

# *Scalar & Pseudoscalar Higgs Couplings with Nucleons*

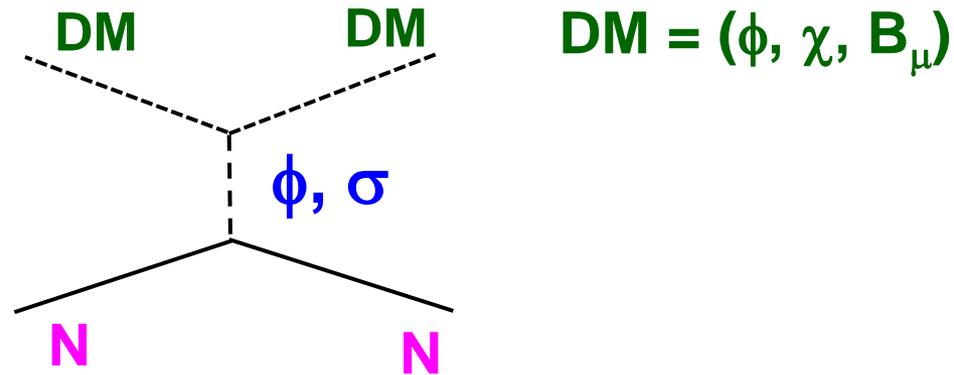
**Hai-Yang Cheng**  
**Academia Sinica**

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Interactions of dark matter with nucleons mediated by scalar Higgs  $\phi$  or pseudoscalar  $\sigma$  can be probed by studying spin-independent & spin-dependent cross sections

# History

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- Shifman, Vainstein, Zakharov ('78) were the first to study Higgs-nucleon couplings  $g_{\phi NN}$  and showed that it is dominated by heavy quarks
- T. P. Cheng ('88), HYC ('89)  $\Rightarrow g_{\phi NN}$  dominated by s quark
- HYC ('89)  $\Rightarrow$  pseudoscalar Higgs coupling with nucleons studied in chiral & large  $N_c$  limits
- Calculations of higher order terms in ChPT available in 90's  
Gasser, Leutwyler, Sainio ('91); Jenkins, Manohar ('92);  
Borasoy, Meissner ('96,'97)
- Intense lattice studies after 2008 on various sigma terms and strange quark fraction in the nucleon
- Many new determinations of pion-nucleon sigma terms from  $\pi$ -N scattering data since 2000
- Study of SU(3) breaking effect on the determination of  $g_A^8$  &  $\Delta s$  in recent years

## Scalar Higgs

$$g_{\phi NN} = (\sqrt{2}G_F)^{1/2} \sum \langle N | \zeta_q m_q \bar{q}q | N \rangle$$

$\zeta_q=1$  in SM, but not necessarily so in BSM. Relevant nucleon matrix elements are related to the trace of energy-momentum tensor  $\Theta^\mu_\mu$   
 Under heavy quark expansion  $m_h \bar{q}_h q_h \rightarrow - (2\alpha_s/8\pi)GG$  SVZ ('78)

$$\begin{aligned} \Theta^\mu_\mu &= m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum m_h \bar{q}_h q_h + (\beta/4\alpha_s)GG \\ &\rightarrow m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s - (9\alpha_s/8\pi)GG \end{aligned}$$

Under light quark expansion, baryon mass is given by

$$m_B = m_0 + \sum_q b_q m_q + \sum_q c_q m_q^{3/2} + \sum_q d_q m_q^2 \ln \frac{m_q}{m_0} + \sum_q e_q m_q^2 + \mathcal{O}(m_q^3)$$

To first order, baryon masses can be expressed in terms of  $B_u = \langle p | \bar{u}u | p \rangle$ ,  $B_d = \langle p | \bar{d}d | p \rangle$ ,  $B_s = \langle p | \bar{s}s | p \rangle$  with SU(3) symmetry

$$B_u - B_s = \frac{2(m_\Xi - m_N)}{2m_s - m_u - m_d} = 3.94 \quad B_d - B_s = \frac{2(m_\Sigma - m_N)}{2m_s - m_u - m_d} = 2.64$$

