ISOSPIN VIOLATION, LIGHT QUARK MASSES and ALL THAT

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CONTENTS

Introduction

ISOSPIN VIOLATION in the STANDARD MODEL

- Isospin violation has two sources (QCD + QED):

$$\mathcal{H}_{\rm QCD}(x) = \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u)(x)$$

$$\mathcal{H}_{\rm QED}(x) = -\frac{1}{2}e^2 \int dy\; D^{\mu\nu}(x-y) T(j_\mu(x) j_\nu(y))$$

$\Rightarrow$ *unique window to quark masses for light quark and heavy-light quark systems*

- Both effects usually small *and* of the same size (e.g. $m_p - m_n$)

$\Rightarrow$ systematic machinery must cope with both these accurately

- Chiral perturbation theory w/ virtual photons is the tool to analyse the strictures of the spontaneously and explicitly broken chiral symmetry of **QCD**

GENERAL REMARKS

- Isospin violation is enhanced in system with nucleons and (neutral) pions

Weinberg 1977

$$a(\pi^0 p) - a(\pi^0 n) = \frac{m_p c_5 B(m_d - m_u)}{\pi(m_p + M_\pi) F_\pi^2} = (-2.3 \pm 0.4) \cdot 10^{-3}/M_\pi$$

$\rightarrow$ precision data from hadronic atoms and threshold pion photoproduction

- further suppression in some processes involving two or more nucleons (CSB)

$\rightarrow$ pioneering data on $\boldsymbol{A_{fb}(np \rightarrow d\pi^0)}$ [TRIUMF] and $\boldsymbol{\sigma(dd \rightarrow \alpha\pi^0)}$ [IUCF]

- high-precision data for eta & kaon decays demand very accurate calculations of chiral *and* electromagnetic corrections

- systems with **heavy** quarks can give addiotional information on $m_u - m_d$ through decays and mass differences

Isospin breaking in the πN scattering lengths

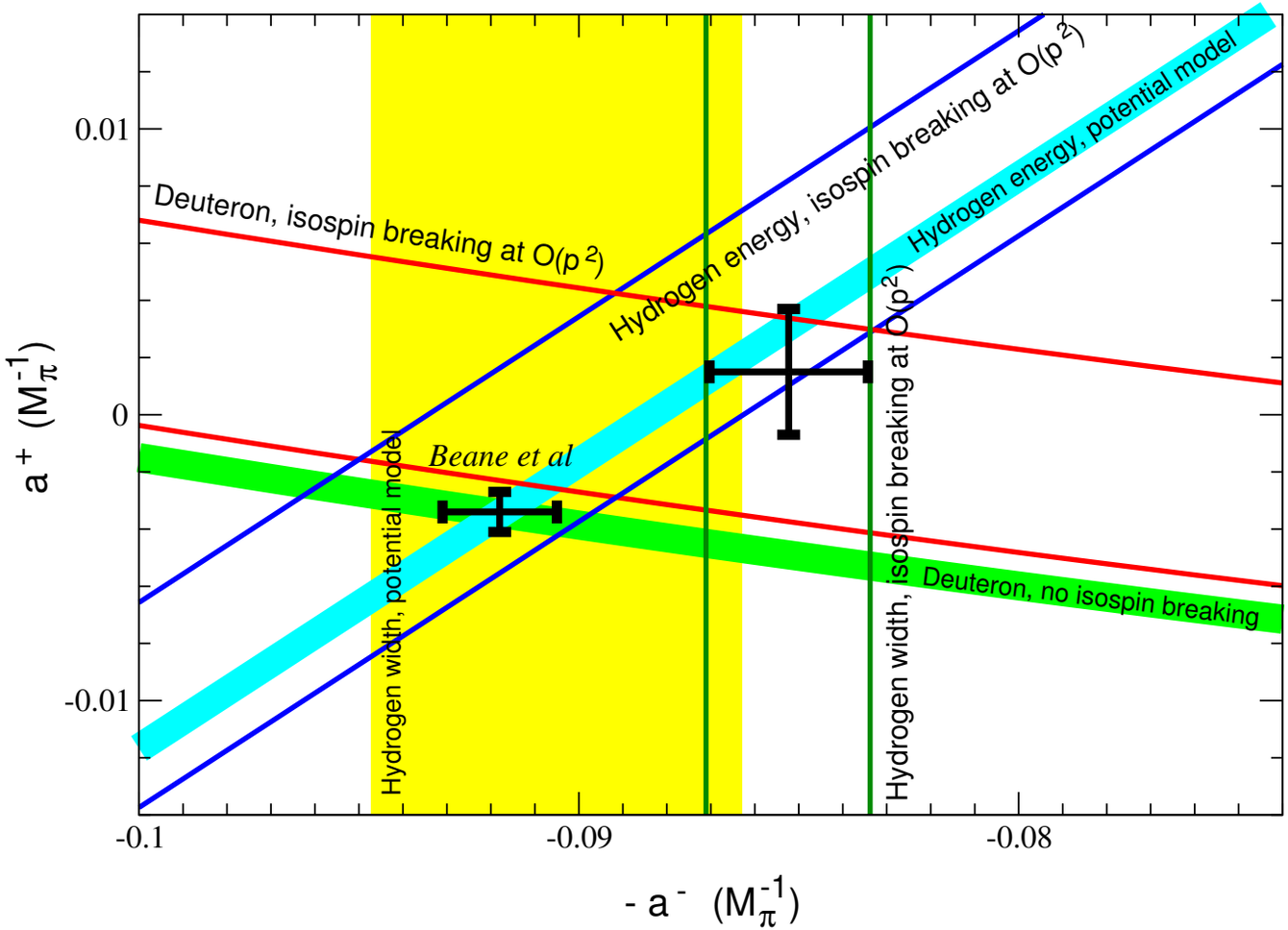
Hoferichter, Kubis, M., Phys. Lett: B **678** (2009) 65 *[arXiv:0903.3890]*

WHY RECONSIDER IV in πN SCATTERING ?

- high precision measurements of pionic hydrogen & deuterium at PSI

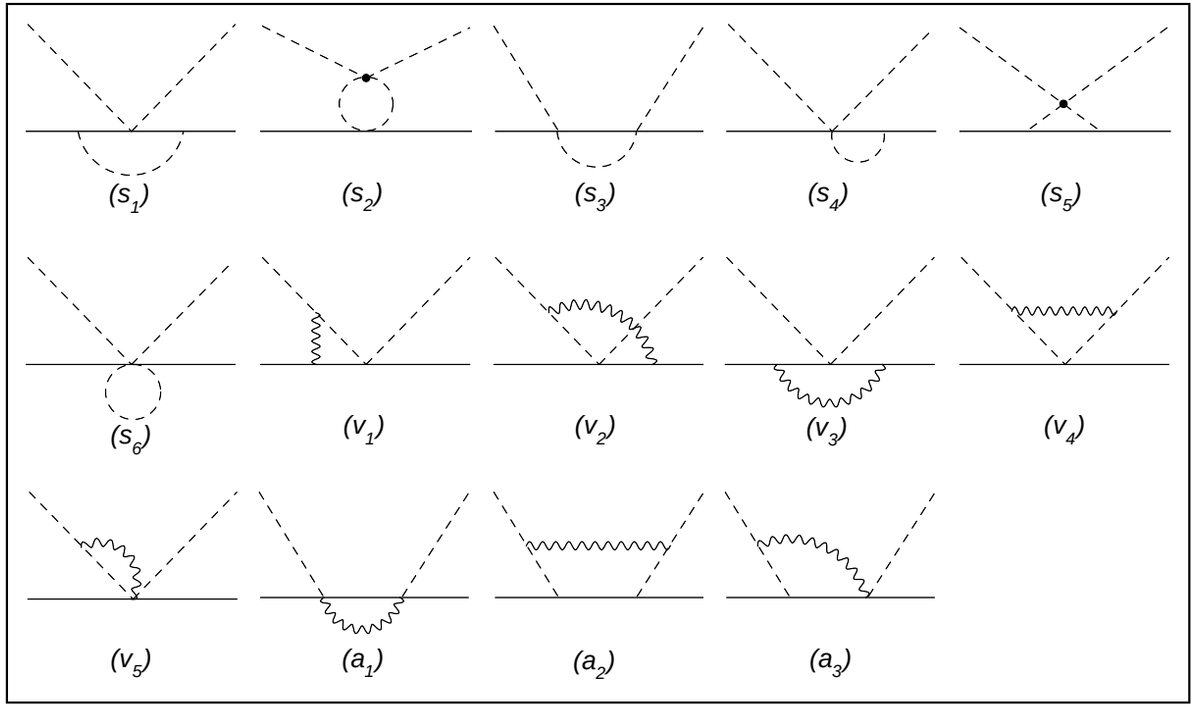
  $\Rightarrow$ need better control of the isospin-breaking corrections

