Status of Tau and QCD Physics at BESIII

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On behalf of

BESIII Collaboration

Outline

- Tau Mass Measurement
  - Motivation
  - Data Analysis and Preliminary Result
- R-value and QCD Physics
  - Motivation
  - Present Status
- Summary

m_τ Tau Mass Measurement

Elementary parameter in SM (PDG2012)

- M_e=0.510998910±0.000000013 (2.6×10^{-8})
- M_μ=105.658367±0.000004 (3.8×10^{-8})
- M_τ=1776.82 ±0.16 (9.0×10^{-5})

Yoshio Koide (1981) equality testing

$$Q=\frac{m_e+m_\mu+m_\tau}{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}=\frac{2}{3}\left(0.666659(10)\right)$$

Lepton universality testing

$$\left(\frac{g_\tau}{g_\mu}\right)^2=\frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5\frac{B(\tau\to e\nu_e\nu_\tau)}{B(\mu\to e\nu_e\nu_\mu)}\left(1+\Delta_e\right)$$

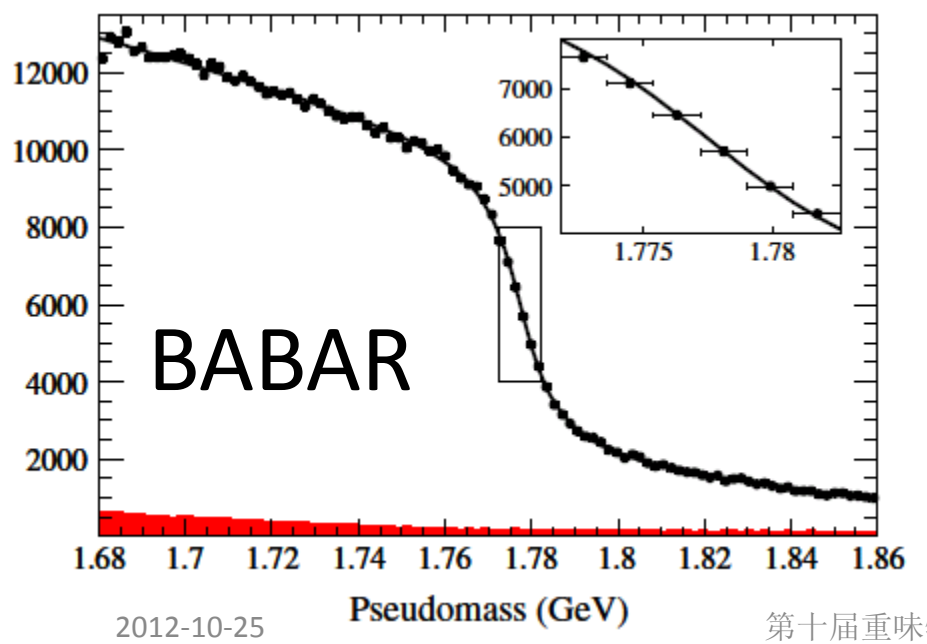
Tau Mass Measurement

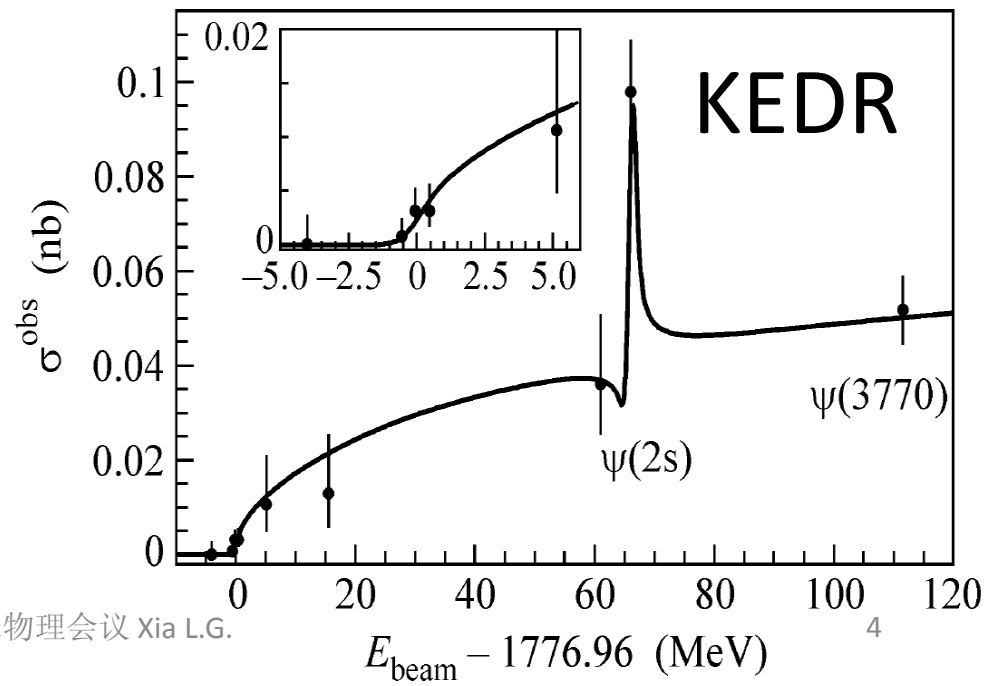
PDG2012:
1776.82 ± 0.16 MeV

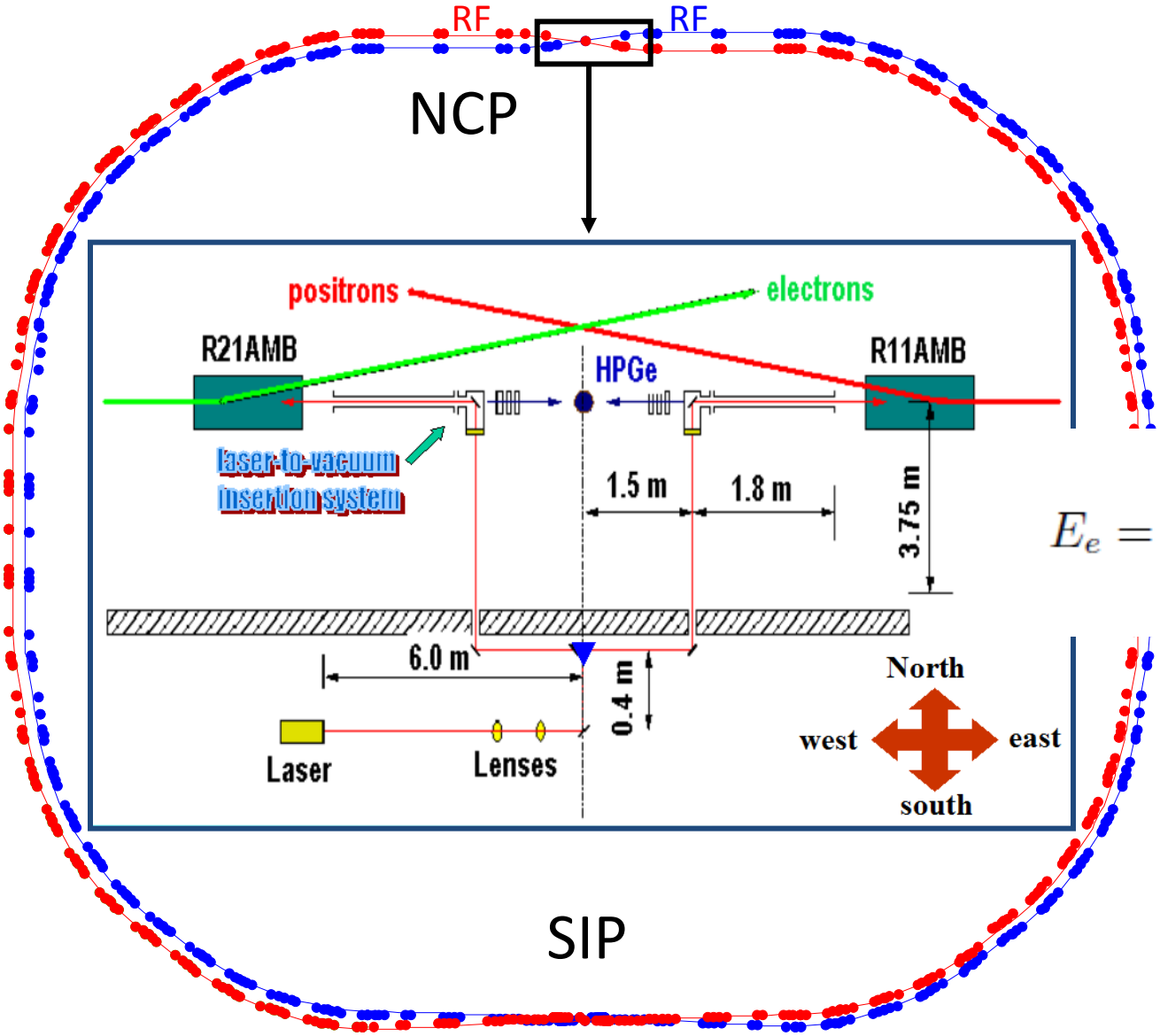
1. Pseudomass Method
2. Threshold Scan Method

Our Goal: $\Delta m_\tau < 0.1$ MeV

$$m_\tau^2 = p_\tau^2 = \left(p_h + p_\nu\right)^2$$
$$\geq m_h^2 + 2\left(E_h - \left|\vec{p}_h\right|\right)\left(E_\tau - E_h\right)$$

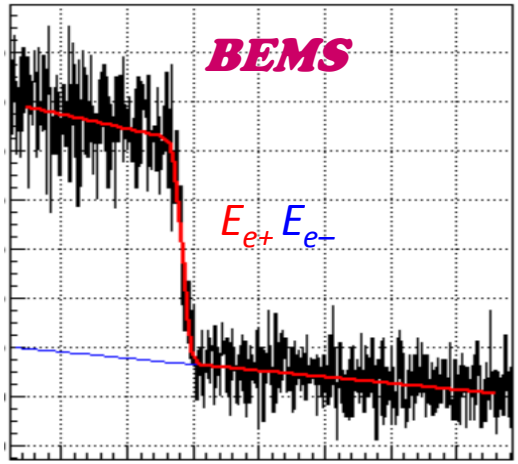


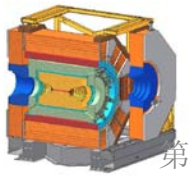




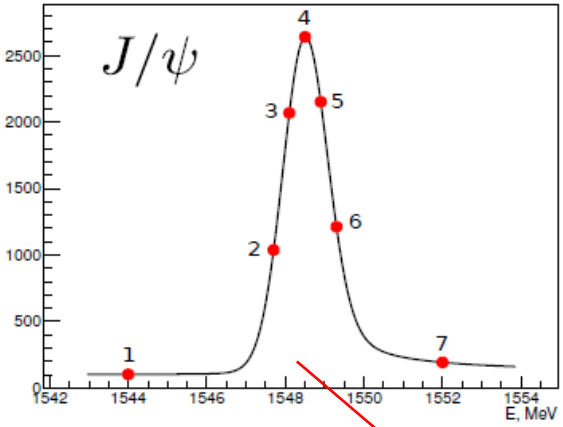
For BEPCII
Energy accuracy
Improvement :
$10^{-3} \rightarrow 5\times 10^{-5}$

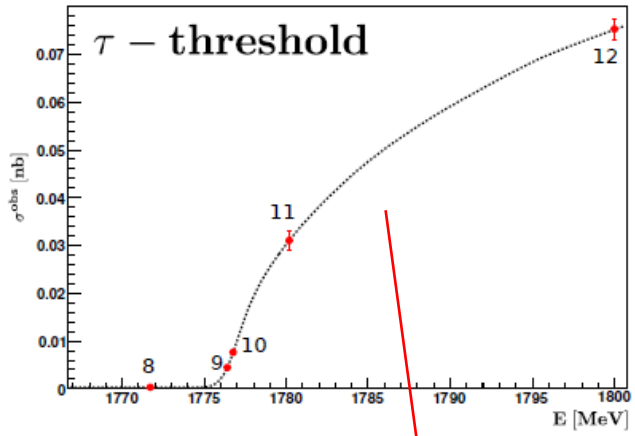
$$E_e = \frac{E_\gamma}{2}\left[1+\sqrt{1+\frac{m_e^2}{\epsilon_\gamma E_\gamma}}\right]$$

