The B_c -> D(D*)T decays in perturbative QCD approach

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Brief introduction

B_c → D(D*) ✚ T

A ground state of two heavy quarks system ($\bar{b}c$)

It provides us an ideal platform to understand the weak interaction of heavy quark flavor.

T denotes a light tensor meson with $J^P = 2^+$:

$a_2(1320)$, $K_2{}^*(1430)$,

$f_2(1270)$, $f_2{}'(1525)$.

$< T \mid J^\mu \mid 0 > = 0$ (J^μ is the (V±A)current or (S±P) density.)

These decays are prohibited in naive factorization.

- We use **the PQCD approach**, which is based on the k_T factorization.

  - This method is usually applied in the hadronic two-body decays of B meson. Due to the heavy mass of B, the process is dominated by the exchange of hard gluons. Thus the process is factorized into the hard part which can be calculated perturbatively and the soft part which can be absorbed into wave function which is universal and nonperturbative.

  - The end-point singularity in collinear factorization can be avoided because of the transverse momentum.

  - The nonfactorizable and annihilation diagrams are calculable by using this approach.

Theoretical framework

- The key step is to calculate the transition matrix elements $\mathcal{M} \propto \langle D^{(*)}T|\mathcal{H}_{eff}|B_c\rangle$

- The weak effective Hamiltonian can be written as

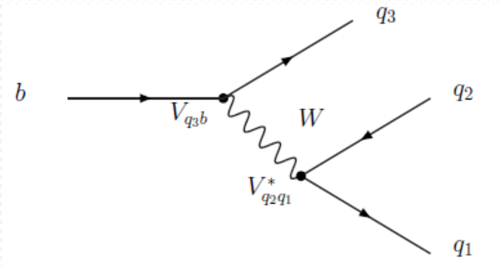
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q=u,c} V_{qb}^* V_{qX} \left[C_1(\mu) O_1^q(\mu) + C_2(\mu) O_2^q(\mu) \right] - V_{tb}^* V_{tX} \left[\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] \right\},$$

$X = d, s$

- O_j (j = 1, …, 10) are the local four-quark operators:
  - current-current (tree) operators

$$O_1^q = (\bar{b}_\alpha q_\beta)_{V-A}(\bar{q}_\beta X_\alpha)_{V-A},$$

$$O_2^q = (\bar{b}_\alpha q_\alpha)_{V-A}(\bar{q}_\beta X_\beta)_{V-A},$$



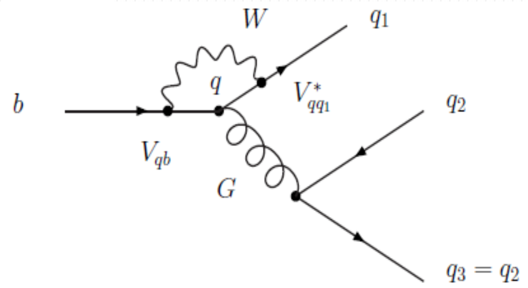
  - QCD penguin operators

$$O_3 = (\bar{b}_\alpha X_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A},$$

$$O_4 = (\bar{b}_\alpha X_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A},$$

$$O_5 = (\bar{b}_\alpha X_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A},$$

$$O_6 = (\bar{b}_\alpha X_\beta)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A},$$



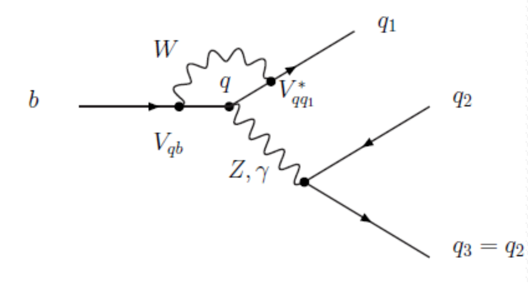
➢ electro-weak penguin operators

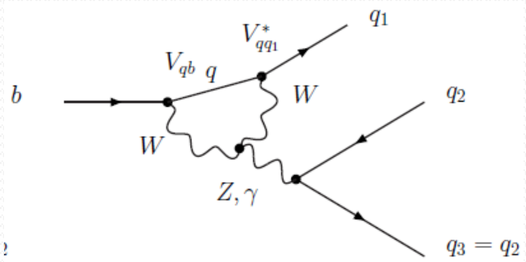
$$O_7 = \frac{3}{2}(\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A},$$

$$O_8 = \frac{3}{2}(\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A},$$

$$O_9 = \frac{3}{2}(\bar{b}_\alpha X_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A},$$

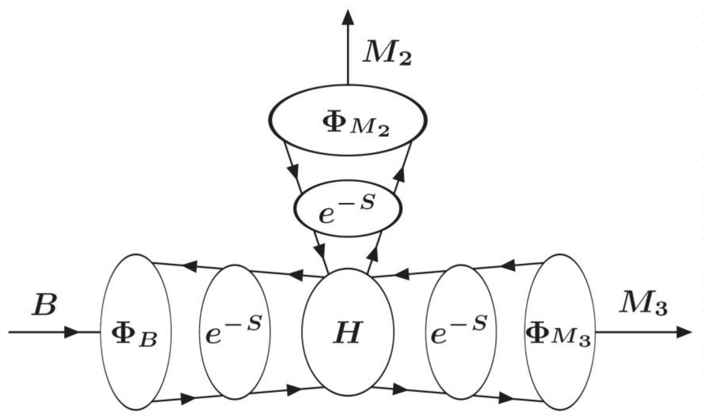
$$O_{10} = \frac{3}{2}(\bar{b}_\alpha X_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A},$$