M., Raha, Rusetsky, Phys. Lett. B **639** (2006):478



NOVEL CALCULATION of the SCATTERING LENGTHS

- third order calculation in covariant baryon CHPT with virtual photons
- largest uncertainty via electromagnetic pion-nucleon LECs f_i, g_i
- pertinent loop graphs



$(s_3) \longrightarrow$ cusp effect

$(s_5) \longrightarrow$ triangle graph

$(v_1) \longrightarrow$ Coulomb pole

$\Rightarrow$ analytical results to $\mathcal{O}(m_u - m_d, e^2)$

ISOSPIN BREAKING CORRECTIONS

- isospin-breaking shifts to $a_{\pi^\pm p}$

$$\Delta a_{\pi^- p} = a_{\pi^- p} - (a^+ + a^-) = \Delta a^+ + \Delta a^- + i\,\mathrm{Im}\, a_{\pi^- p}\,,$$
$$\Delta a_{\pi^+ p} = a_{\pi^+ p} - (a^+ - a^-) = \Delta a^+ - \Delta a^-\,,$$
$$\Delta a^+ = \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{4\Delta_\pi}{F_\pi^2}c_1 - \frac{e^2}{2}\left(4f_1+f_2\right) - \frac{g_\mathrm{A}^2 M_\pi}{32\pi F_\pi^2}\left(\frac{33\Delta_\pi}{4F_\pi^2}+e^2\right)\right\}$$
$$\Delta a^- = -\frac{m_\mathrm{p} M_\pi}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{\Delta_\pi}{32\pi^2 F_\pi^4}\left(3+\log\frac{M_\pi^2}{\mu^2}\right) + \frac{8\Delta_\pi}{F_\pi^2}d_5^\mathrm{r}\right.$$
$$\left. + \frac{e^2 g_\mathrm{A}^2}{16\pi^2 F_\pi^2}\left(1+4\log 2+3\log\frac{M_\pi^2}{\mu^2}\right) - 2e^2\left(g_6^\mathrm{r}+g_8^\mathrm{r}-\frac{5}{9F_\pi^2}\left(k_1^\mathrm{r}+k_2^\mathrm{r}\right)\right)\right\}$$
$$\mathrm{Im}\, a_{\pi^- p} = \frac{m_\mathrm{p}}{4\pi(m_\mathrm{p}+M_\pi)}\left\{\frac{M_\pi^2}{8\pi F_\pi^4}\sqrt{\Delta_\pi - 2M_\pi\Delta_\mathrm{N}} + \frac{e^2 g_\mathrm{A}^2 M_\pi}{4\pi F_\pi^2}\right\} \qquad [\pi^0 n \ \ \mathrm{thr.}]$$

- large contributions from the triangle graph and a cusp effect
- largest uncertainty form badly constrained electromagnetic LECs

RESULTS for the SCATTERING LENGTHS

- isospin-breaking shifts in units of $10^{-3}/M_\pi$

isospin limit	channel	shift	channel	shft
$a^+ + a^-$	$\pi^- p \to \pi^- p$	$-3.4^{+4.3}_{-6.5} + 5.0i$	$\pi^- n \to \pi^- n$	$-4.3^{+4.3}_{-6.5} + 6.0i$
$a^+ - a^-$	$\pi^+ p \to \pi^+ p$	$-5.3^{+4.3}_{-6.5}$	$\pi^+ n \to \pi^+ n$	$-6.2^{+4.3}_{-6.5}$
$-\sqrt{2}a^-$	$\pi^- p \to \pi^0 n$	0.4 ± 0.9	$\pi^+ n \to \pi^0 p$	2.3 ± 0.9
a^+	$\pi^0 p \to \pi^0 p$	-5.2 ± 0.2	$\pi^0 n \to \pi^0 n$	-1.8 ± 0.2

$\Rightarrow$ sizeable effect in elastic channels through triangle graph (s_5)

- small IV in the triangle ratio at threshold

$$R = 2(a_{\pi^+ p} - a_{\pi^- p} - \sqrt{2} a^{\rm cex}_{\pi^- p})/(a_{\pi^+ p} - a_{\pi^- p} + \sqrt{2} a^{\rm cex}_{\pi^- p}) = (1.5 \pm 1.1)\%$$

$\Rightarrow$ consistent with earlier results in heavy baryon CHPT above threshold

$\Rightarrow$ inconsistent with findings of Gibbs et al. and Matsinos at low pion momenta

- large IV in the yet unmeasured $\pi^0 - p$ scattering length $\Rightarrow$ need a measurement

The neutron-proton mass difference from $np \to d\pi^0$

Filin, Baru, Epelbaum, Haidenbauer, Hanhart, Kudryavtsev, M., *arXiv:0907.4671 [nucl-th]*

FORWARD-BACKWARD ASYMMETRY in $np \to d\pi^0$

- CSB fb-asymmetry in $np \to d\pi^0$ @ 279.5 MeV measured at TRIUMF

Opper et al., 2003

$$A_{fb} = \frac{\int_0^{\pi/2} [\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi-\theta)] d\cos\theta}{\int_0^{\pi/2} [\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi-\theta)] d\cos\theta} = (17.2 \pm 8(\mathrm{stat}) \pm 5.5(\mathrm{sys})) \cdot 10^{-4}$$

- Goal: use the measured fb-asymmetry to extract the (strong) proton-neutron mass difference $\boxed{\delta m_N \equiv m_n - m_p}$

- best determination: use the Cottingham sum rule Gasser, Leutwyler, 1982

$$\delta m_N^{\mathrm{str}} = \quad 2.05 \pm 0.30 \ \mathrm{MeV}$$
$$\delta m_N^{\mathrm{em}} = -0.76 \mp 0.30 \ \mathrm{MeV}$$

- pioneering calculations for A_{fb}: Niskanen 1999, van Kolck et al., 2000

THEORY of A_{fb} in $np \to d\pi^0$

- CSB fb-asymmetry through interference of IC and IV amplitudes:

$$\frac{d\sigma}{d\Omega} = A_0 + A_1 P_1(\cos\theta_\pi) + A_2 P_2(\cos\theta_\pi) + \ldots \Longrightarrow \boxed{A_{fb} \simeq \frac{A_1}{2A_0}}$$

- A_0 can be determined from pionic deuterium lifetime measured at PSI:

$$\sigma(np \to d\pi^0) = \tfrac{1}{2}\sigma(nn \to d\pi^-) = 252^{+5}_{-11}\ \eta\ [\mu\mathrm{b}] \to A_0 = 10.0^{+0.2}_{-0.4}\ \eta\ [\mu\mathrm{b}]$$

