Calculation of Equation of State of QCD at Zero Temperature and Finite Chemical Potential

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Outline

1. Introduction
2. Formula for EOS of QCD at Zero Temperature and Finite Chemical Potential
3. Dyson-Schwinger Equation (DSE) of QCD
4. Calculation of EOS with Rainbow-ladder DSE
5. Calculation of EOS with Hard-Dense-Loop Approximation
6. Calculation of EOS with Quasi-particle Model
7. Calculation of EOS with a Nonperturbative Approach Inspired by Chiral Perturbation Theory
8. Summary

⋆ Introduction

The QCD thermodynamical system at finite temperature (T) and finite chemical potential (μ) is described by the density matrix

$$\hat{\rho} = e^{-(\hat{H}_{QCD} - \mu \hat{N}_B)/T}$$

Partition function: $\mathcal{Z}[\mu, T] = \mathrm{Tr}\hat{\rho} = \sum_\alpha \exp\{-\frac{E_\alpha - \mu N_\alpha}{T}\}$

1. The partition function plays an important role in thermal field theory. From the partition function one can obtain all thermodynamical quantities and the equation of state (EOS).

2. A neutron star can be approximately regarded as a zero temperature and high density system. The QCD partition function and EOS at zero T and finite μ is important for the study of neutron stars.

Our approach for studying the problem:

Express the physical quantities we want to calculate in terms of various Green functions of QCD so that the problem reduces to the calculation of these Green functions. These Green functions can be calculated by various QCD methods/models (the Dyson-Schwinger equations, hard-thermal-loop/hard-dense-loop approximation, quasi-particle model, etc.)

⋆ A Formula for the EOS of QCD at Zero T and Finite μ

renormalized QCD partition function at zero T and finite μ:

$$\mathcal{Z}[\mu] \;=\; \int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \;\exp\left\{-S_R[\bar{q}_R, q_R, A_R] + \int d^4x \;\mu Z_2 \bar{q}_R(x)\gamma_4 q_R(x)\right\} \tag{1}$$

$S_R[\bar{q}_R, q_R, A_R]$: renormalized QCD action in Eucldean space

$Z_2 = Z_2(\zeta^2, \Lambda^2)$: quark wave-function renormalization constant

pressure density:

$$\mathcal{P}(\mu) = \frac{1}{\mathcal{V}} \ln \mathcal{Z}[\mu] \tag{2}$$

quark number density:

$$\begin{aligned} \rho(\mu) \;&=\; \frac{\partial \mathcal{P}(\mu)}{\partial \mu} = \frac{1}{\mathcal{V}} \frac{1}{\mathcal{Z}[\mu]} \frac{\partial \mathcal{Z}[\mu]}{\partial \mu} \\ &=\; \frac{1}{\mathcal{V}} \frac{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \int d^4x Z_2 \bar{q}_R(x)\gamma_4 q_R(x) \exp\{-S_R[\bar{q}_R, q_R, A_R; \mu]\}}{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \;\exp\{-S_R[\bar{q}_R, q_R, A_R; \mu]\}} \end{aligned} \tag{3}$$

$S_R[\bar{q}_R, q_R, A_R; \mu] \equiv S_R[\bar{q}_R, q_R, A_R] - \int d^4x \;\mu Z_2 \bar{q}_R(x)\gamma_4 q_R(x)$

dressed quark propagator at finite μ

$$G_{Rij}[\mu](x,y) = \frac{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \; q_{Ri}(x)\bar{q}_{Rj}(y) \exp\{-S_R[\bar{q}_R, q_R, A_R; \mu]\}}{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \; \exp\{-S_R[\bar{q}_R, q_R, A_R; \mu])\}} \tag{4}$$

$$\mathrm{Tr}\left\{G_R[\mu]\gamma_4\right\} = -\frac{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \int d^4x \bar{q}_R(x)\gamma_4 q_R(x) \exp\{-S_R[\bar{q}_R, q_R, A_R; \mu]\}}{\int \mathcal{D}\bar{q}_R \mathcal{D}q_R \mathcal{D}A_R \; \exp\{-S_R[\bar{q}_R, q_R, A_R; \mu]\}} \tag{5}$$

Comparing (3) and (5) gives

$$\rho(\mu) \;=\; -\frac{Z_2}{\mathcal{V}} Tr\left\{G_R[\mu]\gamma_4\right\} = -N_c N_f Z_2 \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left\{G_R[\mu](p)\gamma_4\right\} \tag{6}$$

$\rho(\mu)$ is totally determined by the dressed quark propagator at finite μ

Integrating $\rho(\mu) = \frac{\partial \mathcal{P}(\mu)}{\partial \mu}$ gives

$$\mathcal{P}(\mu) = \mathcal{P}(\mu)|_{\mu=0} + \int_0^\mu d\mu' \rho(\mu') = \mathcal{P}(\mu)|_{\mu=0} - N_c N_f Z_2 \int_0^\mu d\mu' \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\left\{G_R[\mu'](p)\gamma_4\right\} \tag{7}$$

Formula (7) is formally model-independent. When one actually applies it to calculate the EOS, one has to resort to various QCD methods/models.

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标题页
第 5 页 45
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⋆ Using Four Different Methods/Models to Calculate the EOS of QCD at Zero T and Finite μ

- Dyson-Schwinger Equation (DSE) of QCD
- Hard-Dense-Loop (HDL) Approximation
- Quasi-particle Model
- A Nonperturbative Approach Inspired by Chiral Perturbation Theory (χPT)

For details please refer to the following papers:

DSE:

H. S. Zong and W. M. Sun , *The calculation of the equation of state of QCD at finite chemical potential and zero temperature*, Phys. Rev. **D78**, 054001 (2008).

HDL:

Wei-min Sun, Yu Jiang and Hong-shi Zong, *The equation of state of QCD under hard-dense-loop approximation*, to appear in Science in China, Series G.

Quasi-particle Model:

A-meng Zhao, Jing Cao, Liu-jun Luo, Wei-min Sun and Hong-shi Zong, *The equation of state of quasi-particle model of quark-gluon plasma at finite chemical potential*, to appear in Mod. Phys. Lett. A.

A Nonperturbative Approach Inspired by χPT:

Xiao-ya Li, Xiao-fu Lü,Bin Wang, Win-min Sun, Hong-shi Zong, *The evolution properties of nuclear matter with respect to chemical potential at zero temperature in the framework of chiral perturbation theory*, to appear in Phys. Rev. **C**.

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第 6 页 45
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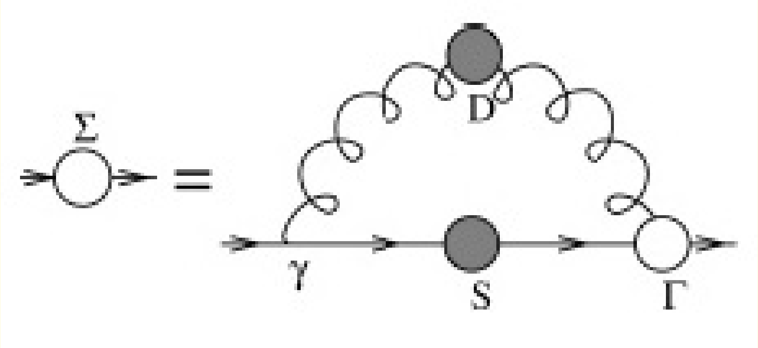
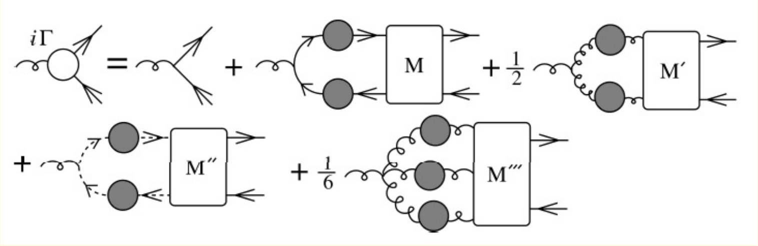
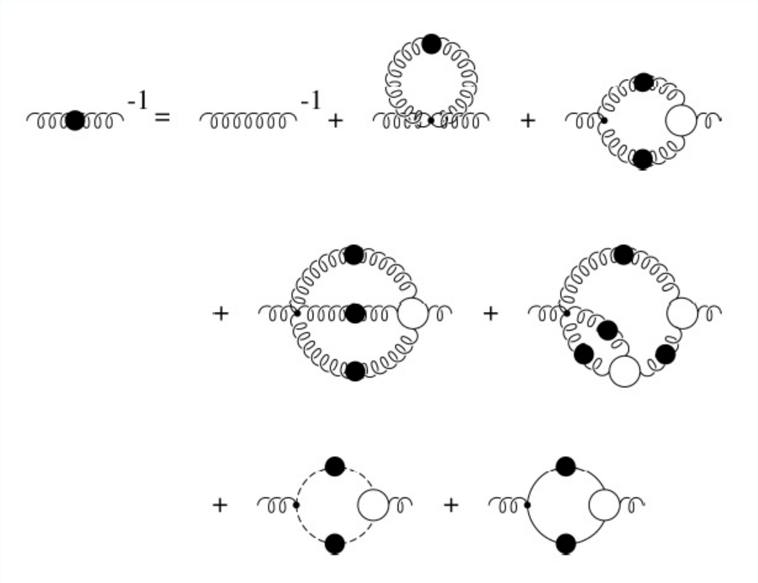
⋆ DSE of QCD