- There are several typical scales
  - When the energy scale is higher than the W boson mass, the physics is the electroweak interaction which can be calculated perturbatively.
  - The physics between W boson mass scale and b quark mass scale can be included in the Wilson coefficients C(t) of the effective four-quark operators.
  - The physics between b quark mass and the factorization scale is included in the calculated hard part H(x,b)
  - The physics below the factorization scale is nonperturbative and described by the hadronic wave functions of mesons, which is universal for all decay modes.

- The decay amplitude can be factorized into the convolution of the Wilson coefficients , the hard scattering kernel and the light-cone wave functions of mesons characterized by different scales.



$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3$$
$$\times Tr\left[C(t)\Phi_B(x_1, b_1)\Phi_{M_2}(x_2, b_2)\Phi_{M_3}(x_3, b_3)H(x_i, b_i, t)S_t(x_i)e^{-S(t)}\right]$$

Wave Functions

- In the PQCD approach, the initial and final state meson wave functions are the most important non-perturbative inputs.
- For B_c meson, we only consider the contribution from the first Lorentz structure, like B_q (q = u, d, s) meson

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}(\not{P} + m_{B_c})\gamma_5\phi_{B_c}(x,b).$$

- For D(D*) meson, in the heavy quark limit, the two-parton LCDAs can be written as

$$\begin{aligned}
\langle D(p)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle &= \frac{i}{\sqrt{2N_c}}\int_0^1 dx\, e^{ixp\cdot z}\left[\gamma_5(\not{P} + m_D)\phi_D(x,b)\right]_{\alpha\beta}, \\
\langle D^*(p)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle &= -\frac{1}{\sqrt{2N_c}}\int_0^1 dx\, e^{ixp\cdot z}\left[\not{\epsilon}_L(\not{P} + m_{D^*})\phi_{D^*}^L(x,b)\right. \\
&\quad \left. + \not{\epsilon}_T(\not{P} + m_{D^*})\phi_{D^*}^T(x,b)\right]_{\alpha\beta},
\end{aligned}$$

Wave Functions

- For tensor mesons, the ± 2 polarizations do not contribute in the B_c -> D(D*)T decays due to the angular momentum conservation argument. Because of the simplification, the wave functions for a generic tensor meson are defined by

$$\Phi_T^L = \frac{1}{\sqrt{6}}\left[m_T \not\epsilon^*_{\bullet L}\phi_T(x) + \not\epsilon^*_{\bullet L}\not P \phi_T^t(x) + m_T^2 \frac{\epsilon_\bullet \cdot v}{P \cdot v}\phi_T^s(x)\right]$$
$$\Phi_T^\perp = \frac{1}{\sqrt{6}}\left[m_T \not\epsilon^*_{\bullet \perp}\phi_T^v(x) + \not\epsilon^*_{\bullet \perp}\not P \phi_T^T(x) + m_T i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu \epsilon^{*\nu}_{\bullet\perp} n^\rho v^\sigma \phi_T^a(x)\right]$$

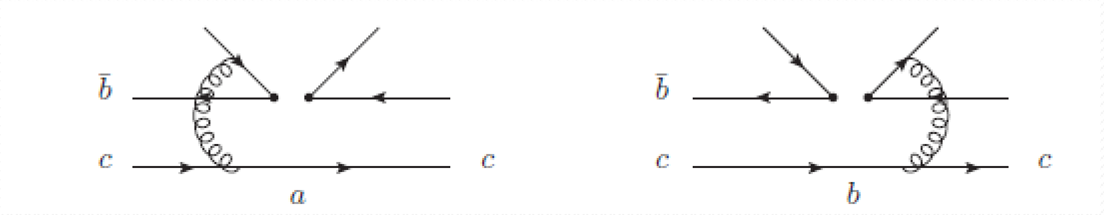
where $\epsilon_\bullet \equiv \frac{\epsilon_{\mu\nu}v^\nu}{P\cdot v}$

and $\epsilon_{\mu\nu}$ is the polarization tensor.

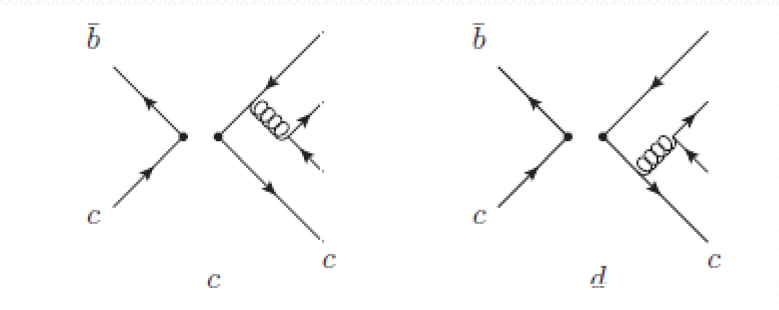
Numerical results & Discussion

- There are 6 types of diagrams contributing to B_c -> D(D*)T decays.