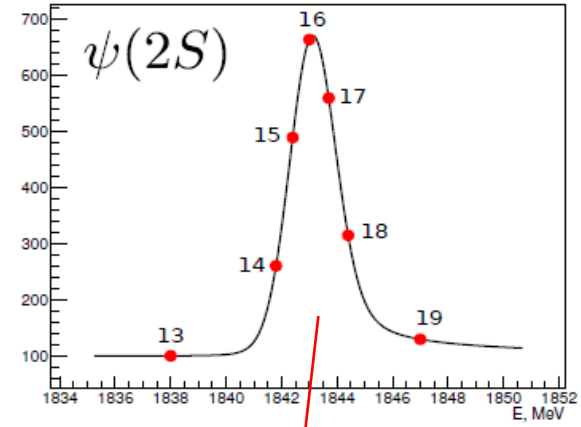




Data Analysis

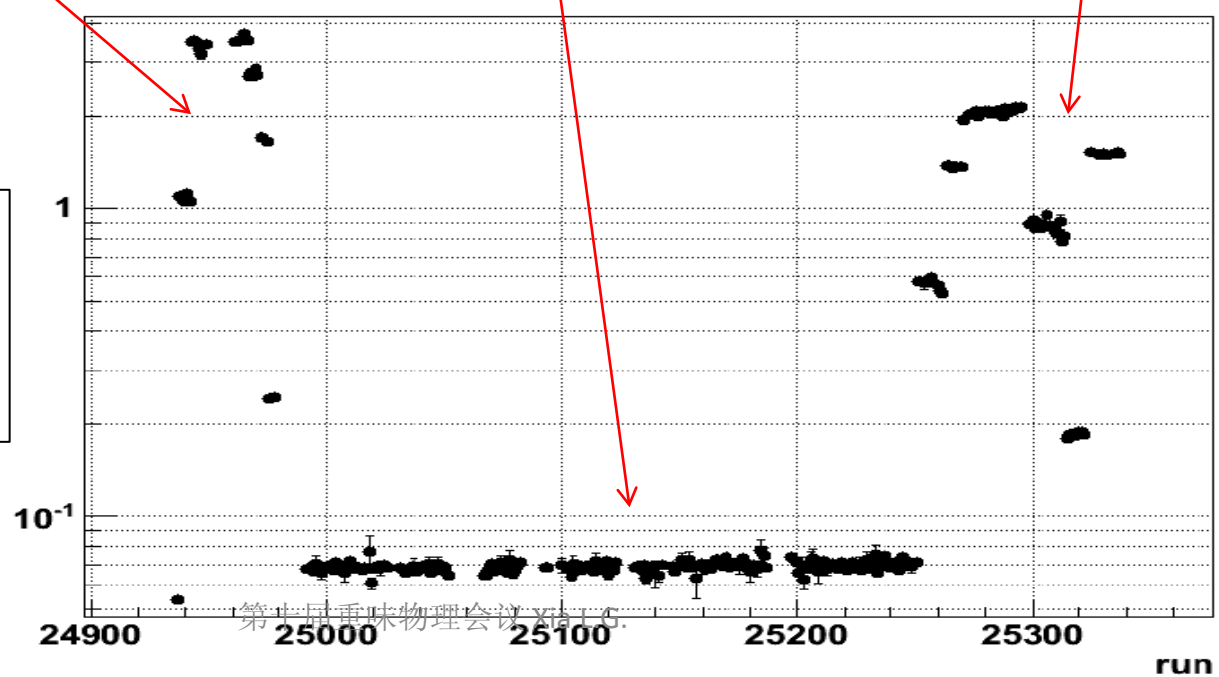




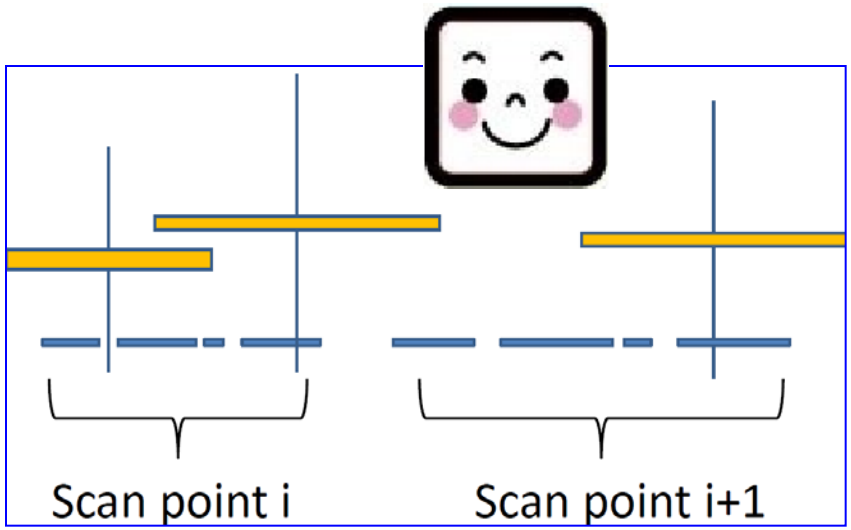


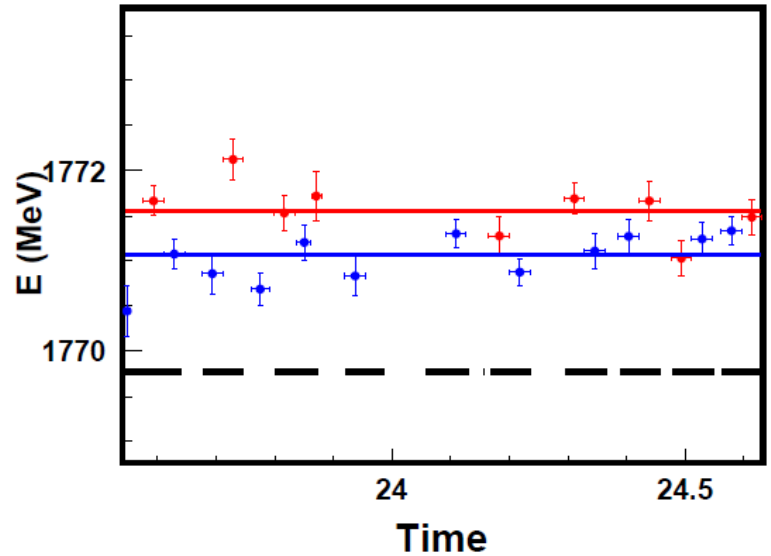
Classification of run by the ratio of N_{had}/N_{ee}