DSE for quark propagator



DSE for quark-gluon vertex



DSE for gluon propagator

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⋆ Calculation of EOS with Rainbow-Ladder DSE

rainbow-ladder quark DSE at zero μ

$$G^{-1}(p) = G_0^{-1}(p) + \frac{4}{3}g_s^2 \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu G(q)\gamma_\nu$$

where $G_0^{-1}(p) = i\gamma \cdot p + m$ is the inverse of free quark propagator.

rainbow-ladder DSE at finite μ

$$G^{-1}[\mu](p) = (i\gamma \cdot p - \mu\gamma_4) + \frac{4}{3}g_s^2 \int \frac{d^4q}{(2\pi)^4} D_{\mu\nu}(p-q)\gamma_\mu G[\mu](q)\gamma_\nu \tag{8}$$

Adopting the approximation that the dressed gluon propagator is independent of μ and assuming the dressed quark propagator is analytic in the neighborhood of $\mu = 0$, one can prove

$$G_R^{-1}[\mu](p) = G_R^{-1}(\tilde{p}) = i\gamma \cdot \tilde{p}A(\tilde{p}^2) + B(\tilde{p}^2) \tag{9}$$

where $\tilde{p} \equiv (\vec{p}, p_4 + i\mu)$, $G_R^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2)$ is the inverse of the dressed quark propagator at zero μ.

For details of the proof, please see

H. S. Zong, L.Chang, F.Y. Hou, W.M. Sun, Y.X. Liu, Phys. Rev. **C 71**, 015205 (2005)

F. Y. Hou, L.Chang, W.M. Sun, H.S. Zong, Y.X. Liu, Phys. Rev. **C 72**, 034901 (2005)

H. T. Feng, F. Y. Hou, X. He, W.M. Sun, H.S. Zong, Phys. Rev. **D 73**, 016004 (2006)

H. T. Feng, W. M. Sun, D. K. He, and H. S. Zong, Phys. Lett. B. **661**, 57 (2008)

In the proof we have used the Taylor expansion of $G_R^{-1}[\mu](p)$ around $\mu = 0$. In the circle of convergence of its Taylor expansion (9) holds. But in fact (9) holds in the whole region in the complex μ plane where $G_R^{-1}[\mu](p)$ is analytic. This conclusion is based on a theorem in complex analysis:

Suppose two functions $f(z)$ and $g(z)$ are analytic in a common region D. If these two functions coincide in some portion $D' \subset D$, then they are equal everywhere in D.

For the dressed quark propagator at $\mu = 0$, we take the following model （R. Alkofer et.al. Phys. Rev. D **70**, 014014 (2004).）

$$G_R(p) = Z_2^{-1}(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \left\{ \frac{r_j}{i \not p + m_j} + \frac{r_j}{i \not p + m_j^*} \right\} \tag{10}$$

where $m_j = a_j + ib_j$ are complex mass scales, r_j are real coefficients. The renormalization is chosen at $\zeta^2 = 16$ GeV^2.

$$\begin{aligned} G_R[\mu](p) &= Z_2^{-1}(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \left\{ \frac{r_j}{i \not{\tilde p} + m_j} + \frac{r_j}{i \not{\tilde p} + m_j^*} \right\} \\ &= Z_2^{-1}(\zeta^2, \Lambda^2) \sum_{j=1}^{n_P} \left\{ \frac{r_j(-i \not{\tilde p} + m_j)}{\tilde p^2 + m_j^2} + \frac{r_j(-i \not{\tilde p} + m_j^*)}{\tilde p^2 + m_j^{*2}} \right\} \end{aligned} \tag{11}$$

From this we obtain the quark number density

$$\begin{aligned} &\rho(\mu) \\ =\ & 4iN_cN_f \int \frac{d^4p}{(2\pi)^4} \sum_{j=1}^{n_P} \left\{ \frac{r_j(p_4 + i\mu)}{\vec p^2 + (p_4 + i\mu)^2 + m_j^2} + \frac{r_j(p_4 + i\mu)}{\vec p^2 + (p_4 + i\mu)^2 + m_j^{*2}} \right\} \\ =\ & \frac{4iN_cN_f}{(2\pi)^4} \sum_{j=1}^{n_P} \left\{ r_j \int d^4p \frac{p_4 + i\mu}{\vec p^2 + (p_4 + i\mu)^2 + m_j^2} + r_j \int d^4p \frac{p_4 + i\mu}{\vec p^2 + (p_4 + i\mu)^2 + m_j^{*2}} \right\} \end{aligned} \tag{12}$$

the calculation of the integral in (12)

$$\int d^4p \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^2} = \int d\vec{p} \int_{-\infty}^{+\infty} dp_4 \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^2}$$

the integral of p_4 can be written as

$$\int_{-\infty}^{+\infty} dp_4 \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^2} = \int_{-\infty+i\mu}^{+\infty+i\mu} dz \frac{z}{z^2 + \vec{p}^2 + m_j^2} \tag{13}$$

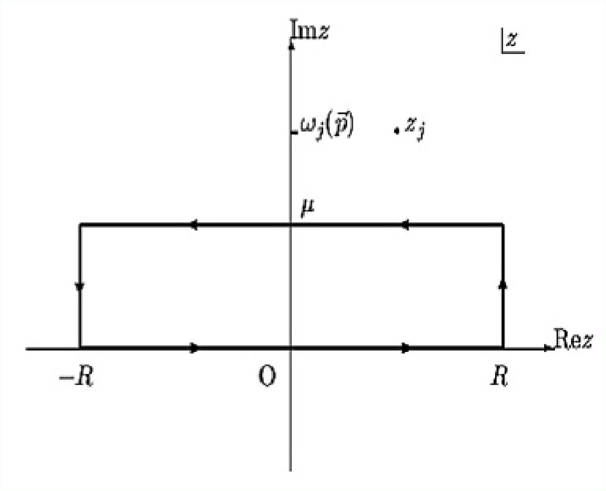
(13) can be calculated by method of contour integral

choose the following rectangular contour

the pole of the function $f(z)=\frac{z}{z^2+\vec{p}^2+m_j^2}$ in the upper complex z plane:

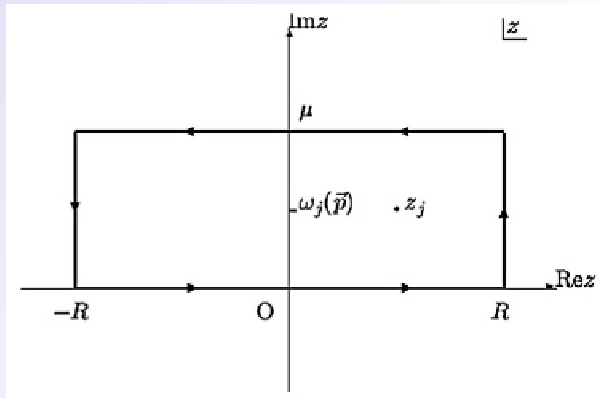
$$
\begin{aligned}
z_j &= \mathrm{sgn}(-\beta_j)\sqrt{\frac{\sqrt{(\vec{p}^2+\alpha_j)^2+\beta_j^2}-\vec{p}^2-\alpha_j}{2}}+i\sqrt{\frac{\sqrt{(\vec{p}^2+\alpha_j)^2+\beta_j^2}+\vec{p}^2+\alpha_j}{2}} \\
&= \chi_j(\vec{p})+i\omega_j(\vec{p})
\end{aligned}
$$

where $m_j^2\equiv\alpha_j+\beta_j i$



（i） $\mu<\omega_j(\vec{p})$ The contour contains no poles, the integral along the contour vanishes. $f(z)$ is an odd function，the integral along $[-R,R]$ vanishes. So one has

$$\int_{-\infty+i\mu}^{+\infty+i\mu} dz f(z)=0,\quad \mu<\omega_j(\vec{p})$$



(ii) $\mu > \omega_j(\vec{p})$ The contour contains one pole. According to residue theorem, one has

$$-\int_{-\infty+i\mu}^{+\infty+i\mu} dz f(z) = 2\pi i \,\mathrm{Res}(f(z), z = z_j) = \pi i, \quad \mu > \omega_j(\vec{p})$$