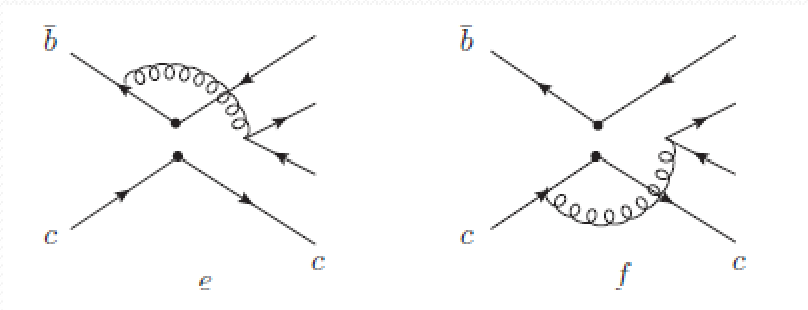
Because the tensor meson can not be produced through local $(V \pm A)$ current and $(S \pm P)$ density.



nonfactorizable emission diagrams



factorizable annihilation diagrams



nonfactorizable annihilation diagrams

- Branching ratios (unit:10^{-6}) and direct CP asymmetries (unit:%) of $B_c \to DT$ decays

Decay Modes	Class	Br	A_{CP}^{dir}
$B_c \to D^0 a_2^+$	A	$2.17^{+0.83}_{-0.71}{}^{+0.17}_{-0.17}{}^{+0.20}_{-0.18}$	$6.47^{+1.35}_{-1.15}{}^{+5.33}_{-1.59}{}^{+0.00}_{-0.74}$
$B_c \to D^0 K_2^{*+}$	A	$31.9^{+10.3}_{-8.76}{}^{+2.81}_{-2.86}{}^{+0.86}_{-0.54}$	$-0.44^{+0.13}_{-0.15}{}^{+0.10}_{-0.22}{}^{+0.10}_{-0.02}$
$B_c \to D^+ a_2^0$	A	$1.10^{+0.42}_{-0.36}{}^{+0.09}_{-0.11}{}^{-0.23}_{-0.26}$	$18.2^{+4.73}_{-3.77}{}^{+10.2}_{-4.65}{}^{+0.00}_{-2.30}$
$B_c \to D^+ K_2^{*0}$	A	$31.6^{+11.3}_{-9.69}{}^{+3.10}_{-2.13}{}^{+1.01}_{-0.63}$	0.0
$B_c \to D^+ f_2$	A	$1.51^{+0.58}_{-0.48}{}^{+0.12}_{-0.09}{}^{+0.14}_{-0.16}$	$-9.71^{+3.45}_{-3.97}{}^{+4.09}_{-5.21}{}^{+2.70}_{-1.59}$
$B_c \to D^+ f_2'$	A,P	$0.012^{+0.006}_{-0.005}{}^{+0.004}_{-0.003}{}^{+0.001}_{-0.002}$	$-47.5^{+16.9}_{-20.1}{}^{+10.2}_{-4.8}{}^{+9.7}_{-9.7}$
$B_c \to D_s^+ a_2^0$	C	$0.0047^{+0.0011}_{-0.0007}{}^{+0.0016}_{-0.0012}{}^{+0.0006}_{-0.0004}$	$-2.04^{+0.34}_{-0.37}{}^{+0.62}_{-1.29}{}^{+0.58}_{-0.28}$
$B_c \to D_s^+ \bar{K}_2^{*0}$	A	$1.90^{+0.67}_{-0.59}{}^{+0.20}_{-0.22}{}^{+0.09}_{-0.07}$	$-1.00^{+0.76}_{-0.82}{}^{+0.72}_{-0.50}{}^{+0.00}_{-0.03}$
$B_c \to D_s^+ f_2$	A,P	$1.87^{+0.43}_{-0.40}{}^{+0.45}_{-0.44}{}^{+0.06}_{-0.06}$	$2.53^{+0.51}_{-0.48}{}^{+1.45}_{-0.72}{}^{+0.10}_{-0.51}$
$B_c \to D_s^+ f_2'$	A	$40.9^{+11.9}_{-10.7}{}^{+4.32}_{-4.17}{}^{+1.20}_{-0.81}$	$-0.11^{+0.02}_{-0.02}{}^{+0.03}_{-0.06}{}^{+0.02}_{-0.00}$

- Branching ratios (unit:10^{-6}), direct CP asymmetries (unit:%) and the percentage of transverse polarizations R_T (unit:%) of $B_c \to D^*T$ decays calculated in the PQCD approach.

