $\Rightarrow \mathbf{k} \cdot \mathbf{v} = \mathcal{O}(1/\mathbf{m}_{\Xi_{cc}^{(*)}})$.

To represent the **lowest-lying** $S$**-wave** $bcq$ **baryons** we use **wavefunctions comprising tensor products of Dirac matrices and spinors,** namely:

$$\mathbf{\Xi'_{bc}} = \left[\frac{(1+\not v)}{2}\gamma_5 \frac{(1-\not v)}{2}\right]_{\alpha\beta} u_\gamma(v,r)$$

$$\mathbf{\Xi_{bc}} = \left[\frac{(1+\not v)}{2}\gamma_\mu \frac{(1-\not v)}{2}\right]_{\alpha\beta} \left[\frac{1}{\sqrt{3}}(v^\mu + \gamma^\mu)\gamma_5 u(v,r)\right]_\gamma$$

$$\mathbf{\Xi^*_{bc}} = \left[\frac{(1+\not v)}{2}\gamma_\mu \frac{(1-\not v)}{2}\right]_{\alpha\beta} u^\mu_\gamma(v,r)$$

$\alpha$, $\beta$, $\gamma$ Dirac indices and $r$ baryon helicity label. These **wavefunctions** can be considered as **matrix elements of the form** $\langle 0|b_\alpha \bar{c}^c_\beta q_\gamma|\Xi_{bc}^{(\prime *)}\rangle$ where $\bar{c}^c = c^T C$ **with** $C$ **the charge-conjugation matrix.**

Under a Lorentz ($\Lambda$), and $b$ and $c$ quark spin ($S_b$ and $S_c$) transformations, **a wavefunction** $\Gamma_{\alpha\beta}\, u_\gamma$ transforms as:

$$\begin{aligned}\Gamma\, u &\rightarrow S(\Lambda)\Gamma S^{-1}(\Lambda)\; S(\Lambda)u \\ \Gamma\, u &\rightarrow S_b\Gamma\, S_c^\dagger\, u\end{aligned}$$

**States** normalised using $\bar{\mathbf{u}}\mathbf{u}\mathrm{Tr}(\mathbf{\Gamma}\overline{\mathbf{\Gamma}})$: **mutually orthogonal and have a common normalisation** ($\overline{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$).

**We use similar wavefunctions for the** $cc$ **baryons.**

... **construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements,**

**SL $\Xi_{bc}^{(\prime *)} \to \Xi_{cc}^{(*)}$** decays $\leftrightarrow$ ME **weak current** $J^\mu = \bar{c}\gamma^\mu(1-\gamma_5)b$

**The most general form for the ME respecting the HQSS is ($j^\mu = \gamma^\mu(1-\gamma_5)$):**

$$\langle \Xi_{cc}^{(*)}, v, k, M' | J^\mu(0) | \Xi_{bc}^{(\prime *)}, v, M \rangle = \bar{u}_{cc}(v,k,M') \boldsymbol{\Omega}\, u_{bc}(v,M) \mathrm{Tr}[\overline{\Gamma}_{cc} j^\mu \Gamma_{bc}]$$

$$\begin{aligned} \Gamma_{bc} &\to S_b \Gamma_{bc} S_c^\dagger, \quad u_{bc} \to u_{bc} \\ \overline{\Gamma}_{cc} &\to S_c \overline{\Gamma}_{cc} S_c^\dagger, \quad \bar{u}_{cc} \to \bar{u}_{cc} \\ \bar{c} j^\mu b : j^\mu &\to S_c j^\mu S_b^\dagger \end{aligned}$$

... **construct spin-invariant and Lorentz covariant amplitudes for the weak transition matrix elements,**

**SL $\Xi_{\mathbf{bc}}^{(\prime *)} \to \Xi_{\mathbf{cc}}^{(*)}$** decays $\leftrightarrow$ ME **weak current** $J^\mu = \bar{c}\gamma^\mu(1-\gamma_5)b$

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$$\Gamma_{bc} \to \mathbf{S_b} \Gamma_{bc} \mathbf{S_c^\dagger}, \quad u_{bc} \to u_{bc}$$

$$\overline{\Gamma}_{cc} \to \mathbf{S_c} \overline{\Gamma}_{cc} \mathbf{S_c^\dagger}, \quad \bar{u}_{cc} \to \bar{u}_{cc}$$

$$\bar{c} j^\mu b : j^\mu \to \mathbf{S_c} j^\mu \mathbf{S_b^\dagger}$$

where $M$ and $M'$ are the helicities of the initial and final states

$$\mathbf{\Omega} \propto \eta(\mathbf{v} \cdot \mathbf{v}')$$

**is the most general Dirac matrix that can be written in terms of the vectors $k$ and $v$.**

- terms with a factor of $\not{v}$ can be omitted because of the equations of motion ($\not{v}u = u$, $\not{v}\Gamma = \Gamma$, $\gamma_\mu u^\mu = 0$, $v_\mu u^\mu = 0$),
- terms with $\not{k}$ will always lead to contributions proportional to $v \cdot k = \mathcal{O}(1/m_{\Xi_{cc}^{(*)}})$.

| | |
|---|---|
| $\Xi_{bc} \longrightarrow \Xi_{cc}$ | $\frac{1}{\sqrt{2}}\eta\,\bar{u}_{cc}\left(2\gamma^\mu - \frac{4}{3}\gamma^\mu\gamma_5\right)u_{bc}$ |
| $\Xi'_{bc} \longrightarrow \Xi_{cc}$ | $-\sqrt{\frac{2}{3}}\eta\,\bar{u}_{cc}\left(-\gamma^\mu\gamma_5\right)u_{bc}$ |
| $\Xi_{bc} \longrightarrow \Xi^*_{cc}$ | $-\sqrt{\frac{2}{3}}\eta\,\bar{u}^\mu_{cc}u_{bc}$ |
| $\Xi'_{bc} \longrightarrow \Xi^*_{cc}$ | $-\sqrt{2}\eta\,\bar{u}^\mu_{cc}u_{bc}$ |
| $\Xi^*_{bc} \longrightarrow \Xi_{cc}$ | $-\sqrt{\frac{2}{3}}\eta\,\bar{u}_{cc}u^\mu_{bc}$ |
| $\Xi^*_{bc} \longrightarrow \Xi^*_{cc}$ | $-\sqrt{2}\eta\,\bar{u}^\lambda_{cc}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)u_{bc\,\lambda}$ |