（i），（ii）can be combined to be written as

$$\int_{-\infty}^{+\infty} dp_4 \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^2} = \int_{-\infty+i\mu}^{+\infty+i\mu} dz f(z) = -\pi i \theta(\mu - \omega_j(\vec{p})) \tag{14}$$

when $\mu < \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}}$, one has $\omega_j(\vec{p}) > \mu$ irrespective the value of $\vec{p}$;
when $\mu \geq \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}}$, $\omega_j(\vec{p}) < \mu$ if and only if $|\vec{p}| < (\mu^2 - \frac{\beta_j^2}{4\mu^2} - \alpha_j)^{1/2}$. So

$$
\begin{aligned}
\int d^4p \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^2} &= -\pi i \int d\vec{p}\, \theta(\mu - \omega_j(\vec{p})) \\
&= \begin{cases} 0, & \text{when } \mu < \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}} \\ -\frac{4\pi^2 i}{3}(\mu^2 - \frac{\beta_j^2}{4\mu^2} - \alpha_j)^{3/2}, & \text{when } \mu \geq \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}} \end{cases} \\
&= -\frac{4\pi^2 i}{3}\theta\left(\mu - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}}\right)\left(\mu^2 - \frac{\beta_j^2}{4\mu^2} - \alpha_j\right)^{3/2} \qquad (15)
\end{aligned}
$$

Make the replacement $\beta_j \to -\beta_j$ in (15), one obtains

$$
\int d^4p \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m_j^{*2}} = -\frac{4\pi^2 i}{3}\theta\left(\mu - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}}\right)\left(\mu^2 - \frac{\beta_j^2}{4\mu^2} - \alpha_j\right)^{3/2} \qquad (16)
$$

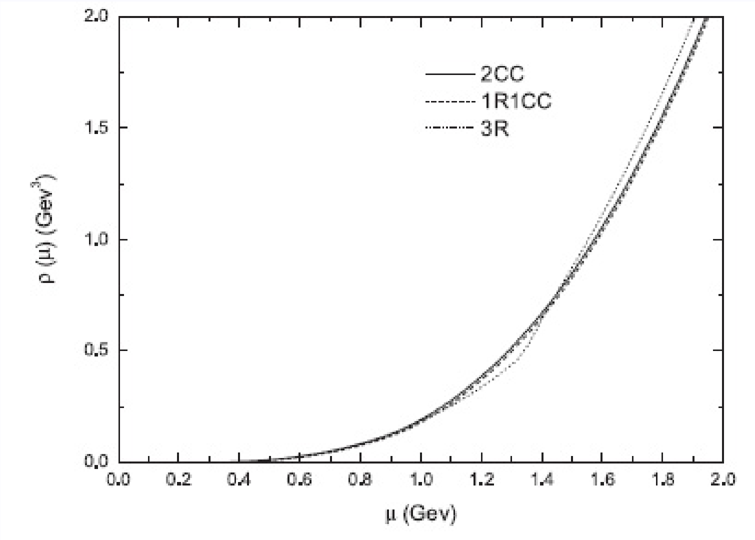
Substituting (15),(16) into (12) gives

$$
\rho(\mu) = \frac{2N_cN_f}{3\pi^2}\sum_{j=1}^{n_P} r_j \theta\left(\mu - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2 + \beta_j^2}}{2}}\right)\left(\mu^2 - \frac{\beta_j^2}{4\mu^2} - \alpha_j\right)^{3/2} \qquad (17)
$$

In numerical calculations, one employes three sets of parameters (taken from R. Alkofer et.al. Phys. Rev. D **70**, 014014 (2004)), representing three forms of the quark propagator: three real poles(3R), two pairs of complex conjugate poles(2CC), one real pole and a pair of complex conjugate poles(1R1CC)

Parametrization	r_1	a_1 (GeV)	b_1 (GeV)	r_2	a_2 (GeV)	b_2 (GeV)	r_3	a_3 (GeV)
2CC	0.360	0.351	0.08	0.140	−0.899	0.463	···	···
1R1CC	0.354	0.377	···	0.146	−0.91	0.45	···	···
3R	0.365	0.341	···	1.2	−1.31	···	−1.06	−1.40

the dependence of the quark number density on μ



when μ is less than a critical value μ_0 (for 2CC, 1R1CC and 3R parametrizations, μ_0 equals 351 MeV, 377 MeV and 341 MeV, repectively), the quark number density vanishes identically. $\mu = \mu_0$ is a singularity.

It is a model-independent result that some singularity exists at $\mu = \mu_0$ and $T = 0$

(see M. A. Halasz et. al. Phys. Rev. **D 58**, 096007 (1998))

Argument：The QCD partition function is

$$Z = \sum_\alpha \exp\{-\frac{E_\alpha - \mu N_\alpha}{T}\}$$

In the limit $T \to 0$, the state with lowest $E_\alpha - \mu N_\alpha$ gives an exponentially dominant contribution to the partition function.

When $\mu = 0$, this is the state with $N = 0$ and $E = 0$, i.e., the vacuum.

Introducing

$$\mu_0 \equiv \min_\alpha \frac{E_\alpha}{|N_\alpha|}$$

When $\mu < \mu_0$, the state with the lowest $E_\alpha - \mu N_\alpha$ is still the vacuum. Therefore, at $T = 0$

$$\rho(\mu) = 0, \quad \mu < \mu_0$$

For pure QCD (electromagnetic interaction being switched off)，μ_0 is estimated to be $\frac{m_N - 16\ MeV}{N_c} = 307$ MeV (for details, please see the above reference).

From (17) one can obtain

$$
\begin{aligned}
\int_0^{\mu} d\mu' \rho(\mu') &= \frac{2N_cN_f}{3\pi^2}\sum_{j=1}^{n_P} r_j \int\limits_0^{\mu} d\mu' \theta\left(\mu' - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2+\beta_j^2}}{2}}\right)(\mu'^2 - \frac{\beta_j^2}{4\mu'^2} - \alpha_j)^{3/2} \\
&= \frac{2N_cN_f}{3\pi^2}\sum_{j=1}^{n_P} r_j\, \theta\left(\mu - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2+\beta_j^2}}{2}}\right) \int\limits_{\sqrt{\frac{\alpha_j+\sqrt{\alpha_j^2+\beta_j^2}}{2}}}^{\mu} d\mu'(\mu'^2 - \frac{\beta_j^2}{4\mu'^2} - \alpha_j)^{3/2} \\
&= \frac{2N_cN_f}{3\pi^2}\sum_{j=1}^{n_P} r_j\, \theta\left(\mu - \sqrt{\frac{\alpha_j + \sqrt{\alpha_j^2+\beta_j^2}}{2}}\right) I(\mu;\alpha_j,\beta_j).
\end{aligned}
\tag{18}
$$

where $I(\mu;\alpha_j,\beta_j)$ is

$$
\begin{aligned}
I(\mu;\alpha_j,\beta_j) &\equiv \int\limits_{\sqrt{\frac{\alpha_j+\sqrt{\alpha_j^2+\beta_j^2}}{2}}}^{\mu} d\mu' \left(\mu'^2 - \frac{\beta_j^2}{4\mu'^2} - \alpha_j\right)^{3/2} \\
&= \frac{3(\alpha_j^2-\beta_j^2)}{16} \ln \frac{\sqrt{\mu^2 - \alpha_j/2 + \sqrt{\alpha_j^2+\beta_j^2}/2} + \sqrt{\mu^2 - \alpha_j/2 - \sqrt{\alpha_j^2+\beta_j^2}/2}}{\sqrt{\mu^2 - \alpha_j/2 + \sqrt{\alpha_j^2+\beta_j^2}/2} - \sqrt{\mu^2 - \alpha_j/2 - \sqrt{\alpha_j^2+\beta_j^2}/2}} \\
&\quad + \frac{3\alpha_j|\beta_j|}{4} \arctan \sqrt{\frac{(\sqrt{\alpha_j^2+\beta_j^2} - \alpha_j)(\mu^2 - \sqrt{\alpha_j^2+\beta_j^2}/2 - \alpha_j/2)}{(\sqrt{\alpha_j^2+\beta_j^2} + \alpha_j)(\mu^2 + \sqrt{\alpha_j^2+\beta_j^2}/2 - \alpha_j/2)}} \\
&\quad + \frac{\mu^2}{4}\sqrt{\mu^4 - \alpha_j\mu^2 - \beta_j^2/4} - \frac{5\alpha_j}{8}\sqrt{\mu^4 - \alpha_j\mu^2 - \beta_j^2/4} + \frac{\beta_j^2}{8}\frac{\sqrt{\mu^4 - \alpha_j\mu^2 - \beta_j^2/4}}{\mu^2}.
\end{aligned}
\tag{19}
$$

$\mathcal{P}(\mu)|_{\mu=0}$ can be calculated from thw CJT effective action.

$$
\begin{aligned}
\mathcal{P}(\mu)|_{\mu=0} &= 2N_cN_f\int\frac{d^4p}{(2\pi)^4}\left\{\ln\left[\frac{A^2(p^2)p^2+B^2(p^2)}{p^2}\right]-\frac{p^2A(p^2)[A(p^2)-1]+B^2(p^2)}{p^2A^2(p^2)+B^2(p^2)}\right\} \\
&= -2N_cN_f\int\frac{d^4p}{(2\pi)^4}\left\{\ln\left[p^2\left(p^2\sigma_v^2(p^2)+\sigma_s^2(p^2)\right)\right]+1+p^2\sigma_v(p^2)\right\} \qquad (20)
\end{aligned}
$$

其中 $G(p)=\frac{1}{i\gamma\cdot pA(p^2)+B(p^2)}\equiv i\gamma\cdot p\sigma_v(p^2)+\sigma_s(p^2)$ is the unrenormalized dressed quark propagator at $\mu=0$.

