ISOSPIN VIOLATION, LIGHT QUARK MASSES
and ALL THAT

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Introduction
Isospin violation has two sources (QCD + QED):

\[ \mathcal{H}_{\text{QCD}}(x) = \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u)(x) \]

\[ \mathcal{H}_{\text{QED}}(x) = -\frac{1}{2}e^2 \int dy \, D^{\mu\nu}(x - y)T(j_{\mu}(x)j_{\nu}(y)) \]

⇒ unique window to quark masses for light quark and heavy-light quark systems

Both effects usually small and of the same size (e.g. \( m_p - m_n \))

⇒ systematic machinery must cope with both these accurately

Chiral perturbation theory w/ virtual photons is the tool to analyse the strictures of the spontaneously and explicitly broken chiral symmetry of QCD
GENERAL REMARKS

• Isospin violation is enhanced in system with nucleons and (neutral) pions

Weinberg 1977

\[
a(\pi^0 p) - a(\pi^0 n) = \frac{m_p c_5 B (m_d - m_u)}{\pi (m_p + M_\pi) F_\pi^2} = (-2.3 \pm 0.4) \cdot 10^{-3} / M_\pi
\]

→ precision data from hadronic atoms and threshold pion photoproduction

• further suppression in some processes involving two or more nucleons (CSB)

→ pioneering data on \( A_{fb}(np \rightarrow d\pi^0) \) [TRIUMF] and \( \sigma(dd \rightarrow \alpha\pi^0) \) [IUCF]

• high-precision data for eta & kaon decays demand very accurate calculations of chiral and electromagnetic corrections

• systems with heavy quarks can give additional information on \( m_u - m_d \) through decays and mass differences
Isospin breaking in the $\pi N$ scattering lengths

WHY RECONSIDER IV in $\pi N$ SCATTERING?

- high precision measurements of pionic hydrogen & deuterium at PSI
  ⇒ need better control of the isospin-breaking corrections

NOVEL CALCULATION of the SCATTERING LENGTHS

- third order calculation in covariant baryon CHPT with virtual photons
- largest uncertainty via electromagnetic pion-nucleon LECs $f_i, g_i$
- pertinent loop graphs

⇒ analytical results to $\mathcal{O}(m_u - m_d, e^2)$
ISOSPIN BREAKING CORRECTIONS

- isospin-breaking shifts to $a_{\pi^\pm p}$

\[
\Delta a_{\pi^-p} = a_{\pi^-p} - (a^+ + a^-) = \Delta a^+ + \Delta a^- + i \text{Im} a_{\pi^-p},
\]
\[
\Delta a_{\pi^+p} = a_{\pi^+p} - (a^+ - a^-) = \Delta a^+ - \Delta a^-,
\]
\[
\Delta a^+ = \frac{m_p}{4\pi(m_p + M_\pi)} \left\{ \frac{4\Delta_\pi}{F_\pi^2} c_1 - \frac{e^2}{2} (4f_1 + f_2) - \frac{g_A^2 M_\pi}{32\pi F_\pi^2} \left( \frac{33\Delta_\pi}{4F_\pi^2} + e^2 \right) \right\}
\]
\[
\Delta a^- = - \frac{m_p M_\pi}{4\pi(m_p + M_\pi)} \left\{ \frac{\Delta_\pi}{32\pi^2 F_\pi^4} \left( 3 + \log \frac{M_\pi^2}{\mu^2} \right) + \frac{8\Delta_\pi}{F_\pi^2} d_5^r \right. + \frac{e^2 g_A^2}{16\pi^2 F_\pi^2} \left( 1 + 4 \log 2 + 3 \log \frac{M_\pi^2}{\mu^2} \right) - 2e^2 \left( g_6^r + g_8^r - \frac{5}{9F_\pi^2} \left( k_1^r + k_2^r \right) \right) \left. \right\}
\]
\[
\text{Im} a_{\pi^-p} = \frac{m_p}{4\pi(m_p + M_\pi)} \left\{ \frac{M_\pi^2}{8\pi F_\pi^4} \sqrt{\Delta_\pi - 2M_\pi \Delta_N} + \frac{e^2 g_A^2 M_\pi}{4\pi F_\pi^2} \right\} \quad [\pi^0 n \text{ thr.}]
\]