Hauser et al., 1998

- A_1 at LO in chiral EFT:

$$A_1 = \frac{1}{128\pi^2}\frac{\eta M_\pi}{p(M_\pi+m_d)^2}\mathrm{Re}\left[\underbrace{\left(M_{^1S_0\to{}^3S_{1,p}} + \frac{2}{3}M_{^1D_2\to{}^3S_{1,p}}\right)}_{\text{IC amplitude calculated at NLO}} M^*_{^1P_1\to{}^3S_{1,p}}\right]$$

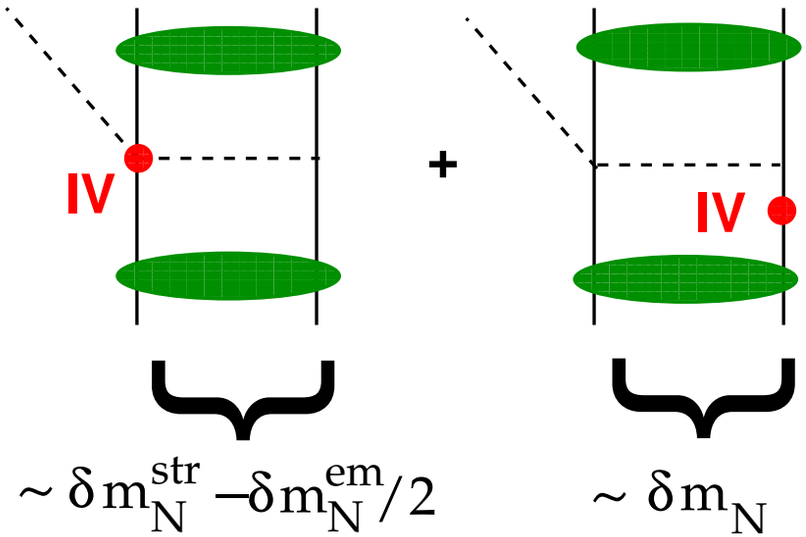
Hanhart et al., 2000
Baru et al., 2009

EXTRACTION of δm_N from $A_{fb}(np \to d\pi^0)$

- Both diagrams combine so that only $\delta m_N^{\rm str}$ survives

- so that we obtain

$$A_{fb}^{\rm LO} = (11.5 \pm 3.5) \cdot 10^{-4} \, \frac{\delta m_N^{\rm str}}{\rm MeV}$$



$$\Longrightarrow \quad \boxed{\delta m_N^{\rm str} = 1.5 \pm 0.8\,({\rm exp}) \pm 0.5\,({\rm th}) \;\; {\rm MeV}}$$

– uncertainty of the expected size

– nice consistency with the earlier determination of $\delta m_N^{\rm str}$ & lattice

– crucial ingredient: A_0 from the precision PSI experiment

– can be improved systematically

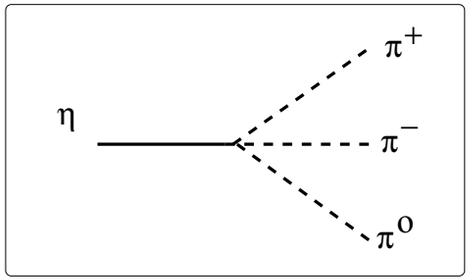
Electromagnetic corrections to $\eta \longrightarrow 3\pi$

Ditsche, Kubis, M., Eur. Phys. J. C **60** (2009) 83 *[arXiv:0812.0344 [hep-ph]]*

ISOSPIN VIOLATION & $\eta \to 3\pi$

Gasser, Leutwyler, Kambor, Wyler, Bijnens, Borasoy. Nißler, . . .

- Isospin violation drives $\eta \to 3\pi$, CHPT analysis:

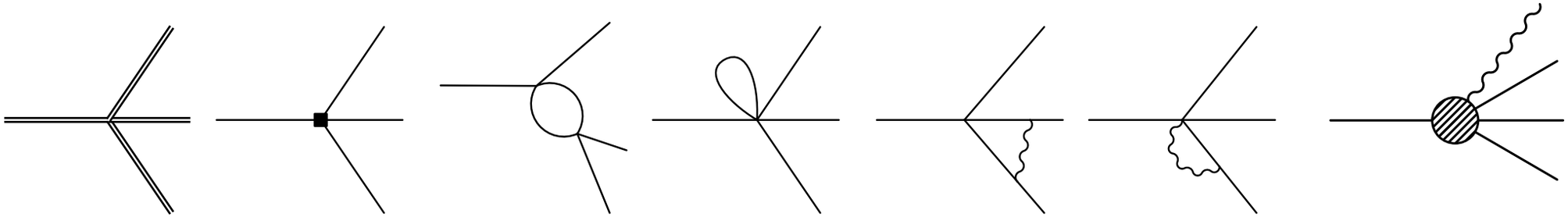


$$
\begin{aligned}
A(\eta \to \pi^+\pi^-\pi^0) &= A^{(2)} + A^{(4)} + \ldots \\
A^{(2)} &= \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s-s_0)}{M_\eta^2 - M_\pi^2}\right\} \\
&\to \Gamma(\eta \to 3\pi) \sim Q^{-4}\,, \quad Q^{-1} = \frac{m_u - m_d}{m_s - \hat{m}}
\end{aligned}
$$

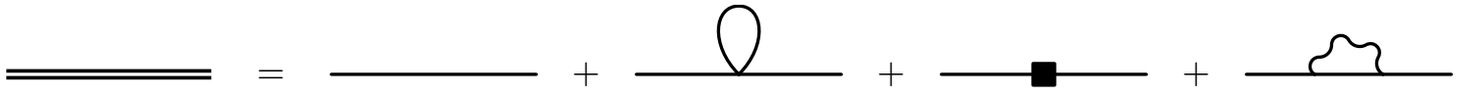
- worked out to tree, one- and two-loop accuracy
- large unitarity corrections (FSI)
  $\to$ can be handled with dispersive machinery or UCHPT
- many testable predictions ($\Gamma(3\pi^0)/\Gamma(\pi^+\pi^-\pi^0)$, Dalitz slopes)
- small em corrections at $\mathcal{O}(e^2\, m_{\rm quark})$ Baur, Kambor, Wyler, 1996
- but how about em corrections $\mathcal{O}(e^2(m_u - m_d))$?

NLO CONTRIBUTIONS

- strong and em diagrams



- full propagator



- LECs: some strong L_i (known), some em K_i (dimensional analysis) ■

RESULTS: GENERAL REMARKS

- neutral and charged amplitudes must be calculated separately

$$\boxed{\mathcal{A}_n(s,t,u) \neq \mathcal{A}_c(s,t,u) + \mathcal{A}_c(t,u,s) + \mathcal{A}_c(u,s,t)}$$

- always compare to the strong one-loop amplitude of Gasser and Leutwyler (GL)

