PION CHARGE FORM FACTOR

AND

CONSTRAINTS FROM SPACE-TIME TRANSLATIONS

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MOTIVATIONS

Form factors of hadronic systems:

an important source of information on the quark dynamics,

especially at high momentum transfers $\rightarrow$ short-range dynamics

High momentum transfers: supposes a relativistic treatment

In relativistic quantum mechanics (RQM) approaches, however:

$\rightarrow$ strong sensitivity to the chosen implementation of relativity,

in the assumption of a one-body current

Present work:

Look at the role of constraints related to space-time translations,

in the case of the pion charge form factor

CONSTRAINTS FROM POINCARÉ
SPACE-TIME TRANSLATION INVARIANCE

A symmetry whose role is often ignored (beyond 4-momentum conservation)

$\longrightarrow$ implies relations such as:

$$e^{iP\cdot a}\; J^\nu(x)\;(S(x))\; e^{-iP\cdot a} = J^\nu(x+a)\;(S(x+a)),$$

and in particular:

$$J^\nu(x)\;(S(x)) = e^{iP\cdot x}\; J^\nu(0)\;(S(0))\; e^{-iP\cdot x}.$$

$\longrightarrow$ matrix element between eigenstates of P^μ

$$< i\;|J^\nu(x)\;(\text{or } S(x))|\; f >= e^{i(P_i-P_f)\cdot x}\;\; < i\;|J^\nu(0)\;(\text{or } (S(0))|\; f > .$$

$\longrightarrow$ 4-momentum conservation when combined with an external field $\propto e^{iq\cdot x}$

and assuming space-time translation invariance or integrating over x

$$(P_f - P_i)^\mu = q^\mu$$

FURTHER RELATIONS

Getting previous relations supposes many-body currents in RQM approaches

$\longrightarrow$ can be checked by considering further relations (Lev):

$$[P^{\mu}\,,\;J^{\nu}(x)]=-i\partial^{\mu}\,J^{\nu}(x),\quad [P^{\mu}\,,\;S(x)]=-i\partial^{\mu}\,S(x),$$

and especially

$$[P_{\mu}\,,[P^{\mu}\,,\;J^{\nu}(x)]]=-\partial_{\mu}\,\partial^{\mu}\,J^{\nu}(x),\quad [P_{\mu}\,,[P^{\mu}\,,\;S(x)]]=-\partial_{\mu}\,\partial^{\mu}\,S(x)\,.$$

$\longrightarrow$ between eigenstates of P^{μ}

$$<\;|q^2\;J^{\nu}(0)\;(\mathrm{or}\;S(0))|\;>=<\;|(p_i-p_f)^2\,J^{\nu}(0)\;(\mathrm{or}\;S(0))|\;>\;.$$

$\longrightarrow$ quite generally, in RQM approaches with the simplest single-particle current:

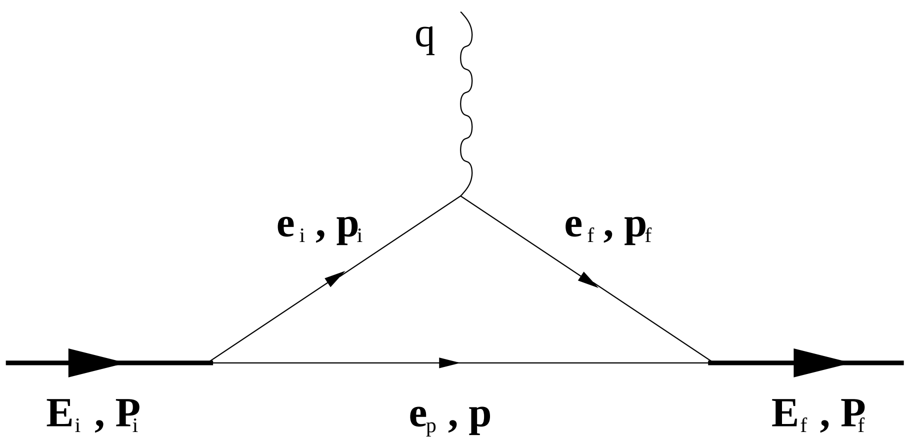
$$q^2\neq(p_i-p_f)^2\,.$$

$\longrightarrow$ the squared momentum transferred to the constituents differs from the one transferred to the whole system: close relationship to discrepancies

IMPLEMENTING CONSTRAINTS FROM POINCARÉ SPACE-TIME TRANSLATION INVARIANCE

Observation:

the coefficient of Q,
in expressions of form factors,
involves a factor $\simeq \frac{2e_k}{M}$
that deviates from 1
by interaction effects
which are here or there
depending on the approach



Proposed:

- modify the coefficient of Q by a factor α so that to fulfill: "$(p_i - p_f)^2$" $= q^2$,

- was done numerically first, can now be done analytically in most cases

$\rightarrow$ **amounts to account for many-body currents at all orders in the interaction**

FURTHER DETAILS

Typical equation to be solved:

$$\begin{aligned} q^2 &= \text{``}[(P_i - P_f)^2 + 2\,(\Delta_i - \Delta_f)\,(P_i - P_f)\cdot\xi + (\Delta_i - \Delta_f)^2\,\xi^2]\text{''} \\ &= \alpha^2 q^2 - 2\,\alpha\,\text{``}(\Delta_i - \Delta_f)\text{''}\; q\cdot\xi + \text{``}(\Delta_i - \Delta_f)^{2}\text{''}\;\xi^2\,, \end{aligned}$$

where Δ is proportional to an interaction effect.