Decay Modes	Class	Br	A_{CP}^{dir}	R_T
$B_c \to D^{*0} a_2^+$	A	$7.34^{+2.05\,+0.99\,+0.24}_{-1.75\,-0.49\,-0.12}$	$5.02^{+0.54\,+1.34\,+0.07}_{-0.54\,-1.37\,-0.51}$	69.8
$B_c \to D^{*0} K_2^{*+}$	A	$151^{+30.1\,+18.2\,+4.69}_{-26.5\,-10.5\,-3.00}$	$-0.15^{+0.02\,+0.05\,+0.03}_{-0.02\,-0.08\,-0.06}$	82.5
$B_c \to D^{*+} a_2^0$	A	$3.75^{+1.05\,+0.49\,+0.05}_{-0.88\,-0.23\,-0.02}$	$7.94^{+1.25\,+4.07\,+0.34}_{-1.23\,-3.87\,-1.26}$	68.2
$B_c \to D^{*+} K_2^{*0}$	A	$158^{+30.6\,+16.0\,+0.00}_{-28.5\,-14.9\,-13.4}$	0.0	80.3
$B_c \to D^{*+} f_2$	A	$3.38^{+1.03\,+0.43\,+0.33}_{-0.90\,-0.22\,-0.26}$	$-2.47^{+1.01\,+1.55\,+0.82}_{-1.11\,-5.11\,-0.00}$	69.7
$B_c \to D^{*+} f_2'$	A	$0.091^{+0.025\,+0.011\,+0.009}_{-0.023\,-0.008\,-0.009}$	$-5.62^{+1.40\,+4.63\,+0.29}_{-1.55\,-6.30\,-0.00}$	45.3
$B_c \to D_s^{*+} a_2^0$	C	$0.0051^{+0.0008\,+0.0022\,+0.0006}_{-0.0006\,-0.0015\,-0.0004}$	$-3.81^{+0.24\,+0.52\,+1.09}_{-0.17\,-0.81\,-0.51}$	12.7
$B_c \to D_s^{*+} \bar{K}_2^{*0}$	A	$8.94^{+1.70\,+0.79\,+0.45}_{-1.58\,-0.92\,-0.28}$	$2.30^{+0.24\,+0.85\,+0.01}_{-0.14\,-0.45\,-0.01}$	82.0
$B_c \to D_s^{*+} f_2$	A	$3.60^{+0.42\,+0.61\,+0.11}_{-0.38\,-0.51\,-0.08}$	$2.09^{+0.15\,+0.39\,+0.10}_{-0.16\,-0.41\,-0.40}$	98.4
$B_c \to D_s^{*+} f_2'$	A	$190^{+30.5\,+19.6\,+6.14}_{-28.1\,-13.2\,-3.88}$	$-0.036^{+0.004\,+0.011\,+0.008}_{-0.003\,-0.012\,-0.001}$	89.5

Theoretical Uncertainties

- For the theoretical uncertainties in our calculations, we estimated three kinds of them:
  - The first errors are caused by the hadronic parameters of mesons' wave functions, such as the decay constants and the shape parameters of light tensor meson, charmed meson and the Bc meson.
  - The second errors are estimated from the uncertainty of $\Lambda_{QCD} = (0.25 \pm 0.05)$ GeV and the choice of the hard scales which vary from 0.8t to 1.2t, which characterize the unknown next-to-leading order QCD corrections.
  - The third error is from the uncertainties of the CKM matrix elements.
- It is easy to see that the most important theoretical uncertainty is caused by the non-perturbative hadronic parameters.

Properties

- For all considered B_c -> D(D*)T decays, the factorizable emission diagrams do not contribute, because the tensor meson can not be produced through local (V $\pm$ A) current and (S $\pm$ P) density. But these decays can get contributions from nonfactorizable and annihilation diagrams.

- In fact, most of these decays are dominant by the W annihilation diagrams (A) as classified in the tables.

(enhanced by the large CKM elements $V_{cs(d)}$)

- There are only four decay channels, which are dominated by the color suppressed (C) or penguin (P) diagrams.

Branching Ratios

- In order to reduce the effects of the choice of input parameters, we define the ratios of the branching ratios between relevant decay modes:

isospin symmetry

$$\frac{Br(B_c \to D^{(*)0} a_2^+)}{Br(B_c \to D^{(*)+} a_2^0)} \sim 2,$$

$$\frac{Br(B_c \to D^{(*)+} K_2^{*0})}{Br(B_c \to D^{(*)0} K_2^{*+})} \sim \frac{Br(B_c \to D^{(*)+} a_2^0)}{Br(B_c \to D^{(*)+} f_2)} \sim 1,$$

$$\frac{Br(B_c \to D_s^{(*)+} \bar{K}_2^{*0})}{Br(B_c \to D_s^{(*)+} f_2')} \sim \left(\frac{f^T_{K_2^*}(f_{K_2^*}) V_{cd}}{f^T_{f_2'}(f_{f_2'}) V_{cs}}\right)^2 \sim \frac{1}{20},$$

$$\frac{Br(B_c \to D^+ f_2)}{Br(B_c \to D^+ K_2^{*0})} \sim \left(\frac{1}{\sqrt{2}} \frac{f^T_{f_2} V_{cd}}{f^T_{K_2^*} V_{cs}}\right)^2 \sim \frac{1}{20},$$

$$\frac{Br(B_c \to D^{*+} f_2)}{Br(B_c \to D^{*+} K_2^{*0})} \sim \left(\frac{1}{\sqrt{2}} \frac{f_{f_2} V_{cd}}{f_{K_2^*} V_{cs}}\right)^2 \sim \frac{1}{40}.$$

