The B to P transition form factors in pQCD approach and the semileptonic decays of B-meson

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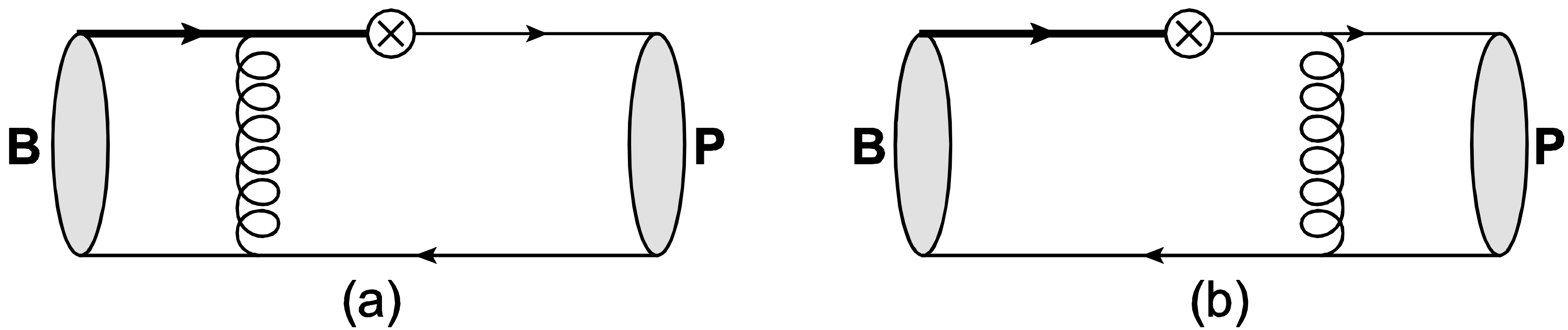
Based on: arXiv:1207.0265

B^0 e e

$$M(p_1,p_2) = \frac{G_F}{\sqrt{2}} V_{ub} \cdot \bar{\nu}_e \gamma_\mu (1-\gamma_5) e \cdot < \pi(p_2) | \bar{u} \gamma^\mu (1-\gamma_5) b | B(p_1) >$$



The strong interaction effects in this decay is parametrized in terms of B to Pi form factors

Form factors

$$\langle P(p_2)|\bar{b}(0)\gamma_\mu q(0)|B(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m_B^2-m_P^2}{q^2}q_\mu\right]F_+(q^2) + \frac{m_B^2-m_P^2}{q^2}q_\mu F_0(q^2),$$

$$\langle P(p_2)|\bar{b}(0)\sigma_{\mu\nu}q(0)|B(p_1)\rangle = i[p_{2\mu}q_\nu - q_\mu p_{2\nu}]\frac{2F_{\rm T}(q^2)}{m_B+m_P},$$

$$\langle P(p_2)|\bar{b}(0)\sigma_{\mu\nu}\gamma_5 q(0)|B(p_1)\rangle = \epsilon_{\mu\nu\alpha\beta}p_2^\alpha q^\beta \frac{2F_{\rm T}(q^2)}{m_B+m_P}.$$

q p_1 p_2 → the momentum of the lepton-pair

Form factors

i) the light cone QCD sum rules (LCSRs)

JHEP 09 (1998) 005, 10 (2001) 019

PRD 71, 014015 (2005)

JHEP 04 (2008) 014

PRD 83, 094031 (2011)

ii) the lattice QCD (LQCD)

PRD 73, 074502 (2006)

PRD 80, 034026 (2009)

PLB 486, 111 (2000)

Form factors

iii) the perturbative QCD (pQCD)

PRD 65, 014007 (2001)

NPB 642, 263 (2002)

EPJC 23, 275 (2002) 28,515 (2003)

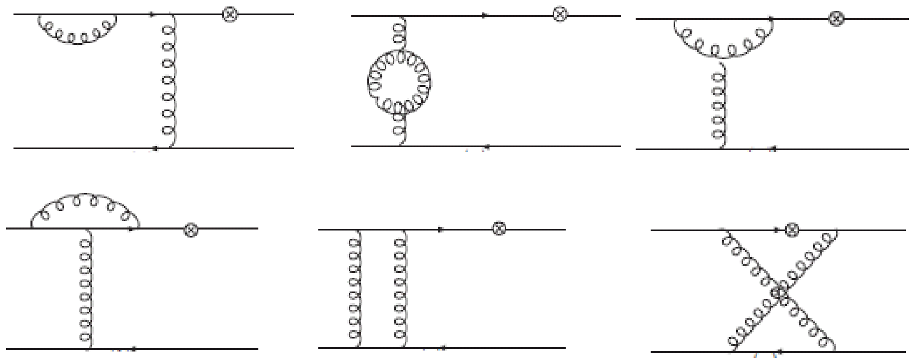
$$\langle P(p_2)|\,\bar{b}(0)\gamma_\mu q(0)|B(p_1)\rangle = g_s^2 C_F N_c \int dx_1 dx_2 d^2k_{1\mathrm{T}} d^2k_{2\mathrm{T}} \frac{dz^+ d^2 z_\mathrm{T}}{(2\pi)^3}\frac{dy^- d^2 y_\mathrm{T}}{(2\pi)^3}$$

$$\times e^{-ik_2\cdot y}\langle P(p_2)|\bar{q}'_\gamma(y) q_\beta(0)|0\rangle e^{ik_1\cdot z}\langle 0|\bar{b}_\alpha(0) q'_\delta(z)|B(p_1)\rangle T_{H\mu}^{\gamma\beta;\alpha\delta},$$

The hard amplitude

Form factors: NLO

H.N. Li, Y.L. Shen & Y.M. Wang
Phys. Rev. D 85, 074004 (2012)



