用QCD求和规则研究核物质中重介子性质

王志刚

华北电力大学物理系
保定 071003

wangzgyiti@yahoo.com.cn.

核物质中重介子的性质

- 核物质中的凝聚
- 核物质中算符乘积展开
- 核物质中重介子质量
- 结论

1 核物质中的凝聚

在核物质密度比较低时，可采取费米气近似，对核物质中的凝聚$\langle\mathcal{O}\rangle_{\rho_N}$ 做如下展开，也就是**对核物质密度做线性近似**，

$$\langle\mathcal{O}\rangle_{\rho_N} = \langle\mathcal{O}\rangle + \frac{\rho_N}{2M_N}\langle\mathcal{O}\rangle_N\,, \tag{1}$$

$$\langle\mathcal{O}\rangle_{\rho_N} = \langle\Psi_0|O|\Psi_0\rangle = \langle 0|\mathcal{O}|0\rangle + \frac{\rho_N}{2M_N}\langle N|\mathcal{O}|N\rangle\,, \tag{2}$$

$|\Psi_0\rangle$ 核物质，$|0\rangle$真空，ρ_N 核物质密度，M_N质子或中子质量，$|N\rangle$单核子态。

在核物质中，常见凝聚如夸克凝聚、胶子凝聚，混合凝聚，会有不同程度修正，例如：
$\langle\bar{q}q\rangle_{\rho_N} = \langle\bar{q}q\rangle + \frac{\sigma_N}{m_u+m_d}\rho_N$, $\langle\frac{\alpha_s GG}{\pi}\rangle_{\rho_N} = \langle\frac{\alpha_s GG}{\pi}\rangle - (0.65\pm 0.15)\,\mathrm{GeV}\rho_N$。

此外，出现许多新凝聚项，这些项来表征核物质的性质，
$\langle q^\dagger q\rangle_{\rho_N} = \frac{3}{2}\rho_N$，$\langle s^\dagger s\rangle_{\rho_N} = 0$，$\langle q^\dagger iD_0 q\rangle_{\rho_N} = 0.18\,\mathrm{GeV}\rho_N$，
$\langle q^\dagger iD_0 iD_0 q\rangle_{\rho_N} + \frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N} = 0.031\,\mathrm{GeV}^2\rho_N$，$\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N} = -0.33\,\mathrm{GeV}^2\rho_N$。

核物质中的关联函数

基于核物质中夸克凝聚、胶子凝聚、混合凝聚等凝聚的展开形式，**对核物质中的关联函数做相似展开**。例如，核物质中矢量流$J_\mu(x)$的关联函数$\Pi_{\mu\nu}(q)$

$$\Pi_{\mu\nu}(q) \;=\; \Pi^0_{\mu\nu}(q) + \frac{\rho_N}{2M_N} T^N_{\mu\nu}(q)\,, \tag{3}$$

where

$$\begin{aligned} \Pi^0_{\mu\nu}(q) &= i\int d^4x e^{iq\cdot x}\langle 0|T\left\{J_\mu(x)J^\dagger_\nu(0)\right\}|0\rangle\,, \\ T^N_{\mu\nu}(\omega,\boldsymbol{q}) &= i\int d^4x e^{iq\cdot x}\langle N(p)|T\left\{J_\mu(x)J^\dagger_\nu(0)\right\}|N(p)\rangle\,, \end{aligned} \tag{4}$$

the $|N(p)\rangle$ denotes the isospin and spin averaged static nucleon state with the four-momentum $p=(M_N,0)$, and normalized as $\langle N(\boldsymbol{p})|N(\boldsymbol{p}')\rangle = (2\pi)^3 2p_0\delta^3(\boldsymbol{p}-\boldsymbol{p}')$. The $T^N_{\mu\nu}(q)$ happen to be the current-nucleon forward scattering amplitudes.