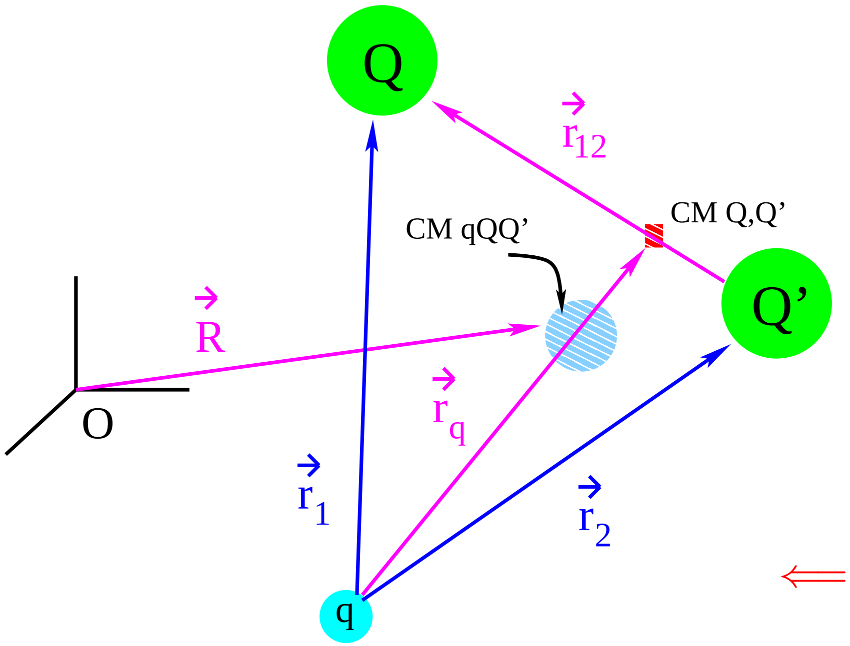
## Remarks:

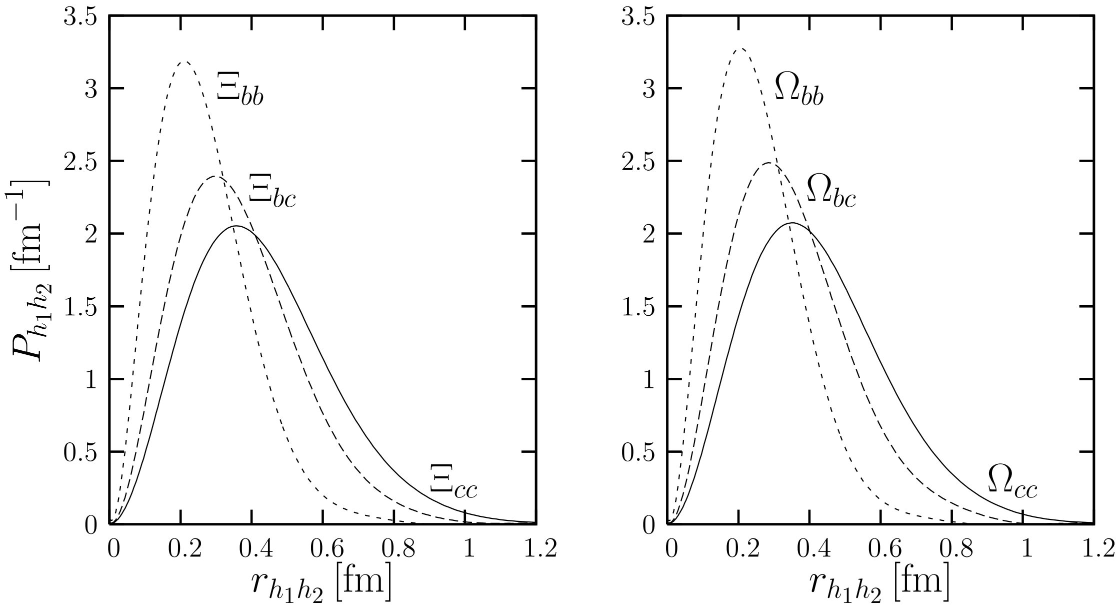
- If the $b$ and $c$ quarks become degenerate, then vector current conservation ensures that $\eta(\mathbf{1}) = \mathbf{1}$.
- **Savage and White (PLB 271 (1991) 410)** found similar results: approach where the two heavy quarks bind into a colour antitriplet which appears as a pointlike colour source to the light degrees of freedom + **"superflavor"** formalism of Georgi and Wise. We find two differences to their results.
- Our approach, where we consider the spin transformations of each heavy quark explicitly, is **straightforward and similar** to that used to describe $B_c$ **meson decays:** Jenkins, Luke, Manohar and Savage, NPB 390 (1993) 463.

- **Spin symmetry** for both the $b$ and $c$ quarks **enormously simplifies** the description of all $\boldsymbol{\Xi_{bc}^{(\prime *)} \rightarrow \Xi_{cc}^{(*)}\, l\, \bar{\nu}_l}$ decays in the heavy quark limit and near the zero recoil point. **All the weak transition matrix elements are given in terms of a single universal function.** Lorentz covariance alone allows a large number of form factors (six form factors to describe $\Xi_{bc} \rightarrow \Xi_{cc}$, another six for $\Xi'_{bc} \rightarrow \Xi_{cc}$, eight each for $\Xi_{bc} \rightarrow \Xi^*_{cc}$, $\Xi'_{bc} \rightarrow \Xi^*_{cc}$ and $\Xi^*_{bc} \rightarrow \Xi_{cc}$, and even more for $\Xi^*_{bc} \rightarrow \Xi^*_{cc}$).