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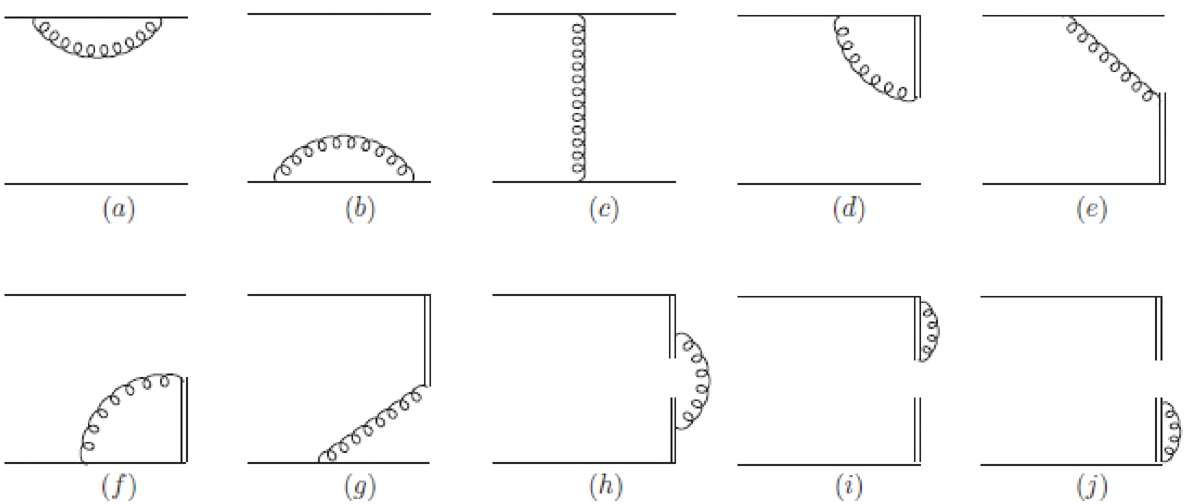
Quark diagrams



Effective diagrams

Form factors: NLO

H.N. Li, Y.L. Shen & Y.M. Wang
Phys. Rev. D 85, 074004 (2012)

$$
\begin{aligned}
H^{(1)} &= F(x_1, x_2, \eta, \mu_f, \mu, \zeta_1) H^{(0)} \\
&= \frac{\alpha_s(\mu_f) C_F}{4\pi} \left[\frac{21}{4} \ln \frac{\mu^2}{m_B^2} - \left(\ln \frac{m_B^2}{\zeta_1^2} + \frac{13}{2} \right) \ln \frac{\mu_f^2}{m_B^2} + \frac{7}{16} \ln^2(x_1 x_2) \right. \\
&\quad + \frac{1}{8} \ln^2 x_1 + \frac{1}{4} \ln x_1 \ln x_2 + \left(2 \ln \frac{m_B^2}{\zeta_1^2} + \frac{7}{8} \ln \eta - \frac{1}{4} \right) \ln x_1 \\
&\quad + \left(\frac{7}{8} \ln \eta - \frac{3}{2} \right) \ln x_2 + \left(\frac{15}{4} - \frac{7}{16} \ln \eta \right) \ln \eta \\
&\quad \left. - \frac{1}{2} \ln \frac{m_B^2}{\zeta_1^2} \left(3 \ln \frac{m_B^2}{\zeta_1^2} + 2 \right) + \frac{101}{48} \pi^2 + \frac{219}{16} \right] H^{(0)}.
\end{aligned}
$$

$$
H = H^{(0)} + H^{(1)} = (1 + F) \times H^{(0)}
$$

$$F_+(q^2) = \frac{1}{2}[f_1(q^2) + f_2(q^2)],$$
$$F_0(q^2) = \frac{1}{2}f_1(q^2)\left[1 + \frac{q^2}{m_B^2 - m_P^2}\right] + \frac{1}{2}f_2(q^2)\left[1 - \frac{q^2}{m_B^2 - m_P^2}\right].$$

$$\begin{aligned} f_1(q^2) = 16\pi C_F m_B^2 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \psi_B(x_1, b_1) \\ \times\Big\{ \left[r_0\left(\phi^p(x_2) - \phi^t(x_2)\right) \cdot h_1(x_1, x_2, b_1, b_2) - r_0 x_1 \eta m_B^2 \phi^\sigma(x_2) \cdot h_2(x_1, x_2, b_1, b_2)\right] \\ \cdot\alpha_s(t_1) \exp\left[-S_{ab}(t_1)\right] \\ + \left[x_1\left(\eta\phi^a(x_2) - 2r_0\phi^p(x_2)\right) + 4r_0 x_1 \phi^p(x_2)\right] \cdot h_1(x_2, x_1, b_2, b_1) \\ \cdot\alpha_s(t_2) \exp\left[-S_{ab}(t_2)\right]\Big\}, \end{aligned}$$

$$\begin{aligned} f_2(q^2) = 16\pi C_F m_B^2 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \psi_B(x_1, b_1) \\ \times\Big\{ \Big[\Big[(x_2\eta + 1)\,\phi^a(x_2) + 2r_0\Big(\Big(\frac{1}{\eta} - x_2\Big)\phi^t(x_2) - x_2\phi^p(x_2) + 3\phi^\sigma(x_2)\Big)\Big] \\ \cdot h_1(x_1, x_2, b_1, b_2) - r_0 x_1 m_B^2\,(1 + x_2\eta)\,\phi^\sigma(x_2) \cdot h_2(x_1, x_2, b_1, b_2)\Big] \cdot \alpha_s(t_1) \exp\left[-S_{ab}(t_1)\right] \\ + 2r_0\left(\frac{x_1}{\eta} + 1\right)\phi^p(x_2) \cdot h_1(x_2, x_1, b_2, b_1) \cdot \alpha_s(t_2) \exp\left[-S_{ab}(t_2)\right]\Big\}, \end{aligned}$$

$$
\begin{aligned}
F_{\mathrm{T}}(q^2) = 8\pi C_F m_B^2 \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 (1+r_P)\psi_B(x_1,b_1) \\
\times\Big\{\Big[\ r_0 x_1 m_B^2 \phi^\sigma(x_2)\cdot h_2(x_1,x_2,b_1,b_2) \\
+\left[\phi^a(x_2) - r_0 x_2 \phi^p(x_2) + r_0\left(\frac{2}{\eta}+x_2\right)\phi^t(x_2) + r_0\phi^\sigma(x_2)\right]\cdot h_1(x_1,x_2,b_1,b_2)\Big] \\
\cdot\alpha_s(t_1)\exp\left[-S_{ab}(t_1)\right] \\
+2r_0\phi^p(x_2)\left(1+\frac{x_1}{\eta}\right)\cdot h_1(x_2,x_1,b_2,b_1)\cdot\alpha_s(t_2)\exp\left[-S_{ab}(t_2)\right]\Big\},
\end{aligned}
$$

$$
\mathbf{H} = \mathbf{H}^{(0)} + \mathbf{H}^{(1)} = (1+\mathbf{F})\times\mathbf{H}^{(0)}
$$

$$F_0(q^2) = \frac{F_0(0)}{1 - a \times \frac{q^2}{m_B^2} + b \times \left(\frac{q^2}{m_B^2}\right)^2}$$