此方法突出的优点是把真空部分$\Pi^0_{\mu\nu}(q)$和核物质诱导部分$T^N_{\mu\nu}(\omega,\mathrm{q})$干净分开，彼此没有影响。微扰项贡献一般来说是主要的，体现在真空部分$\Pi^0_{\mu\nu}(q)$，不会对核物质诱导部分产生影响。

核物质中的关联函数的算符乘积展开

We carry out the operator product expansion to the condensates up to dimension-5 at the large space-like region in the nuclear matter, and obtain the analytical expressions of the correlation functions at the level of quark-gluon degree's of freedom,

$$\Pi_{\mu\nu}(q_0,\vec{q}) \;=\; \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\sum_n C_n(q_0,\vec{q})\langle\mathcal{O}_n\rangle_{\rho_N}+\cdots\,, \tag{5}$$

where the $C_n(q_0,\vec{q})$ are the Wilson coefficients, the in-medium condensates $\langle\mathcal{O}_n\rangle_{\rho_N}=\langle\mathcal{O}_n\rangle+\frac{\rho_N}{2M_N}\langle\mathcal{O}_n\rangle_N$ at the low nuclear density, the $\langle\mathcal{O}_n\rangle$ and $\langle\mathcal{O}_n\rangle_N$ denote the vacuum condensates and nuclear matter induced condensates, respectively. Then we collect the terms proportional to ρ_N (or the nuclear matter induced condensates),

$$T^N_{\mu\nu}(\omega,\vec{q}) \;=\; \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\sum_n C_n(\omega,\vec{q})\langle\mathcal{O}_n\rangle_N+\cdots\,, \tag{6}$$

$T^N_{\mu\nu}(\omega,\vec{q})$ 完全由核物质诱导而产生，表征核物质的效应。事实上，强子层次上和夸克层次上，这个关联函数都可以计算，可以彼此验证。

2 核物质中重介子的性质

The $J_\mu(x)$ denotes the isospin averaged currents $\eta_5(x)$, $\eta_0(x)$, $\eta_\mu(x)$ and $\eta_{5\mu}(x)$,

$$\begin{aligned}
\eta_5(x) &= \eta_5^\dagger(x) = \frac{\bar{c}(x)i\gamma_5 q(x) + \bar{q}(x)i\gamma_5 c(x)}{2}\,, \\
\eta_0(x) &= \eta_0^\dagger(x) = \frac{\bar{c}(x)q(x) + \bar{q}(x)c(x)}{2}\,, \\
\eta_\mu(x) &= \eta_\mu^\dagger(x) = \frac{\bar{c}(x)\gamma_\mu q(x) + \bar{q}(x)\gamma_\mu c(x)}{2}\,, \\
\eta_{5\mu}(x) &= \eta_{5\mu}^\dagger(x) = \frac{\bar{c}(x)\gamma_\mu\gamma_5 q(x) + \bar{q}(x)\gamma_\mu\gamma_5 c(x)}{2}\,,
\end{aligned} \tag{7}$$

which interpolate the pseudoscalar scalar, scalar, vector and axialvector mesons D, D_0, D^* and D_1, respectively, the q denotes the u or d quark.

下面以矢量和轴矢介子为例说明问题

We can decompose the correlation functions $T^N_{\mu\nu}(\omega, \boldsymbol{q})$ as

$$T^N_{\mu\nu}(\omega, \boldsymbol{q}) = T_N(\omega, \boldsymbol{q})\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \Pi^0_N(\omega, \boldsymbol{q})\frac{q_\mu q_\nu}{q^2}\,, \tag{8}$$

according to Lorentz covariance, where the $T_N(\omega, \boldsymbol{q})$ denotes the contributions from the vector and axialvector mesons, and the $T^0_N(\omega, \boldsymbol{q})$ denotes the contributions from the scalar and pseudoscalar mesons.