Time: Dec. 2012
Data: ~ 20 pb^{-1}
BOSS: 662



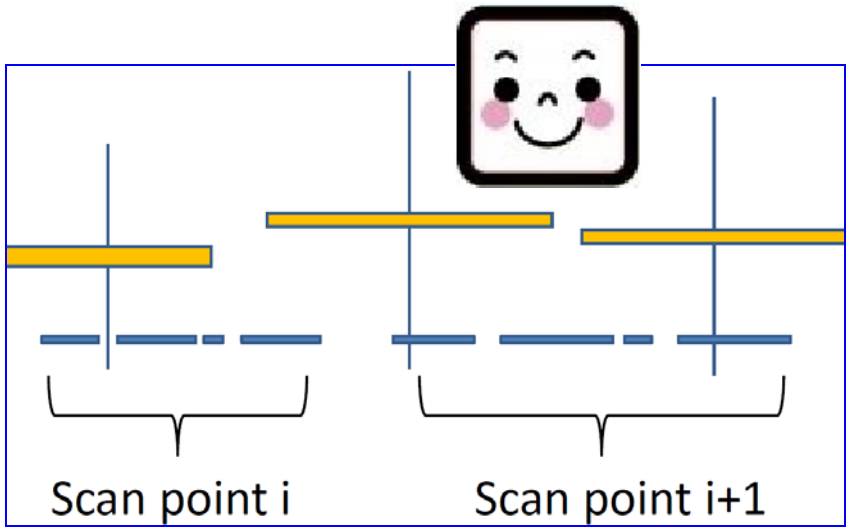
Time matching

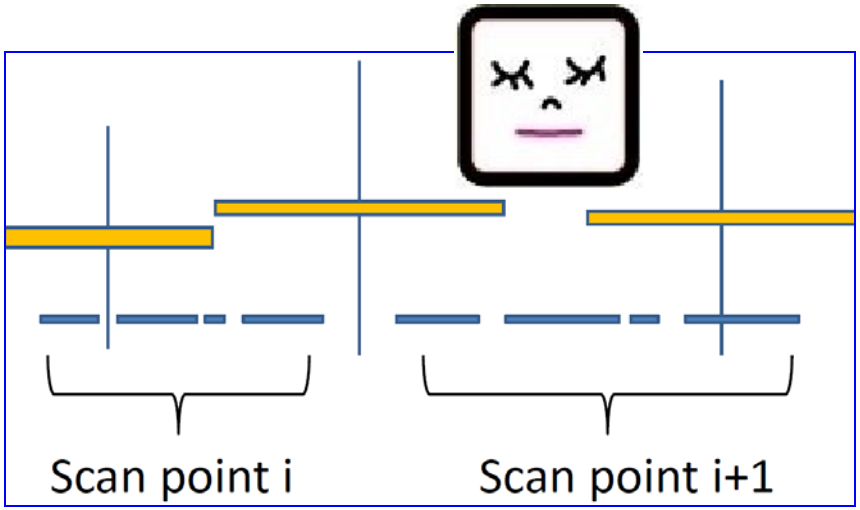




BEMS d.t time

BES d.t. time





	Scan point	Run number	$E_{e^-}(MeV)$	E_{e^+} (MeV)	E_{cm} (MeV)
τ	1	24983-25015	1771.558 ± 0.067	1771.069 ± 0.053	3542.413 ± 0.085
	2	25016-25094	1777.307 ± 0.047	1776.730 ± 0.046	3553.822 ± 0.075
	3	25100-25141	1780.926 ± 0.055	1780.431 ± 0.065	3561.142 ± 0.085
	4	25143-25243	1800.526 ± 0.044	1799.878 ± 0.044	3600.186 ± 0.062
J/ψ	1	24937-24937	1544.542 ± 0.135	1544.312 ± 0.217	3088.667 ± 0.256
	2	24938-24942	1547.917 ± 0.099	1547.548 ± 0.106	3095.278 ± 0.145
	3	24943-24949	1548.692 ± 0.103	1548.171 ± 0.086	3096.676 ± 0.135
	4	24959-24966	1549.079 ± 0.109	1548.714 ± 0.075	3097.606 ± 0.133
	5	24967-24971	1549.451 ± 0.081	1549.014 ± 0.114	3098.278 ± 0.140
	6	24972-24975	1549.566 ± 0.101	1549.438 ± 0.083	3098.817 ± 0.131
	7	24976-24978	1552.186 ± 0.088	1551.936 ± 0.107	3103.934 ± 0.139
ψ'	1	25245-25251	1838.183 ± 0.256	1837.940 ± 0.157	3675.901 ± 0.300
	2	25252-25262	1842.177 ± 0.090	1841.279 ± 0.220	3683.653 ± 0.303
	3	25264-25270	1842.755 ± 0.153	1842.489 ± 0.087	3685.113 ± 0.230
	4	25271-25295	1843.402 ± 0.075	1842.893 ± 0.110	3686.337 ± 0.189
	5	25299-25314	1844.787 ± 0.125	1844.137 ± 0.107	3688.819 ± 0.226
	6	25315-25322	1846.832 ± 0.138	1846.487 ± 0.108	3693.515 ± 0.245
	7	25325-25337	1844.130 ± 0.091	1843.396 ± 0.088	3687.573 ± 0.158

$$E_{cm}^{\sqrt{s}} \approx 2E_{beam}\left(1-\frac{\alpha^2}{8}\right)$$

Bhabha

$e^+e^- \rightarrow e^+e^-$

Di-gamma

$e^+e^- \rightarrow \gamma\gamma$

	Scan point	N_2^{obs}	N_1^{obs}	L_{bhabha} (nb^{-1})	N^{obs}	$L_{digamma}$ (nb^{-1})
τ	1	1575827	58018	4502.89	74240	4252.17
	2	2043538	75371	5877.14	96570	5566.82
	3	1413321	52432	4082.30	67192	3889.29
	4	3411037	126081	10068.29	161482	9553.18
J/ψ	1	38143	1393	81.79	1804	78.52
	2	114205	7191	239.19	5016	219.26
	3	137995	21744	260.07	5557	243.13
	4	109972	17947	206.00	4718	206.55
	5	116221	15593	225.34	5104	223.53
	6	106130	10079	215.17	4950	216.87
	7	150860	6618	324.23	7218	317.31
ψ'	1	269201	9878	830.58	12763	787.04
	2	284362	10995	879.30	13291	823.10
	3	285762	12775	878.75	13432	832.47
	4	414291	20998	1266.84	19097	1184.34
	5	565681	27641	1734.27	26761	1660.77
	6	265322	11889	817.48	12366	767.97
	7	501530	19215	1559.59	23624	1470.75

Event Selection

Partial information, not the full list !

PID	p (GeV/c)	EMC	TOF	MUC	other
e	$p_{min} < p < p_{max}$	$0.8 < E/p < 1.05$	$\lvert\Delta tof(e)\rvert < 0.2$ $0 < tof < 4.5$		
μ	$p_{min} < p < p_{max}$	$E/p < 0.7$ $0.1 < E < 0.3$	$\lvert\Delta tof(\mu)\rvert < 0.2$	($depth > 80 \times p$-50 or $depth > 40$) and $numhits > 1$	
π	$p_{min} < p < p_{max}$	$E/p < 0.6$	$\lvert\Delta tof(\pi)\rvert < 0.2$ $0 < tof < 4.5$		not μ
K	$p_{min} < p < p_{max}$	$E/p < 0.6$	$\lvert\Delta tof(K)\rvert < 0.2$ $0 < tof < 4.5$		not μ

$$PTEM = \frac{P_T}{E_{miss}^{max}} = \frac{(\vec{P}_1 + \vec{P}_2)_T}{W - |\vec{P}_1| - |\vec{P}_2|}$$

No good photon: N_γ=0
Good photon:
1) 0<TDC<14, (unit: 50ns)
2) |cosθ|<0.8, E>25MeV
3) 0.84<| cosθ |<0.92, E>50MeV
4) θγc >20

6

The detection efficiency for different final states at different scan points

scan point	Efficiency (%)								
	ee	$e\mu$	eh	$\mu\mu$	μh	hh	$e\rho$	$\mu\rho$	$\pi\rho$
2	17.1	21.8	32.4	14.2	15.3	25.6	9.9	5.5	9.1
3	17.6	23.2	34.9	14.0	16.9	29.3	10.4	6.1	8.9
4	17.8	23.1	36.2	13.9	17.7	34.5	10.8	5.3	12.8