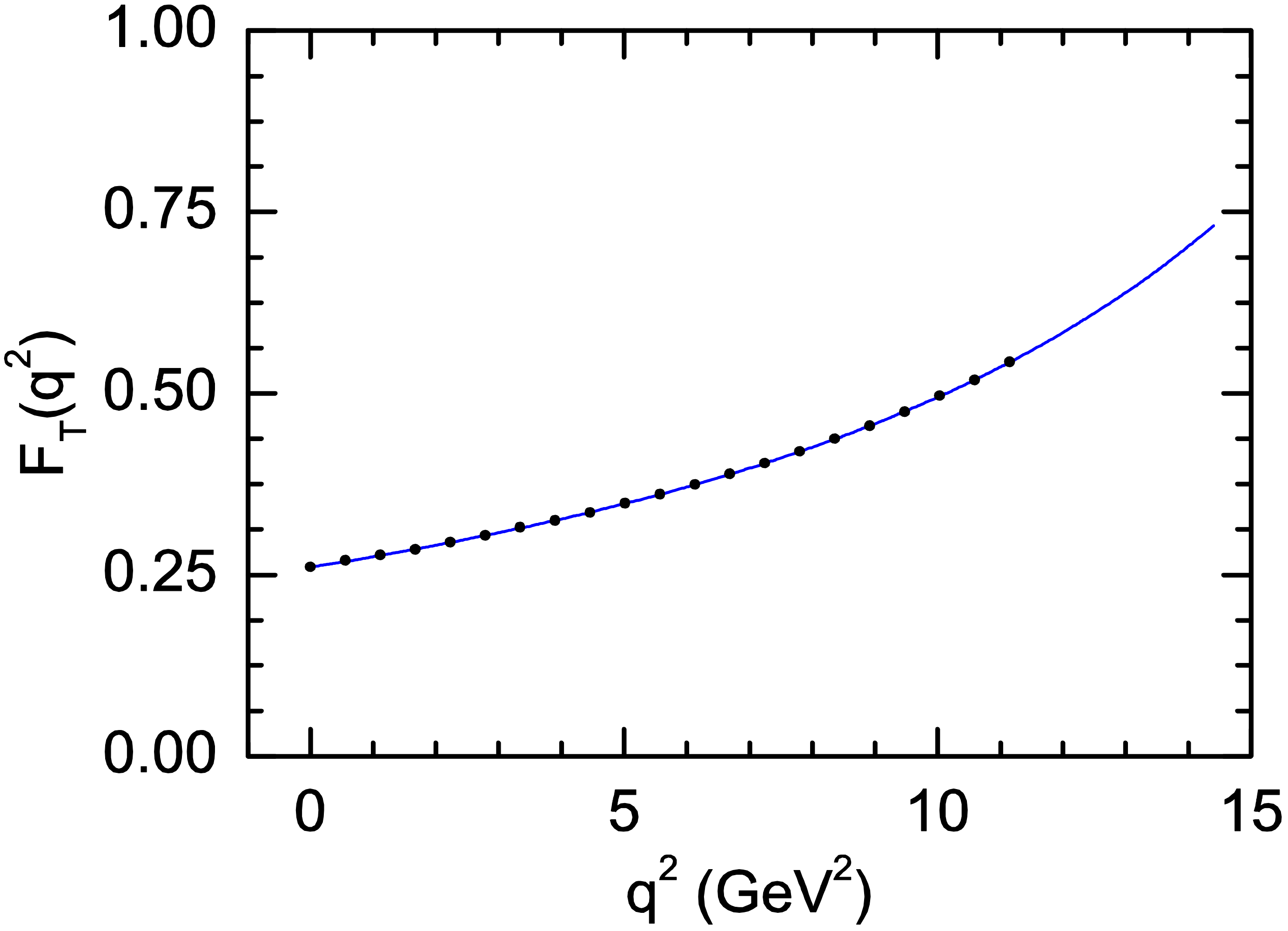
pole model

$$F_{+,T}(q^2) = F_{+,T}(0) \times \left(\frac{1}{1 - \frac{q^2}{m_{B*}^2}} - \frac{r \times \frac{q^2}{m_{B*}^2}}{\left(1 - \frac{q^2}{m_{B*}^2}\right) \times \left(1 - \alpha \times \frac{q^2}{m_B^2}\right)} \right)$$

Ball Zwicky (BZ)

m_B ⟹ **The mass of B or Bs meson**

m_B { **5.325GeV for B meson**; **5.415GeV for Bs meson**

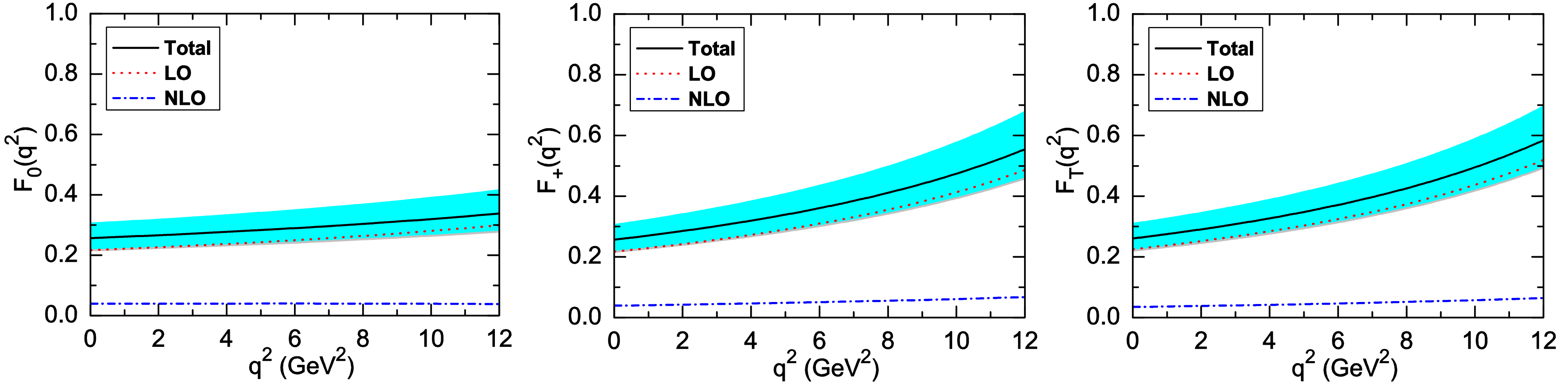

1.00
0.75
0.50
0.25
0.00
0
5
10
15
$F_T(q^2)$
q^2 (GeV^2)