In the limit of the 3-vector $\boldsymbol{q} \to \mathbf{0}$, the correlation functions $T_N(\omega, \boldsymbol{q})$ can be related to the D^*N and D_1N scattering T-matrixes, i.e. $\mathcal{T}_{D^*N}(M_{D^*}, 0) = 8\pi(M_N + M_{D^*})a_{D^*}$ and $\mathcal{T}_{D_1N}(M_{D_1}, 0) = 8\pi(M_N + M_{D_1})a_{D_1}$, where the a_{D^*} and a_{D_1} are the D^*N and D_1N scattering lengths, respectively. Near the pole positions of the D^* and D_1 mesons, the phenomenological spectral densities $\rho(\omega, 0)$ can be parameterized with three unknown parameters a, b and c,

$$\rho(\omega, 0) = -\frac{f^2_{D^*/D_1} M^2_{D^*/D_1}}{\pi} \mathrm{Im}\left[\frac{\mathcal{T}_{D^*/D_1N}(\omega, \mathbf{0})}{(\omega^2 - M^2_{D^*/D_1} + i\varepsilon)^2}\right] + \cdots , \tag{9}$$

$$= a\,\frac{d}{d\omega^2}\delta(\omega^2 - M^2_{D^*/D_1}) + b\,\delta(\omega^2 - M^2_{D^*/D_1}) + c\,\delta(\omega^2 - s_0)\,, \tag{10}$$

the terms denoted by $\cdots$ represent the continuum contributions. The first term denotes the double-pole term, and corresponds to the on-shell (i.e. $\omega^2 = M^2_{D^*/D_1}$) effects of the T-matrixes,

$$a = -8\pi(M_N + M_{D^*/D_1})a_{D^*/D_1} f^2_{D^*/D_1} M^2_{D^*/D_1}\,, \tag{11}$$

and related with the mass-shifts of the D^* and D_1 mesons through the relation

$$\delta M_{D^*/D_1} = -\frac{\rho_N}{4M_N f^2_{D^*/D_1} M^3_{D^*/D_1}} a\,; \tag{12}$$

the second term denotes the single-pole term, and corresponds to the off-shell (i.e. $\omega^2 \neq M^2_{D^*/D_1}$) effects of the T-matrixes; and the third term denotes the continuum term or the remaining effects, where the s_0 is the continuum threshold.

In the limit $\omega \to 0$, the $T_N(\omega, \mathbf{0})$ is equivalent to the Born term $T^{\rm Born}_{D^*/D_1 N}(\omega, \mathbf{0})$. We take into account the Born term at the phenomenological side,

$$T_N(\omega^2) \;=\; T^{\rm Born}_{D^*/D_1 N}(\omega^2) + \frac{a}{(M^2_{D^*/D_1} - \omega^2)^2} + \frac{b}{M^2_{D^*/D_1} - \omega^2} + \frac{c}{s_0 - \omega^2}\,, \tag{13}$$

with the constraint

$$\frac{a}{M^4_{D^*/D_1}} + \frac{b}{M^2_{D^*/D_1}} + \frac{c}{s_0} \;=\; 0\,. \tag{14}$$

波恩项也就是

$$
\begin{aligned}
D^*(-q)N(p) &\rightarrow \Lambda_c/\Sigma_c(p-q) \rightarrow D^*(-q)N(p) \\
D_1(-q)N(p) &\rightarrow \Lambda_c/\Sigma_c(p-q) \rightarrow D_1(-q)N(p)
\end{aligned} \tag{15}
$$

The contributions from the intermediate spin-$\frac{3}{2}$ charmed baryons are zero in the soft-limit $q_\mu \rightarrow 0$, and we only take into account the intermediate spin-$\frac{1}{2}$ charmed baryons in calculating the Born terms, and parameterize the hadronic matrix elements as

$$
\begin{aligned}
\langle \Lambda_c/\Sigma_c(p-q)|D^*(-q)N(p)\rangle &= \bar{U}_{\Lambda_c/\Sigma_c}(p-q)\left[g_{\Lambda_c/\Sigma_c D^* N}\not{\epsilon} + i\frac{g^T_{\Lambda_c/\Sigma_c D^* N}}{M_N + M_{\Lambda_c/\Sigma_c}}\sigma^{\alpha\beta}\epsilon_\alpha q_\beta\right]U_N(p)\,, \\
\langle \Lambda_c/\Sigma_c(p-q)|D_1(-q)N(p)\rangle &= \bar{U}_{\Lambda_c/\Sigma_c}(p-q)\left[g_{\Lambda_c/\Sigma_c D_1 N}\not{\epsilon} + i\frac{g^T_{\Lambda_c/\Sigma_c D_1 N}}{M_N + M_{\Lambda_c/\Sigma_c}}\sigma^{\alpha\beta}\epsilon_\alpha q_\beta\right]\gamma_5 U_N(p)\,,
\end{aligned} \tag{16}
$$