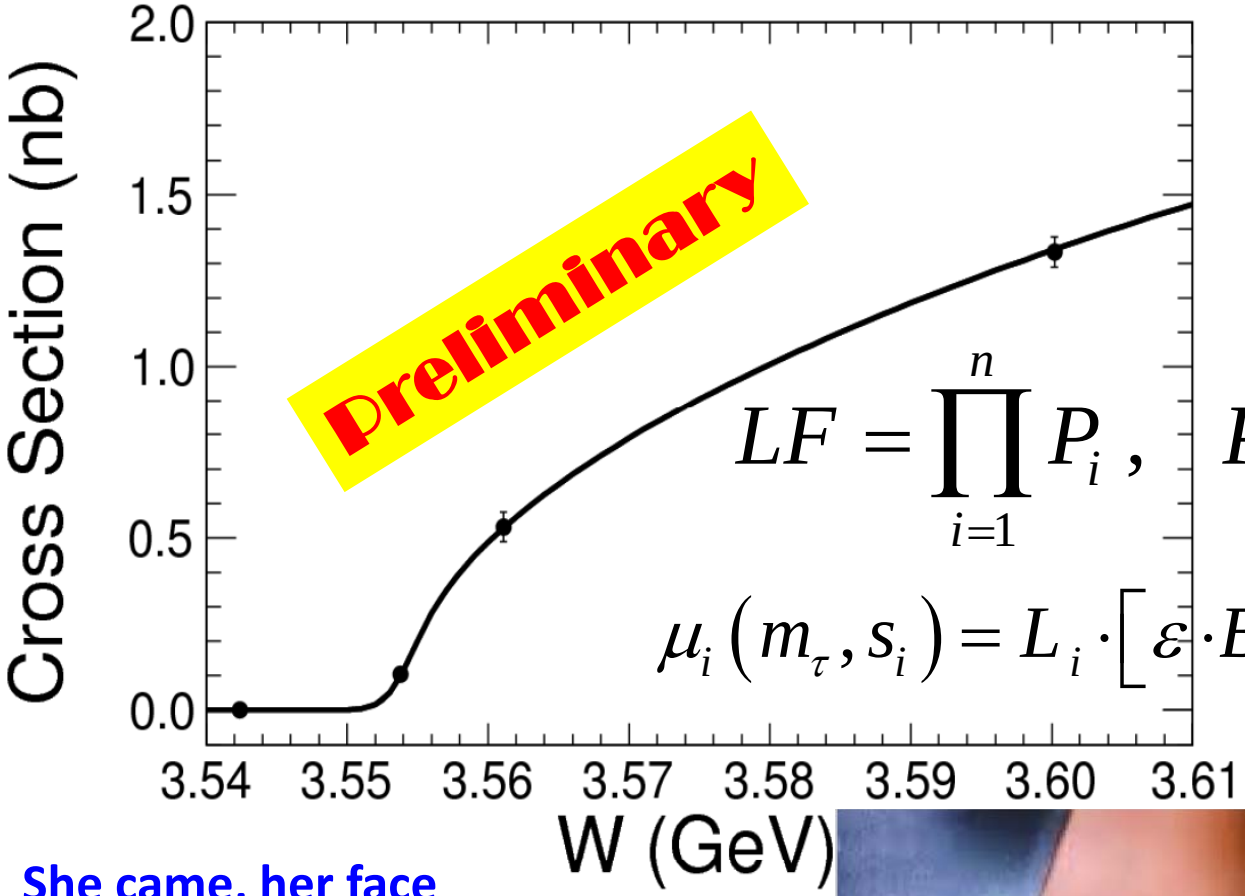
The number of observed events and that of normalized MC samples are consistent within errors.

final state	1		2		3		4		total	
	Data	MC	Data	MC	Data	MC	Data	MC	Data	MC
ee	0	0	4	3.7	13	12.2	84	76.1	101	91.9
$e\mu$	0	0	8	9.2	35	31.3	168	192.7	211	233.1
$e\pi$	0	0	8	8.6	33	29.6	202	184.5	243	222.7
ek	0	0	0	0.5	2	1.8	16	16.9	18	19.3
$\mu\mu$	0	0	2	2.9	8	9.2	49	56.3	59	68.4
$\mu\pi$	0	0	4	3.9	11	14.0	89	86.7	104	104.7
μk	0	0	0	0.2	3	0.8	7	9.0	10	10.1
$\pi\pi$	0	0	1	2.0	5	7.7	57	54.0	63	63.8
πk	0	0	1	0.3	0	0.8	10	8.2	11	9.3
kk	0	0	0	0.0	1	0.1	1	0.3	2	0.4
$e\rho$	0	0	3	6.1	19	20.6	142	132.0	164	158.7
$\mu\rho$	0	0	8	3.3	18	11.8	52	62.3	68	78.5
$\pi\rho$	0	0	5	3.4	15	10.8	97	96.0	117	110.2
Total	0	0	44	44.2	153	150.8	974	976.1	1171	1171.1

Preliminary!

Total consistency is fairly well!

τ mass measurement



~0.15 MeV or 9×10⁻⁵,
~0.1 MeV or 6×10⁻⁵.

$$LF = \prod_{i=1}^{n} P_i \,, \quad P_i = \frac{\mu_i^{N_i} e^{-\mu_i}}{N_i!}$$

$$\mu_i\left(m_\tau, s_i\right) = L_i \cdot \left[\varepsilon \cdot B \cdot \sigma_{obs}\left(m_\tau, s_i\right) + \sigma_{BG}\right]$$

She came, her face half hid behind a pipa still.