For the model quark propagator we use

$$
\sigma_v(p^2)=-\sum_{j=1}^{n_P}\left(\frac{r_j}{p^2+m_j^2}+\frac{r_j}{p^2+m_j^{*2}}\right),\qquad \sigma_s(p^2)=\sum_{j=1}^{n_P}\left(\frac{r_jm_j}{p^2+m_j^2}+\frac{r_jm_j^*}{p^2+m_j^{*2}}\right) \qquad (21)
$$

From (20), (21) and the model parameters given before, one can calculate $\mathcal{P}(\mu)|_{\mu=0}$.

Comparison of our EOS with the cold, perturbative EOS of QCD

In E. S. Fraga, R. D. Pisarski, J. Schaffner-Bielich, Phys. Rev. **D 63**, 121702(R) (2001), the following EOS was obtained:

$$\mathcal{P}_{FPS}(\mu) = \frac{N_f \mu^4}{4\pi^2}\left\{1 - 2\left(\frac{\alpha_s}{\pi}\right) - \left[G + N_f \ln\frac{\alpha_s}{\pi} + \left(11 - \frac{2}{3}N_f\right)\ln\frac{\bar{\Lambda}}{\mu}\right]\left(\frac{\alpha_s}{\pi}\right)^2\right\} \tag{22}$$

Here we adopt $\overline{MS}$ scheme， $G = G_0 - 0.536N_f + N_f \ln N_f$， $G_0 = 10.374 \pm 0.13$， $\bar{\Lambda}$ is the renormalization subtraction point.

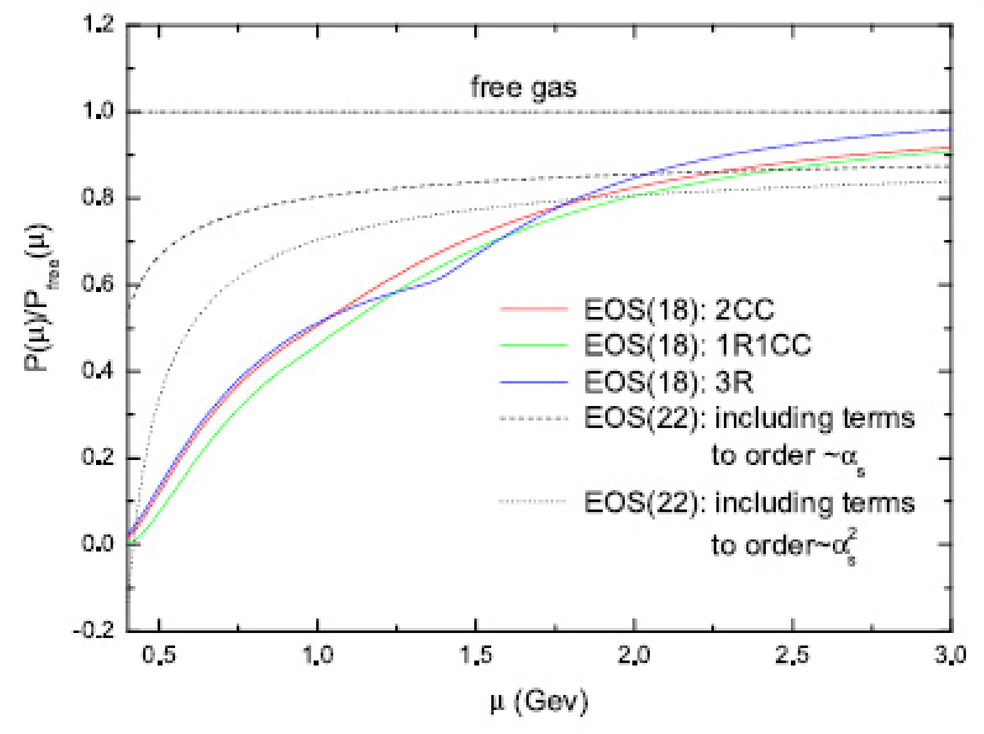
$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 u}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln(u)}{u} + \frac{4\beta_1^2}{\beta_0^4 u^2}\left(\left(\ln(u) - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]$$

where $u = \ln(\bar{\Lambda}^2/\Lambda_{\overline{MS}}^2)$, $\beta_0 = 11 - 2N_f/3$, $\beta_1 = 51 - 19N_f/3$, $\beta_2 = 2857 - 5033N_f/9 + 325N_f^2/27$.

For $N_f = 3$, $\Lambda_{\overline{MS}} = 365$ MeV. The ratio $\bar{\Lambda}/\mu$ is taken to be 2.

EOS (22) is only valid in the chirally symmetric phase ($\mu > \mu_\chi$)

Comparison of our EOS with the perturbative EOS (when making the comparison we have neglected the constant term $\mathcal{P}(\mu)|_{\mu=0}$)



$\star$ Calculation of EOS with HDL Approximation

At sufficiently high chemical potential, the HDL approximation is a good approximation to QCD.

quark propagator in HDL approximation

$$G_R(p) = -Z_2^{-1}\frac{1}{D_+(p)}\frac{\gamma_4 + i\hat{\mathbf{p}}\cdot\vec{\gamma}}{2} - Z_2^{-1}\frac{1}{D_-(p)}\frac{\gamma_4 - i\hat{\mathbf{p}}\cdot\vec{\gamma}}{2}, \tag{23}$$

where $\hat{\mathbf{p}} = \vec{p}/|\vec{p}|$, $p_4 = (2n+1)\pi T$ $(n \in \mathbf{Z})$ are the fermion Matsubara freqencies

$$D_\pm(p) = -ip_4 \pm |\vec{p}| + \frac{m_q^2}{|\vec{p}|}\left[Q_0\left(\frac{ip_4}{|\vec{p}|}\right) \mp Q_1\left(\frac{ip_4}{|\vec{p}|}\right)\right] \tag{24}$$

$m_q \equiv g\sqrt{(T^2+\mu^2/\pi^2)/6}$ is the quark thermal mass (g is the strong coupling constant)

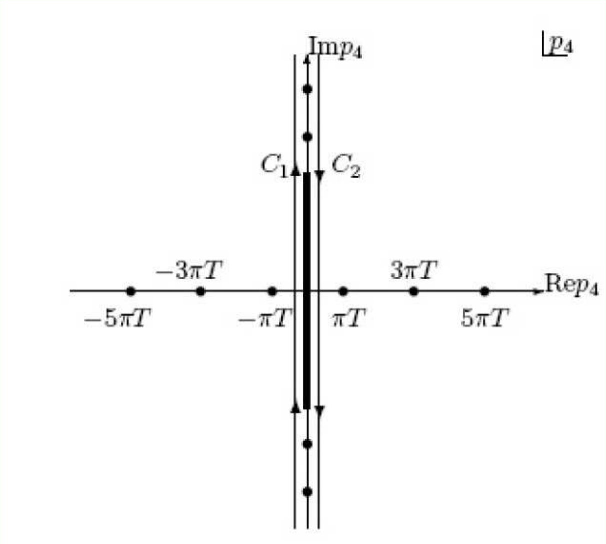
From this one obtains the baryon number density under HDL approximation

$$\rho(\mu,T) \;=\; 2N_cN_f\int\frac{d^3\vec{p}}{(2\pi)^3}T\sum_n\left[\frac{1}{D_+}+\frac{1}{D_-}\right] \tag{25}$$

The summation over Matsubara frequencies can be calculated by contour integral method

$$T\sum_n\left[\frac{1}{D_+}+\frac{1}{D_-}\right]=\int_{C_1\cup C_2}\frac{dp_4}{2\pi}\left[\frac{1}{D_+}+\frac{1}{D_-}\right]\frac{1}{2}\tanh\frac{ip_4}{2T} \tag{26}$$

the integral contour



The calculation of the integral:

Define

$$f(p_4) \;:=\; \left[\frac{1}{D_+}+\frac{1}{D_-}\right]\frac{1}{2}\tanh\frac{ip_4}{2T}$$