## Test: QM [EPJA 32 (2007) 183 ]

$$\eta(\mathbf{v}\cdot\mathbf{v}') = \int d^3r_1 d^3r_2 \exp[-i\vec{k}\cdot\vec{r}_{12}/2][\boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{cc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}})]^* \,\boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{bc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}})$$

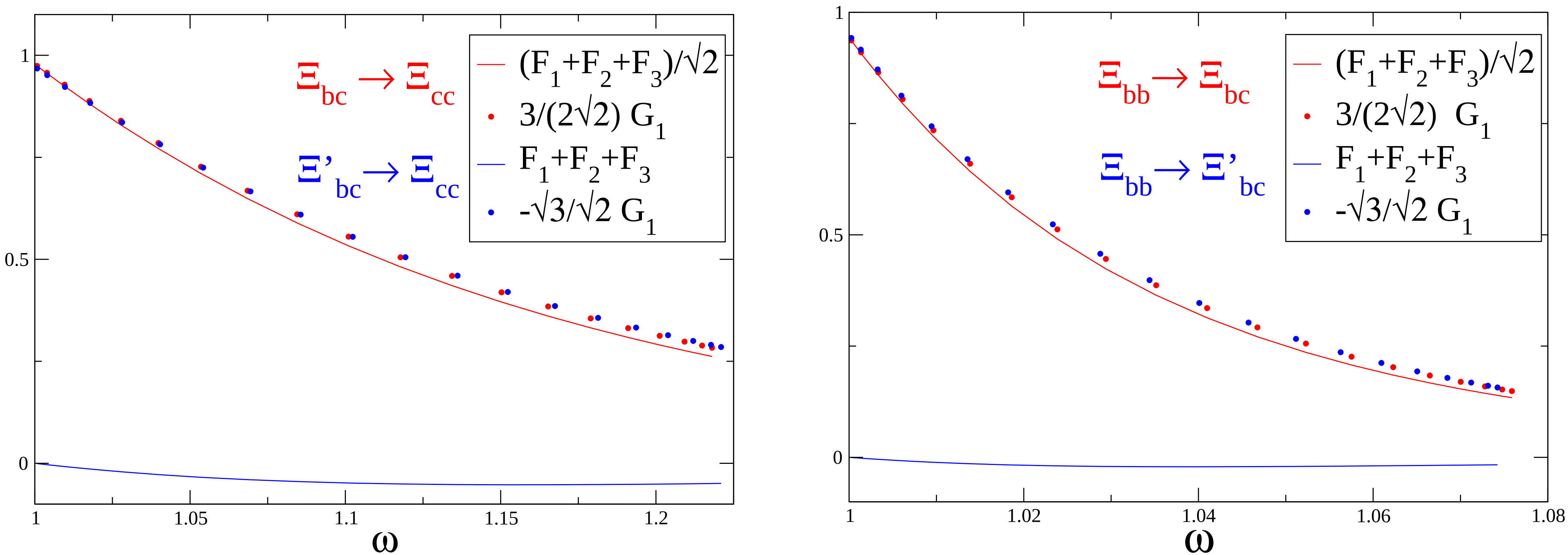



$\Longleftarrow$ Jacobi's coordinates, $Q, Q' = c, b$.

$$r_{12} \ll r_1, r_2 \rightarrow \boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{Qc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}}) \approx \overbrace{\Phi_{Qc}(r_{12})}^{\mathbf{Qc}\ \text{DIQUARK}} \underbrace{\phi(r_q)}_{\text{RELATIVE MOTION OF } \mathbf{q} \text{ AND A POINTLIKE } \mathbf{Qc} \text{ DIQUARK}} \overbrace{\varphi_{Qc}(\vec{r}_{12}\cdot\vec{r}_q)}^{\text{VARIATIONAL}}$$

$$\left\langle \mathbf{\Xi_{cc}}, r'\ \vec{p}\,' \,|\, \bar{c}\gamma^\mu(1-\gamma_5)b(0) \,|\, \mathbf{\Xi_{bc}^{(\prime)}}, r\ \vec{p} \right\rangle = \bar{u}_{r'}^{\Xi_{cc}}(\vec{p}\,')\Big\{\gamma^\mu\left(\mathbf{F_1(w)} - \gamma_5 \mathbf{G_1(w)}\right)$$
$$+v^\mu\left(\mathbf{F_2(w)} - \gamma_5 \mathbf{G_2(w)}\right) + v'^\mu\left(\mathbf{F_3(w)} - \gamma_5 \mathbf{G_3(w)}\right)\Big\} u_r^{\Xi_{bc}^{(\prime)}}(\vec{p})$$



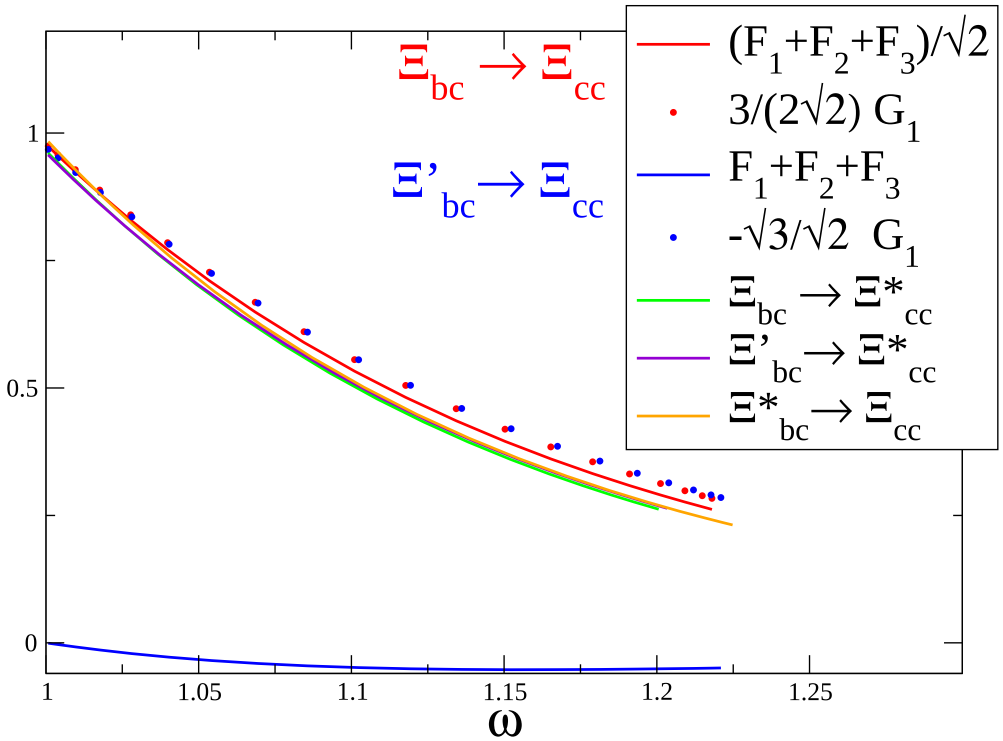
**... $1/2 \rightarrow 3/2$ spin transitions**