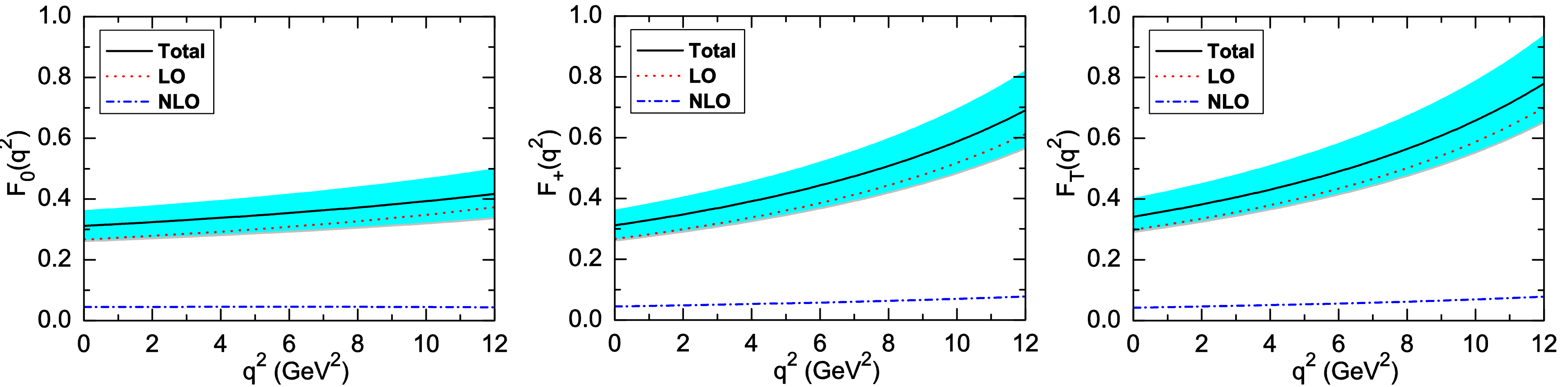
	$F_+(0)_{LO}$	α_{LO}	r_{LO}
$B \to \pi$	$0.22^{+0.03}_{-0.02} \pm 0.01$	$0.61^{+0.00}_{-0.01} \pm 0.01$	$0.51 \pm 0.00 \pm 0.03$
$B \to K$	$0.27^{+0.04}_{-0.03} \pm 0.01$	$0.62^{+0.00}_{-0.01} \pm 0.01$	$0.58 \pm 0.00 \pm 0.03$
$B_s \to K$	$0.22 \pm 0.03 \pm 0.01$	$0.64 \pm 0.00 \pm 0.01$	$0.56 \pm 0.00 \pm^{+0.04}_{-0.03}$
	$F_{\rm T}(0)_{LO}$	α_{LO}	r_{LO}
$B \to \pi$	$0.23 \pm 0.03 \pm 0.01$	$0.69^{+0.00}_{-0.01} \pm 0.01$	$0.55 \pm 0.01 \pm 0.03$
$B \to K$	$0.30^{+0.04}_{-0.03} \pm 0.01$	$0.71^{+0.00}_{-0.01} \pm 0.01$	$0.58 \pm 0.01 \pm 0.03$
$B_s \to K$	$0.25^{+0.04}_{-0.03} \pm 0.01$	$0.71^{+0.01}_{-0.00} \pm 0.01$	$0.59 \pm 0.00 \pm 0.03$
	$F_+(0)_{NLO}$	α_{NLO}	r_{NLO}
$B \to \pi$	$0.26^{+0.04}_{-0.03} \pm 0.02$	$0.52 \pm 0.01 \pm 0.03$	$0.45 \pm 0.00^{+0.05}_{-0.04}$
$B \to K$	$0.31 \pm 0.04 \pm 0.02$	$0.54 \pm 0.01^{+0.02}_{-0.03}$	$0.50 \pm 0.00 \pm 0.05$
$B_s \to K$	$0.26^{+0.04}_{-0.03} \pm 0.02$	$0.57 \pm 0.01 \pm 0.02$	$0.50 \pm 0.01 \pm 0.05$
	$F_{\rm T}(0)_{NLO}$	α_{NLO}	r_{NLO}
$B \to \pi$	$0.26^{+0.04}_{-0.03} \pm 0.02$	$0.65^{+0.01}_{-0.02} \pm 0.01$	$0.50 \pm 0.00 \pm 0.00$
$B \to K$	$0.34^{+0.05}_{-0.04} \pm 0.02$	$0.67 \pm 0.01 \pm 0.01$	$0.53 \pm 0.00^{+0.05}_{-0.04}$
$B_s \to K$	$0.28 \pm 0.04 \pm 0.02$	$0.69 \pm 0.01 \pm 0.01$	$0.53^{+0.01+0.04}_{-0.00-0.02}$

	$F_0(0)_{LO}$	a_{LO}	b_{LO}
$B \to \pi$	$0.22^{+0.03}_{-0.02} \pm 0.01$	$0.58 \pm 0.01 \pm 0.03$	$-0.15 \pm 0.01 \pm 0.01$
$B \to K$	$0.27^{+0.04}_{-0.03} \pm 0.01$	$0.60 \pm 0.01 \pm 0.03$	$-0.15^{+0.01+0.01}_{-0.00-0.02}$
$B_s \to K$	$0.22 \pm 0.03 \pm 0.01$	$0.61 \pm 0.01^{+0.04}_{-0.03}$	$-0.16 \pm 0.00^{+0.02}_{-0.01}$
	$F_0(0)_{NLO}$	a_{NLO}	b_{NLO}
$B \to \pi$	$0.26^{+0.04}_{-0.03} \pm 0.02$	$0.50 \pm 0.01^{+0.05}_{-0.04}$	$-0.13 \pm 0.01 \pm 0.01$
$B \to K$	$0.31 \pm 0.04 \pm 0.02$	$0.53 \pm 0.01^{+0.05}_{-0.04}$	$-0.13 \pm 0.01^{+0.02}_{-0.01}$
$B_s \to K$	$0.26^{+0.04}_{-0.03} \pm 0.02$	$0.54 \pm 0.00 \pm 0.05$	$-0.15 \pm 0.01 \pm 0.01$

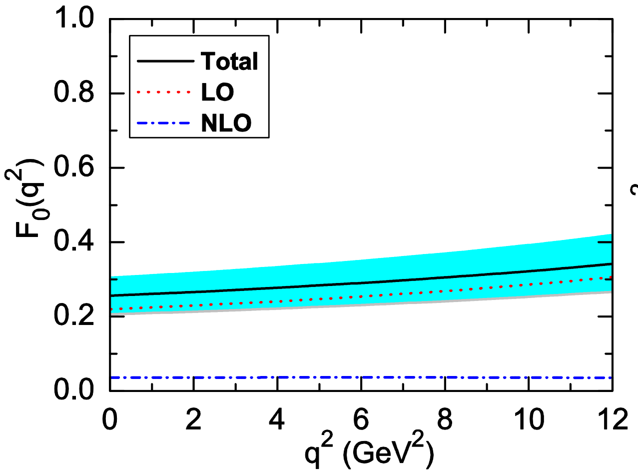
	$F_0(0)=F_+(0)$	$F_T(0)$
B	~ 18%	~ 13%
B K	~ 15%	~ 13%
B_s K	~ 18%	~ 12%