Closing the contour C_1, C_2 at upper and lower infinity of the imaginary axis, one obtains

$$\begin{aligned}
&\int_{C_1\cup C_2}\frac{dp_4}{2\pi}\left[\frac{1}{D_+}+\frac{1}{D_-}\right]\frac{1}{2}\tanh\frac{ip_4}{2T}\\
=\;&\frac{1}{2\pi}(-2\pi i)\sum_j \mathrm{Res}\{f(\chi_j)\}+\int_{-|\vec{p}|}^{|\vec{p}|}\frac{d\omega}{2\pi}i\frac{1}{2}\tanh\frac{-\omega}{2T}\left[\frac{1}{D_+(i\omega-0^+)}-\frac{1}{D_+(i\omega+0^+)}\right]\\
&+\int_{-|\vec{p}|}^{|\vec{p}|}\frac{d\omega}{2\pi}i\frac{1}{2}\tanh\frac{-\omega}{2T}\left[\frac{1}{D_-(i\omega-0^+)}-\frac{1}{D_-(i\omega+0^+)}\right] \qquad (27)
\end{aligned}$$

where χ_j are poles of $f(p_4)$ on the imaginary axis poles of $1/D_+$:

$$\chi_1 = -i\omega_+\,, \quad \mathrm{Res}\left\{\frac{1}{D_+(\chi_1)}\right\} = iZ_+ \tag{28}$$

$$\chi_2 = i\omega_-\,, \quad \mathrm{Res}\left\{\frac{1}{D_+(\chi_2)}\right\} = iZ_- \tag{29}$$

poles of $1/D_-$:

$$\chi_3 = i\omega_+\,, \quad \mathrm{Res}\left\{\frac{1}{D_-(\chi_3)}\right\} = iZ_+ \tag{30}$$

$$\chi_4 = -i\omega_-\,, \quad \mathrm{Res}\left\{\frac{1}{D_-(\chi_4)}\right\} = iZ_- \tag{31}$$

where $\omega_\pm(|\vec{p}|)(> |\vec{p}|)$ are solutions of the following equations

$$\frac{|\vec{p}|(\omega_+ - |\vec{p}|)}{m_q^2} - 1 = \frac{1}{2}\left(1 - \frac{\omega_+}{|\vec{p}|}\right)\ln\frac{\omega_+ + |\vec{p}|}{\omega_+ - |\vec{p}|} \tag{32}$$

$$\frac{|\vec{p}|(\omega_- + |\vec{p}|)}{m_q^2} + 1 = \frac{1}{2}\left(1 + \frac{\omega_-}{|\vec{p}|}\right)\ln\frac{\omega_- + |\vec{p}|}{\omega_- - |\vec{p}|} \tag{33}$$

where $Z_\pm(|\vec{p}|)$ are

$$Z_\pm(|\vec{p}|) = \frac{\omega_\pm - |\vec{p}|^2}{2m_q^2} \tag{34}$$

From the above results the first term on the right of (27) is calculated to be

$$
\begin{aligned}
& \frac{1}{2\pi}(-2\pi i)\sum_j \mathrm{Res}\{f(\chi_j)\} \\
= & -\frac{i}{2}\left[iZ_+\tanh\frac{\omega_+}{2T} + iZ_-\tanh\frac{-\omega_-}{2T} + iZ_+\tanh\frac{-\omega_+}{2T} + iZ_-\tanh\frac{\omega_-}{2T}\right] \\
= & \ 0 \qquad (35)
\end{aligned}
$$

The contribution of the branch cut:

First we have

$$
\begin{aligned}
& \frac{1}{D_\pm(i\omega-0^+)} - \frac{1}{D_\pm(i\omega+0^+)} \\
= & \frac{-\pi i(m_q^2/|\vec{p}|)(1\pm\omega/|\vec{p}|)}{\left[\omega \pm |\vec{p}| \pm (m_q^2/|\vec{p}|) + (m_q^2/|\vec{p}|)(1/2)(1\pm\omega/|\vec{p}|)\ln\frac{|\vec{p}|-\omega}{|\vec{p}|+\omega}\right]^2 + \left[m_q^2(1\pm\omega/|\vec{p}|)\pi/(2|\vec{p}|)\right]^2} \\
= & -i\rho_\pm(-\omega), \qquad (36)
\end{aligned}
$$

where $\rho_\pm$ are the spectral density of $1/D_\pm$. From this one obtains

$$
\begin{aligned}
& T\sum_n\left[\frac{1}{D_+}+\frac{1}{D_-}\right] \\
= & \frac{1}{2}\int_{-|\vec{p}|}^{|\vec{p}|}\frac{d\omega}{2\pi}\tanh\frac{\omega}{2T}[\rho_+(\omega)+\rho_-(\omega)] = \frac{1}{2}\int_{-|\vec{p}|}^{|\vec{p}|}\frac{d\omega}{2\pi}[1-2n_F(\omega)][\rho_+(\omega)+\rho_-(\omega)] \qquad (37)
\end{aligned}
$$

where $n_F(\omega)$ is the fermion distribution function: $n_F(\omega) = \frac{1}{\exp(\omega/T)+1}$

Taking the limit $T \to 0$, then $n_F(\omega) \to 0$, so one has

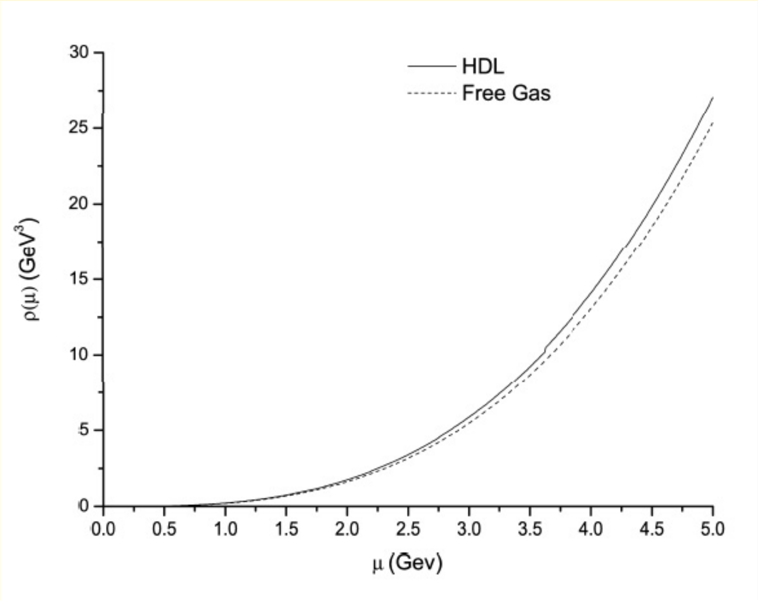
$$T\sum_n \left[\frac{1}{D_+} + \frac{1}{D_-}\right] \stackrel{T\to 0}{=} \frac{1}{2}\int_{-|\vec{p}|}^{|\vec{p}|} \frac{d\omega}{2\pi}[\rho_+(\omega) + \rho_-(\omega)] \tag{38}$$

baryon number density at zero T and finite μ:

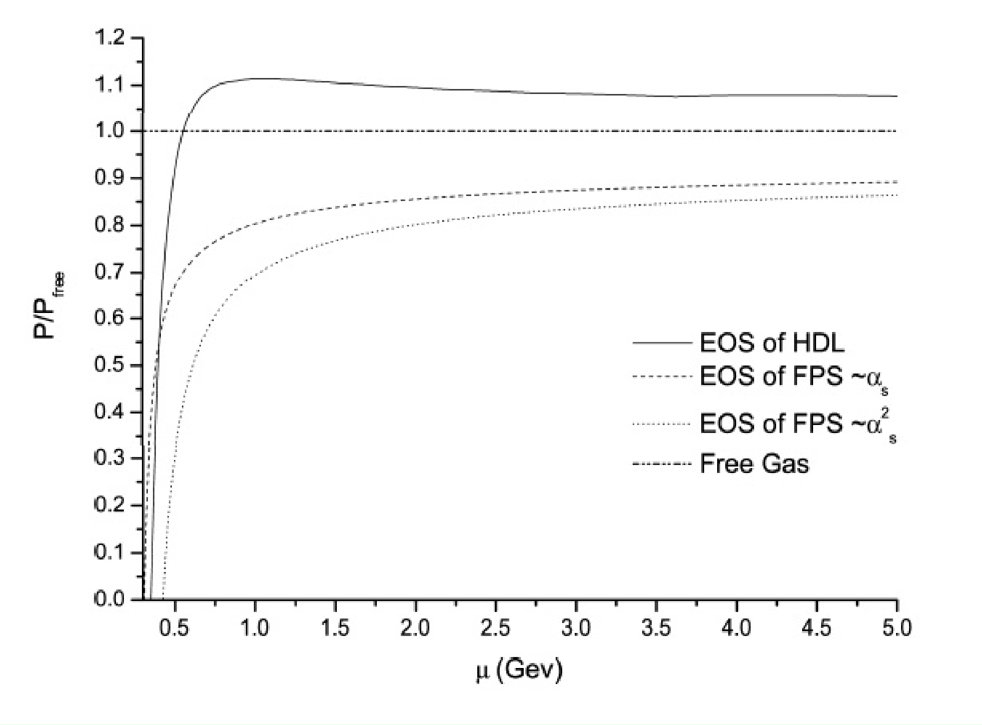
$$\rho(\mu) = N_c N_f \int \frac{d^3\vec{p}}{(2\pi)^3} \int_{-|\vec{p}|}^{|\vec{p}|} \frac{d\omega}{2\pi}[\rho_+(\omega) + \rho_-(\omega)] \tag{39}$$