SU(3) symmetry

Decay Modes	Br
$B_c \to D^0 a_2^+$	$2.17^{+0.83+0.17+0.20}_{-0.71-0.17-0.18}$
$B_c \to D^0 K_2^{*+}$	$31.9^{+10.3+2.81+0.86}_{-8.76-2.86-0.54}$
$B_c \to D^+ a_2^0$	$1.10^{+0.42+0.09-0.23}_{-0.36-0.11-0.26}$
$B_c \to D^+ K_2^{*0}$	$31.6^{+11.3+3.10+1.01}_{-9.69-2.13-0.63}$
$B_c \to D^+ f_2$	$1.51^{+0.58+0.12+0.14}_{-0.48-0.09-0.16}$
$B_c \to D^+ f_2'$	$0.012^{+0.006+0.004+0.001}_{-0.005-0.003-0.002}$
$B_c \to D_s^+ a_2^0$	$0.0047^{+0.0011+0.0016+0.0006}_{-0.0007-0.0012-0.0004}$
$B_c \to D_s^+ \bar{K}_2^{*0}$	$1.90^{+0.67+0.20+0.09}_{-0.59-0.22-0.07}$
$B_c \to D_s^+ f_2$	$1.87^{+0.43+0.45+0.06}_{-0.40-0.44-0.06}$
$B_c \to D_s^+ f_2'$	$40.9^{+11.9+4.32+1.20}_{-10.7-4.17-0.81}$

Experiment Test

- Most of the predicted branching ratios are in the order of 10^{-6} or even bigger.
- The most promising decay modes for experiment

$B_c \to D^+ K_2^{*0}$ Br $= 3 \times 10^{-5}$

$\mathcal{B}(D^+ \to K^- \pi^+ \pi^+) \sim 10\%$

$\mathcal{B}(K_2^{*0} \to K\pi) \sim 50\%$

$B_c \to D_s^+ f_2'$ Br $= 4 \times 10^{-5}$

$\mathcal{B}(D_s^+ \to K^+ K^- \pi^+) \sim 6\%$

$\mathcal{B}(f_2' \to K\bar{K}) \sim 89\%$

$B_c \to D^{*+} K_2^{*0}$ $B_c \to D_s^{*+} f_2'$ Br $\sim 10^{-4}$

Direct CP Asymmetries

- The direct CP asymmetries are all very small, because the contributions from penguin operators are too small to compare with the contributions from tree operators.
- But there is one exception

Decay Modes	Class	Br	A_{CP}^{dir}
$B_c \to D^+ f_2'$	A,P	$0.012^{+0.006\,+0.004\,+0.001}_{-0.005\,-0.003\,-0.002}$	$-47.5^{+16.9\,+10.2\,+9.7}_{-20.1\,-4.8\,-9.7}$

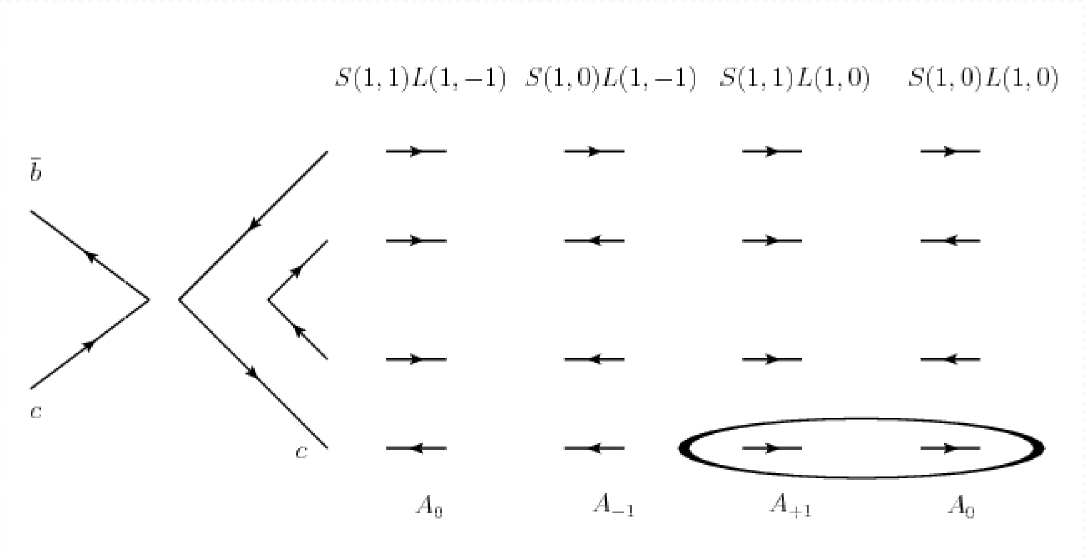
$$f_2' = f_2^q \sin\theta - f_2^s \cos\theta, \quad \text{with } f_2^q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}),\ f_2^s = s\bar{s}$$

The tree contributions from $f_2{}^q$ term are suppressed by the mixing angle , to be at the same level with penguin contributions from $f_2{}^s$ term. The interference is sizable.