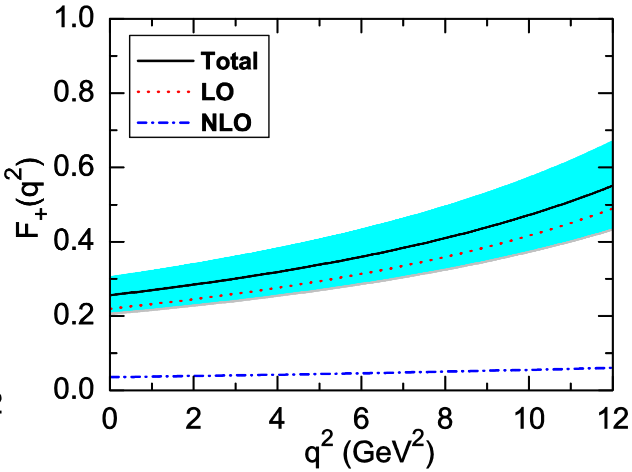
Form factors: the pQCD result

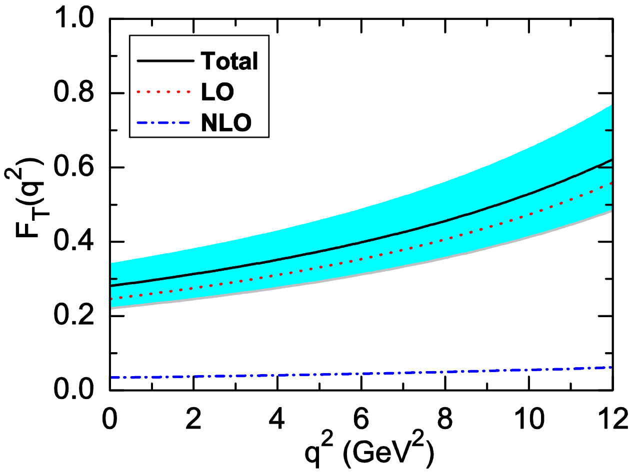




Form factors: the pQCD result







The $b \to u l^- \bar{\nu}_l$ transitions

$$\mathcal{H}_{eff}(b \to u l \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub}\, \bar{u}\gamma_\mu(1-\gamma_5)b \cdot \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

$$\frac{d\Gamma(b \to u l \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \frac{q^2 - m_l^2}{(q^2)^2} \sqrt{\frac{(q^2 - m_l^2)^2}{q^2}} \sqrt{\frac{(m_B^2 - m_P^2 - q^2)^2}{4q^2} - m_P^2}$$
$$\times \Big\{ (m_l^2 + 2q^2) \left[q^2 - (m_B - m_P)^2\right] \left[q^2 - (m_B + m_P)^2\right] F_+^2(q^2)$$
$$+ 3 m_l^2 \left(m_B^2 - m_P^2\right)^2 F_0^2(q^2) \Big\},$$

$$\boxed{\frac{d\Gamma(b \to u l \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) |F_+(q^2)|^2}$$

$$\lambda(q^2) = (m_B^2 + m_P^2 - q^2)^2 - 4 m_B^2 m_P^2$$

The $b \to (s,d)l^- l^+$ transitions (i)

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

$$O_1 = (\bar{q}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}, \quad O_2 = (\bar{q}_\alpha c_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A},$$

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \left(\bar{q}'_\beta q'_\beta\right)_{V-A}, \quad O_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} \left(\bar{q}'_\beta q'_\beta\right)_{V+A}, \quad O_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_7 = \frac{e m_b}{8\pi^2} \, \bar{q} \sigma^{\mu\nu} (1+\gamma_5) b \, F_{\mu\nu},$$

$$O_9 = \frac{\alpha_{\rm em}}{8\pi} \left(\bar{l} \gamma_\mu l\right) \left[\bar{q} \gamma^\mu (1-\gamma_5) b\right], \quad O_{10} = \frac{\alpha_{\rm em}}{8\pi} \left(\bar{l} \gamma_\mu \gamma_5 l\right) \left[\bar{q} \gamma^\mu \left(1-\gamma_5\right) b\right],$$

The $b \to (s,d) l^- l^+$ transitions (ii)

$$\frac{d\Gamma(b \to s l^+ l^-)}{dq^2} =$$

$$\frac{G_F^2 \alpha_{em}^2 |V_{tb}|^2 |V_{ts}^*|^2 \sqrt{\lambda(q^2)}}{512 m_B^3 \pi^5} \sqrt{\frac{q^2 - 4m_l^2}{q^2}} \frac{1}{3q^2}$$

$$\times \Bigg[6m_l^2 |C_{10}|^2 (m_B^2 - m_P^2)^2 F_0^2(q^2)$$

$$+ (q^2 + 2m_l^2) \lambda(q^2) \left| C_9^{eff} \; F_+(q^2) + \frac{2C_7^{eff} \; (m_b - m_s) F_{\mathrm{T}}(q^2)}{m_B + m_P} \right|^2$$

$$+ |C_{10}|^2 (q^2 - 4m_l^2) \; \lambda(q^2) \; F_+^2(q^2) \Bigg],$$

The $b \to s\nu\bar{\nu}$ transitions

$$H_{b\to s\nu\bar{\nu}} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi \sin^2(\theta_W)} V_{tb} V_{ts}^* \eta_X X(x_t) \; [\bar{s}\gamma^\mu(1-\gamma_5)b] \, [\bar{\nu}\gamma_\mu(1-\gamma_5)\nu]$$
$$= C_L^{b\to s} \, O_L^{b\to s},$$

$$\frac{d\Gamma(b\to s\nu\bar{\nu})}{dq^2} = 3 \frac{|C_L^{b\to s}|^2 \lambda^{3/2}(m_B^2, m_P^2, q^2)}{96 m_B^3 \pi^3} |F_+(q^2)|^2.$$

sum of the three generations

Branching ratios

$$Br(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l) = (1.42^{+0.40}_{-0.30}(\omega_b) \pm 0.15(a_2^\pi) \pm 0.12(m_0^\pi)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau) = (0.90^{+0.25}_{-0.19}(\omega_b)^{+0.09}_{-0.08}(a_2^\pi) \pm 0.08(m_0^\pi)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(B^- \to \pi^0 l^- \bar{\nu}_l) = (7.63^{+2.16}_{-1.61}(\omega_b)^{+0.82}_{-0.78}(a_2^\pi)^{+0.66}_{-0.63}(m_0^\pi)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-5},$$