- large contributions from the triangle graph and a cusp effect
- largest uncertainty form badly constrained electromagnetic LECs
RESULTS for the SCATTERING LENGTHS

- isospin-breaking shifts in units of $10^{-3}/M_\pi$

<table>
<thead>
<tr>
<th>isospin limit</th>
<th>channel</th>
<th>shift</th>
<th>channel</th>
<th>shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^+ + a^-$</td>
<td>$\pi^- p \to \pi^- p$</td>
<td>$-3.4^{+4.3}_{-6.5} + 5.0i$</td>
<td>$\pi^- n \to \pi^- n$</td>
<td>$-4.3^{+4.3}_{-6.5} + 6.0i$</td>
</tr>
<tr>
<td>$a^+ - a^-$</td>
<td>$\pi^+ p \to \pi^+ p$</td>
<td>$-5.3^{+4.3}_{-6.5}$</td>
<td>$\pi^+ n \to \pi^+ n$</td>
<td>$-6.2^{+4.3}_{-6.5}$</td>
</tr>
<tr>
<td>$-\sqrt{2}a^-$</td>
<td>$\pi^- p \to \pi^0 n$</td>
<td>$0.4 \pm 0.9$</td>
<td>$\pi^+ n \to \pi^0 p$</td>
<td>$2.3 \pm 0.9$</td>
</tr>
<tr>
<td>$a^+$</td>
<td>$\pi^0 p \to \pi^0 p$</td>
<td>$-5.2 \pm 0.2$</td>
<td>$\pi^0 n \to \pi^0 n$</td>
<td>$-1.8 \pm 0.2$</td>
</tr>
</tbody>
</table>

⇒ sizeable effect in elastic channels through triangle graph ($s_5$)

- small IV in the triangle ratio at threshold

$$R = 2(a_{\pi^+ p} - a_{\pi^- p} - \sqrt{2}a_{\pi^- p}^{cex})/(a_{\pi^+ p} - a_{\pi^- p} + \sqrt{2}a_{\pi^- p}^{cex}) = (1.5 \pm 1.1)\%$$

⇒ consistent with earlier results in heavy baryon CHPT above threshold
⇒ inconsistent with findings of Gibbs et al. and Matsinos at low pion momenta

- large IV in the yet unmeasured $\pi^0 - p$ scattering length ⇒ need a measurement
The neutron-proton mass difference from $np \rightarrow d\pi^0$

FORWARD-BACKWARD ASYMMETRY in \( np \rightarrow d\pi^0 \)

- CSB fb-asymmetry in \( np \rightarrow d\pi^0 \) @ 279.5 MeV measured at TRIUMF
  
  Opper et al., 2003

\[
A_{fb} = \frac{\int_{0}^{\pi/2} \left[ \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right] d\cos\theta}{\int_{0}^{\pi/2} \left[ \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) \right] d\cos\theta} = (17.2 \pm 8\text{(stat)} \pm 5.5\text{(sys)}) \cdot 10^{-4}
\]

- Goal: use the measured fb-asymmetry to extract the (strong) proton-neutron mass difference \( \delta m_N \equiv m_n - m_p \)

- best determination: use the Cottingham sum rule
  
  \( \delta m_{N}^{\text{str}} = 2.05 \pm 0.30 \text{ MeV} \)
  
  \( \delta m_{N}^{\text{em}} = -0.76 \mp 0.30 \text{ MeV} \)

- pioneering calculations for \( A_{fb} \):
  
  Niskanen 1999, van Kolck et al., 2000
THEORY of $A_{fb}$ in $np \rightarrow d\pi^0$

- CSB fb-asymmetry through interference of IC and IV amplitudes:

$$\frac{d\sigma}{d\Omega} = A_0 + A_1 P_1(\cos \theta_\pi) + A_2 P_2(\cos \theta_\pi) + \ldots \implies A_{fb} \simeq \frac{A_1}{2A_0}$$