EOS under HDL approximation (here the constant term $\mathcal{P}(\mu)|_{\mu=0}$ is neglected)

$$\mathcal{P}(\mu) = N_c N_f \int_0^\mu d\mu' \int \frac{d^3\vec{p}}{(2\pi)^3} \int_{-|\vec{p}|}^{|\vec{p}|} \frac{d\omega}{2\pi}[\rho_+(\omega) + \rho_-(\omega)]. \tag{40}$$



comparison of HDL EOS with the perturbative EOS



⋆ Calculation of EOS with the Quasi-particle Model

• Introduction to quasi-particle model

In quasi-particle model, the interacting particle system is effectively regarded as an ideal gas of non-interacting quasi-particles with temperature and density dependent effective mass.

In quasi-particle model, the quark propagator has the form of a free fermion propagator with the effective mass

$$G^{-1}[\mu](\vec{p},\omega_n) = i\vec{\gamma}\cdot\vec{p} + i\gamma_4(\omega_n + i\mu) + m(\mu,T)$$

$\omega_n = (2n+1)\pi T, n \in \mathbf{Z}$ are the fermion Matsubara frequencies.

The effective mass is taken to be (X.P. Zhang, M. Kang, X.W. Liu, and S.H. Yang, Phys. Rev. **C** **72**, 025809 (2005)):

$$m^2(T,\mu) = aT^2 + \frac{N_c^2-1}{8N_c}(T^2 + \frac{\mu^2}{\pi^2})g^2(T,\mu) \tag{41}$$

$g^2(T,\mu)$ is the effective strong coupling constant.

At $T=0$, the effective mass and effective strong coupling constant are

$$m^2(\mu) = \frac{\mu^2}{3\pi^2}g^2(0,\mu), \quad g^2(0,\mu) = \frac{16\pi^2}{9\ln(\frac{\mu+T_s}{T_c\pi\backslash\lambda})^2} \tag{42}$$

where the parameters are $\lambda = 6.6$， $T_c = 170$ MeV， $T_s = -0.78T_c$

In the limit $T \to 0$, discrete Matsubara frequency $\omega_n \Rightarrow$ continuous variable p_4

baryon number densdity at zero T and finite μ

$$\rho(\mu) = 4iN_cN_f \int \frac{d^4p}{(2\pi)^4} \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m^2(\mu)} \tag{43}$$

the calculation of the integral

$$\int d^4p \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m(\mu)^2} = \int d\vec{p} \int_{-\infty}^{+\infty} dp_4 \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m(\mu)^2}$$

The integral of p_4 can be calculated by the contour integral method. The result is

$$\int_{-\infty}^{+\infty} dp_4 \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m(\mu)^2} = -\pi i \theta(\mu - \omega_j(\vec{p}, \mu)), \quad \omega_j(\vec{p}, \mu) = \sqrt{\vec{p}^2 + m(\mu)^2}$$

Using

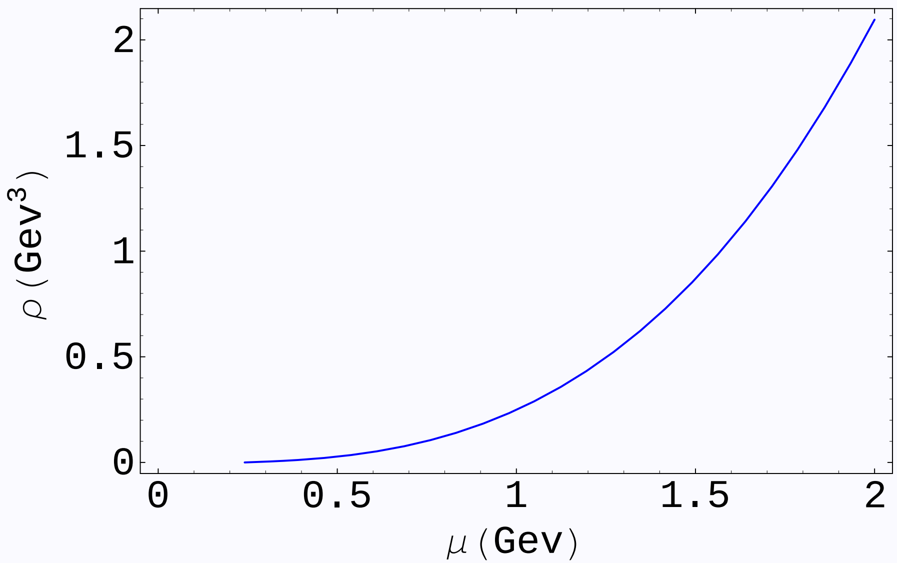
$$\theta(\mu - \omega_j(\vec{p}, \mu)) = \begin{cases} 1, & \text{when } |\vec{p}| < (\mu^2 - m(\mu)^2)^{1/2} \\ 0, & \text{when } |\vec{p}| > (\mu^2 - m(\mu)^2)^{1/2}. \end{cases}$$

one obtains

$$\begin{aligned} \int d^4p \frac{p_4 + i\mu}{\vec{p}^2 + (p_4 + i\mu)^2 + m(\mu)^2} &= -\pi i \int d\vec{p}\, \theta(\mu - \omega_j(\vec{p}, \mu)) \\ &= \begin{cases} 0, & \text{when } \mu < m(\mu) \\ -\frac{4\pi^2 i}{3}(\mu^2 - m(\mu)^2)^{3/2}, & \text{when } \mu \geq m(\mu) \end{cases} \\ &= -\frac{4\pi^2 i}{3}\theta(\mu - m(\mu))(\mu^2 - m(\mu)^2)^{3/2} \end{aligned}$$

Thus one has

$$\rho(\mu) = \frac{3}{\pi^2}\theta(\mu - m(\mu))(\mu^2 - m(\mu)^2)^{3/2} \tag{44}$$



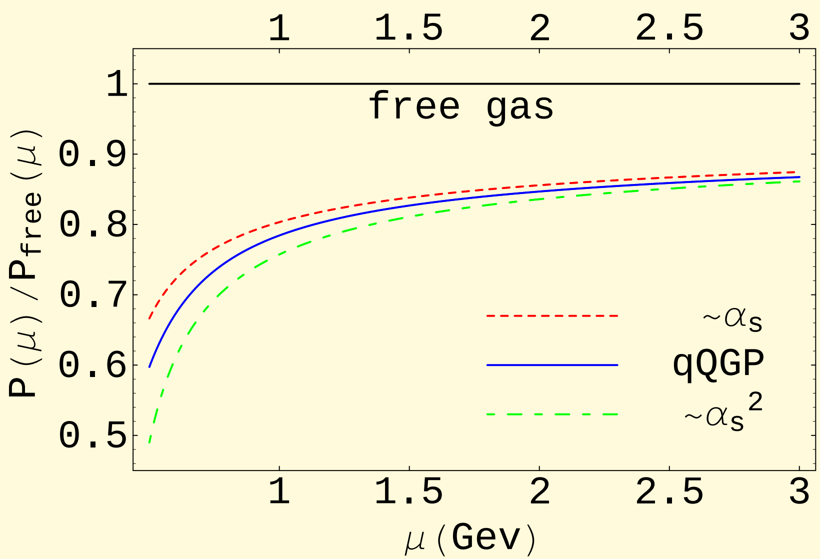
The obtained baryon number density differs significantly with the free fermion gas result. This difference comes from the chemical potential dependence of the effective mass.

critical chemical potential: $\mu_0 = 241$ MeV， This is comparable to the result $\mu_0 = 307$ MeV in M. A. Halasz et. al. Phys. Rev. **D 58**, 096007 (1998).

EOS:

$$\mathcal{P}(\mu) = \mathcal{P}(\mu)|_{\mu=0} + \frac{3}{\pi^2}\int_0^{\mu} d\mu' \theta(\mu' - m(\mu'))(\mu'^2 - m(\mu')^2)^{3/2} \tag{45}$$

Comparison of the EOS in the quasi-particle model with the perturbative EOS of QCD (the constant term $\mathcal{P}(\mu)|_{\mu=0}$ is neglected)



The form of the EOS in the quasi-particle model depends on the form of the effective mass in the quasi-particle model and the parameters therein. The parameters in the effective mass employed in our calculation has some arbitrariness. It is expected that the result of EOS can give some constraints on the choice of these parameters.