$$Br(B^- \to \pi^0 \tau^- \bar{\nu}_\tau) = (4.85^{+1.34}_{-1.01}(\omega_b)^{+0.47}_{-0.41}(a_2^\pi)^{+0.45}_{-0.43}(m_0^\pi)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-5},$$

$$Br(\bar{B}_s^0 \to K^+ l^- \bar{\nu}_l) = (1.27^{+0.46}_{-0.26}(\omega_{bs})^{+0.14}_{-0.13}(a_i^K)^{+0.10}_{-0.09}(m_0^K)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(\bar{B}_s^0 \to K^+ \tau^- \bar{\nu}_\tau) = (7.78^{+2.51}_{-1.81}(\omega_{bs})^{+0.69}_{-0.66}(a_i^K)^{+0.62}_{-0.59}(m_0^K)) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-5},$$

$$Br(B^- \to \pi^0 l^- \bar{\nu}_l) = (7.78 \pm 0.28) \times 10^{-5}, \quad \mathrm{PDG2012}$$

$$Br\left(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l\right) = \begin{cases} (1.41 \pm 0.05(syst.) \pm 0.07(stat.)) \times 10^{-4}, \\ (1.44 \pm 0.05) \times 10^{-4}, \quad \mathrm{PDG2012} \end{cases}$$

$|V_{ub}|$

PRD-83-052011 & 83-032007(2011) BaBar

$$Br\left(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l\right) = \begin{cases} (1.41 \pm 0.05(syst.) \pm 0.07(stat.)) \times 10^{-4}, \\ (1.44 \pm 0.05) \times 10^{-4}, \quad \mathrm{PDG2012} \end{cases}$$

$$|V_{ub}| = \left(3.80^{+0.50}_{-0.43}(\omega_b) \pm 0.20(a_2^\pi)^{+0.16}_{-0.15}(m_0^\pi)\right) \times 10^{-3}$$

$$|V_{ub}| = \left(3.78 \pm 0.13(exp.)^{+0.55}_{-0.40}(theor.)\right) \times 10^{-3},$$

BaBar

Decay modes	NLO pQCD predictions	Data
$Br(\bar{B}^0 \to \pi^0 l^+ l^-)$	$(0.91^{+0.26+0.18}_{-0.19-0.17} \pm 0.10 \pm 0.08) \times 10^{-8}$	$< 1.2 \times 10^{-7}$
$Br(\bar{B}^0 \to \pi^0 \tau^+ \tau^-)$	$(0.28^{+0.07+0.06}_{-0.06-0.05} \pm 0.02 \pm 0.03) \times 10^{-8}$	
$Br(\bar{B}^0 \to \pi^0 \nu \bar{\nu})$	$(7.30^{+2.07+1.45+0.79+0.63}_{-1.54-1.32-0.74-0.61}) \times 10^{-8}$	$< 2.2 \times 10^{-4}$
$Br(B^- \to \pi^- l^+ l^-)$	$(1.95^{+0.55+0.39+0.21+0.17}_{-0.41-0.35-0.20-0.16}) \times 10^{-8}$	$(2.3 \pm 0.6 \pm 0.1) \times 10^{-8}$
$Br(B^- \to \pi^- \tau^+ \tau^-)$	$(0.60^{+0.16+0.12+0.04}_{-0.12-0.11-0.03} \pm 0.06) \times 10^{-8}$	
$Br(B^- \to \pi^- \nu \bar{\nu})$	$(1.57^{+0.44+0.31+0.17+0.14}_{-0.33-0.28-0.16-0.13}) \times 10^{-7}$	$< 1.0 \times 10^{-4}$
$Br(\bar{B}^0 \to \bar{K}^0 l^+ l^-)$	$(5.1^{+1.5+1.02+0.5+0.4}_{-1.1-0.93-0.5-0.4}) \times 10^{-7}$	$(4.7^{+0.6}_{-0.2}) \times 10^{-7}$
$Br(\bar{B}^0 \to \bar{K}^0 \tau^+ \tau^-)$	$(1.20^{+0.32+0.24+0.07+0.11}_{-0.25-0.22-0.07-0.10}) \times 10^{-7}$	
$Br(\bar{B}^0 \to \bar{K}^0 \nu \bar{\nu})$	$(4.1^{+1.2+0.82+0.4+0.3}_{-0.9-0.74-0.4-0.3}) \times 10^{-6}$	$< 5.6 \times 10^{-5}$
$Br(B^- \to K^- l^+ l^-)$	$(5.50^{+1.59+1.10+0.57+0.42}_{-1.18-1.00-0.55-0.41}) \times 10^{-7}$	$(5.1 \pm 0.5) \times 10^{-7}$
$Br(B^- \to K^- \tau^+ \tau^-)$	$(1.29^{+0.35+0.26}_{-0.26-0.23} \pm 0.08 \pm 0.11) \times 10^{-7}$	
$Br(B^- \to K^- \nu \bar{\nu})$	$(4.42^{+1.28+0.88+0.46+0.34}_{-0.95-0.80-0.44-0.33}) \times 10^{-6}$	$< 1.3 \times 10^{-5}$
$Br(\bar{B}^0_s \to K^0 l^+ l^-)$	$(1.63^{+0.54+0.44+0.18+0.12}_{-0.38-0.39-0.17-0.12}) \times 10^{-8}$	
$Br(\bar{B}^0_s \to K^0 \tau^+ \tau^-)$	$(0.43^{+0.13+0.12+0.03+0.04}_{-0.10-0.10-0.03-0.04}) \times 10^{-8}$	
$Br(\bar{B}^0_s \to K^0 \nu \bar{\nu})$	$(1.31^{+0.43+0.35+0.14+0.10}_{-0.31-0.31-0.13-0.10}) \times 10^{-7}$	

Form factors:

Feldmann, Kroll & Stech **PRD 58, 114006 (1998) PLB 449, 339 (1999)**

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = U(\phi, \phi_G) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |\eta_g\rangle \end{pmatrix} \qquad U(\phi_{FKS}) = \begin{pmatrix} \cos\phi_{FKS} & -\sin\phi_{FKS} \\ \sin\phi_{FKS} & \cos\phi_{FKS} \end{pmatrix}$$