- $A_0$ can be determined from pionic deuterium lifetime measured at PSI:

$$\sigma(np \rightarrow d\pi^0) = \frac{1}{2}\sigma(nn \rightarrow d\pi^-) = 252^{+5}_{-11}\eta [\mu b] \implies A_0 = 10.0^{+0.2}_{-0.4}\eta [\mu b]$$

Hauser et al., 1998

- $A_1$ at LO in chiral EFT:

$$A_1 = \frac{1}{128\pi^2} \frac{\eta M_\pi}{p(M_\pi+m_d)^2} \text{Re} \left[ \left( M_{1S_0 \rightarrow 3S_{1,p}} + \frac{2}{3} M_{1D_2 \rightarrow 3S_{1,p}} \right) M_{1P_1 \rightarrow 3S_{1,p}}^* \right]$$

IC amplitude calculated at NLO

Hanhart et al., 2000
Baru et al., 2009
EXTRACTION of $\delta m_N$ from $A_{fb}(np \rightarrow d\pi^0)$

- Both diagrams combine so that only $\delta m_N^{str}$ survives

- so that we obtain

$$A_{fb}^{LO} = (11.5 \pm 3.5) \cdot 10^{-4} \frac{\delta m_N^{str}}{\text{MeV}}$$

$$\implies \delta m_N^{str} = 1.5 \pm 0.8 \text{ (exp)} \pm 0.5 \text{ (th) MeV}$$

- uncertainty of the expected size
- nice consistency with the earlier determination of $\delta m_N^{str}$ & lattice
- crucial ingredient: $A_0$ from the precision PSI experiment
- can be improved systematically
Electromagnetic corrections to $\eta \rightarrow 3\pi$

**ISOSPIN VIOLATION & $\eta \to 3\pi$**

- Isospin violation drives $\eta \to 3\pi$, CHPT analysis:

\[
A(\eta \to \pi^+\pi^-\pi^0) = A^{(2)} + A^{(4)} + \ldots
\]

\[
A^{(2)} = \frac{B_0(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s - s_0)}{M_\eta^2 - M_\pi^2}\right\}
\]

\[
\Gamma(\eta \to 3\pi) \sim Q^{-4}, \quad Q^{-1} = \frac{m_u - m_d}{m_s - \hat{m}}
\]

- worked out to tree, one- and two-loop accuracy
- large unitarity corrections (FSI)
  → can be handled with dispersive machinery or UCHPT
- many testable predictions ($\Gamma(3\pi^0)/\Gamma(\pi^+\pi^-\pi^0)$, Dalitz slopes)
- small em corrections at $\mathcal{O}(e^2 m_{\text{quark}})$  
  Baur, Kambor, Wyler, 1996
- but how about em corrections $\mathcal{O}(e^2(m_u - m_d))$?
NLO CONTRIBUTIONS

- strong and em diagrams

- full propagator

\[ \begin{array}{c}
\text{(diagram)} \\
= \text{(diagram)} + \text{(diagram)} + \text{(diagram)} + \text{(diagram)} \\
\end{array} \]

- LECs: some strong $L_i$ (known), some em $K_i$ (dimensional analysis)
RESULTS: GENERAL REMARKS

• neutral and charged amplitudes must be calculated separately

\[ A_n(s, t, u) \neq A_c(s, t, u) + A_c(t, u, s) + A_c(u, s, t) \]

• always compare to the strong one-loop amplitude of Gasser and Leutwyler (GL)


• EM corrections in general small (but need to be accounted for)

• corrections of order \( e^2 (m_d - m_u) \) (DKM) as big (or bigger) as \( e^2 m_q \) (BKW)

• Coulomb-pole at the edge of the physical region in the charged amplitude

• cusps in the neutral amplitude due to rescattering \( \pi^0 \pi^0 \rightarrow \pi^+ \pi^- \rightarrow \pi^0 \pi^0 \)


• timely: WASA-at-COSY, CB at MAMI, …
AMPLITUDES for $\eta \rightarrow \pi^0 \pi^+ \pi^-$

- One-loop representation with em corrections: real and imaginary part
- Uncertainties from varying the $K_i \rightarrow K_i \pm \frac{\Sigma_i}{16\pi^2}$ (hardly visible)

\[ \text{GL} \quad \text{BKW} \quad \text{DKM} \]
AMPLITUDES for $\eta \rightarrow 3\pi^0$