GL, Nucl. Phys. **B 250** (1985) 539

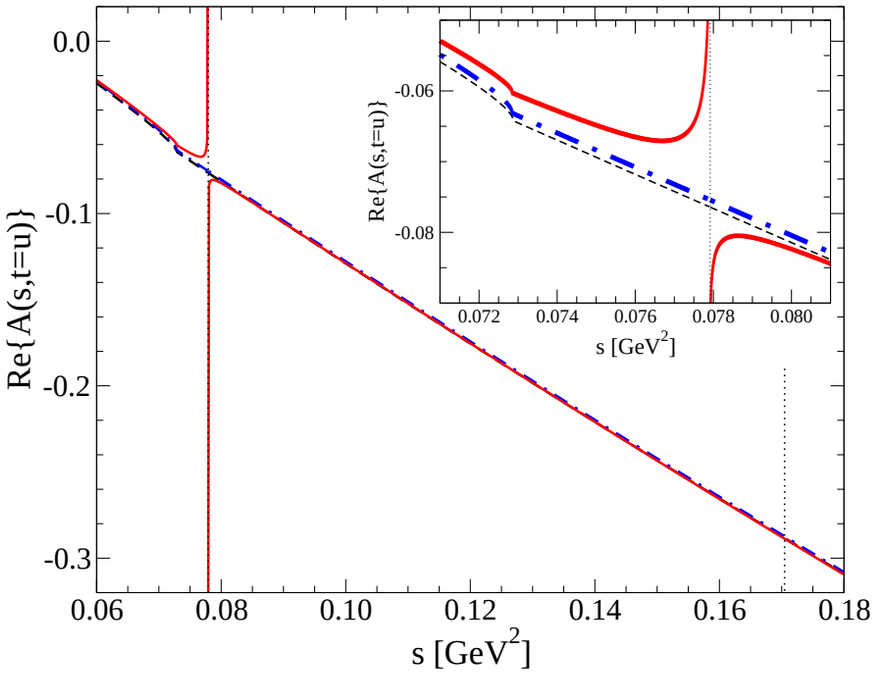
- EM corrections in general small (but need to be accounted for)
- corrections of order $e^2(m_d - m_u)$ (DKM) as big (or bigger) as $e^2 m_q$ (BKW)
- Coulomb-pole at the edge of the physical region in the charged amplitude
- cusps in the neutral amplitude due to rescattering $\pi^0\pi^0 \to \pi^+\pi^- \to \pi^0\pi^0$

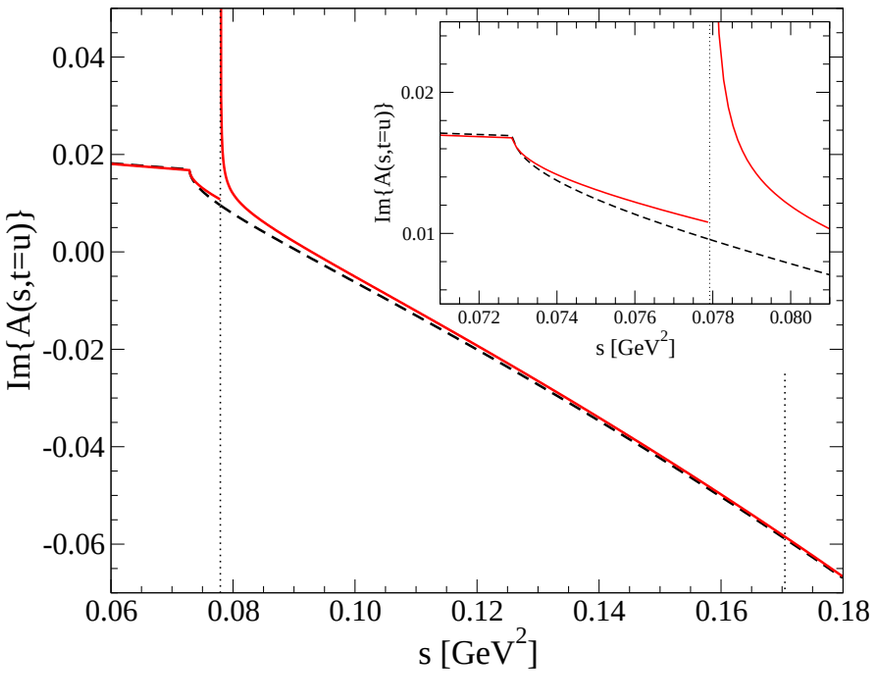
M., Müller, Steininger, Phys. Lett. **B406** (1997) 154

- timely: WASA-at-COSY, CB at MAMI, ...

AMPLITUDES for $\eta \to \pi^0\pi^+\pi^-$

- One-loop representation with em corrections: real and imaginary part
- uncertainties from varying the $K_i \to K_i \pm \frac{\Sigma_i}{16\pi^2}$ (hardly visible)

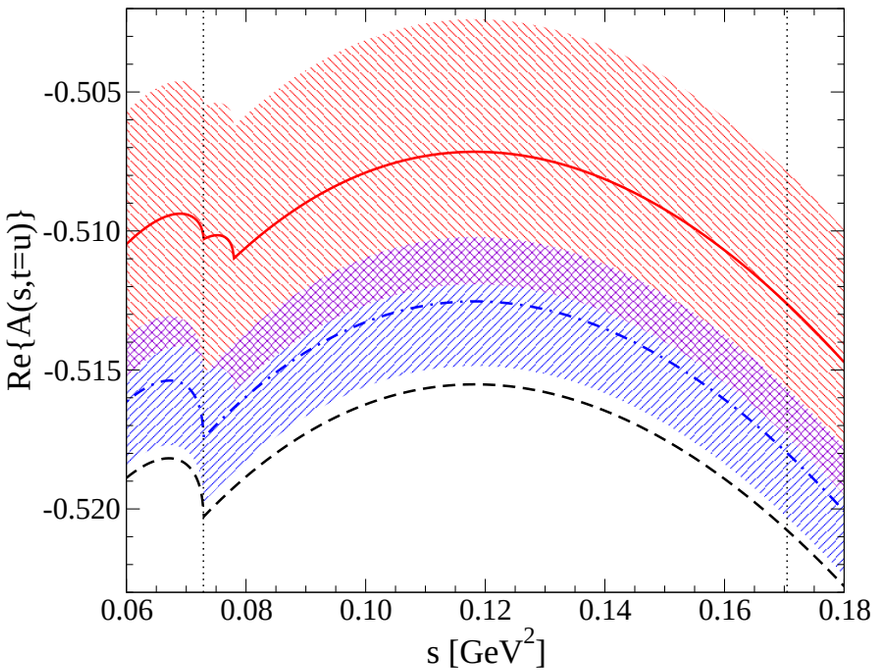


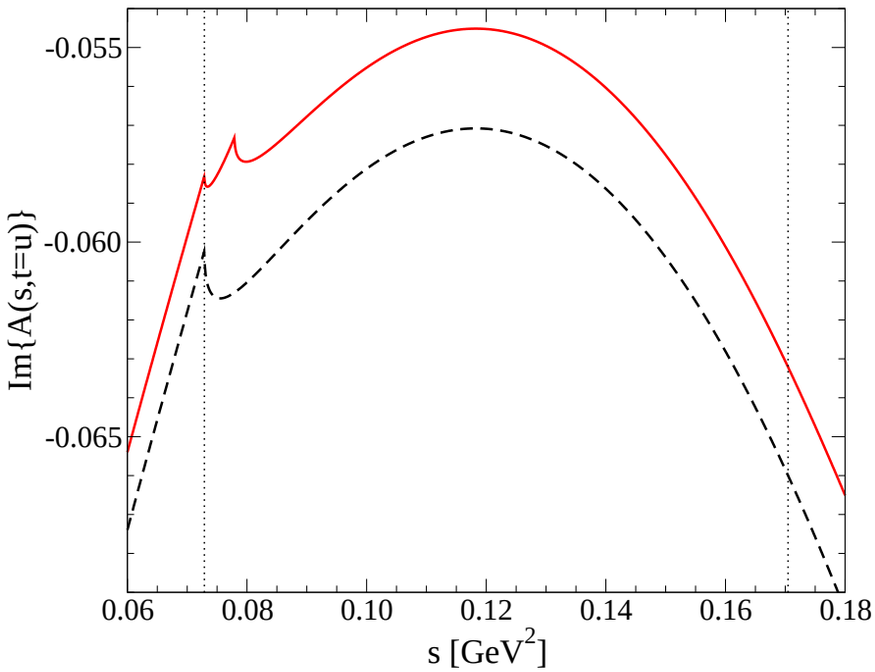


GL BKW DKM

AMPLITUDES for $\eta \to 3\pi^0$

- One-loop representation with em corrections: real and imaginary part
- uncertainties from varying the $K_i \to K_i \pm \frac{\Sigma_i}{16\pi^2}$