⋆ Calculation of EOS with a Nonperturbative Approach inspired by χPT

χPT is an efficient method of studying the evolution properties of cold, dense strongly interacting matter with the chemical potential in low energy regime (where chiral symmetry is spontaneously broken)

Limitation of χPT： it is only valid in the low energy regime. J.A.Oller and E.Oset developed a nonperturbative method, extending the idea of χPT beyond its range of validity. (J.A.Oller and E.Oset, Nucl. Phys. A 620, 438 (1997))

- Effective chiral Lagrangian at zero T and finite baryon chemical potential

baryon number density operator： $\frac{1}{3}q^\dagger q$

baryon number density： $\langle\frac{1}{3}q^\dagger q\rangle$

The hadron system can be described by an effective Lagrangian containing all possible terms consistent with the symmetry principles. One can calculate the vacuum expectation value of the quark operators from χPT.

The calculation of $\langle\bar{q}\gamma_\mu q\rangle$, $\langle\bar{q}\gamma_5\gamma_\mu q\rangle$, $\langle\bar{q}q\rangle$,$\langle\bar{q}\gamma_5 q\rangle$ in χPT：

Introducing external fields： a_μ, v_μ, s, p

$$\mathcal{L}_{ext}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q - \bar{q}\gamma^\mu(s - i\gamma_5 p)q$$

In low energy regime the effective Lagrangian contains all possible terms consistent with symmetry principles. The generating functional derived from this Lagrangian is equivalent to the generating functional derived from QCD. The vacuum expectation value of the quark current can be obtained by differentiation of the generating functional with respect to the external fields.

The calculation of baryon number density $\langle\frac{1}{3}q^\dagger q\rangle$

Introduction of baryon chemical potential：

Introduce the external field b_μ and do the replacement $\partial_\mu q \to \partial_\mu q - B^q b_\mu q = \partial_\mu q - \frac{i}{3}b_\mu q$

In the last step of calculation we set $b_\mu = \delta_{\mu 0}\mu_b$, where μ_b is the baryon chemical potential.

Because of $SU(2)_f$ symmetry, u, d quark have equal mass and chemical potential.

baryon number density can be obtained by differentiation of the generating functional:

$\langle \frac{1}{3}q^+q\rangle = \frac{\partial Z[b_\mu]}{\partial b_0}|_{b_0=\mu_b, b_i=0} = \frac{\partial Z[\mu_b]}{\partial \mu_b}$

introduction of chemical potential at the nucleon level： in $\mathcal{L}_{\pi N}$ replace $\partial_\mu \psi$ by

$$\partial_\mu \psi \to \partial_\mu \psi - iB^N b_\mu \psi = \partial_\mu \psi - i\delta_{0\mu}\mu_b \psi$$

Effective Lagrangian： for pionic part one only needs to consider the leading order term:

$$\mathcal{L}_\pi^{(2)} = \frac{F_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^+) + \frac{F_\pi^2}{4} Tr(\chi U^+ + U\chi^+)$$

$$U = exp(\frac{i\vec{\pi}\cdot\vec{\tau}}{F_\pi}), \quad \chi = 2B_0(s+ip)$$

in our calculation $p=0$, $s=$ mass matrix of u, d quark

under $SU(2)_f$, $\chi = 2B_0 M_q = 2B_0 \hat{m} = M_\pi^2$

for πN part, to $O(p^4)$ order

$$\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= \bar{\psi}(i \not{D} - m)\psi + \tfrac{1}{2} g_A \bar{\psi} \not{u}\gamma_5 \psi, \\
\mathcal{L}_{\pi N}^{(2)} &= c_1 \langle \chi_+ \rangle \bar{\psi}\psi - \tfrac{c_2}{4m^2}\langle u_\mu u_\nu \rangle (\bar{\psi} D^\mu D^\nu \psi + h.c.) + \tfrac{c_3}{2} u_\mu u^\mu \bar{\psi}\psi + \cdots, \\
\mathcal{L}_{\pi N}^{(4)} &= -\tfrac{e_1}{16}\langle \chi_+ \rangle^2 \bar{\psi}\psi + \cdots
\end{aligned}$$

where

$$\begin{aligned}
D_\mu \psi &= \partial_\mu \psi - i\delta_{0\mu}\mu_b \psi + \Gamma_\mu \psi, \\
\Gamma_\mu &= \tfrac{1}{2}[u^+, \partial_\mu u], \\
u^2 &= U, u_\mu = iu^+ \partial_\mu U u^+
\end{aligned}$$

The numerical values of m, M_π, F_π, g_A, c_1, c_2, c_3,e_1 used in our calculation. The values of c_1, c_2,c_3 are the same as those in V. Bernard, Prog. Part. Nucl. Phys. **60**, 82, 2007. The values of m and e_1 are obtained through fitting the nucleon mass.

$M_\pi[MeV]$	$m[MeV]$	$F_\pi[MeV]$	g_A	$c_1[GeV^{-3}]$	$c_2[GeV^{-3}]$	$c_3[GeV^{-3}]$	$e_1[GeV^{-2}]$
137	896	92.4	1.27	-0.90×10^{-3}	3.3×10^{-3}	-4.7×10^{-3}	1×10^{-9}

- The calculation of baryon number density and EOS

$$\mathcal{L}_{\pi N} = \bar{\psi}\hat{K}\psi, \quad \hat{K} = i\gamma_\mu\partial_\mu + \gamma_0\mu_b - m + \hat{O}$$

baryon number density

$$\begin{aligned} n(\mu_b) &= \langle \tfrac{1}{3}q^+q\rangle = \tfrac{\partial}{\partial\mu_b}\int D\bar{\psi}D\psi DU e^{i\int d^4x\mathcal{L}_{eff}} \\ &= \tfrac{\partial}{\partial\mu_b}\int DU(DetK)e^{i\int d^4x\mathcal{L}_\pi} \\ &= \int DUTr(K^{-1}\tfrac{\partial}{\partial\mu_b}K)e^{i\int d^4x\mathcal{L}_\pi} \end{aligned}$$

where $\mathcal{L}_{eff} = \mathcal{L}_{\pi N} + \mathcal{L}_\pi$.

K,O are the results of the path integral after integrating out the nucleon field.

The most important contribution to baryon number density is from $Tr(K^{-1}\gamma_0)$, the contribution of $Tr(K^{-1}\frac{\partial}{\partial\mu_b}O)$ can be neglected.

$$\begin{aligned} n(\mu_b) &\doteq \int DUTr(K^{-1}\gamma_0)e^{i\int d^4x\mathcal{L}_\pi} \\ &= \frac{-i}{(2\pi^4)}\int d^4pTr(\gamma_0 S) \end{aligned}$$

S: nucleon propagator

The chemical potential is introduced in Euclidean space, p_0 is imaginary.

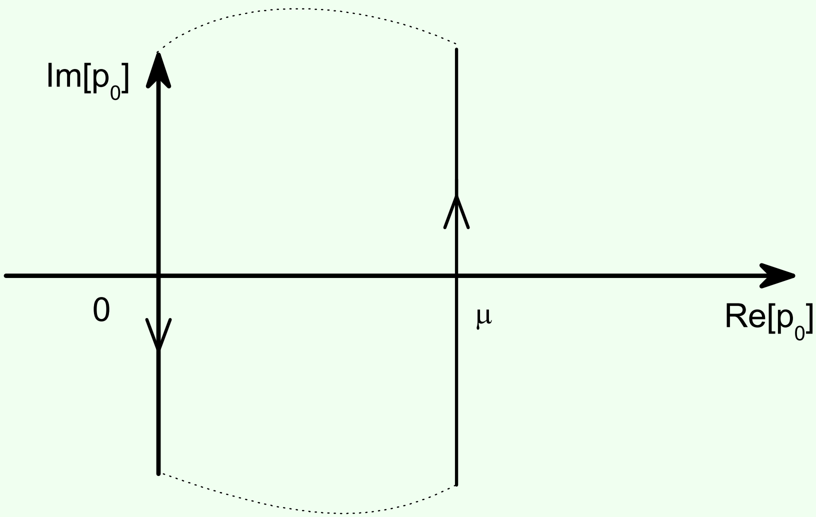
The integral has the form

$$\int d^4p = \int d^3\vec{p} \int_{-i\infty}^{i\infty} dp_0$$

When $\mu = 0$ the baryon number density vanishes, so

$$n(\mu_b) = \frac{-i}{(2\pi^4)} \int d^3\vec{p} (\int_{-i\infty+\mu_b}^{i\infty+\mu_b} dp_0' - \int_{-i\infty}^{i\infty} dp_0') Tr(\frac{\gamma_0}{\not{p}' - m - \Sigma(p')})$$

the integral path in the complex p_0 plane:



The usual χPT requires real momentum. It is unreasonable to extend the results to the complex plane when singularity apppears according to perturbation theory. So nonperturbative method is necessary. Here we employ the same method as that in

J.A. Oller and E. Oset, Nucl. Phys. **A** 620, 438 (1997)

J.A.Oller, E.Oset and J.R.Pelaez, Phys.Rev.Lett.80:3452 (1998)

to calculate the nucleon propagator.