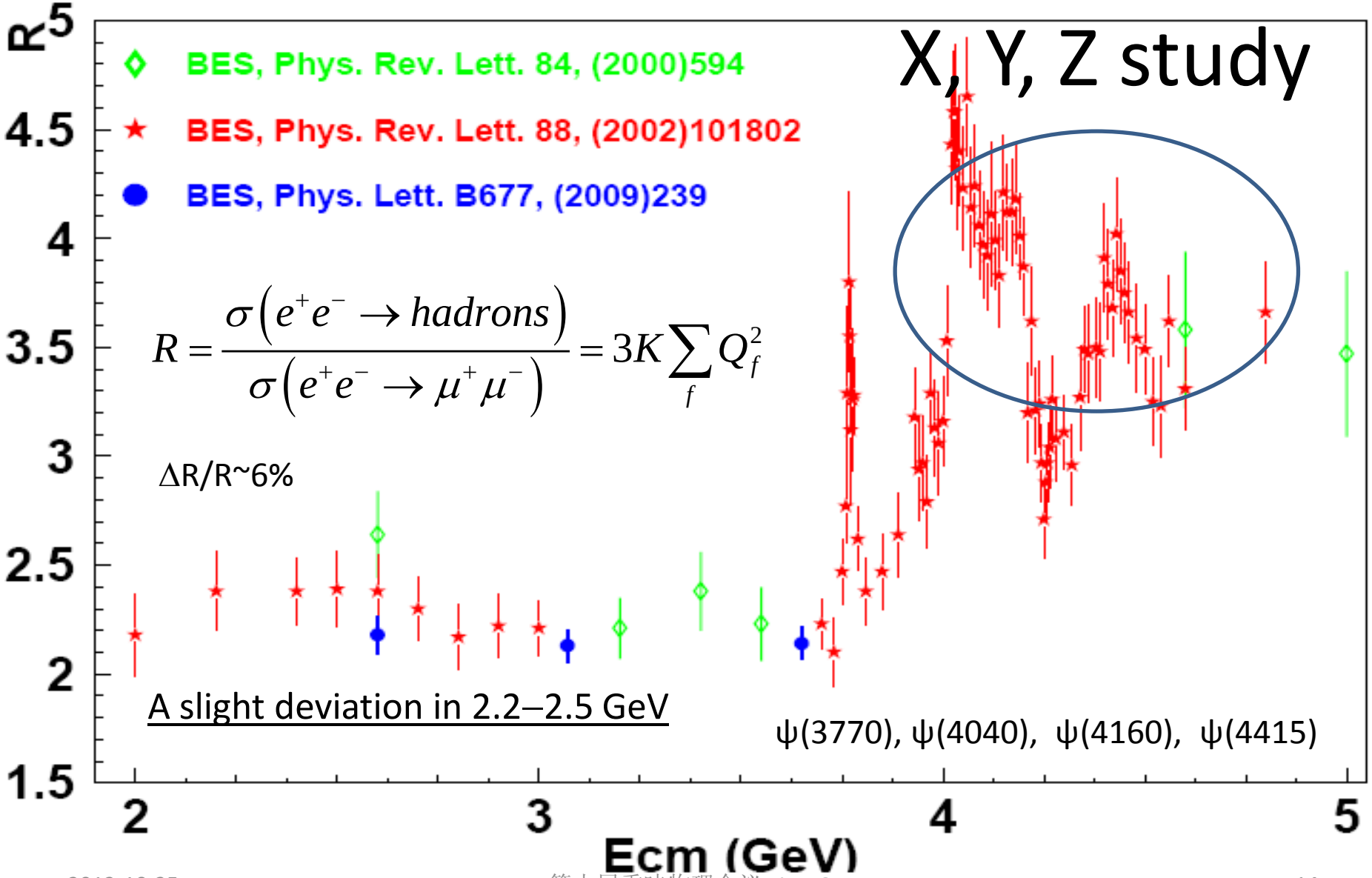
M_τ = 1776.? MeV

R-value and QCD Physics

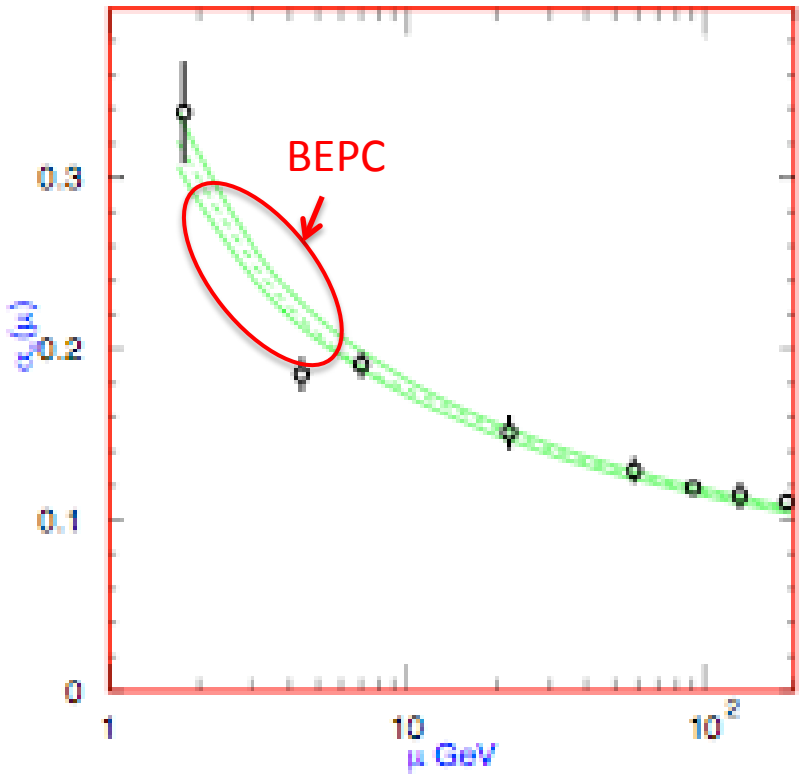
Motivations

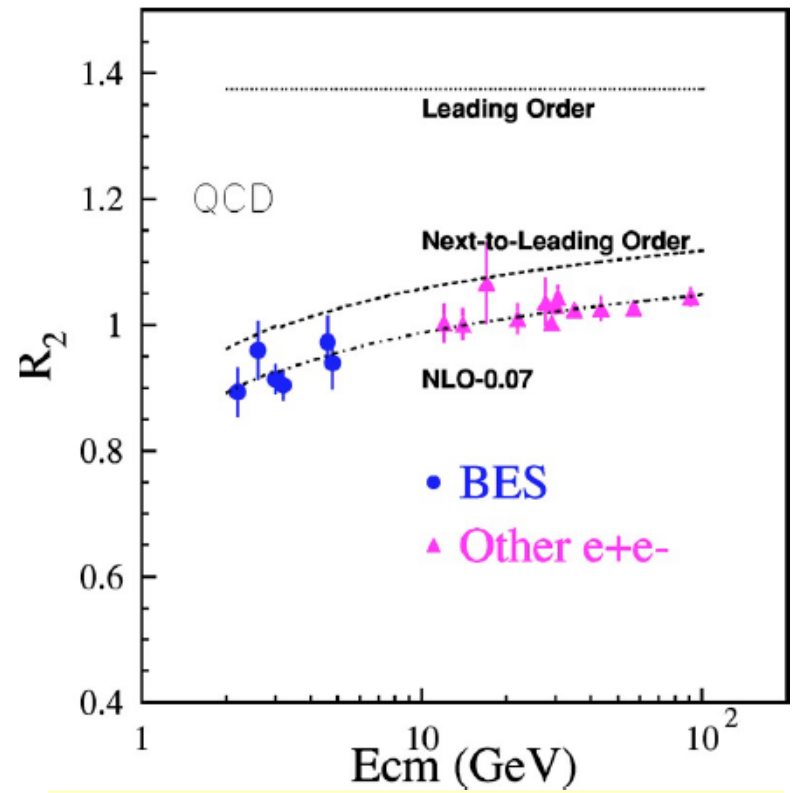
- R-value measurement;
- Hadronic contribution to
  - QED running coupling constant $\alpha_{QED}(M_Z)$
  - Anomalous magnet moment of the muon a_μ
- Resonance structures in open charm region;
- Strong coupling constant α_s determination;
- Baryon form factor (p, n, Λ …);
- Charm quark mass m_c determination;
- X, Y, Z particles and other possible new resonances
- Physics with D_s, Charmed baryons, ……

R Measurements at BESII



α_s and R_2





PDG10: $\alpha_s(M_Z)=0.1184\pm0.0007$

BESII √s=2.60, 3.07, 3.65GeV

$\alpha_s(M_Z)=0.117^{+0.012}_{-0017}$

$$R_2 \equiv \frac{\langle n_{ch}(n_{ch}-1)\rangle}{\langle n_{ch}\rangle^2} = \frac{11}{8}(1-c\sqrt{\alpha_s(s)})$$

$$c=(4455-40n_f)/1728\sqrt{6\pi}$$

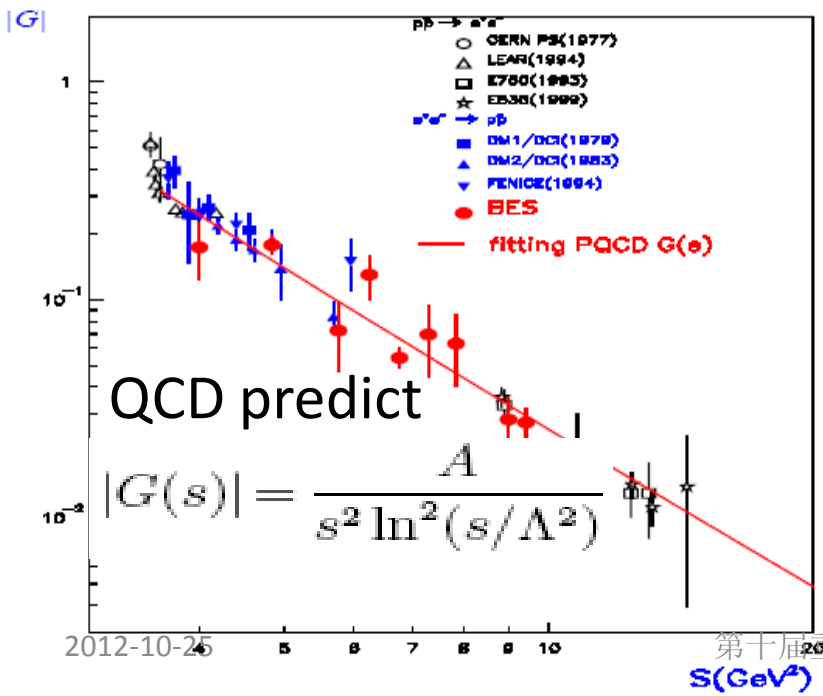
Proton Form Factor: $e^+e^- \rightarrow p^+p^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s} C[|G_M(s)|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E(s)|^2\sin^2\theta]$$