To determine  $B_u$ ,  $B_d$ ,  $B_s$ , we define several quantities:

◆ strange quark content of the proton:  $y=2B_s/(B_u+B_d)$

◆ pion-nucleon sigma term:  $\sigma_{\pi N} \equiv \hat{m} \langle N | \bar{u}u + \bar{d}d | N \rangle$ ,  $\hat{m} = (m_u + m_d)/2$

◆ sigma term  $\sigma_0$ :  $\sigma_0 \equiv \hat{m} \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle$

◆ quark sigma term:  $\sigma_q = m_q \langle p | \bar{q}q | p \rangle$

$\sigma_{\pi N}$  &  $\sigma_0$  are related by  $\sigma_{\pi N} = \sigma_0 / (1-y)$  and  $\sigma_0$  can be fixed by baryon masses to be  $\sim 24$  MeV.

In 80's  $\sigma_{\pi N} = 55 \sim 60$  MeV  $\Rightarrow$  large strange quark content  $y \sim 0.5-0.6$   
and  $\sigma_u \sim 20$  MeV,  $\sigma_d \sim 33$  MeV,  $\sigma_s \sim 390$  MeV

Strange quark content  $y$  and sigma terms  $\sigma_{\pi N}$  and  $\sigma_s$  are reduced in 90's:

1. higher order expansion terms in  $m_q^{3/2}$  and  $m_q^2$

$$\sigma_0 \sim 36 \text{ MeV} \quad \text{Borasoy, Missner ('97); Jenksin, Manohar ('92); Gasser, Leutwyler, Saino ('91)}$$

2.  $\Sigma_{\pi N} = 64 \pm 8 \text{ MeV}$  extracted from  $\pi N$  scattering data Koch ('82)

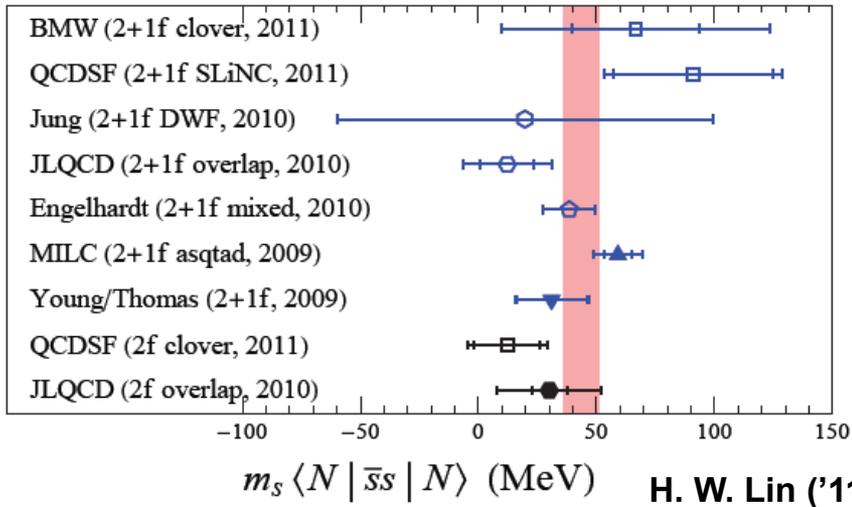
$$\Rightarrow \sigma_{\pi N} = \Sigma_{\pi N} - \Delta_\sigma - \Delta_R \sim 45 \text{ MeV}$$

2.  $y = \sigma_0 / \sigma_{\pi N} \sim 0.2 \Rightarrow \sigma_s \sim 130 \text{ MeV}$

**Sigma terms and strange quark content in 90's:**

$$\sigma_{\pi N} \sim 45 \text{ MeV}, \quad \sigma_0 \sim 36 \text{ MeV}, \quad \sigma_s \sim 130 \text{ MeV}, \quad y \sim 0.2$$

