Spin Symmetry for Anti-Lambda Spectrum in atomic nucleus

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Introduction

- Symmetries in the single particle spectra of nuclei,
  - violation of spin-symmetry \textit{Mayer:1955}
  - with $\tilde{l} = l \pm 1$, the approximation of pseudo-spin symmetry \textit{Arima:1969, Hecht:1969}


- RMF been used to investigate pseudospin symmetry in nuclei spectrum.
  - The relation was first noted by Bahri et al.; found that the origin related to the strength of the scalar and vector potentials. \textit{Bahri:1992, Blokhin:1995}
  - Ginocchio show that pseudo-orbital angular momentum is nothing but the ”orbital angular momentum” of the lower component of the Dirac wave function. \textit{Ginocchio:1997 & 2005}
  - Meng reveal that the quality of pseudospin symmetry. \textit{Meng:1998 & 1999}
The spin symmetry in antinucleon spectrum been investigated with RMF; a well developed spin symmetry in the antinucleon spectrum has been found. Zhou:2003


**the present work**

- How good is the spin symmetry in the single $\bar{\Lambda}$ spectrum?
Framework I

For nuclei system contains $\bar{\Lambda}$, the Lagrangian density,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{\Lambda}},$$  \hspace{1cm} (1)

The Lagrangian density for $\bar{\Lambda}$, $\mathcal{L}_{\bar{\Lambda}}$ can be written as,

$$\mathcal{L}_{\bar{\Lambda}} = \bar{\psi}_{\bar{\Lambda}} \left( i \gamma^\mu \partial_\mu - M_{\bar{\Lambda}} - g_{\sigma\bar{\Lambda}} \sigma - g_{\omega\bar{\Lambda}} \gamma^\mu \omega_\mu \right) \psi_{\bar{\Lambda}}. \hspace{1cm} (2)$$

as $\bar{\Lambda}$ is charge neutral and isoscalar, it couples only to the $\sigma$ and $\omega$ mesons.

From the Euler-lagrange equation, the Dirac equation for $\bar{\Lambda}$ hyperon can be obtained,

$$\{ \alpha \cdot p + V_{\bar{\Lambda}}(r) + \beta [M_{\bar{\Lambda}} + S_{\bar{\Lambda}}(r)] \} \psi_{\bar{\Lambda}}(r) = \epsilon_{\bar{\Lambda}} \psi_{\bar{\Lambda}}(r), \hspace{1cm} (3)$$

where $M_{\bar{\Lambda}} = 1115.7$ MeV, $\epsilon_{\bar{\Lambda}}$ is the single-particle energy.
The scalar and vector potentials, $S_{\bar{\Lambda}}(r)$ and $V_{\bar{\Lambda}}(r)$ can be written as,

\begin{align*}
S_{\bar{\Lambda}}(r) &= g_{\sigma\bar{\Lambda}}\sigma(r), \\
V_{\bar{\Lambda}}(r) &= g_{\omega\bar{\Lambda}}\omega(r),
\end{align*}

According to the charge conjugation transformation,

\begin{align*}
g_{\sigma\bar{\Lambda}} &= g_{\sigma\Lambda}, \\
g_{\omega\bar{\Lambda}} &= -g_{\omega\Lambda}.
\end{align*}
For a spherical system, the Dirac spinor of $\bar{\Lambda}$ has the form

$$\psi_{\bar{\Lambda}}(r) = \frac{1}{r} \left( \begin{array}{c} iG_{n\kappa}(r) Y_{jm}^l(\theta, \phi) \\ -F_{\bar{n}\kappa}(r) Y_{jm}^\bar{l}(\theta, \phi) \end{array} \right), \quad j = l \pm \frac{1}{2},$$

(8)

The Schrödinger-like equations for the upper (dominant) component can be obtained from Eq.(3),

$$\left[ -\frac{1}{2M_+} \left( \frac{d^2}{dr^2} + \frac{1}{2M_+} \frac{dV_-}{dr} \frac{dr}{d} - \frac{l(l+1)}{r^2} \right) - \frac{1}{4M_+^2} \frac{\kappa}{r} \frac{dV_-}{dr} + M_{\bar{\Lambda}} - V_+ \right] G(r) = \epsilon_{\bar{\Lambda}} G(r).$$

(9)

where $M_+ = M_{\bar{\Lambda}} + \epsilon_{\bar{\Lambda}} - V_-$ and $V_\pm(r) = V_{\bar{\Lambda}}(r) \pm S_{\bar{\Lambda}}(r)$.

$$\frac{1}{4M_+^2} \frac{\kappa}{r} \frac{dV_-}{dr} \sim$$

the spin-orbit term, which determines the energy difference between the spin doublets.
Potential and spectrum

Taking $^{16}\text{O}$ as an example, with PK1 parameters for the nucleon part, and $g_{\sigma\bar{\Lambda}} = \frac{2}{3}g_{\sigma N}$; $g_{\omega\bar{\Lambda}} = -\frac{2}{3}g_{\omega N}$, the spectrum of $\bar{\Lambda}$ have been calculated.

Figure: Potential and spectrum of $\bar{\Lambda}$ in $^{16}\text{O}$. For each pair of the spin doublets, the left levels are with $\kappa < 0$ and the right one with $\kappa > 0$. The inset gives the potential and spectrum of $\Lambda$ in $^{16}\text{O}$. 
Spin-orbit splitting

The \( \tilde{\Lambda} \) spin-orbit splitting, \( \Delta E_{s.o} = \left( \epsilon_{\tilde{\Lambda}(nl_j=l-1/2)} - \epsilon_{\tilde{\Lambda}(nl_j=l+1/2)} \right)/(2l + 1) \),

is plotted as a function of the the average energy,

\[
E_{av} = \frac{1}{2}(\epsilon_{\tilde{\Lambda}(nl_j=l-1/2)} + \epsilon_{\tilde{\Lambda}(nl_j=l+1/2)})
\]

**Figure:** The spin-orbit splitting \( \Delta E_{s.o} \) for \( \tilde{\Lambda} \) and antineutron in \( ^{16}\text{O} \) as a function of the average energy \( E_{av} \).
3 Radial wave functions

The dominant component $G(r)$ of the wave functions of the spin doublets are expected to be almost identical,

**Figure:** Radial wave functions of $\bar{\Lambda}$ spin doublets with different orbit quantum numbers in $^{16}$O.
Summary and Perspective

Summary

- The spin symmetry in the single $\bar{\Lambda}$ spectrum in $^{16}\text{O}$ has been studied within the RMF theory.
- The spin-orbit splittings in the $\bar{\Lambda}$ spectrum have been found to be around 0.03-0.07 MeV, which are much smaller than those in antineutron spectrum 0.06-0.20 MeV.
- The dominant components of the Dirac spinor for the $\bar{\Lambda}$ spin doublets are found to be nearly identical.

Perspective

- What’s the polarization effects due to the $\bar{\Lambda}$?
- How deep is the antinucleon potential in the nuclei?
- ...
Thank you!