- One-loop representation with em corrections: real and imaginary part
- uncertainties from varying the $K_i \rightarrow K_i \pm \frac{\Sigma_i}{16\pi^2}$
RECENT EXPERIMENTS

- WASA-at-COSY and CRYSTAL BALL at MAMI B & C
  Adolph et al., PLB 677 ('09) 24; Unverzagt et al, EPJA 39 ('09) 169; Prakhov et al., PRC 79 ('09) 035204

- Extraction of the slope parameter $\alpha$ in $\eta \to 3\pi^0$: $|A(z)|^2 = c^2(1 + 2\alpha z + \ldots)$

  $\alpha = -0.027 \pm 0.008 \pm 0.005$

- Poses a challenge to CHPT: $\alpha = +0.013 \pm 0.032$ at two loops
  Bijnens and Ghorbani, JHEP 0711 (2007) 030
The cusp in $\eta' \rightarrow \eta \pi \pi$

• Neutral pion–pion scattering at one-loop including isospin breaking
• Pion mass difference $\sim \bullet$ induces a cusp in $\pi^0\pi^0 \rightarrow \pi^0\pi^0$ in the threshold region

$\Rightarrow$ Long believed to be an unobservable curiosity
REOCCURRENCE OF THE CUSP IN $K \rightarrow 3\pi$

- Rescattering graph in $K^+ \rightarrow \pi^+ \pi^0 \pi^0$:
  - $\Rightarrow$ gives access to the $\pi\pi$ S-wave scattering lengths $a_0 - a_2$
  - $\Rightarrow \pi^0\pi^0$ inv. mass distribution $d\Gamma/dM_{\pi\pi}$ sensitive to $a_0 - a_2$
  - $a_0 - a_2 = 0.265/M_{\pi^+}$
  - $a_0 - a_2 = 0$

- Large data sample from NA 48/2 @ CERN, $\sim 10^8$ events

- Two-loop rescattering contributions:
  - $\Rightarrow$ Systematic EFT approach w/ kin. energy resummations and systematic electromagnetic corrections
  - $\Rightarrow$ promising alternative to extract $a_0$ and $a_2$
  - $a_0 - a_2 = 0.273 \pm 0.005_{st} \pm 0.002_{sy} \pm 0.001_{ex}$
  - $a_2 = -0.065 \pm 0.015_{st} \pm 0.010_{sy} \pm 0.002_{ex}$
THE CUSP IN $\eta' \rightarrow \eta\pi^0\pi^0$

- NREFT calculation to two loops
- input: $\pi\pi$ and $\pi\eta$ scattering parameters
  $\pi\pi$ from Roy equations
  $\pi\eta$ from CHPT w/ large uncertainties
- match EFT parameters to the Dalitz plot parameters from VES assuming isospin invariance


⇒ cusp reduces the number of events in the pertinent $s_3$ region by 8%

compare: 13% for $K^+ \rightarrow \pi^+\pi^0\pi^0$ and 2% for $\eta \rightarrow 3\pi^0$

⇒ should be measured at COSY, ELSA, MAMI, BES-III, ...!
The light quark mass ratio $m_u/m_d$
from $\psi'$ decays

THE PUZZLE of $m_u/m_d$ EXTRactions

• Extract $m_u/m_d$ from $\psi' \to J/\psi \pi^0(\eta)$

  $R_{\pi^0/\eta} = \frac{\mathcal{B}(\psi' \to J/\psi \pi^0)}{\mathcal{B}(\psi' \to J/\psi \eta)} = 3 \left( \frac{m_d - m_u}{m_d + m_u} \right)^2 \frac{F_{\pi}^2 M_{\pi}^4}{F_{\eta}^2 M_{\eta}^4} \left| \vec{q}_{\pi} \vec{q}_{\eta} \right|^3 (1 + \Delta)$

⇒ using CLEO data (2008): $\frac{m_u}{m_d} = 0.40 \pm 0.01$

• Extraction from the Goldstone bosons mass ratios:

  $\frac{m_u}{m_d} = \frac{M_{K^+}^2 - M_{K^0}^2 - 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} (1 + \Delta^{str} + \Delta^{em}) = 0.553 \pm 0.043$