# Intense lattice studies of $\sigma_s$ in past few years



$\sigma_s = 43 \pm 8$  MeV (2011)  
 $\sim 20 - 60$  MeV circa 2012

$y$  lies in the range 0.05 ~ 0.20

$\sigma_{\pi N} = 37 \pm 10$       QCDSF ('12)  
 $39 \pm 4^{+18}_{-7}$       Durr et al ('11)

For  $\sigma_s = 50$  MeV as an example,  $\Rightarrow \sigma_{\pi N} - \sigma_0 = 4$  MeV &  $y \sim 0.1$

$$g_{\phi NN} = (\sqrt{2}G_F)^{1/2} \left( \sigma_u + \sigma_d + \sigma_s + \frac{2}{27} (m_N - \sigma_u - \sigma_d - \sigma_s) \right) \quad \text{for } \zeta_q=1 \text{ \& three heavy quarks}$$

**dominated by heavy quark contributions**

$\sigma_s$	$\sigma_u$	$\sigma_d$	$\sigma_{\pi N}$	$\langle p   \frac{\alpha_s}{4\pi} GG   p \rangle$	$y$	$g_{\phi NN}$
60	15.9	22.3	40.5	-187	0.11	$1.2 \times 10^{-3}$
50	15.6	21.8	39.8	-189	0.09	$1.1 \times 10^{-3}$
40	15.4	21.3	39.0	-191	0.08	$1.1 \times 10^{-3}$

## $\sigma_{\pi N}$ from $\pi N$ scattering data

Consider  $\pi N \rightarrow \pi N$  scattering amplitude & define

$$\Sigma_{\pi N} \equiv F_{\pi}^2 \underline{D}^+ \text{ at Cheng-Dashen point, } s = u = m_N^2, t = 2m_{\pi}^2$$

$$\Sigma_{\pi N} = \sigma_{\pi N}(2m_{\pi}^2) + \Delta_R = \sigma_{\pi N}(0) + \Delta_{\sigma} + \Delta_R \text{ with } \Delta_{\sigma}=15 \text{ MeV, } \Delta_R=2 \text{ MeV}$$

Since Cheng-Dashen point is outside of physical region, one relies on dispersion relation to extract  $\Sigma$  from the data

- Koch ('82) :  $\Sigma = 64 \pm 8 \Rightarrow \sigma_{\pi N} \sim 45 \text{ MeV}$ , benchmark in 90's
  - Olsson ('00):  $\Sigma = 73 \pm 9$
  - Pavan et al. ('02):  $\Sigma = 79 \pm 7$
  - Hite et al. ('05):  $\Sigma = 81 \pm 6$
  - Alarcon et al. ('11):  $\Sigma = 76 \pm 7$  (a different approach)
- $\Rightarrow \sigma_{\pi N} \sim 60\text{-}65 \text{ MeV}$  after 2000

- A conflict between lattice and expt'l results for  $\sigma_{\pi N}$  ?
- Decouplets will enhance  $\sigma_{u,d}$ , and hence  $\sigma_{\pi N}$ ,  $\sigma_0$  and suppress  $\gamma$  ?

# Pseudoscalar Higgs

$$g_{\sigma NN} = (\sqrt{2}G_F)^{1/2} \sum \langle N | \xi_q m_q \bar{q} i \gamma_5 q | N \rangle$$

**Heavy quark expansion:**  $m_h \bar{q}_h i \gamma_5 q_h \rightarrow -\frac{\alpha_s}{8\pi} G\tilde{G}$        $G\tilde{G} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{a\mu\nu} G^{a\alpha\beta}$

$E_q \equiv \langle N | \bar{q} i \gamma_5 q | N \rangle$  are related to nucleon m.e. of axial-vector currents

$$\begin{cases} A_\mu^3 = \bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d, \\ A_\mu^8 = \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d - 2\bar{s} \gamma_\mu \gamma_5 s \\ A_\mu^0 = \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s, \end{cases} \xrightarrow{\text{EOM}} \begin{cases} g_A^3 m_N = m_u E_u - m_d E_d, \\ g_A^8 m_N = m_u E_u + m_d E_d - 2m_s E_s, \\ g_A^0 m_N = m_u E_u + m_d E_d + m_s E_s + 3 \left\langle N \left| \frac{\alpha_s}{8\pi} G\tilde{G} \right| N \right\rangle \end{cases}$$