$$\left\langle \Xi^*_{cc}, r'\ \vec{p}\,' \,|\, \bar{c}\gamma^\mu(1-\gamma_5)b(0) |\, \Xi^{(\prime)}_{bc}, r\ \vec{p} \right\rangle = \bar{u}^{\Xi^*_{cc}}_{\lambda r'}(\vec{p}\,')\Gamma^{\lambda\mu} u^{\Xi^{(\prime)}_{bc}}_r(\vec{p})$$

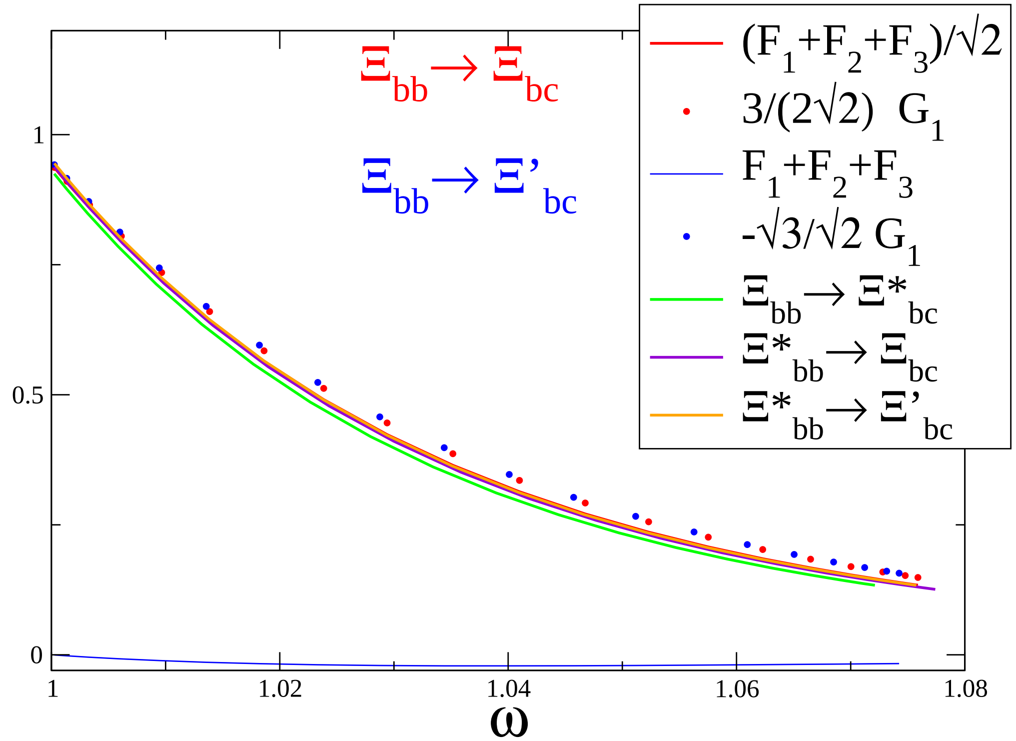
$$\Gamma^{\lambda\mu} = \left( \frac{C^V_3(\omega)}{m_{\Xi^{(\prime)}_{bc}}}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^V_4(\omega)}{m^2_{\Xi^{(\prime)}_{bc}}}(g^{\lambda\mu}qp' - q^\lambda p'^\mu) + \frac{C^V_5(\omega)}{m^2_{\Xi^{(\prime)}_{bc}}}(g^{\lambda\mu}qp - q^\lambda p^\mu) + C^V_6(\omega) g^{\lambda\mu} \right)\gamma_5 + \left( \frac{C^A_3(\omega)}{m_{\Xi^{(\prime)}_{bc}}}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^A_4(\omega)}{m^2_{\Xi^{(\prime)}_{bc}}}(g^{\lambda\mu}qp' - q^\lambda p'^\mu) + C^A_5(\omega) g^{\lambda\mu} + \frac{C^A_6(\omega)}{m^2_{\Xi^{(\prime)}_{bc}}} q^\lambda q^\mu \right)$$

**and $3/2 \rightarrow 1/2$ transitions...**

$$\langle \Xi_{cc}, r'\ \vec{p}\,' \,|\, \bar{c}\gamma^\mu(1-\gamma_5)b(0) |\, \Xi^*_{bc}, r\ \vec{p} \rangle = \bar{u}^{\Xi_{cc}}_{r'}(\vec{p}\,')\hat{\Gamma}^{\lambda\mu} u^{\Xi^*_{bc}}_{\lambda,r}(\vec{p})$$