FKS **39.3°** **1.0°**

Hai-Yang Cheng, Hsiang-nan Li & Keh-Fei Liu **PRD 79, 014024 (2009)**

$$U(\phi, \phi_G) = U_3(\theta) U_1(\phi_G) U_3(\theta_i) \; ,$$
$$= \begin{pmatrix} \cos\phi + \sin\theta \sin\theta_i \Delta_G & -\sin\phi + \sin\theta \cos\theta_i \Delta_G & -\sin\theta \sin\phi_G \\ \sin\phi - \cos\theta \sin\theta_i \Delta_G & \cos\phi - \cos\theta \cos\theta_i \Delta_G & \cos\theta \sin\phi_G \\ -\sin\theta_i \sin\phi_G & -\cos\theta_i \sin\phi_G & \cos\phi_G \end{pmatrix}$$

$$\mathrm{R}_s^{\mathrm{Exp}} = \frac{\mathrm{BR}(B_s \to J\psi\eta')}{\mathrm{BR}(B_s \to J\psi\eta)} = 0.73 \pm 0.14(stat.) \pm 0.02(syst.)$$

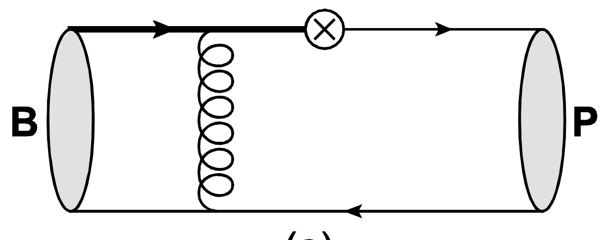
---- PRL 108, 181808 (2012) by Belle Collaboration

$$\mathrm{R}_s \equiv \frac{\mathrm{BR}(B_s \to J\psi\eta')}{\mathrm{BR}(B_s \to J\psi\eta)} > 1$$

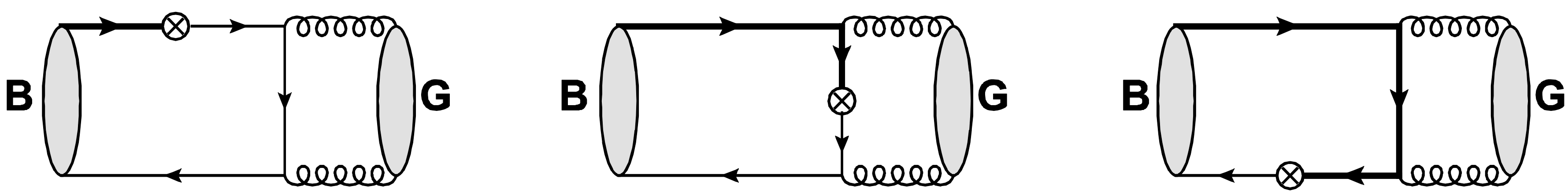
---- PLB 529,93 (2002) & PRD 85, 013016 (2012)

X. Liu, H.N. Li & Z.J. Xiao Phy.Rev. D86, 011501(R) (2012)

Form factors:



(a) (b)



(a) (b) (c)

Form factors:

	$F_0(0)$
$B \to \eta_q$	0.17
$B_s \to \eta_s$	0.27
	$F_+(0)$
$B \to \eta_q$	0.17
$B_s \to \eta_s$	0.27
	$F_T(0)$
$B \to \eta_q$	0.15
$B_s \to \eta_s$	0.27

	$F_0(0)$
$B \to \eta_g$	0.14×10^{-2}
$B_s \to \eta_g$	0.11×10^{-2}
	$F_+(0)$
$B \to \eta_g$	0.14×10^{-2}
$B_s \to \eta_g$	0.11×10^{-2}
	$F_T(0)$
$B \to \eta_g$	0.10×10^{-2}
$B_s \to \eta_g$	0.08×10^{-2}

$$F_i(q^2) = \frac{F_i(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2},$$

Branching ratios

$$Br(B^- \to \eta l^- \bar{\nu}_l) = (0.33) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(B^- \to \eta \tau^- \bar{\nu}_\tau) = (0.19) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(B^- \to \eta' l^- \bar{\nu}_l) = (0.16) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-4},$$

$$Br(B^- \to \eta' \tau^- \bar{\nu}_\tau) = (0.79) \times \frac{|V_{ub}|^2}{|0.0038|^2} \times 10^{-5},$$

$$Br(B^- \to \eta l^- \bar{\nu}_l) = (0.36 \pm 0.05 \pm 0.04) \times 10^{-4},$$

---- Babar PRD83,052011(2011)

Decay modes	pQCD predictions
$Br(\bar{B}^0 \to \eta l^+ l^-)$	$(0.39) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta \tau^+ \tau^-)$	$(0.84) \times 10^{-9}$
$Br(\bar{B}^0 \to \eta \nu \bar{\nu})$	$(3.07) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta' l^+ l^-)$	$(0.18) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta' \tau^+ \tau^-)$	$(0.21) \times 10^{-9}$
$Br(\bar{B}^0 \to \eta' \nu \bar{\nu})$	$(1.44) \times 10^{-8}$
$Br(\bar{B}_s^0 \to \eta l^+ l^-)$	$(1.68) \times 10^{-7}$
$Br(\bar{B}_s^0 \to \eta \tau^+ \tau^-)$	$(0.39) \times 10^{-7}$
$Br(\bar{B}_s^0 \to \eta \nu \bar{\nu})$	$(1.34) \times 10^{-6}$
$Br(\bar{B}_s^0 \to \eta' l^+ l^-)$	$(1.77) \times 10^{-7}$
$Br(\bar{B}_s^0 \to \eta' \tau^+ \tau^-)$	$(0.23) \times 10^{-7}$
$Br(\bar{B}_s^0 \to \eta' \nu \bar{\nu})$	$(1.39) \times 10^{-6}$