⇒ serious discrepancy

Isospin violation. light quark masses and all that – Ulf-G. Meiβner – QNP09, Beijing, September 24, 2009
INCLUSION of CHARMED MESON LOOPS

- consider intermediate charmed mesons

- HQEFT (velocity) expansion:

\[ v \sim \sqrt{\frac{2M_D - M_{\psi'}}{M_D}} \approx 0.5 \]

- direct versus charmed meson loop contribution:

\[ M(\psi' \to J/\psi\pi^0)_{\text{direct}} \sim (m_d - m_u) |\vec{q}_\pi| \]

\[ M(\psi' \to J/\psi\pi^0)_{\text{D-loops}} \sim (m_d - m_u) \frac{|\vec{q}_\pi|}{v} \rightleftharpoons \text{enhancement} \]

- charmed meson loop saturation:

\[ R_{\pi^0/\eta} = 0.031 \pm 0.016 \]

\[ R_{\pi^0/\eta}^{\text{exp}} = 0.0388 \pm 0.023 \pm 0.005 \]

\[ \implies \text{need an NLO calculation to see how accurately } m_u/m_d \text{ can be extracted from these decays} \]
Mass splittings in heavy baryon multiplets

**INTRA-MULTIPLETS SPLITTINGS**

- *up* quarks are lighter than *down* quarks:

  \[ m_u = 1.5 - 3.3 \text{ MeV}, \quad m_d = 3.5 - 6.0 \text{ MeV} \]  
  \[ \text{[using } \overline{\text{MS}} \text{ at } \mu = 2 \text{ GeV]} \]

  \[ \Rightarrow \]  
  the more *down* quarks in a state, the heavier a state in a multiplet is

  e.g. \( n(udd) > p(uud), \quad K^0(d\bar{s}) > K^+(u\bar{s}), \ldots \)

- **stunning exception:**  
  \( \Sigma_c^{++}(cucc) > \Sigma_c^0(cddd) > \Sigma_c^+(cud) \)

  \[ 2454.0 \pm 0.2 \quad 2453.8 \pm 0.2 \quad 2452.9 \pm 0.6 \text{ MeV} \]

- **natural order restored for the bottom cousins:**  
  \( \Sigma_b^-(bddd) > \Sigma_b^+(buuu) \)

  \[ 5815.2 \pm 2.0 \quad 5807.8 \pm 2.7 \text{ MeV} \]

**how do these patterns arise?**  
**heavy quark symmetry?**
EFFECTIVE LAGRANGIAN at TREE LEVEL

• basic ingredients:

Goldstone boson octet, symm. sextet and anti-symm. triplet in SU(3)

\[ \phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
K^-
\end{pmatrix} \begin{pmatrix}
\pi^+ \\
-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^0
\end{pmatrix} \begin{pmatrix}
K^+ \\
K^0 \\
-\frac{2}{\sqrt{6}} \eta
\end{pmatrix} ,
\]

\[ B_{6c} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} \Sigma^{++}_c \\
\Sigma^+_c \\
\Xi^{'+}_c
\end{pmatrix} \begin{pmatrix}
\Sigma^+_c \\
\sqrt{2} \Sigma^0_c \\
\Xi^{'0}_c
\end{pmatrix} \begin{pmatrix}
\Xi^+_c \\
\Xi^0_c \\
\sqrt{2} \Omega^0_c
\end{pmatrix} , \quad B_{3c} = \begin{pmatrix}
0 & \Lambda^+_c & \Xi^+_c \\
-\Lambda^+_c & 0 & \Xi^0_c \\
-\Xi^+_c & -\Xi^0_c & 0
\end{pmatrix} .
\]

• construction of the effective Lagrangian: non-linear \( SU(3)_L \times SU(3)_R \to SU(3)_V \)

• isospin splittings through quark masses and virtual photons \( \to \) well behaved

• technology developed in the late 90ties for \( \pi N \) scattering

• same for \textit{bottom} cousins \( (c \to b) \)
EFFECTIVE LAGRANGIAN at TREE LEVEL cont’d

• symmetry breaking terms at order $p^2$ as in the pion-nucleon case!