where the U_N and $\bar{U}_{\Lambda_c/\Sigma_c}$ are the Dirac spinors of the nucleon and the charmed baryons Λ_c/Σ_c, respectively; the $g_{\Lambda_c/\Sigma_c D^* N}$, $g_{\Lambda_c/\Sigma_c D_1 N}$, $g^T_{\Lambda_c/\Sigma_c D^* N}$ and $g^T_{\Lambda_c/\Sigma_c D_1 N}$ are the strong coupling constants in the vertexes. In the limit $q_\mu \rightarrow 0$, the strong coupling constants $g^T_{\Lambda_c/\Sigma_c D^* N}$ and $g^T_{\Lambda_c/\Sigma_c D_1 N}$ have no contributions.

We draw the Feynman diagrams, calculate the Born terms and obtain the results

$$
\begin{aligned}
T^{\rm Born}_{D^*N}(\omega,\mathbf{0}) &= \frac{2f^2_{D^*}M^2_{D^*}M_N(M_H+M_N)g^2_{HD^*N}}{\left[\omega^2-(M_H+M_N)^2\right]\left[\omega^2-M^2_{D^*}\right]^2}\,,\\
T^{\rm Born}_{D_1N}(\omega,\mathbf{0}) &= \frac{2f^2_{D_1}M^2_{D_1}M_N(M_H-M_N)g^2_{HD_1N}}{\left[\omega^2-(M_H-M_N)^2\right]\left[\omega^2-M^2_{D_1}\right]^2}\,,
\end{aligned}
\tag{17}
$$

where the H means either Λ_c^+, Σ_c^+, Σ_c^{++} or Σ_c^0. The masses $M_{\Lambda_c}=2.286\,\mathrm{GeV}$ and $M_{\Sigma_c}=2.454\,\mathrm{GeV}$ from the Particle Data Group, we can take $M_H\approx 2.4\,\mathrm{GeV}$ as the average value. On the other hand, there are no inelastic channels for the $\bar{D}^*N$ and $\bar{D}_1N$ interactions in the case of the charmed mesons $\bar{c}q$.

Two QCD sum rules:

$$a\left\{\frac{1}{M^2}e^{-\frac{M_{D^*}^2}{M^2}}-\frac{s_0}{M_{D^*}^4}e^{-\frac{s_0}{M^2}}\right\}+b\left\{e^{-\frac{M_{D^*}^2}{M^2}}-\frac{s_0}{M_{D^*}^2}e^{-\frac{s_0}{M^2}}\right\}+\frac{2f_{D^*}^2M_{D^*}^2M_N(M_H+M_N)g_{HD^*N}^2}{(M_H+M_N)^2-M_{D^*}^2}$$
$$\left\{\left[\frac{1}{(M_H+M_N)^2-M_{D^*}^2}-\frac{1}{M^2}\right]e^{-\frac{M_{D^*}^2}{M^2}}-\frac{1}{(M_H+M_N)^2-M_{D^*}^2}e^{-\frac{(M_H+M_N)^2}{M^2}}\right\}=\left\{-\frac{m_c\langle\bar{q}q\rangle_N}{2}\right.$$
$$\left.-\frac{2\langle q^\dagger iD_0q\rangle_N}{3}+\frac{m_c^2\langle q^\dagger iD_0q\rangle_N}{M^2}+\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle_N}{3M^2}+\frac{8m_c\langle\bar{q}iD_0iD_0q\rangle_N}{3M^2}-\frac{m_c^3\langle\bar{q}iD_0iD_0q\rangle_N}{M^4}\right\}e^{-\frac{m}{M}}$$
$$-\frac{1}{24}\langle\frac{\alpha_sGG}{\pi}\rangle_N\int_0^1dx\left(1+\frac{\widetilde{m}_c^2}{2M^2}\right)e^{-\frac{\widetilde{m}_c^2}{M^2}}+\frac{1}{48M^2}\langle\frac{\alpha_sGG}{\pi}\rangle_N\int_0^1\frac{1-x}{x}\left(\widetilde{m}_c^2-\frac{\widetilde{m}_c^4}{M^2}\right)e^{-\frac{\widetilde{m}_c^2}{M^2}}\,,\quad(18$$