To be noticed

$\longrightarrow$ No modification required for the standard front form ($q^+ = 0$),

where the above equality is trivially fulfilled with $\alpha^2 = 1$

(using $q\cdot\xi = q\cdot\omega = 0$, $\xi^2 = \omega^2 = 0$)

$\longrightarrow$ one tractable solution has been found but there may be less tractable other ones

RESULTS INCORPORATING EFFECTS MOTIVATED BY SPACE-TIME TRANSLATION INVARIANCE

Sample of results:

Pion charge form factor for the following approaches:

- the front form with $q^+ = 0$ ("perpendicular" momentum configuration; F.F. (perp.))
- the instant form (I.F.) in the Breit frame
- the point-form (Dirac inspired; D.P.F.)
- the instant and front forms in a "parallel" momentum configuration (I.F.+F.F.(parallel))
- the "point-form" (Bakamjian, Sokolov; "P.F.")

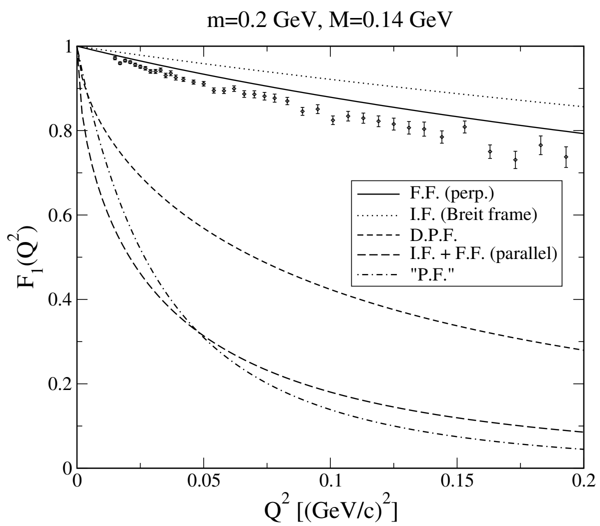
Mass operator used for RQM calculations:

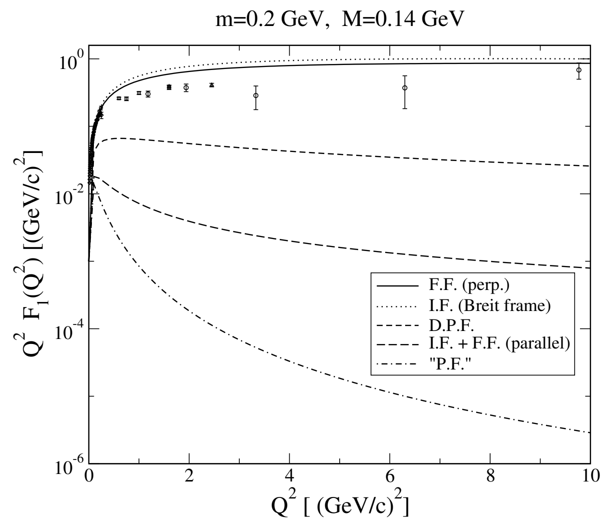
confinement and instantaneous one-gluon exchange,
with one constant parameter fitted to reproduce the pion mass

NUMERICAL RESULTS FOR THE PION CHARGE FORM FACTOR

Uncorrected results

- Standard front and instant forms do well
- Role of M
- Lorentz invariance does not ensure good results





Corrected results

- all results can be made identical to the standard front-form one
- the effects due to a small value of M have disappeared
- some discrepancy with measurements $\rightarrow$ solution of the mass operator?

A FEW OBSERVATIONS

- Similar results for the scalar pion form factor
- Disappearance of some paradoxes ($r_{ch.} \to \infty$ with $M \to 0$)
- Support for the standard front-form results (unchanged)
- Dispersion-relation approach (Anisovich *et al.*, Krutov and Troitsky): convergence point for RQM ones
- Some space for extra many-body currents (required for the asymptotic pion charge form factor)
- Generalizations to non-zero spin systems, time-like momentum transfers, $\cdots$

CONCLUSION AND OUTLOOK

Large discrepancies between predicted form factors in different RQM approaches

- could be due to the underlying physics?
- or an incomplete implementation of relativity?

$\rightarrow$ **present results ascribe them to missing properties from Poincaré space-time translation invariance**

$\rightarrow$ **allow one to concentrate on the physics**

In the present case, from comparison with measurements:

- results better than those obtained by Cardarelli *et al.*
- nevertheless pointing to an excessively too large role of high-momentum components in the solution of the mass operator