（the mixing angle θ is small)

Transverse Polarization

- The percentages of the transverse polarization are large because of
  - W annihilation diagrams dominant decays
  - Tensor meson S=J=1



except

Decay Modes	Class	Br	A_{CP}^{dir}	R_T
$B_c \to D_s^{*+} a_2^0$	C	$0.0051^{+0.0008}_{-0.0006}{}^{+0.0022}_{-0.0015}{}^{+0.0006}_{-0.0004}$	$-3.81^{+0.24}_{-0.17}{}^{+0.52}_{-0.81}{}^{+1.09}_{-0.51}$	12.7

Summary

- In this work, we investigate those B_c -> D(D*)T decays in the perturbative QCD approach, based on kT factorization, where T denotes a light tensor meson.
- We find that the annihilation amplitudes are dominant in these decays, which are calculable in the pQCD approach.
- The branching ratios of many decays are in the order of 10^{-6} or even bigger, which can be detected in the ongoing experiments.

Summary

- Most of the direct CP asymmetries are very small because the penguin contributions are too small to compare with the tree contributions.

- We also predict large ratios of transverse polarizations around 70% or even bigger for those W annihilation dominant decays.

Thank you!

Backup

$$\phi_{B_c}(x,b) = \frac{f_{B_c}}{2\sqrt{2N_c}}\delta(x - m_c/m_{B_c})\exp\left[-\frac{1}{2}w^2b^2\right],$$

$$\phi_T(x) = \frac{f_T}{2\sqrt{2N_c}}\phi_\parallel(x),\ \phi_T^t = \frac{f_T^\perp}{2\sqrt{2N_c}}h_\parallel^{(t)}(x),$$
$$\phi_T^s(x) = \frac{f_T^\perp}{4\sqrt{2N_c}}\frac{d}{dx}h_\parallel^{(s)}(x),\ \phi_T^T(x) = \frac{f_T^\perp}{2\sqrt{2N_c}}\phi_\perp(x),$$
$$\phi_T^v(x) = \frac{f_T}{2\sqrt{2N_c}}g_\perp^{(v)}(x),\ \phi_T^a(x) = \frac{f_T}{8\sqrt{2N_c}}\frac{d}{dx}g_\perp^{(a)}(x).$$

$$\phi_{\parallel,\perp}(x) = 30x(1-x)(2x-1),$$
$$h_\parallel^{(t)}(x) = \frac{15}{2}(2x-1)(1-6x+6x^2),\ h_\parallel^{(s)}(x) = 15x(1-x)(2x-1),$$
$$g_\perp^{(a)}(x) = 20x(1-x)(2x-1),\ \ g_\perp^{(v)}(x) = 5(2x-1)^3.$$

$$\phi_D(x,b) = \frac{1}{2\sqrt{2N_c}} f_D\, 6x(1-x)\,[1 + C_D(1-2x)]\exp\left[\frac{-\omega^2 b^2}{2}\right],$$

with $C_D = 0.5 \pm 0.1, \omega = 0.1$ GeV and $f_D = 207$ MeV [64] for $D(\bar{D})$ meson and $C_D = 0.4 \pm 0.1, \omega = 0.2$ GeV and $f_{D_s} = 241$ MeV [64] for $D_s(\bar{D}_s)$ meson. For D^* meson, we take the same model as the D meson and determine the decay constant by using the following relation based on heavy quark effective theory (HQET) [65].

$$f_{D^*_{(s)}} = \sqrt{\frac{m_{D_{(s)}}}{m^*_{D_{(s)}}}}\, f_{D_{(s)}}$$

$$\epsilon^{\mu\nu}(\pm 2) \equiv \epsilon(\pm 1)^{\mu}\epsilon(\pm 1)^{\nu},$$
$$\epsilon^{\mu\nu}(\pm 1) \equiv \sqrt{\frac{1}{2}}\left[\epsilon(\pm 1)^{\mu}\epsilon(0)^{\nu} + \epsilon(0)^{\mu}\epsilon(\pm 1)^{\nu}\right],$$
$$\epsilon^{\mu\nu}(0) \equiv \sqrt{\frac{1}{6}}\left[\epsilon(+1)^{\mu}\epsilon(-1)^{\nu} + \epsilon(-1)^{\mu}\epsilon(+1)^{\nu}\right] + \sqrt{\frac{2}{3}}\epsilon(0)^{\mu}\epsilon(0)^{\nu}.$$

$$\epsilon^{\mu}(0) = \frac{1}{\sqrt{2}m_T}(k_0 + k_3,\, k_0 - k_3,\, 0,\, 0),\ \epsilon^{\mu}(\pm 1) = \frac{1}{\sqrt{2}}(0,\, 0,\, 1,\, \pm i),$$