**Axial-vector couplings  $g_A^3$ ,  $g_A^8$  determined from neutron and hyperon  $\beta$  decays,  $g_A^0$  from polarized DIS experiment**

$$\int_0^1 g_1^p(x, Q^2) dx = C_{NS}(Q^2) \left( \frac{1}{12} g_A^3 + \frac{1}{36} g_A^8 \right) + \frac{1}{9} C_S(Q^2) g_A^0(Q^2)$$

$$g_A^3(Q^2) = \Delta u(Q^2) - \Delta d(Q^2),$$

$$g_A^8(Q^2) = \Delta u(Q^2) + \Delta d(Q^2) - 2\Delta s(Q^2)$$

$$g_A^0(Q^2) = \Delta u(Q^2) + \Delta d(Q^2) + \Delta s(Q^2),$$

$\Delta q$ : net quark helicity component

# Need one more relation to determine $\langle N | \underline{q} i \gamma_5 q | N \rangle$

Some proposals before:

◆ Large- $N_c$  and chiral limits  $\Rightarrow E_u + E_d + E_s = 0$  **HYC ('89)**

◆  $\langle p | G\tilde{G} | p \rangle$  is related to gluon spin component

$$2m_N \Delta G \bar{u} i \gamma_5 u = \langle p | G\tilde{G} | p \rangle \quad \text{Cheng, Li ('89); Geng, Ng ('90)}$$

However, (i) there is no twist-2 local operator definition for  $\Delta G$

in OPE, (ii)  $\langle N | G\tilde{G} | N \rangle$  violates isospin symmetry

**Gross, Treiman,  
Wilczek ('79)**

◆ Strong CP-odd operator replaced by quark operator

$$-(g^2 / 32\pi^2) \theta G\tilde{G} \rightarrow \theta \bar{m} (\bar{u} i \gamma_5 u + \bar{d} i \gamma_5 d + \bar{s} i \gamma_5 d), \quad \bar{m} = (1/m_u + 1/m_d + 1/m_s)^{-1}$$

**Baluni ('79)**

$$\bar{m} (E_u + E_d + E_s) = -\langle N | \frac{\alpha_s}{8\pi} G\tilde{G} | N \rangle$$

**No solutions for  $E_u, E_d, E_s$  !**

◆ **U(1) Goldberger-Treiman relation**

$$g_A^0 = \frac{\sqrt{3} f_\pi}{2m_N} g_{\eta_0 NN} + \dots \quad \text{in chiral limit} \quad \Rightarrow \quad \left\langle N \left| \frac{\alpha_s}{8\pi} G\tilde{G} \right| N \right\rangle = \frac{f_\pi}{2\sqrt{3}} g_{\eta_0 NN} + \dots$$

## Large- $N_c$ and chiral relation $\langle p | \underline{u} i \gamma_5 \underline{u} + \underline{d} i \gamma_5 \underline{d} + \underline{s} i \gamma_5 \underline{s} | p \rangle = 0$

$$E_u = \frac{1}{2} \frac{m_N}{m_d m_s (1+z+w)} \left[ (m_d + 2m_s) g_A^3 + m_d g_A^8 \right],$$

$$E_d = \frac{1}{2} \frac{m_N}{m_d m_s (1+z+w)} \left[ -(m_u + 2m_s) g_A^3 + m_u g_A^8 \right],$$

$$E_s = \frac{1}{2} \frac{m_N}{m_d m_s (1+z+w)} \left[ (m_u - m_d) g_A^3 - (m_u + m_d) g_A^8 \right],$$

$$\begin{aligned} \left\langle N \left| \frac{\alpha_s}{8\pi} G \tilde{G} \right| N \right\rangle &= \frac{1}{2} \frac{m_N}{m_d m_s (1+z+w)} \left\{ \frac{2}{3} m_d m_s (1+z+w) g_A^0 + m_s (m_d - m_u) g_A^3 \right. \\ &\quad \left. + \frac{1}{3} [m_s (m_u + m_d) - 2m_u m_d] g_A^8 \right\} \quad z = m_u/m_d, w = m_u/m_s \end{aligned}$$