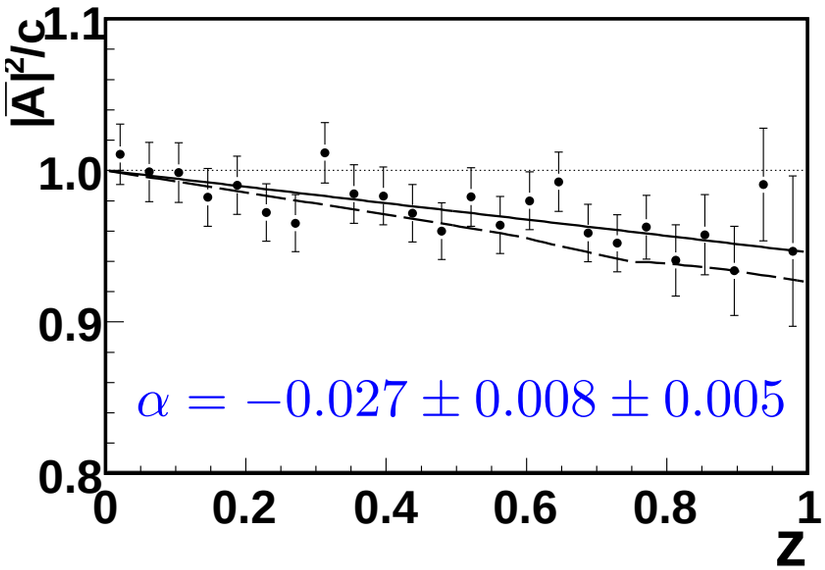
GL BKW DKM

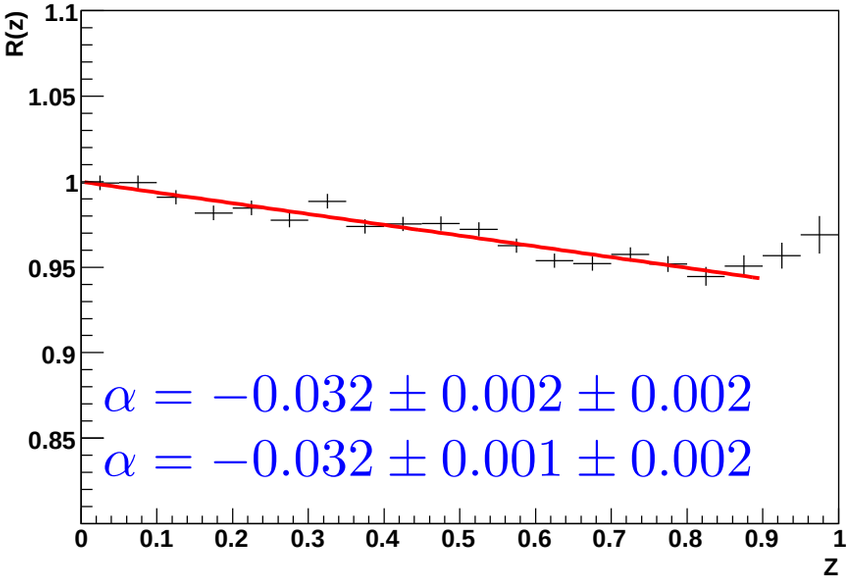
RECENT EXPERIMENTS

- WASA-at-COSY and CRYSTAL BALL at MAMI B & C

Adolph et al., PLB 677 ('09) 24; Unverzagt et al, EPJA 39 ('09) 169; Prakhov et al., PRC 79 ('09) 035204

- extraction of the slope parameter α in $\eta \to 3\pi^0$: $|A(z)|^2 = c^2(1 + 2\alpha z + \ldots)$





- poses a challenge to CHPT: $\alpha = +0.013 \pm 0.032$ at two loops

Bijnens and Ghorbani, JHEP 0711 (2007) 030

The cusp in $\eta' \to \eta\pi\pi$

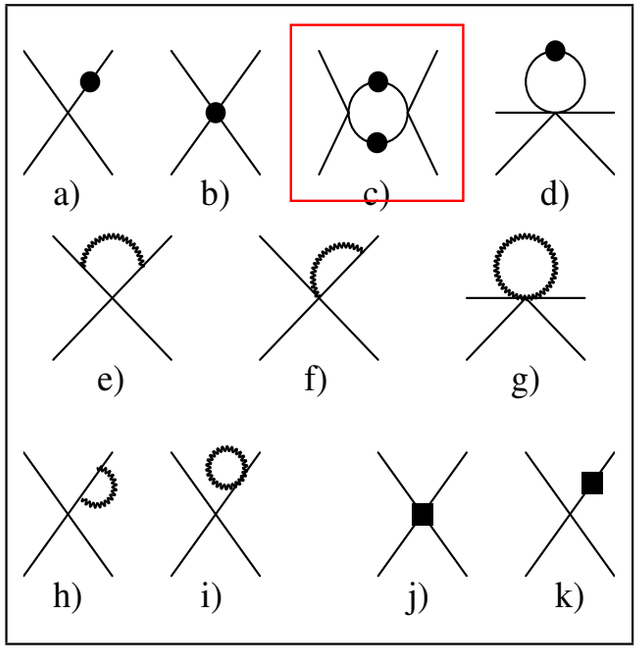
Kubis, Schneider, Eur. Phys. J. C **62** (2009) 511 *[arXiv:0904.1320 [hep-ph]]*

AN INTRIGUING CUSP EFFECT

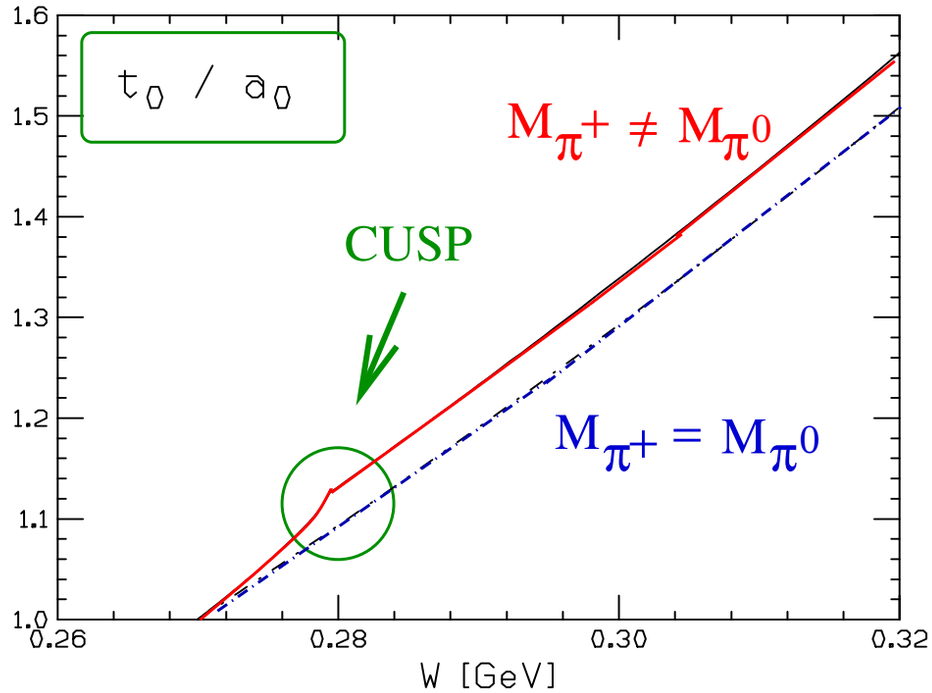
M., Müller, Steininger, Phys. Lett. B 406 (1997) 154 [hep-ph/9704377]
M., Nucl. Phys. A 629 (1998) 72c [hep-ph/9706367]

- Neutral pion–pion scattering at one-loop including isospin breaking
- Pion mass difference $\sim$ • induces a cusp in $\pi^0\pi^0 \to \pi^0\pi^0$ in the threshold region