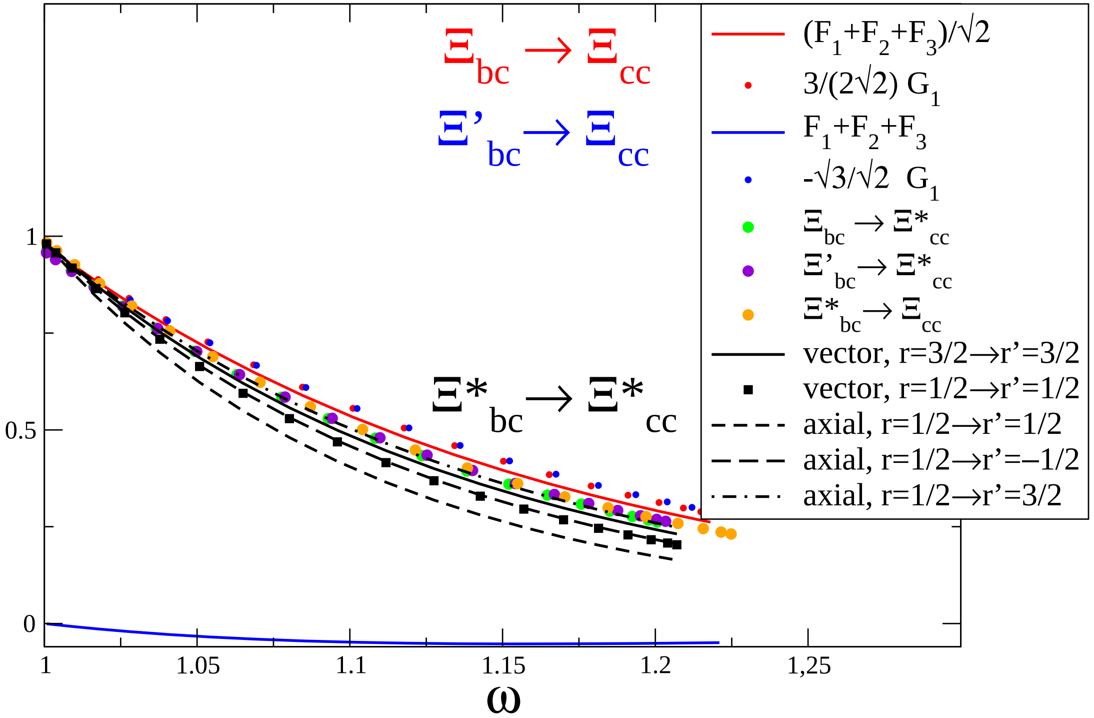


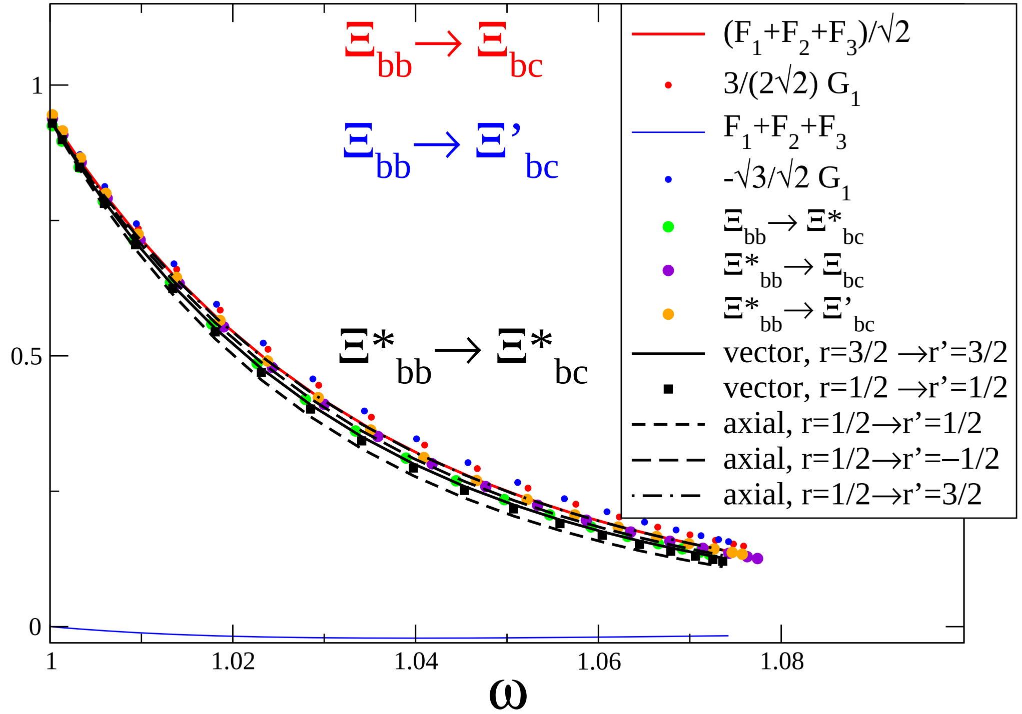
$\Xi_{bc} \rightarrow \Xi_{cc}$
$\Xi'_{bc} \rightarrow \Xi_{cc}$
$(F_1+F_2+F_3)/\sqrt{2}$
$3/(2\sqrt{2})\ G_1$
$F_1+F_2+F_3$
$-\sqrt{3}/\sqrt{2}\ G_1$
$\Xi_{bc} \rightarrow \Xi^*_{cc}$
$\Xi'_{bc} \rightarrow \Xi^*_{cc}$
$\Xi^*_{bc} \rightarrow \Xi_{cc}$
1
0.5
0
1
1.05
1.1
1.15
1.2
1.25
ω

$\Xi_{bb} \rightarrow \Xi_{bc}$
$\Xi_{bb} \rightarrow \Xi'_{bc}$
$(F_1+F_2+F_3)/\sqrt{2}$
$3/(2\sqrt{2})\ G_1$
$F_1+F_2+F_3$
$-\sqrt{3}/\sqrt{2}\ G_1$
$\Xi_{bb} \rightarrow \Xi^*_{bc}$
$\Xi^*_{bb} \rightarrow \Xi_{bc}$
$\Xi^*_{bb} \rightarrow \Xi'_{bc}$
1
0.5
0
1
1.02
1.04
1.06
1.08
ω