$$
\begin{aligned}
&a\left\{\frac{1}{M^2}e^{-\frac{M_{D_1}^2}{M^2}}-\frac{s_0}{M_{D_1}^4}e^{-\frac{s_0}{M^2}}\right\}+b\left\{e^{-\frac{M_{D_1}^2}{M^2}}-\frac{s_0}{M_{D_1}^2}e^{-\frac{s_0}{M^2}}\right\}+\frac{2f_{D_1}^2M_{D_1}^2M_N(M_H-M_N)g_{HD_1N}^2}{(M_H-M_N)^2-M_{D_1}^2}\\
&\left\{\left[\frac{1}{(M_H-M_N)^2-M_{D_1}^2}-\frac{1}{M^2}\right]e^{-\frac{M_{D_1}^2}{M^2}}-\frac{1}{(M_H-M_N)^2-M_{D_1}^2}e^{-\frac{(M_H-M_N)^2}{M^2}}\right\}=\left\{\frac{m_c\langle\bar{q}q\rangle_N}{2}\right.\\
&\left.-\frac{2\langle q^\dagger iD_0q\rangle_N}{3}+\frac{m_c^2\langle q^\dagger iD_0q\rangle_N}{M^2}-\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle_N}{3M^2}-\frac{8m_c\langle\bar{q}iD_0iD_0q\rangle_N}{3M^2}+\frac{m_c^3\langle\bar{q}iD_0iD_0q\rangle_N}{M^4}\right\}e^{-\frac{m}{M}}\\
&-\frac{1}{24}\langle\frac{\alpha_sGG}{\pi}\rangle_N\int_0^1dx\left(1+\frac{\widetilde{m}_c^2}{2M^2}\right)e^{-\frac{\widetilde{m}_c^2}{M^2}}+\frac{1}{48M^2}\langle\frac{\alpha_sGG}{\pi}\rangle_N\int_0^1\frac{1-x}{x}\left(\widetilde{m}_c^2-\frac{\widetilde{m}_c^4}{M^2}\right)e^{-\frac{\widetilde{m}_c^2}{M^2}},\quad(19
\end{aligned}
$$

where $\widetilde{m}_c^2=\frac{m_c^2}{x}$.

对这两个求和规则用$\frac{d}{d\frac{1}{M^2}}$求导，得到另外两个求和规则，可以消去参数b，这样得到质量修正$\delta M_{D^*/D_1}$ 和散射长度a_{D^*/D_1}.

- **质量修正和散射长度随波恩项耦合常数的变化。**

g^2	0	10	20	30	40	50
δM_{D^*} (MeV)	-75	-72	-70	-67	-65	-62
δM_{B^*} (MeV)	-382	-381	-380	-380	-379	-378
δM_{D_1} (MeV)	70	71	73	74	76	78
δM_{B_1} (MeV)	262	263	264	265	266	267
a_{D^*N} (fm)	-1.13	-1.09	-1.05	-1.02	-0.98	-0.94
a_{B^*N} (fm)	-7.20	-7.18	-7.17	-7.15	-7.14	-7.13
a_{D_1N} (fm)	1.11	1.14	1.16	1.19	1.21	1.24
a_{B_1N} (fm)	5.00	5.02	5.04	5.05	5.07	5.09

具体计算时，波恩项的耦合常数的值，来自光锥QCD求和规则，即$g \approx 3.86$，得到

质量修正 $\delta M_{D^*} = -71\,\mathrm{MeV}$，$\delta M_{B^*} = -380\,\mathrm{MeV}$，$\delta M_{D_1} = 72\,\mathrm{MeV}$，$\delta M_{B_1} = 264\,\mathrm{MeV}$，**和散射长度** $a_{D^*N} = -1.07\,\mathrm{fm}$，$a_{B^*N} = -7.17\,\mathrm{fm}$，$a_{D_1N} = 1.15\,\mathrm{fm}$，$a_{B_1N} = 5.03\,\mathrm{fm}$。