Decay modes	$\phi = 43.7^\circ$ & $\phi_G = 33^\circ$	$\phi = 40^\circ$ & $\phi_G = 22^\circ$
$Br(B^- \to \eta l^- \bar{\nu}_l)$	$(0.27) \times 10^{-4}$	$(0.31) \times 10^{-4}$
$Br(B^- \to \eta \tau^- \bar{\nu}_\tau)$	$(0.16) \times 10^{-4}$	$(0.18) \times 10^{-4}$
$Br(B^- \to \eta' l^- \bar{\nu}_l)$	$(0.12) \times 10^{-4}$	$(0.13) \times 10^{-4}$
$Br(B^- \to \eta' \tau^- \bar{\nu}_\tau)$	$(0.62) \times 10^{-5}$	$(0.67) \times 10^{-5}$
$Br(\bar{B}^0 \to \eta l^+ l^-)$	$(0.32) \times 10^{-8}$	$(0.37) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta \tau^+ \tau^-)$	$(0.68) \times 10^{-9}$	$(0.79) \times 10^{-9}$
$Br(\bar{B}^0 \to \eta \nu \bar{\nu})$	$(2.49) \times 10^{-8}$	$(2.89) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta' l^+ l^-)$	$(0.15) \times 10^{-8}$	$(0.16) \times 10^{-8}$
$Br(\bar{B}^0 \to \eta' \tau^+ \tau^-)$	$(0.16) \times 10^{-9}$	$(0.18) \times 10^{-9}$
$Br(\bar{B}^0 \to \eta' \nu \bar{\nu})$	$(1.13) \times 10^{-8}$	$(1.23) \times 10^{-8}$
$Br(\bar{B}^0_s \to \eta l^+ l^-)$	$(2.11) \times 10^{-7}$	$(1.79) \times 10^{-7}$
$Br(\bar{B}^0_s \to \eta \tau^+ \tau^-)$	$(0.50) \times 10^{-7}$	$(0.42) \times 10^{-7}$
$Br(\bar{B}^0_s \to \eta \nu \bar{\nu})$	$(1.67) \times 10^{-6}$	$(1.42) \times 10^{-6}$
$Br(\bar{B}^0_s \to \eta' l^+ l^-)$	$(1.18) \times 10^{-7}$	$(1.56) \times 10^{-7}$
$Br(\bar{B}^0_s \to \eta' \tau^+ \tau^-)$	$(0.16) \times 10^{-7}$	$(0.21) \times 10^{-7}$
$Br(\bar{B}^0_s \to \eta' \nu \bar{\nu})$	$(0.93) \times 10^{-6}$	$(1.22) \times 10^{-6}$

$$|\eta\rangle \simeq (\cos\phi + \sin\theta \sin\theta_i \Delta_G)\,|\eta_q\rangle - (\sin\phi - \sin\theta\cos\theta_i\Delta_G)\,|\eta_s\rangle,$$
$$|\eta'\rangle \simeq (\sin\phi - \cos\theta\sin\theta_i\Delta_G)\,|\eta_q\rangle + (\cos\phi - \cos\theta\cos\theta_i\Delta_G)\,|\eta_s\rangle$$
$$|G\rangle \simeq -(\sin\theta_i \sin\phi_G)\,|\eta_q\rangle - (\cos\theta_i\sin\phi_G)\,|\eta_s\rangle,$$

$$Br(B^- \to G l^- \bar{\nu}_l) = 0.21 \times 10^{-8}$$
$$Br(B^- \to G \tau^- \bar{\nu}_\tau) = 0.11 \times 10^{-8}$$
$$Br(\bar{B}^0 \to G l^+ l^-) = 0.28 \times 10^{-12}$$
$$Br(\bar{B}^0 \to G \tau^+ \tau^-) = 0.13 \times 10^{-13}$$
$$Br(\bar{B}^0 \to G \nu\bar{\nu}) = 0.21 \times 10^{-11}$$
$$Br(\bar{B}_s^0 \to G l^+ l^-) = 0.37 \times 10^{-11}$$
$$Br(\bar{B}_s^0 \to G \tau^+ \tau^-) = 0.25 \times 10^{-12}$$
$$Br(\bar{B}_s^0 \to G \nu\bar{\nu}) = 0.29 \times 10^{-10}$$

Decay modes	$\phi_G = 33°$	$\phi_G = 22°$
$Br(B^- \to G l^- \bar{\nu}_l)$	$(0.50) \times 10^{-5}$	$(0.23) \times 10^{-5}$
$Br(B^- \to G \tau^- \bar{\nu}_\tau)$	$(2.02) \times 10^{-6}$	$(0.96) \times 10^{-6}$
$Br(\bar{B}^0 \to G l^+ l^-)$	$(0.59) \times 10^{-9}$	$(0.28) \times 10^{-9}$
$Br(\bar{B}^0 \to G \tau^+ \tau^-)$	$(1.52) \times 10^{-10}$	$(0.72) \times 10^{-10}$
$Br(\bar{B}^0 \to G \nu \bar{\nu})$	$(4.58) \times 10^{-9}$	$(2.17) \times 10^{-9}$
$Br(\bar{B}_s^0 \to G l^+ l^-)$	$(1.90) \times 10^{-8}$	$(0.90) \times 10^{-8}$
$Br(\bar{B}_s^0 \to G \tau^+ \tau^-)$	$(0.74) \times 10^{-9}$	$(0.35) \times 10^{-9}$
$Br(\bar{B}_s^0 \to G \nu \bar{\nu})$	$(1.48) \times 10^{-7}$	$(0.70) \times 10^{-7}$

Some ratios

$$R_\nu = \frac{Br(\bar{B}^0 \to \pi^0 \nu\bar{\nu})}{Br(\bar{B}^0 \to \pi^0 l^+ l^-)} \approx \frac{Br(\bar{B}^0 \to \bar{K}^0 \nu\bar{\nu})}{Br(\bar{B}^0 \to \bar{K}^0 l^+ l^-)} \approx \frac{Br(B^- \to \pi^- \nu\bar{\nu})}{Br(B^- \to \pi^- l^+ l^-)}$$

$$\approx \frac{Br(B^- \to K^- \nu\bar{\nu})}{Br(B^- \to K^- l^+ l^-)} \approx \frac{Br(\bar{B}_s^0 \to K^0 \nu\bar{\nu})}{Br(\bar{B}_s^0 \to K^0 l^+ l^-)} \approx 8,$$

$$R_{N1} = \frac{Br(\bar{B}^0 \to \pi^0 l^+ l^-)}{Br(\bar{B}^0 \to \pi^0 \tau^+ \tau^-)} \approx \frac{Br(B^- \to \pi^- l^+ l^-)}{Br(B^- \to \pi^- \tau^+ \tau^-)} \approx 3.3,$$

$$R_{N2} = \frac{Br(\bar{B}^0 \to \bar{K}^0 l^+ l^-)}{Br(\bar{B}^0 \to \bar{K}^0 \tau^+ \tau^-)} \approx \frac{Br(B^- \to K^- l^+ l^-)}{Br(B^- \to K^- \tau^+ \tau^-)} \approx 4.3,$$

$$R_{N3} = \frac{Br(\bar{B}_s^0 \to K^0 l^+ l^-)}{Br(\bar{B}_s^0 \to K^0 \tau^+ \tau^-)} \approx 3.8,$$

Summary

- B/Bs to P form factors in pQCD at NLO.
- Branching ratios of $B_{(s)} \to P(ll, l\nu, \nu\bar{\nu})$
- Extract out the $|V_{ub}|$
- Some ratios are defined

Thank you !