Feynman graphs



Normalized scattering amplitude

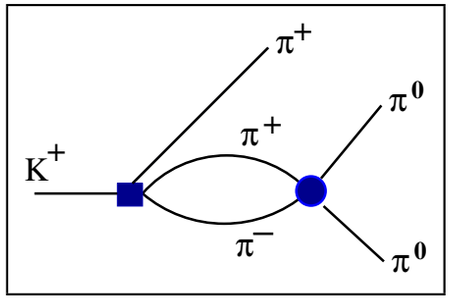


$\Rightarrow$ Long believed to be an unobservable curiosity

REOCCURRENCE OF THE CUSP IN $K \to 3\pi$

Cabibbo, Phys. Rev. Lett. 93 (2005) 121801 [hep-ph/0405001]

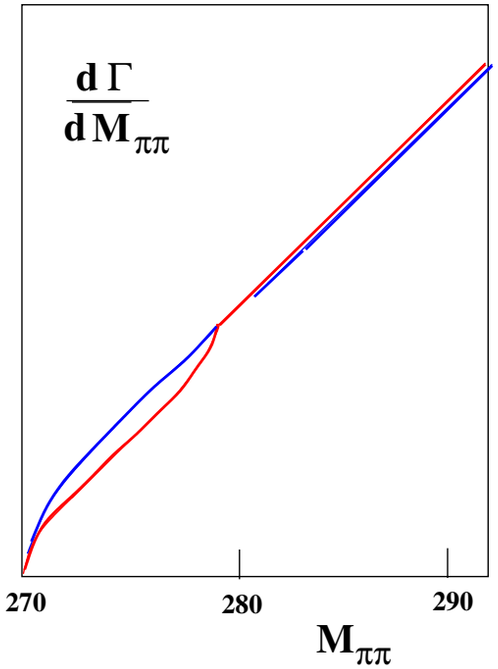
- Rescattering graph in $K^+ \to \pi^+\pi^0\pi^0$:



$\Rightarrow$ gives access to the $\pi\pi$ S-wave scattering lengths $a_0 - a_2$

$\Rightarrow$ $\pi^0\pi^0$ inv. mass distribution $d\Gamma/dM_{\pi\pi}$ senstive to $a_0 - a_2$

$$a_0 - a_2 = 0.265/M_{\pi^+} \qquad a_0 - a_2 = 0$$



- Large data sample from NA 48/2 @ CERN, $\sim 10^8$ events
- Two-loop rescattering contributions:

  $\Rightarrow$ Systematic EFT approach w/ kin. energy resummations and systematic electromagnetic corrections

  Colangelo, Gasser, Kubis, Rusetsky, Phys. Lett. B **638** (2006) 187

$\Rightarrow$ promising alternative to extract a_0 and a_2

$$\Rightarrow a_0 - a_2 = 0.273 \pm 0.005_{\rm st} \pm 0.002_{\rm sy} \pm 0.001_{\rm ex}$$
$$a_2 = -0.065 \pm 0.015_{\rm st} \pm 0.010_{\rm sy} \pm 0.002_{\rm ex}$$

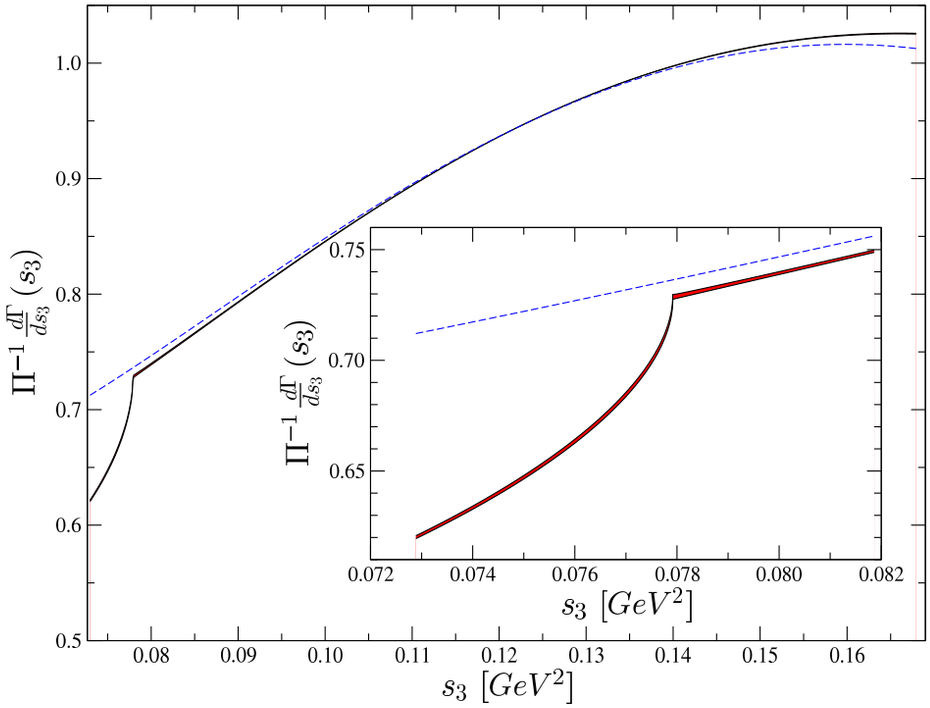
THE CUSP IN $\eta' \to \eta\pi^0\pi^0$

- NREFT calculation to two loops
- input: $\pi\pi$ and $\pi\eta$ scattering parameters

  $\pi\pi$ from Roy equations

  $\pi\eta$ from CHPT w/ large uncertainties
- match EFT parameters to the Dalitz plot parameters from VES assuming isospin invariance

Dorofeev et al., Phys. Lett. B **651** (2007) 22



$\Rightarrow$ cusp reduces the number of events in the pertinent s_3 region by 8%

compare: 13% for $K^+ \to \pi^+\pi^0\pi^0$ and 2% for $\eta \to 3\pi^0$

$\Rightarrow$ **should be measured at COSY, ELSA, MAMI, BES-III,. . . !**

The light quark mass ratio m_u/m_d from ψ' decays

Guo, Hanhart, M., Phys. Rev. Lett. **103** (2009) 082003 *[arXiv:0907.0521 [hep-ph]]*

THE PUZZLE of m_u/m_d EXTRACTIONS

- Extract m_u/m_d from $\psi' \to J/\psi\pi^0(\eta)$

Ioffe, Shifman, 1980; Donoghue et al., 1982

$$R_{\pi^0/\eta} = \frac{\mathcal{B}(\psi' \to J/\psi\pi^0)}{\mathcal{B}(\psi' \to J/\psi\eta)} = 3\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \frac{F_\pi^2}{F_\eta^2}\frac{M_\pi^4}{M_\eta^4}\left|\frac{\vec{q}_\pi}{\vec{q}_\eta}\right|^3 (1+\Delta)$$

$\Rightarrow$ using CLEO data (2008): $\dfrac{m_u}{m_d} = 0.40 \pm 0.01$

- Extraction from the Goldstone bosons mass ratios:

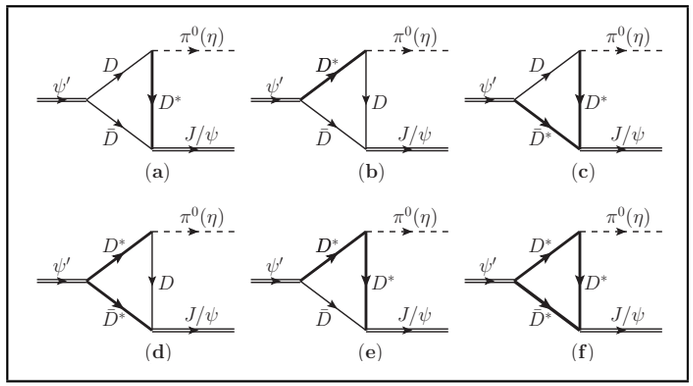
Weinberg, 1977; Gasser and Leutwyler, 1982; Leutwyler, 1996

$$\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2}(1 + \Delta^{\rm str} + \Delta^{\rm em}) = 0.553 \pm 0.043$$