$$\epsilon_T(\lambda) = \frac{1}{m_B}\epsilon_{\mu\nu}(\lambda)P_B^{\nu},$$

$$\epsilon_{T\mu}(\pm 2) = 0, \quad \epsilon_{T\mu}(\pm 1) = \frac{\epsilon(0)\cdot P_B\epsilon_{\mu}(\pm 1)}{\sqrt{2}m_B}, \quad \epsilon_{T\mu}(0) = \frac{\sqrt{\frac{2}{3}}\epsilon(0)\cdot P_B\epsilon(0)}{m_B}.$$

$$\langle T(P,\lambda)|V_\mu|0\rangle = a\epsilon^{*(\lambda)}_{\mu\nu}P^\nu + b\epsilon^{*(\lambda)\nu}_{\nu}P_\mu = 0,$$

$$\langle T(P,\lambda)|A_\mu|0\rangle = \varepsilon_{\mu\nu\rho\sigma}P^\nu\epsilon^{\rho\sigma *}_{(\lambda)} = 0,$$

$$\epsilon^{(\lambda)}_{\alpha\beta}P^\beta = 0$$

$$\epsilon^{\mu\nu}(\lambda) = \epsilon^{\nu\mu}(\lambda),$$

$$\mathcal{A}(B_c \to a_2^+ D^0) = \frac{G_F}{\sqrt{2}} \{V_{ub}^* V_{ud} \mathcal{M}_{enf}^{LL} C_1 + V_{cb}^* V_{cd} (\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1)$$
$$-V_{tb}^* V_{td} [\mathcal{M}_{enf}^{LL} (C_3 + C_9) + \mathcal{M}_{enf}^{LR} (C_5 + C_7) + \mathcal{M}_{af}^{LL} (a_4 + a_{10})$$
$$+\mathcal{M}_{af}^{SP} (a_6 + a_8) + \mathcal{M}_{anf}^{LL} (C_3 + C_9) + \mathcal{M}_{anf}^{LR} (C_5 + C_7)]\} ,$$

$$\mathcal{A}(B_c \to K_2^{*+} D^0) = \mathcal{A}(B_c \to a_2^+ D^0) \,|_{V_{ud} \to V_{us}, V_{cd} \to V_{cs}, V_{td} \to V_{ts}, a_2^+ \to K_2^{*+}} ,$$

$$\mathcal{A}(B_c \to a_2^0 D^+) = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} \{V_{ub}^* V_{ud} \mathcal{M}_{enf}^{LL} C_2 - V_{cb}^* V_{cd} (\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1)$$
$$-V_{tb}^* V_{td} [\mathcal{M}_{enf}^{LL} (-C_3 + 3a_{10}/2) + \mathcal{M}_{enf}^{LR} (-C_5 + C_7/2)$$
$$+\mathcal{M}_{enf}^{SP} (3C_8/2) - \mathcal{M}_{af}^{LL} (a_4 + a_{10}) - \mathcal{M}_{af}^{SP} (a_6 + a_8)$$
$$-\mathcal{M}_{anf}^{LL} (C_3 + C_9) - \mathcal{M}_{anf}^{LR} (C_5 + C_7)]\} ,$$

$$
\begin{aligned}
\mathcal{A}(B_c \to K_2^{*0} D^+) \;=\; & \frac{G_F}{\sqrt{2}} \{V_{cb}^* V_{cs} (\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1) \\
& - V_{tb}^* V_{ts} [\mathcal{M}_{enf}^{LL} (C_3 - C_9/2) + \mathcal{M}_{enf}^{LR} (C_5 - C_7/2) + \mathcal{M}_{af}^{LL} (a_4 + a_{10} \\
& + \mathcal{M}_{af}^{SP} (a_6 + a_8) + \mathcal{M}_{anf}^{LL} (C_3 + C_9) + \mathcal{M}_{anf}^{LR} (C_5 + C_7)]\} ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}(B_c \to f_2^q D^+) \;=\; & \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} \{V_{ub}^* V_{ud} \mathcal{M}_{enf}^{LL} C_2 + V_{cb}^* V_{cd} (\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1) \\
& - V_{tb}^* V_{td} [\mathcal{M}_{enf}^{LL} (C_3 + 2C_4 - C_9/2 + C_{10}/2) + \mathcal{M}_{enf}^{LR} (C_5 - C_7/2) \\
& + \mathcal{M}_{enf}^{SP} (2C_6 + C_8/2) + \mathcal{M}_{af}^{LL} (a_4 + a_{10}) + \mathcal{M}_{af}^{SP} (a_6 + a_8) \\
& + \mathcal{M}_{anf}^{LL} (C_3 + C9) + \mathcal{M}_{anf}^{LR} (C_5 + C_7)]\} ,
\end{aligned}
$$

$$\mathcal{A}(B_c \to f_2^s D^+) = \frac{G_F}{\sqrt{2}} \{-V_{tb}^* V_{td}[\mathcal{M}_{enf}^{LL}(C_4 - C_{10}/2) + \mathcal{M}_{enf}^{SP}(C_6 - C_8/2)]\} ,$$

$$\mathcal{A}(B_c \to a_2^0 D_s^+) = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} \{V_{ub}^* V_{us} \mathcal{M}_{enf}^{LL} C_2 - V_{tb}^* V_{ts}[\mathcal{M}_{enf}^{LL} 3C_{10}/2 + \mathcal{M}_{enf}^{SP} 3C_8/2]\} ,$$