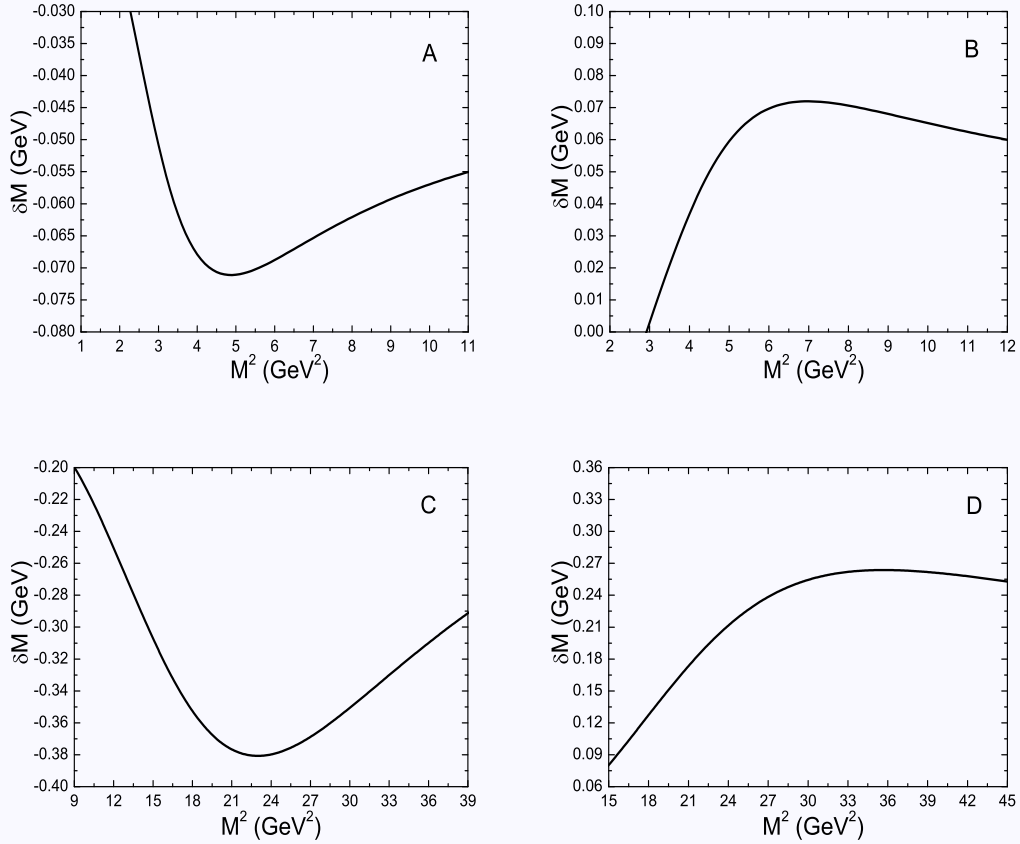

Figure 1: The mass-shifts δM versus the Borel parameter M^2, the A, B, C and D denote the D^* D_1, B^* and B_1 mesons, respectively.

需要说明，如果不分离真空部分和核物质诱导部分，整体上参数化强子谱密度

$$\frac{\mathrm{Im}\Pi(\omega,0)}{\pi} = F_+\delta(\omega - M_+) - F_-\delta(\omega + M_-)\,, \tag{20}$$

where $M_\pm = M_{D^*/D_1} \pm \delta M_{D^*/D_1}$ and $F_\pm = F \pm \delta F$, the mass center M_{D^*/D_1} and the mass splitting $\delta M_{D^*/D_1}$ can also be obtained.

但得到的质量修正和本文预言，即使在定性上也不符合，谁对谁错，尚无结论。

3 结论

- 我们比较系统的计算了核物质中重介子的质量修正，优点在于把关联函数清楚分离为真空部分和核物质诱导部分。
- 如果不分离真空部分和核物质诱导部分，由于真空部分贡献很大，会出现大数吃掉小数现象，核物质影响不明显。