$\Longrightarrow$ serious discrepancy

INCLUSION of CHARMED MESON LOOPS

- consider intermediate charmed mesons



- HQEFT (velocity) expansion:

$$v \sim \sqrt{(2M_{\hat{D}} - M_{\hat{\psi}})/M_{\hat{D}}} \simeq 0.5$$

- direct versus charmed meson loop contribution:

$$\mathcal{M}(\psi' \to J/\psi\pi^0)_{\rm direct} \sim (m_d - m_u)\,|\vec{q}_\pi|$$

$$\mathcal{M}(\psi' \to J/\psi\pi^0)_{\rm D-loops} \sim (m_d - m_u)\frac{|\vec{q}_\pi|}{v} \quad \Leftarrow \text{ enhancement}$$

- charmed meson loop saturation: $R_{\pi^0/\eta} = 0.031 \pm 0.016$

$$R^{\rm exp}_{\pi^0/\eta} = 0.0388 \pm 0.023 \pm 0.005$$

$\Longrightarrow$ need an NLO calculation to see how accurately m_u/m_d can be extracted from these decays

Mass splittings in heavy baryon multiplets

Guo, Hanhart, M., JHEP **0809** (2008) 136 *[arXiv:0809.2359 [hep-ph]]*

INTRA-MULTIPLET SPLITTINGS

- *up* quarks are lighter than *down* quarks:

$$m_u = 1.5 - 3.3\,\mathrm{MeV},\ m_d = 3.5 - 6.0\,\mathrm{MeV} \quad \text{[using } \overline{\mathrm{MS}} \text{ at } \mu = 2\,\text{GeV]}$$

$\Rightarrow$ the more *down* quarks in a state, the heavier a state in a multiplet is

e.g. $n(udd) > p(uud),\ K^0(d\bar{s}) > K^+(u\bar{s}), \ldots$

- stunning exception: $\Sigma_c^{++}(cuu) > \Sigma_c^0(cdd) > \Sigma_c^+(cud)$

$$2454.0 \pm 0.2 \quad 2453.8 \pm 0.2 \quad 2452.9 \pm 0.6 \quad \mathrm{MeV}$$

- natural order restored for the bottom cousins: $\Sigma_b^-(bdd) > \Sigma_b^+(buu)$

$$5815.2 \pm 2.0 \quad 5807.8 \pm 2.7 \quad \mathrm{MeV}$$

how do these patterns arise? heavy quark symmetry?

EFFECTIVE LAGRANGIAN at TREE LEVEL

- basic ingredients:

  Goldstone boson octet, symm. sextet and anti-symm. triplet in SU(3)

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B_{6c} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\Sigma_c^{++} & \Sigma_c^+ & \Xi_c'^+ \\ \Sigma_c^+ & \sqrt{2}\Sigma_c^0 & \Xi_c'^0 \\ \Xi_c'^+ & \Xi_c'^0 & \sqrt{2}\Omega_c^0 \end{pmatrix}, \quad B_{\bar{3}c} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}.$$

- construction of the effective Lagrangian: non-linear $SU(3)_L \times SU(3)_R \to SU(3)_V$

- isospin splittings through quark masses and virtual photons $\to$ well behaved

- technology developed in the late 90ties for πN scattering Fettes, M., Müller, Steininger, . . .

- same for *bottom* cousins ($c \to b$)

EFFECTIVE LAGRANGIAN at TREE LEVEL cont'd

- symmetry breaking terms at order p^2 as in the pion-nucleon case !

$$\mathcal{L}^{(2)}_{\rm str.} = -\langle \bar{B}_Q \left(\alpha_1 \chi_+ + \alpha_2 \langle \chi_+ \rangle\right) B_Q \rangle$$

$$\mathcal{L}^{(2)}_{QQ} = -F_\pi^2 \langle \bar{B}_{6Q} \big[\beta_0 \left(Q_+^2 - Q_-^2\right) + \beta_1 Q_+ \langle Q_+ \rangle + \beta_2 \langle Q_+^2 - Q_-^2 \rangle + \beta_3 \langle Q_+^2 + Q_-^2 \rangle\big] B_{6Q} \rangle - F_\pi^2 \beta_4 \langle Q_+^T \bar{B}_{6Q} Q_+ B_{6Q} \rangle$$

- new symmetry breaking term at order p^2

$$\boxed{\mathcal{L}^{(2)}_{\rm em} = \mathcal{L}^{(2)}_{QQ} - F_\pi^2 \beta_{1h} \left\langle \bar{B}_{6Q} Q_+ \langle q_h \mathbb{I} \rangle B_{6Q} \right\rangle}$$

- physics behind this new term: static heavy quark charge q_h

$$Q_B = 2Q + q_h \mathbb{I} = \begin{cases} e \cdot {\rm diag}\left\{2, 0, 0\right\}, & \text{for the charm baryons,} \\ e \cdot {\rm diag}\left\{1, -1, -1\right\}, & \text{for the bottom baryons,} \end{cases}$$

LOWEST ORDER MASS SPLITTINGS

- mass splittings at order p^2

$$\left(m_{\Sigma_c^+} - m_{\Sigma_c^0}\right)^{(2)} = 2\alpha_1 B_0(m_u - m_d) + \tfrac{1}{6}F_\pi^2 e^2(\beta_0 - 2\beta_4 + 6\beta_{1h})$$

$$\left(m_{\Sigma_c^{++}} - m_{\Sigma_c^0}\right)^{(2)} = 4\alpha_1 B_0(m_u - m_d) + \tfrac{1}{3}F_\pi^2 e^2(\beta_0 + \beta_4 + 6\beta_{1h})$$

$$\left(m_{\Xi_c'^+} - m_{\Xi_c'^0}\right)^{(2)} = 2\alpha_1 B_0(m_u - m_d) + \tfrac{1}{6}F_\pi^2 e^2(\beta_0 - 2\beta_4 + 6\beta_{1h})$$

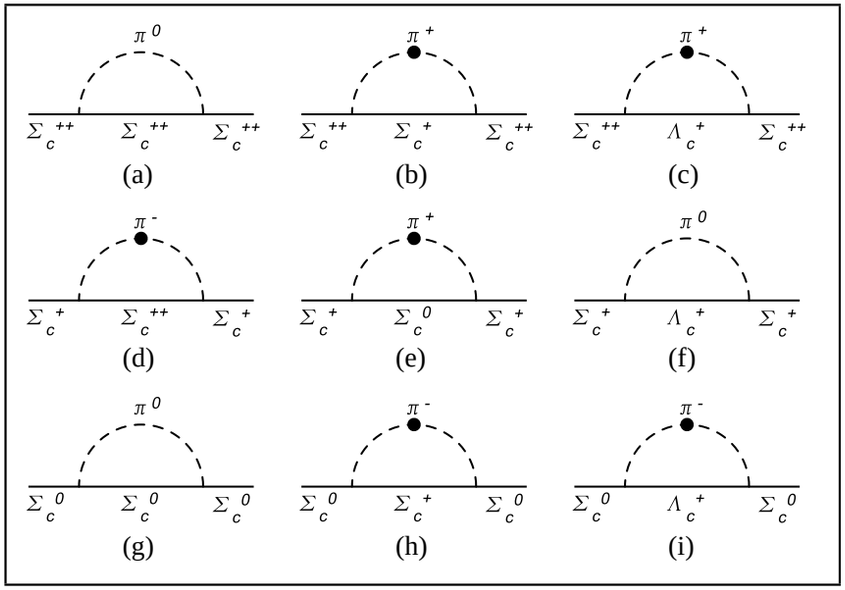
$$\boxed{\Rightarrow \left(m_{\Xi_c'^+} - m_{\Xi_c'^0}\right) = \left(m_{\Sigma_c^+} - m_{\Sigma_c^0}\right) + \mathcal{O}(p^3)}$$