## and $3/2 \to 3/2$ transitions, $\Xi^*_{bc} \to \Xi^*_{cc}$ $\sim$ 15 FF's



## HQSS constraints on semileptonic decay widths

$$\Gamma = \frac{G_F^2}{2\pi^4}|V_{cb}|^2 m^3_{\Xi^{(*)}_{cc}} \int_1^{\omega_{max}} d\omega \sqrt{\omega^2-1}\, \mathcal{L}^{\mu\nu} \underbrace{\mathcal{H}_{\mu\nu}}_{\text{hadron FF's}}$$

$$\begin{aligned}\mathcal{L}^{\mu\nu} &= \int \frac{d^3k_1}{2E_1}\frac{d^3k_2}{2E_2}\,\delta^{(4)}(q-k_1-k_2)\left(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu} k_1\cdot k_2 + \mathrm{i}\epsilon^{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}\right)\\ &= A(q^2)\, g^{\mu\nu} + B(q^2)\,\frac{q^\mu q^\nu}{q^2}\end{aligned}$$

For the actual doubly heavy baryon masses $\omega_{max} \approx 1.22\,(1.08)$ for $bc \to cc\,(bb \to bc)$ transitions. The different differential decay widths $d\Gamma/d\omega$ show a maximum at around $\omega \approx 1.05\,(1.01) \Longrightarrow$

$\boxed{\eta(\omega) \to \mathcal{H}_{\mu\nu}}$ and approximating

$$m_{\Xi_{bb}} \approx m_{\Xi^*_{bb}} \;;\; m_{\Xi_{bc}} \approx m_{\Xi'_{bc}} \approx m_{\Xi^*_{bc}} \;;\; m_{\Xi_{cc}} \approx m_{\Xi^*_{cc}}$$

**predict that some ratios between different decay widths should be approximately 1...**

$bc \to cc$

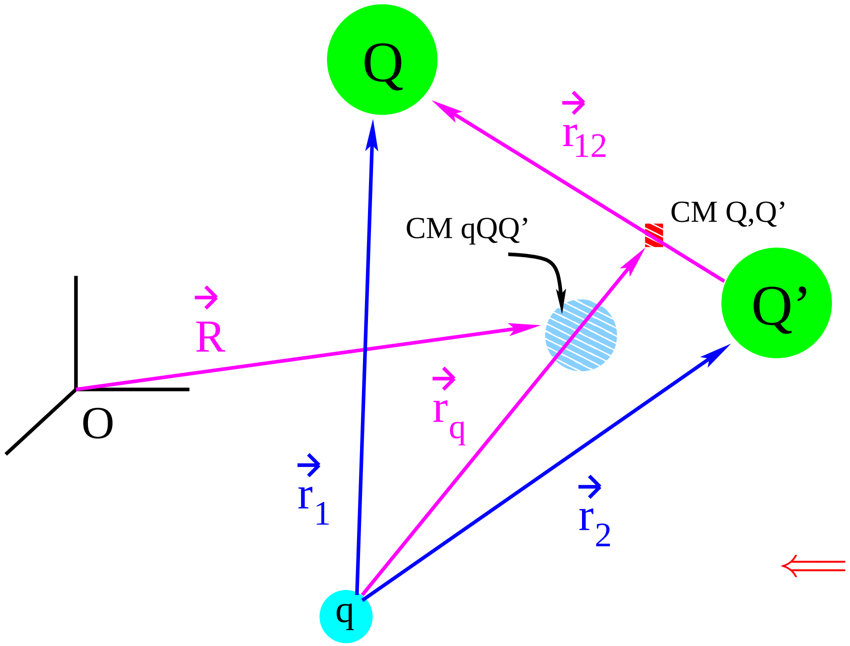
| | Hernández et al. PLB 663 234 | | Ebert et al. PRD70 014018 | | Guo et al. PRD58 114007 | | Faessler et al. PRD80 034025 | |
|---|---|---|---|---|---|---|---|---|
| | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ |
| $\frac{\Gamma(B'_{bc}\to B^*_{cc}\, l\bar\nu_l)}{3\,\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}$ | $1.04^{+0.03}_{-0.01}$ | $1.04_{-0.03}$ | 0.79 | 0.82 | 0.68 | — | $1.1^{+0.6}_{-0.4}$ | $1.0^{+0.6}_{-0.4}$ |
| $\frac{\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}{\frac{2}{3}\,\Gamma(B'_{bc}\to B_{cc}\, l\bar\nu_l)}$ | $0.82^{+0.06}_{-0.01}$ | $0.84^{+0.13}_{-0.01}$ | 1.22 | 1.17 | **2.72** | — | $0.9^{+0.5}_{-0.3}$ | $1.0^{+0.5}_{-0.3}$ |
| $\frac{\Gamma(B^*_{bc}\to B_{cc}\, l\bar\nu_l)}{\frac{1}{3}\,\Gamma(B'_{bc}\to B_{cc}\, l\bar\nu_l)}$ | $0.94^{+0.11}$ | $0.97^{+0.10}_{-0.01}$ | 1.28 | 1.26 | **10.6** | — | $0.8^{+0.4}_{-0.3}$ | $0.8^{+0.4}_{-0.3}$ |
| $\frac{\Gamma(B^*_{bc}\to B^*_{cc}\, l\bar\nu_l)}{\Gamma(B_{bc}\to B_{cc}\, l\bar\nu_l)+\frac{1}{2}\,\Gamma(B_{bc}\to B^*_{cc}\, l\bar\nu_l)}$ | $0.89^{+0.11}$ | $0.94^{+0.13}_{-0.01}$ | 1.01 | 1.01 | 1.08 | — | $1.0^{+0.5}_{-0.4}$ | $1.1^{+0.5}_{-0.4}$ |