\[
\mathcal{L}^{(2)}_{\text{str.}} = -\langle \bar{B}_Q (\alpha_1 \chi_+ + \alpha_2 \langle \chi_+ \rangle) B_Q \rangle
\]

\[
\mathcal{L}^{(2)}_{QQ} = -F_\pi^2 \langle \bar{B}_6 Q \left[ \beta_0 \left( Q_+^2 - Q_-^2 \right) + \beta_1 Q_+ \langle Q_+ \rangle + \beta_2 \langle Q_+^2 - Q_-^2 \rangle + \beta_3 \langle Q_+^2 + Q_-^2 \rangle \right] B_6 Q \rangle - F_\pi^2 \beta_4 \langle Q_T^T \bar{B}_6 Q Q + B_6 Q \rangle
\]

• new symmetry breaking term at order $p^2$

\[
\mathcal{L}^{(2)}_{\text{em}} = \mathcal{L}^{(2)}_{QQ} - F_\pi^2 \beta_{1h} \langle \bar{B}_6 Q Q + \langle q_h \Pi \rangle B_6 Q \rangle
\]

• physics behind this new term: static heavy quark charge $q_h$

\[
Q_B = 2Q + q_h \Pi = \begin{cases} \ e \cdot \text{diag} \{2, 0, 0\}, & \text{for the charm baryons,} \\ \ e \cdot \text{diag} \{1, -1, -1\}, & \text{for the bottom baryons,} \end{cases}
\]
LOWEST ORDER MASS SPLITTINGS

- mass splittings at order $p^2$

\[
\left( m_{\Sigma^+_c} - m_{\Sigma^0_c} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 \left( \beta_0 - 2\beta_4 + 6\beta_{1h} \right)
\]

\[
\left( m_{\Sigma^{++}_c} - m_{\Sigma^0_c} \right)^{(2)} = 4\alpha_1 B_0 (m_u - m_d) + \frac{1}{3} F_\pi^2 e^2 \left( \beta_0 + \beta_4 + 6\beta_{1h} \right)
\]

\[
\left( m_{\Xi^{'}_c} - m_{\Xi^{'0}_c} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 \left( \beta_0 - 2\beta_4 + 6\beta_{1h} \right)
\]

\[
\Rightarrow \left( m_{\Xi^{'}_c} - m_{\Xi^{'0}_c} \right) = \left( m_{\Sigma^+_c} - m_{\Sigma^0_c} \right) + \mathcal{O}(p^3)
\]

- experiment

- $0.9 \pm 0.4$  \quad - $2.3 \pm 4.2$  \quad MeV

- good starting point, need to know the corrections
LEADING LOOP CORRECTIONS

- pion-baryon loops at order $p^3$
- no photon-baryon loops at order $p^3$
- resulting isospin splittings

\[
m_{\Sigma^+_c} - m_{\Sigma^0_c} = \Delta^{(2)}_{1c} + \Delta^{\text{loop}}_{1c}(m_{\Sigma_c}, m_{\Lambda_c}) + \mathcal{O}(p^4)
\]
\[
m_{\Sigma^{++}_c} - m_{\Sigma^0_c} = \Delta^{(2)}_{2c} + \mathcal{O}(p^4)
\]
\[
m_{\Xi^{'+}_c} - m_{\Xi^{0'}_c} = \Delta^{(2)}_{1c} + \mathcal{O}(p^4)
\]

- loop function depends on 3 unknown LECs \(\rightarrow\) need three splittings as input
- symmetry breaking LECs come out of natural size
- extend to the bottom sector utilizing heavy quark symmetry (axial couplings)
PREDICTIONS and more

- We predict:

\[ m_{\Xi_c^+} - m_{\Xi_c^0} = m_{\Sigma_c^+} - m_{\Sigma_c^0} - \Delta_{1c}^{\text{loop}} = -0.2 \pm 0.6 \text{ MeV} \quad [-2.3 \pm 4.2] \]

\[ m_{\Sigma_b^0} = \frac{1}{2} \left( m_{\Sigma_b^+} + m_{\Sigma_b^-} - \tilde{\beta}_4 \right) + \Delta_{1b}^{\text{loop}} = 5810.3 \pm 1.9 \text{ MeV} \]

\[ m_{\Xi_b^0} - m_{\Xi_b^-} = \frac{1}{2} \left( m_{\Sigma_b^+} - m_{\Sigma_b^-} - \tilde{\beta}_4 \right) = -4.0 \pm 1.9 \text{ MeV} \]