$G_E = F_1 + \tau F_2$ $\quad$ $G_M = F_1 + F_2$

$$\sigma_0 = \frac{4\pi\alpha^2\beta}{3s}(1+\frac{2M^2}{s})|G(s)|^2$$

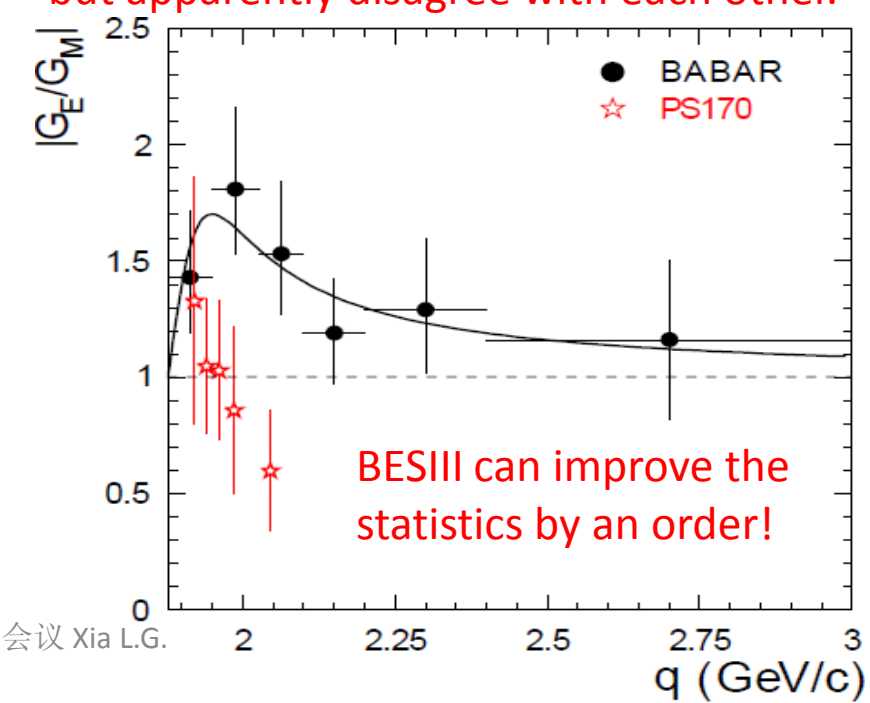
Most measurements assume $G_E=G_M$.



QCD predict

$$|G(s)| = \frac{A}{s^2\ln^2(s/\Lambda^2)}$$

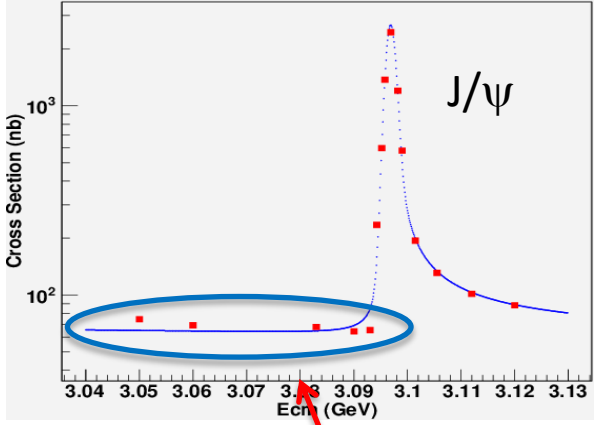
Only 2 experiments measured $|G_E/G_M|$, but apparently disagree with each other.

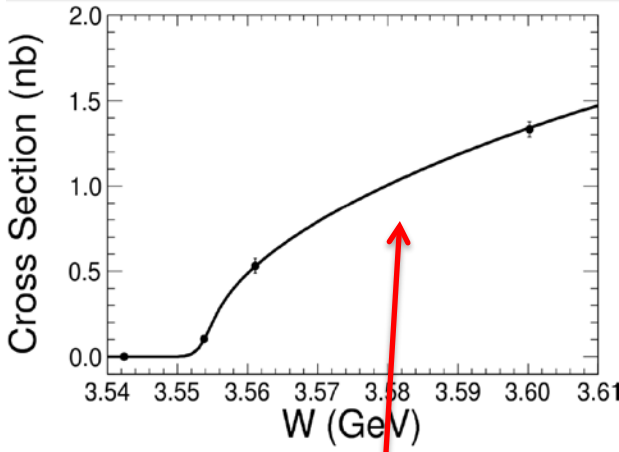


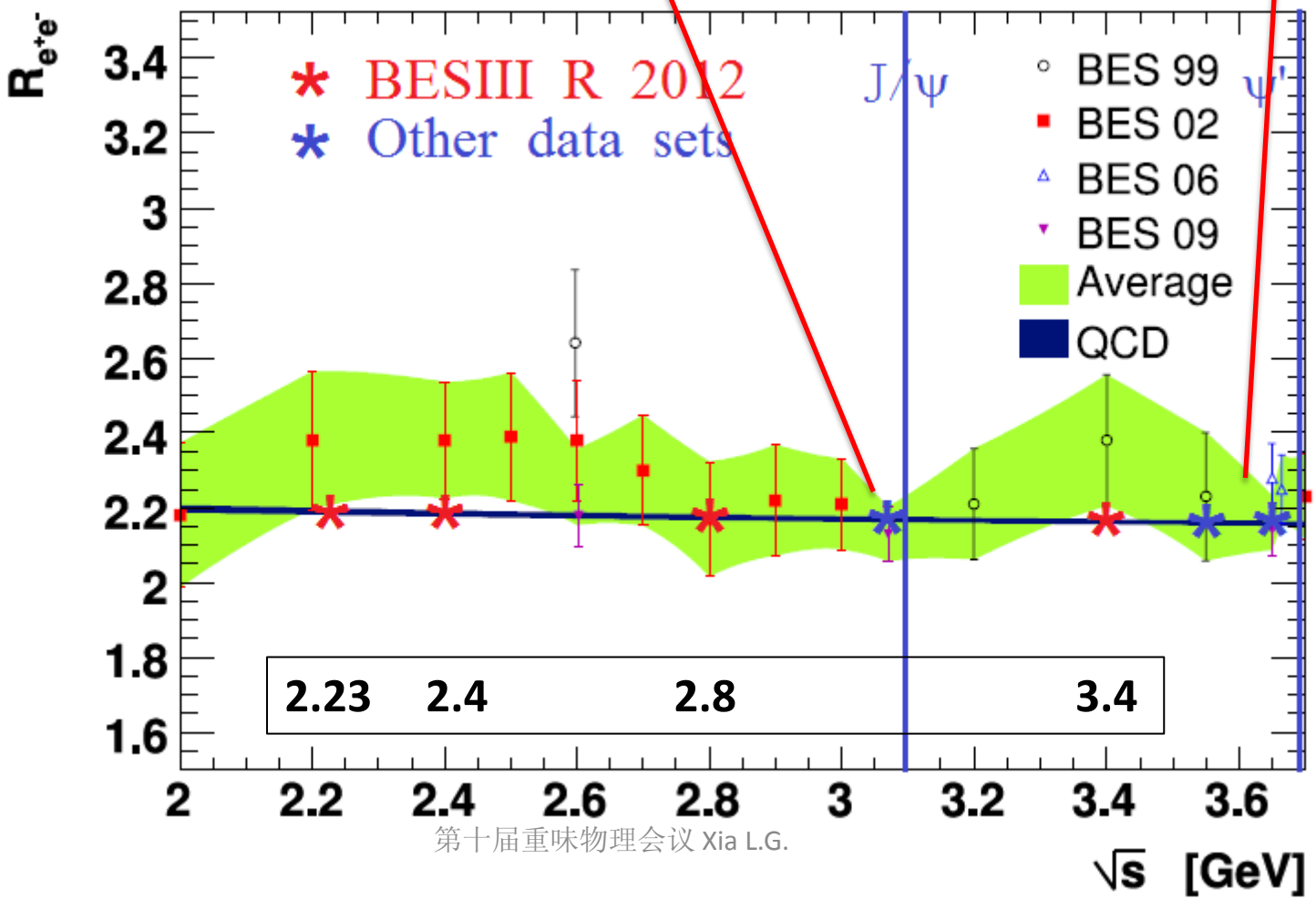
BESIII can improve the statistics by an order!