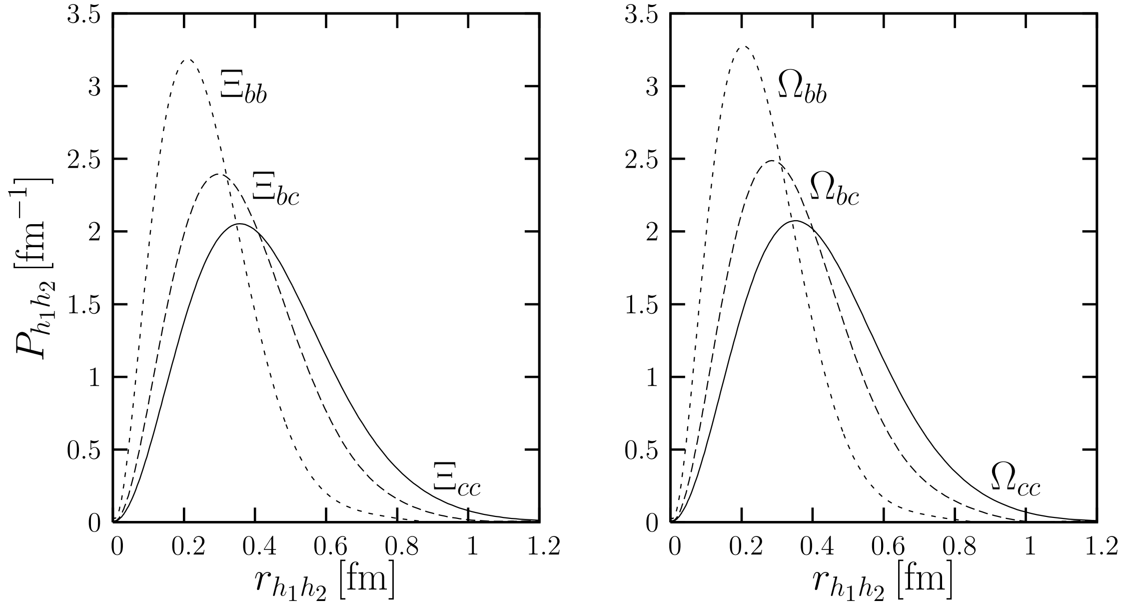
## $bb \rightarrow bc$

| | Hernández et al. | | Ebert et al. | | Guo et al. | | Faessler et al. | |
|---|---|---|---|---|---|---|---|---|
| | PLB 663 234 | | PRD70 014018 | | PRD58 114007 | | PRD80 034025 | |
| | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ | $\Xi$ | $\Omega$ |
| $\frac{\Gamma(B^*_{bb}\to B'_{bc}\, l\bar\nu_l)}{3\,\Gamma(B^*_{bb}\to B_{bc}\, l\bar\nu_l)}$ | $1.00^{+0.01}_{-0.04}$ | $1.00^{+0.03}_{-0.01}$ | 0.99 | 0.99 | **0.05** | — | $0.9^{+0.5}_{-0.3}$ | $0.9^{+0.6}_{-0.4}$ |
| $\frac{\Gamma(B_{bb}\to B^*_{bc}\, l\bar\nu_l)}{\frac{2}{3}\,\Gamma(B_{bb}\to B'_{bc}\, l\bar\nu_l)}$ | $0.86^{+0.08}_{-0.06}$ | $0.86^{+0.05}$ | 0.96 | 0.99 | **9.53** | — | $0.9^{+0.5}_{-0.3}$ | $0.9^{+0.5}_{-0.3}$ |
| $\frac{\Gamma(B^*_{bb}\to B_{bc}\, l\bar\nu_l)}{\frac{1}{3}\,\Gamma(B_{bb}\to B'_{bc}\, l\bar\nu_l)}$ | $0.98^{+0.09}_{-0.03}$ | $0.97^{+0.06}_{-0.14}$ | 1.01 | 1.03 | **36.4** | — | $1.0^{+0.5}_{-0.3}$ | $0.9^{+0.5}_{-0.4}$ |
| $\frac{\Gamma(B^*_{bb}\to B^*_{bc}\, l\bar\nu_l)}{\Gamma(B_{bb}\to B_{bc}\, l\bar\nu_l)+\frac{1}{2}\,\Gamma(B_{bb}\to B^*_{bc}\, l\bar\nu_l)}$ | $0.94^{+0.07}_{-0.06}$ | $0.93^{+0.11}_{-0.10}$ | 1.01 | 1.01 | **0.31** | — | $1.1^{+0.8}_{-0.5}$ | $1.1^{+0.8}_{-0.5}$ |

## Diquark Picture and link to $B_c$ Meson Decays

$$\eta(\mathbf{v}\cdot\mathbf{v}') = \int d^3r_1 d^3r_2 \exp[-i\vec{k}\cdot\vec{r}_{12}/2][\boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{cc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}})]^* \boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{bc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}})$$




$\Longleftarrow$ Jacobi's coordinates, $Q, Q' = c, b$.

$$r_{12} \ll r_1, r_2 \rightarrow \boldsymbol{\Psi}^{\boldsymbol{\Xi}}_{\mathbf{Qc}}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_{12}}) \approx \overbrace{\Phi_{Qc}(r_{12})}^{\textbf{Qc}\text{ DIQUARK}}\ \underbrace{\phi(r_q)}_{\text{RELATIVE MOTION OF } \mathbf{q} \text{ AND A POINTLIKE } \mathbf{Qc} \text{ DIQUARK}}\ \overbrace{\varphi_{Qc}(\vec{r}_{12}\cdot\vec{r}_q)}^{\text{VARIATIONAL}\approx 1}$$