$$\mathcal{A}(B_c \to \bar{K}_2^{*0} D_s^+) = \frac{G_F}{\sqrt{2}} \{V_{cb}^* V_{cd}(\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1) - V_{tb}^* V_{td}[\mathcal{M}_{af}^{LL}(a_4 + a_{10}) + \mathcal{M}_{af}^{SP}(a_6 + a_8) + \mathcal{M}_{anf}^{LL}(C_3 + C_9) + \mathcal{M}_{anf}^{LR}(C_5 + C_7)]\} ,$$

$$\mathcal{A}(B_c \to f_2^q D_s^+) = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{2}} \{V_{ub}^* V_{us} \mathcal{M}_{enf}^{LL} C_2 - V_{tb}^* V_{ts}[\mathcal{M}_{enf}^{LL}(2C_4 + C_{10}/2) + \mathcal{M}_{enf}^{SP}(2C_6 + C_8/2)]\} ,$$

$$\mathcal{A}(B_c \to f_2^s D_s^+) = \frac{G_F}{\sqrt{2}}\{V_{cb}^* V_{cs}(\mathcal{M}_{af}^{LL} a_1 + \mathcal{M}_{anf}^{LL} C_1) - V_{tb}^* V_{ts}[\mathcal{M}_{enf}^{LR}(C_5 - C_7/2)$$
$$+\mathcal{M}_{enf}^{LL}(C_3 + C_4 - C_9/2 - C_{10}/2) + \mathcal{M}_{enf}^{SP}(C_6 - C_8/2)$$
$$+\mathcal{M}_{af}^{LL}(a_4 + a_{10}) + \mathcal{M}_{af}^{SP}(a_6 + a_8) + \mathcal{M}_{anf}^{LL}(C_3 + C_9)$$
$$+\mathcal{M}_{anf}^{LR}(C_5 + C_7)]\},$$

$$\mathcal{A}(B_c \to D^{(*)} f_2) = \mathcal{A}(B_c \to D^{(*)} f_2^q)\cos\theta + \mathcal{A}(B_c \to D^{(*)} f_2^s)\sin\theta,$$
$$\mathcal{A}(B_c \to D^{(*)} f_2') = \mathcal{A}(B_c \to D^{(*)} f_2^q)\sin\theta - \mathcal{A}(B_c \to D^{(*)} f_2^s)\cos\theta,$$

$$a_1 = C_2 + C_1/3, \qquad a_2 = C_1 + C_2/3,$$
$$a_i = C_i + C_{i+1}/3,\ i = 3,5,7,9, \quad a_j = C_j + C_{j-1}/3,\ j = 4,6,8,10.$$

$$\Gamma(B_c \to DT) = \frac{|\overrightarrow{P}|}{8\pi m_{B_c}^2} |\mathcal{A}(B_c \to DT)|^{\,2}$$

$$|\overrightarrow{P}| = \frac{1}{2m_{B_c}} \sqrt{\left[m_{B_c}^2 - (m_D + m_T)^2\right] \left[m_{B_c}^2 - (m_D - m_T)^2\right]}.$$

The amplitudes of $B_c \to D^*T$ decay can be decomposed as

$$\mathcal{A}(\epsilon_D, \epsilon_T) = i\mathcal{A}^N + i(\epsilon_D^{T*} \cdot \epsilon_T^{T*})\mathcal{A}^s + (\epsilon_{\mu\nu\alpha\beta} n^\mu v^\nu \epsilon_D^{T*\alpha} \epsilon_T^{T*\beta})\mathcal{A}^p,$$

$$\Gamma(B_c \to D^*T) = \frac{|\overrightarrow{P}|}{8\pi m_B^2} (|\ \mathcal{A}^N\ |^2 + 2(|\ \mathcal{A}^s\ |^2 + |\ \mathcal{A}^p\ |^2)).$$

$$R_T = \frac{2(|\mathcal{A}^s|^2 + |\mathcal{A}^p|^2)}{|\mathcal{A}^N|^2 + 2(|\mathcal{A}^s|^2 + |\mathcal{A}^p|^2)}.$$

$$\Lambda_{\overline{MS}}^{f=4} = 0.25,\ m_{B_c} = 6.286,\ f_{B_c} = 0.489,$$

$$\tau_{B_c} = 0.46ps,\ \omega_{B_c} = 0.6,\ m_b = 4.8,\ m_c = 1.5.$$

Tensor(mass(MeV))	f_T(MeV)	$f_T^{\perp}$(MeV)
$f_2(1270)$	102 ± 6	117 ± 25
$f_2'(1525)$	126 ± 4	65 ± 12
$a_2(1320)$	107 ± 6	105 ± 21
$K_2^*(1430)$	118 ± 5	77 ± 14

$B_c \to D_s^{*+} f_2'$

↳ $D_s{}^+ \gamma$

$B_c \to D^{*+} K_2^{*0}$ Br $\sim 10^{-3}$

↳ $D^0 \pi^+ \to K^- \pi^+ \pi^+$