## Pseudoscalar-nucleon coupling:

$$g_{\sigma NN} = (\sqrt{2} G_F)^{1/2} m_N \left\{ \left[ \xi_u - \frac{\bar{m}}{m_u} \sum_q \xi_q \right] \Delta u + \left[ \xi_d - \frac{\bar{m}}{m_d} \sum_q \xi_q \right] \Delta d + \left[ \xi_s - \frac{\bar{m}}{m_s} \sum_q \xi_q \right] \Delta s \right\}$$

This is precisely the general axion-nucleon coupling. Note that physical axion is free of anomalous  $G\tilde{G}$  interaction

**Light quark contributions to scalar-nucleon coupling vanish in chiral limit. By contrast,  $g_{\sigma NN}$  receives significant light quark contributions even in chiral limit because pseudoscalar Higgs has admixture with  $\pi^0, \eta_8, \eta_0$  which couple strongly with nucleon**

**Zero anomalous dimension for  $A_{\mu^3}, A_{\mu^8} \Rightarrow g_A^3$  &  $g_A^8$  can be determined from low-energy neutron & hyperon  $\beta$  decays.**

**SU(3) symmetry implies  $g_A^8 = 3F-D = 0.585$  while  $g_A^3 = F+D = 1.2701$**

$$\int_0^1 g_1^p(x, Q^2) dx = C_{NS}(Q^2) \left( \frac{1}{12} g_A^3 + \frac{1}{36} g_A^8 \right) + \frac{1}{9} C_S(Q^2) g_A^0(Q^2)$$

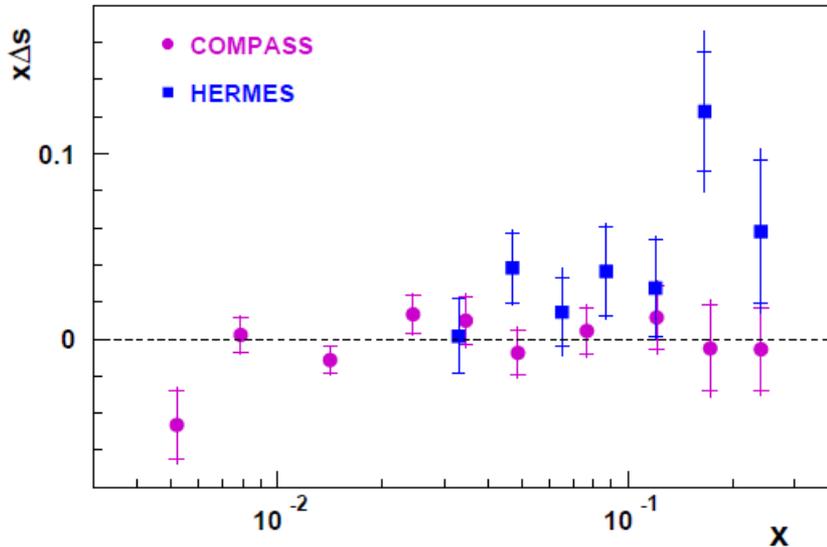
**Using  $g_A^8=0.585$ , COMPASS ('07) & HERMES ('07) obtained**

$$g_A^0(3 \text{ GeV}^2) = 0.35 \pm 0.03 \pm 0.05 \quad (\text{COMPASS})$$

$$g_A^0(5 \text{ GeV}^2) = 0.330 \pm 0.0011 \pm 0.025 \pm 0.028 \quad (\text{HERMES})$$

$$\Rightarrow \Delta u = 0.85, \quad \Delta d = -0.42, \quad \Delta s = -0.08 \quad \text{at } Q^2 \sim 4 \text{ GeV}^2$$