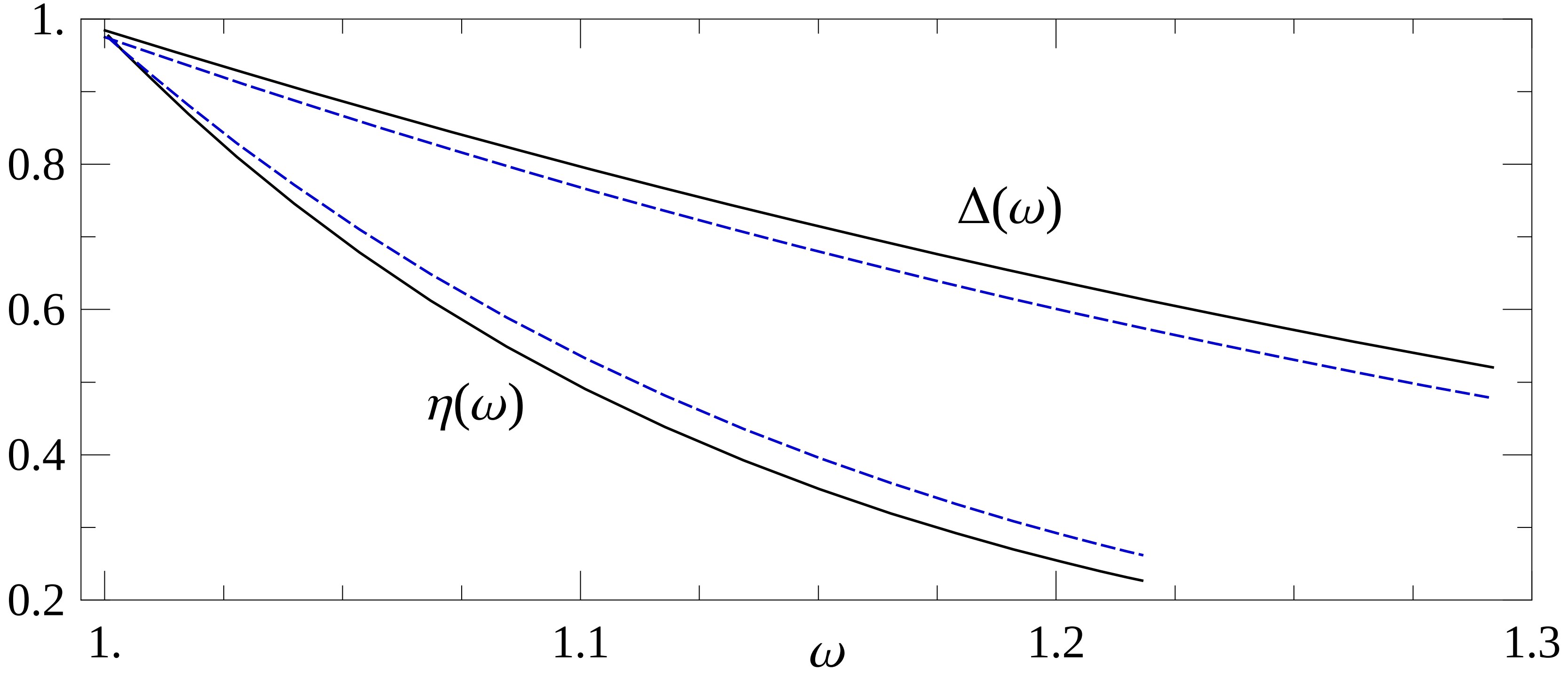
$$\eta(\mathbf{v}\cdot\mathbf{v}') = \int \mathbf{d^3r_{12}}\,\exp[-\mathbf{i}\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}_{\mathbf{12}}/\mathbf{2}][\mathbf{\Phi_{cc}}(\mathbf{r_{12}})]^*\mathbf{\Phi_{bc}}(\mathbf{r_{12}})\underbrace{\int d^3r\phi^*(r)\phi(r)}_{\mathbf{1}}$$

where $\vec{r} = \vec{r}_{ccq}$ and in the $d^3r$ integral we have replaced $\phi(r_{bcq})$ by $\phi(r)$ since $\vec{r}_{bcq} = \vec{r}_{ccq} + \mathcal{O}(\vec{r}_{12})$. **This approximation leads to uncertainties of $\mathcal{O}(r_{12}^2)$ after integration,**

$$\eta(\mathbf{v}\cdot\mathbf{v}') = \int \mathbf{d^3r_{12}}\,\exp[-\mathbf{ik}\cdot\mathbf{r_{12}}/\mathbf{2}][\mathbf{\Phi_{cc}}(\mathbf{r_{12}})]^*\mathbf{\Phi_{bc}}(\mathbf{r_{12}})$$

which has an **identical form to the expression of the form factor $\Delta$, unique form factor which describes the $B_c$ to $\eta_c$ and $J/\psi$ semileptonic decays,** in terms of wavefunctions of the $\bar{b}c$ and $\bar{c}c$ bound states (Jenkins et al., NPB 390 (1993) 463).

**This does not mean that $\eta$ and $\Delta$ are identical because the $QQ$ and $Q\bar{Q}$ potentials used to compute the diquark and meson wavefunctions are not the same. For example a $\lambda_i\lambda_j$ colour dependence ($\lambda_i$ are the usual Gell-Mann matrices) would lead to $V^{QQ} = V^{Q\bar{Q}}/2$.** [ approx wfnt overlaps (solid lines) vs IW funcs (dahsed lines)]



The $\omega^2$ **slope of the $\Delta$ form factor is indeed smaller than that of $\eta$**, but the ratio is around 1 to 3 rather than 1 to 6, so there are **significant corrections to the Coulomb wavefunction description.**

# Conclusions

- Separate HQSS make it possible to describe all SL

$$\Xi_{bc}^{(\prime *)} \to \Xi_{cc}^{(*)}\, l\, \bar{\nu}_l, \qquad \Omega_{bc}^{(\prime *)} \to \Omega_{cc}^{(*)}\, l\, \bar{\nu}_l$$

  decays **using a single form factor**. Similarly for $bb \to bc$ **decays.**

- We have discussed the **resemblance of the $bc$ baryon decays to those of $B_c$ mesons to $\eta_c$ and $J/\psi$ mesons**

- **Lattice QCD** simulations work best near the zero-recoil point and thus are well-suited to **check the validity of the results.**

- **QM calculations consistent with HQSS ?**
  - Results by **Hernández et al.** (FF's and decay width ratios), **Ebert et al. and Faessler et al.** (decay width ratios), **compare well**, within expectations **with HQSS**
  - **We detect problems** either in the model or in the calculation performed by Guo et al.

- **HQSS and triply heavy baryon SL decays ?**