The full necleon propagator satisfy the following DSE:

$$S = S_0 + S_0 \Sigma S$$

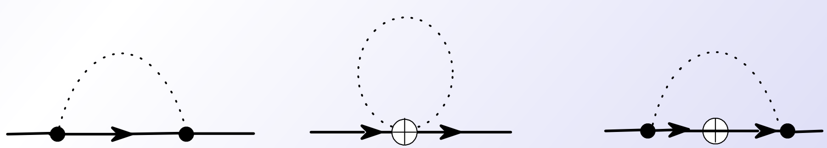
S_0: free nucleon propagator. Σ: self-energy

Replacing the full nucleon propagator in the self-energy by the free one, one obtains

$$S = (1 - S_0\Sigma)^{-1} S_0 = (S_0^{-1}(1 - S_0\Sigma))^{-1} = \frac{1}{S_0^{-1} - \Sigma}$$

Σ can be calculated perturbatively with the chiral effective Lagrangian.

the one loop diagrams contributing to Σ to $O(p^4)$ order:



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explicit expression of the self-energy:

$$\begin{aligned}
\Sigma &= -4c_1 M^2 + \Sigma_a + \Sigma_b + \Sigma_c + e_1 M^4 + O(p^5), \\
\Sigma_a &= \frac{3g_A^2}{4F_\pi^2}(m + \not{p}')\{M_\pi^2 I + (m - \not{p}')\not{p}' I^{(1)}\}, \\
\Sigma_b &= \frac{3M_\pi^2\Delta_\pi}{F_\pi^2}\{2c_1 - \frac{p'^2}{m^2 d}c_2 - c_3\}, \\
\Sigma_c &= -4c_1 M_\pi^2 \frac{\partial \Sigma_a}{\partial m}
\end{aligned}$$

where $d = 4$, $\not{p}' = \not{p} + \gamma_0 \mu_b$.

$$\begin{aligned}
I &= -\frac{1}{8\pi^2}\frac{\alpha\sqrt{1-\Omega^2}}{1+2\alpha\Omega+\alpha^2} ArcCos(-\frac{\Omega+\alpha}{\sqrt{1+2\alpha\Omega+\alpha^2}}) - \frac{1}{16\pi^2}\frac{\alpha(\Omega+\alpha)}{1+2\alpha\Omega+\alpha^2}(2ln\alpha - 1), \\
I^{(1)} &= \frac{1}{2p'^2}\{(p'^2 - m^2 + M_\pi^2)I + \Delta_\pi\}, \\
\Delta_\pi &= \frac{M_\pi^2}{8\pi^2} ln \frac{M_\pi}{m}
\end{aligned}$$

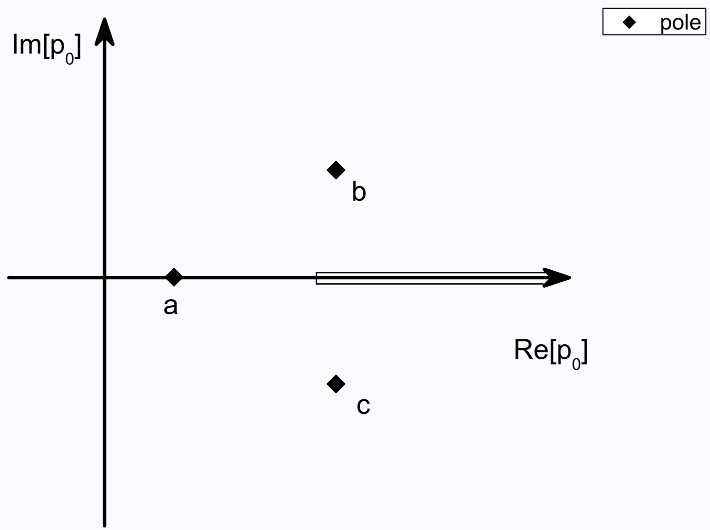
$$\begin{aligned}
\alpha &= \frac{M_\pi}{m}, \\
\Omega &= \frac{p'^2 - m^2 - M_\pi^2}{2mM_\pi}
\end{aligned}$$

The renormalization scheme we employed is the one based on ”Infrared Regularization” proposed by Leutwyler et al. (Euro. Phys. J. **C** 9, 643 (1999)).

baryon number density：

$$n(\mu_b) = \frac{-i}{(2\pi^4)} \int d^4 p Tr(\frac{\gamma_0}{\not p' - m - \Sigma(\not p')})$$

The integrand has three poles and one branch cut in the complex p_0 plane.



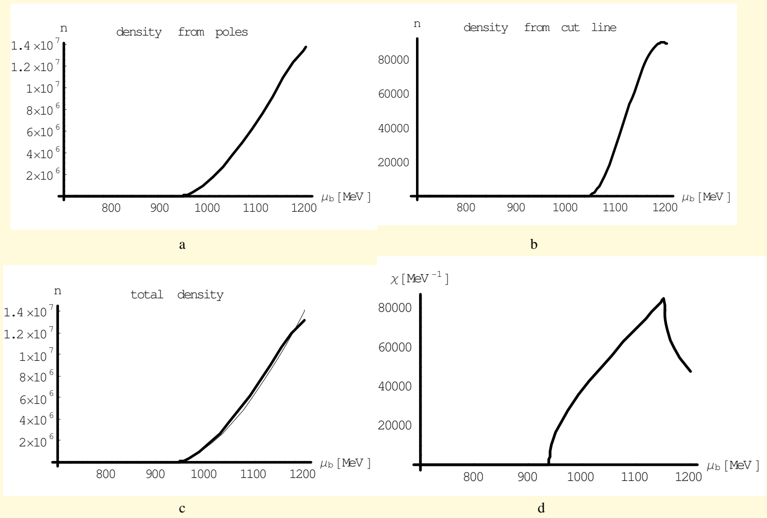
Position of poles: $p_0 = \sqrt{\vec{p}^2 + p_n^2}$， $n = a, b, c$。 $p_a = 938, p_b = 1152 + i337, p_c = 1152 - i337$ (unit: MeV)

The branch cut starts at d, where $p_0 = \sqrt{\vec{p}^2 + (m + M_\pi)^2}$.

baryon number susceptibility:

$$\chi(\mu_b) = \frac{\partial}{\partial \mu_b} n(\mu_b)$$

results for baryon number density and baryon number susceptibility:



critical chemical potential: $\mu = m_N = 938$ MeV

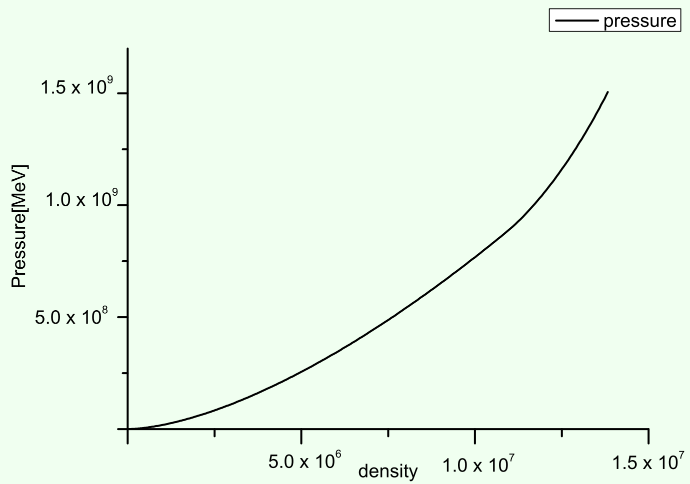
Discussion: Physically, after considering the binding energy, the value of the critical chemical potential will be less than 938 MeV. In our calculation the pion diagrams are not considered, so we have not revealed this contribution. It is expected that when calculating to higher orders, the result will be better.

When μ_b starts to be larger than 938 MeV, our result for the baryon number density is larger than the free nucleon result. When μ_b reaches 1152 MeV, the rate of the increasing of $n(\mu_b)$ begins to slow down. This value coincides with the chemical potential at which the $\chi(\mu_b) - \mu_b$ curve shows a peak.

EOS

$$n(\mu_b) = \frac{\partial P(\mu_b)}{\partial \mu_b}$$

$P - n$ curve



Our results on the baryon number density and EOS are very similar to the free nucleon approximation results. We think this explains why one can adopt the almost independent particle model in the study of nuclear matter.

⋆ Summary

- We give a direct method for calculating the EOS of QCD at zero T and finite μ
- We calculate the EOS of QCD at zero T and finite μ using four different QCD methods/models (rainbow-ladder DSE, HDL approximation, quasi-particle model, a nonperturbative approach inspired by χPT)

Thank You!