**Semi-inclusive DIS data of COMPASS & HERMES show no evidence of large negative  $\Delta s$ :  $\Delta s = -0.02 \pm 0.03$  by COMPASS**



**Also  $\Delta G/G$  is small from RHIC, COMPASS, HERMES**

$$\Delta q_{GI}(Q^2) - \Delta q_{CI}(Q^2) = -\frac{1}{2\pi} \alpha_s(Q^2) \Delta G(Q^2)$$

**Sea polarization should be small due to smallness of  $\Delta G$**

**Three lattice calculations in 2012 :**

**1. QCDSF  $\Delta s = -0.020 \pm 0.010 \pm 0.004$  at  $Q = 2.7$  GeV**

**2. Engelhardt  $\Delta s = -0.031 \pm 0.017$  at  $Q = 2$  GeV**

**3. Babich et al  $\Delta s = G_A^s(0) = -0.019 \pm 0.017$  not renormalized yet**

When SU(3) symmetry is broken,  $g_A^8$  may be reduced. For example,  $g_A^8 = 0.46 \pm 0.05$  in cloudy bag model Bass, Thomas ('10)

$$\int_0^1 g_1^p(x, Q^2) dx = C_{NS}(Q^2) \left( \frac{1}{12} g_A^3 + \frac{1}{36} g_A^8 \right) + \frac{1}{9} C_S(Q^2) g_A^0(Q^2)$$

$$g_A^8 \downarrow \Rightarrow g_A^0 \uparrow \Rightarrow -\Delta s = 1/3 (g_A^8 - g_A^0) \downarrow$$

For  $g_A^8 = 0.46$ ,  $\Delta s = -0.03$  sensitive to SU(3) breaking

$(g_A^3)_n = - (g_A^3)_p \Rightarrow$  large isospin violation

$N$	$m_u E_u$	$m_d E_d$	$m_s E_s$	$\langle N   \frac{\alpha_s}{8\pi} G\tilde{G}   N \rangle$	$\Delta u$	$\Delta d$	$\Delta s$	$g_{\sigma NN}$
$p$	405	-787	-466	389	0.85	-0.42	-0.08	$-8.2 \times 10^{-3}$
$n$	-396	796	-75	-2	-0.42	0.85	-0.08	$1.3 \times 10^{-3}$
$p$	404	-788	-408	380	0.84	-0.44	-0.03	$-7.8 \times 10^{-3}$
$n$	-397	795	-17	-11	-0.44	0.84	-0.03	$1.7 \times 10^{-3}$

## Comparison with other works

$$f_{T_q} \equiv \frac{\langle N | m_q \bar{q} q | N \rangle}{m_N} = \frac{\sigma_q}{m_N}$$

	DarkSUSY	Ellis et al	Our
$f_{T_u}^{(p)}$	0.023	0.020	0.017
$f_{T_u}^{(n)}$	0.019	0.014	0.012
$f_{T_d}^{(p)}$	0.034	0.026	0.023
$f_{T_d}^{(n)}$	0.041	0.036	0.033
$f_{T_s}^{(p)}$	0.14	0.118	0.053
$f_{T_s}^{(n)}$	0.14	0.118	0.053
$\Delta u$	0.77	0.78	0.84
$\Delta d$	-0.40	-0.48	-0.44
$\Delta s$	-0.12	-0.15	-0.03

## Conclusions

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- Intensive lattice calculations of sigma terms of strange quark content in the nucleon  $\Rightarrow$  scalar-nucleon coupling dominated by heavy quarks
- A conflict between lattice and expt'l results for  $\sigma_{\pi N}$  ?
- Due to axial anomaly, additional assumption is needed to solve the nucleon matrix elements of pseudoscalar densities. We rely on chiral and large- $N_c$  limits to derive the extra constraint. Chiral and  $1/N_c$  corrections need to be studied.