- Explanation for the ordering:

The heavy-light photon exchange has a different sign for the charm and the bottom baryons. It is the interference of this term with the others that drives the behavior of the \( \Sigma_c \) iso-triplet. Also seen in D-meson splittings.
SUMMARY & OUTLOOK

- Precision calculations in the light quark sector = important QCD tests
  → isospin-breaking in the pion-nucleon scattering lengths
  → strong neutron-proton mass difference from $A_{fb}(np \rightarrow d\pi^0)$
  → higher order electromagnetic corrections in $\eta \rightarrow 3\pi$
  → the cusp in $\eta' \rightarrow \eta\pi\pi$

- Intriguing isospin-breaking effects in heavy-light systems
  → $m_u/m_d$ from $\psi'$ decays? (charm meson loops)
  → interference effects in mass splittings in heavy baryon multiplets

⇒ do not miss these great experimental opportunities!
Spares
THE CUSP in $E_{0+}$

- Electric dipole amplitude: $E_{0+}(\omega) = -a - b\sqrt{1 - \omega^2/\omega_c^2}$

- Cusp parameter: $b = M_\pi \alpha^{\text{cex}}_{\pi+n} E_{0+}^{\gamma p \rightarrow \pi^+ n} = (3.43 \pm 0.08) \cdot 10^{-3}/M_\pi$ [data]
  $= 3.63 \cdot 10^{-3}/M_\pi$ [c.v. CHPT]

- But: large IV in the CEX $\Delta a^{\text{cex}}_{\pi+n} = (-1.9 \pm 0.8)\%$

- Better th’y analysis needed

- Measure precisely the polarized target asymmetry
  $T \propto \text{Im}[E_{0+}(P_3 - P_2)]$

$\Rightarrow$ MAMI A2 proposal!
TRIANGLE RATIO

• triangle ratio $R$ vanishes in the isospin limit:

$$R = 2 \frac{a_\pi + p - a_\pi - p - \sqrt{2a_{\pi}^{\text{cex}}}}{a_\pi + p - a_\pi - p + \sqrt{2a_{\pi}^{\text{cex}}}}$$

• analytical expression including electromagnetic LECs

$$R = \frac{m_p}{4\pi(m_p + M_\pi)\alpha} - \frac{e^2 f_2}{2} + \frac{g_A^2 \Delta_\pi}{4F_\pi^2 m_p} - \frac{M_\pi \Delta_N}{4F_\pi^2 m_p} \left(1 + 2g_A^2\right) - \frac{3M_\pi \Delta_\pi}{16F_\pi^2 m_p^2} + \frac{M_\pi \Delta_\pi}{4m_p^2} B_{\text{thr}}$$

$$- \frac{M_\pi \Delta_\pi}{48\pi^2 F_\pi^4} \left(4 + \log \frac{M_\pi^2}{\mu^2}\right) - \frac{g_A^2 M_\pi \Delta_\pi}{192\pi^2 F_\pi^4} \left(7 + 5 \log \frac{M_\pi^2}{\mu^2}\right) + \frac{e^2 M_\pi}{32\pi^2 F_\pi^2} \left(5 + 3 \log \frac{M_\pi^2}{\mu^2}\right)$$

$$- \frac{e^2 g_A^2 M_\pi}{16\pi^2 F_\pi^2} \left(1 + 4 \log 2 + 3 \log \frac{M_\pi^2}{\mu^2}\right) + \frac{e^2 M_\pi}{2} \left(4g_6^r + g_7^r + 4g_8^r\right) + \frac{e^2 M_\pi}{2F_\pi} \left(k_4^r - 2k_3^r\right)$$

$$\Delta_\pi = M_\pi^2 - M_\pi^2_0, \quad \Delta_N = m_n - m_p, \quad B_{\text{thr}} = \ldots$$