- experiment $\quad -0.9 \pm 0.4 \qquad -2.3 \pm 4.2 \quad \mathrm{MeV}$

- good starting point, need to know the corrections

LEADING LOOP CORRECTIONS

- pion-baryon loops at order p^3
- no photon-baryon loops at order p^3
- resulting isospin splittings



$$m_{\Sigma_c^+} - m_{\Sigma_c^0} = \Delta_{1c}^{(2)} + \Delta_{1c}^{\rm loop}(m_{\Sigma_c}, m_{\Lambda_c}) + \mathcal{O}(p^4)$$

$$m_{\Sigma_c^{++}} - m_{\Sigma_c^0} = \Delta_{2c}^{(2)} + \mathcal{O}(p^4)$$

$$m_{\Xi_c^{\prime +}} - m_{\Xi_c^{\prime 0}} = \Delta_{1c}^{(2)} + \mathcal{O}(p^4)$$

- loop function depends on 3 unknown LECs $\rightarrow$ need three splittings as input
- symmetry breaking LECs come out of *natural size*
- extend to the *bottom* sector utilizing heavy quark symmmetry (axial couplings)

PREDICTIONS and more

- We predict:

$$m_{\Xi_c^{\prime +}} - m_{\Xi_c^{\prime 0}} = m_{\Sigma_c^+} - m_{\Sigma_c^0} - \Delta_{1c}^{\rm loop} = -0.2 \pm 0.6 \ {\rm MeV} \quad [-2.3 \pm 4.2]$$

$$m_{\Sigma_b^0} = \frac{1}{2}\left(m_{\Sigma_b^+} + m_{\Sigma_b^-} - \tilde{\beta}_4\right) + \Delta_{1b}^{\rm loop} = 5810.3 \pm 1.9 \ {\rm MeV}$$

$$m_{\Xi_b^{\prime 0}} - m_{\Xi_b^{\prime -}} = \frac{1}{2}\left(m_{\Sigma_b^+} - m_{\Sigma_b^-} - \tilde{\beta}_4\right) = -4.0 \pm 1.9 \ {\rm MeV}$$

- Explanation for the ordering:

The heavy-light photon exchange has a different sign for the *charm* and the *bottom* baryons. It is the interference of this term with the others that drives the behavior of the Σ_c iso-triplet. Also seen in D-meson splittings.

SUMMARY & OUTLOOK

- Precision calculations in the light quark sector = important QCD tests
  - $\longrightarrow$ isospin-breaking in the pion-nucleon scattering lenghts
  - $\longrightarrow$ strong neutron-proton mass difference from $A_{fb}(np \to d\pi^0)$
  - $\longrightarrow$ higher order electromagentic orrections in $\eta \to 3\pi$
  - $\longrightarrow$ the cusp in $\eta' \to \eta\pi\pi$
- Intriguing isospin-breaking effects in heavy-light systems
  - $\longrightarrow$ m_u/m_d from ψ' decays? (charm meson loops)
  - $\longrightarrow$ interference effects in mass splittings in heavy baryon multiplets

$\Rightarrow$ **do not miss these great experimental opportunities !**

Spares

THE CUSP in E_{0+}

- Electric dipole amplitude: $E_{0+}(\omega) = -a - b\sqrt{1-\omega^2/\omega_c^2}$

- cusp parameter: $b = M_\pi \, a^{\rm cex}_{\pi^+ n} \, E_{0+}^{\gamma p \to \pi^+ n} = (3.43 \pm 0.08) \cdot 10^{-3}/M_\pi$ [data]

  $= 3.63 \cdot 10^{-3}/M_\pi$ [c.v. CHPT]

Bernard, Kaiser, M., Eur. Phys. J. A **11** (2001) 209

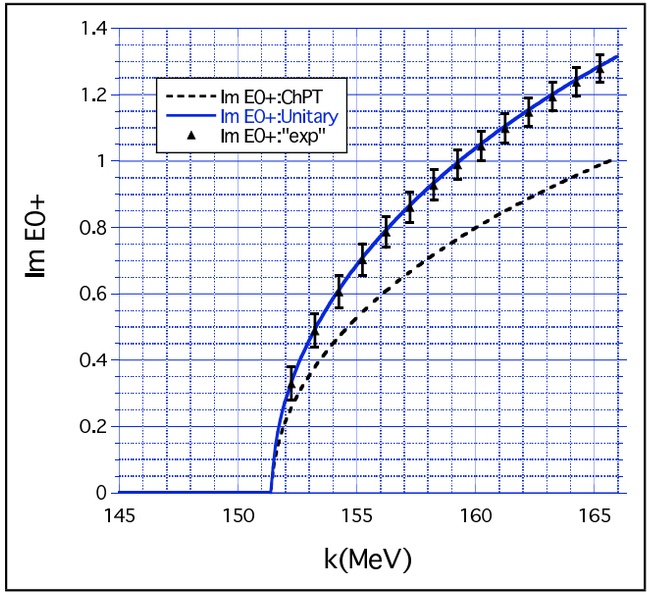
- but: large IV in the CEX $\Delta a^{\rm cex}_{\pi^+ n} = (-1.9 \pm 0.8)\%$

- better th'y analysis needed

- measure precisely the polarized target asymmetry

  $T \propto \mathrm{Im}[E_{0+}(P_3 - P_2)]$

$\Rightarrow$ MAMI A2 proposal!



TRIANGLE RATIO

- triangle ratio R vanishes in the isospin limit:

$$\boxed{R = 2\,\frac{a_{\pi^+p} - a_{\pi^-p} - \sqrt{2}\,a^{\rm cex}_{\pi^-p}}{a_{\pi^+p} - a_{\pi^-p} + \sqrt{2}\,a^{\rm cex}_{\pi^-p}}}$$

- analytical expression including electromagnetic LECs

$$R = \frac{m_{\rm p}}{4\pi(m_{\rm p}+M_\pi)a^-}\left\{\frac{e^2 f_2}{2} + \frac{g_{\rm A}^2\Delta_\pi}{4F_\pi^2 m_{\rm p}} - \frac{M_\pi\Delta_{\rm N}}{4F_\pi^2 m_{\rm p}}(1+2g_{\rm A}^2) - \frac{3M_\pi\Delta_\pi}{16F_\pi^2 m_{\rm p}^2} + \frac{M_\pi\Delta_\pi}{4m_{\rm p}^2}B^-_{\rm thr}\right.$$
$$- \frac{M_\pi\Delta_\pi}{48\pi^2F_\pi^4}\left(4+\log\frac{M_\pi^2}{\mu^2}\right) - \frac{g_{\rm A}^2 M_\pi\Delta_\pi}{192\pi^2F_\pi^4}\left(7+5\log\frac{M_\pi^2}{\mu^2}\right) + \frac{e^2M_\pi}{32\pi^2F_\pi^2}\left(5+3\log\frac{M_\pi^2}{\mu^2}\right)$$
$$\left. - \frac{e^2g_{\rm A}^2M_\pi}{16\pi^2F_\pi^2}\left(1+4\log 2+3\log\frac{M_\pi^2}{\mu^2}\right) + \frac{e^2M_\pi}{2}(4g_6^{\rm r}+g_7^{\rm r}+4g_8^{\rm r}) + \frac{e^2M_\pi}{2F_\pi^2}(k_4^{\rm r}-2k_3^{\rm r})\right\}$$
$$\Delta_\pi = M_\pi^2 - M_{\pi^0}^2\,,\quad \Delta_{\rm N} = m_{\rm n} - m_{\rm p}\,,\quad B^-_{\rm thr} = \ldots$$