BESIII continuum data overview

~8.3pb^{-1}







Works ongoing

$$R = \frac{1}{\sigma_{\mu+\mu-}} \cdot \frac{N_{had} - N_{bg}}{L \cdot \varepsilon_{had} \cdot (1+\delta)}$$

N_{had}, N_{bg} : observed hadronic events, backgrounds

L: integrated luminosity

ε_{had}: detection efficiency for N_{had}

δ: radiative correction factor

- Luminosity measurement
- J/ψ line shape fitting
- $e^+e^- \rightarrow$ppbar cross section
- $e^+e^- \rightarrow \pi^+\pi^-$ cross section

Summary

- BESIII has measured Tau mass with a precision better than PDG value. Preliminary results is under internal review. And the future goal is less than 0.1 MeV/c^2 with more data taken.
- R measurement and QCD studies with data below 3.7 GeV are in progress, and R scan in the high energy region is expected.

Thank you!

- BACK UP

Definition of R

- At lowest order

Number of quark colors

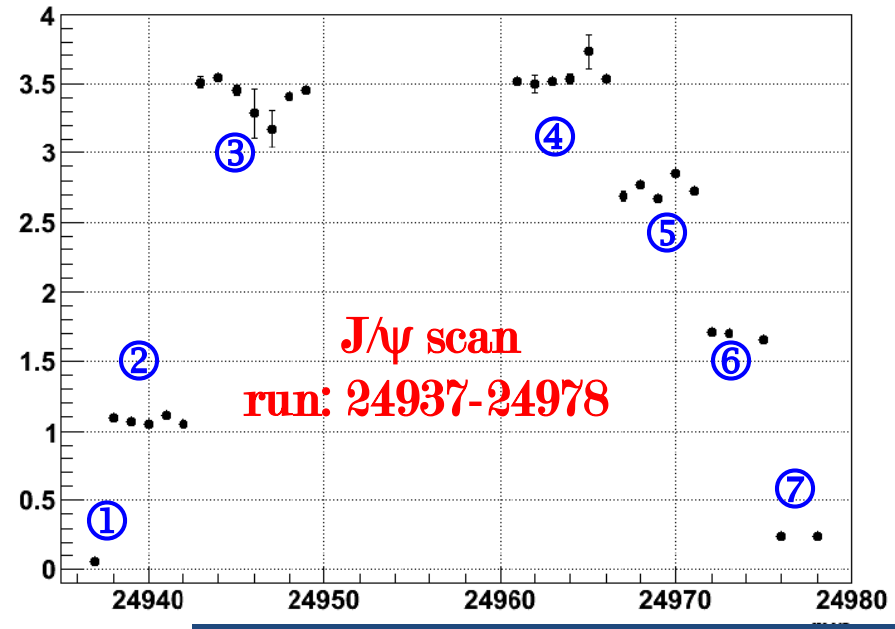
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

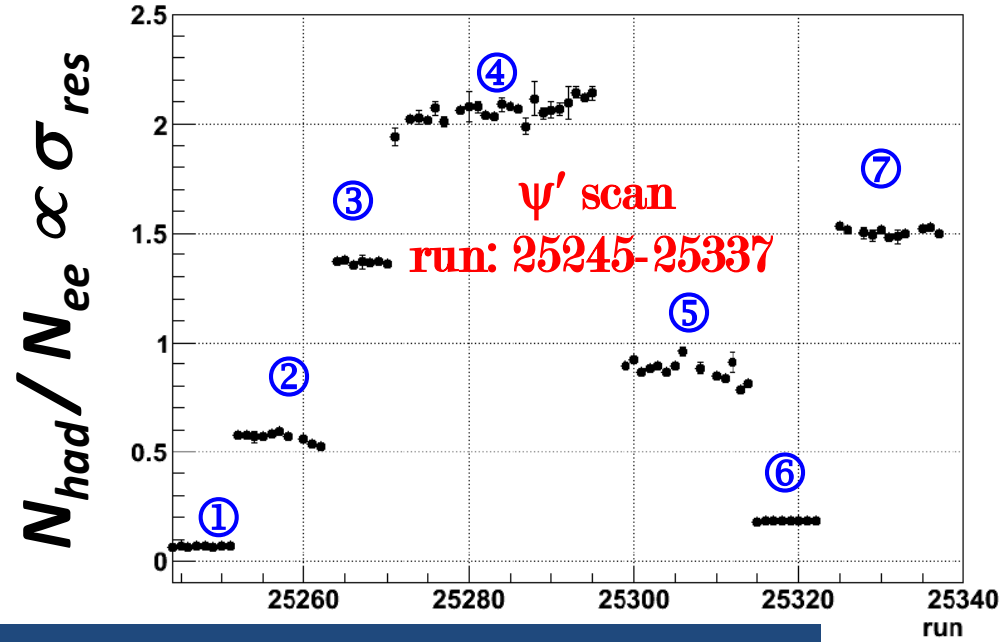
- At higher order

$$R = 3\, K_{QCD} \sum_q Q_q^2 ,$$

$$K_{QCD} = 1 + \frac{\alpha_S(\mu^2)}{\pi} + \sum_{n\geq 2} C_n\left(\frac{s}{\mu^2}\right)\left(\frac{\alpha_S(\mu^2)}{\pi}\right)^n$$

- R is one of the most fundamental quantities in particle physics that directly reflect the flavor and color of the quarks.
- Directly test of quark model and QCD, and discover new particles.





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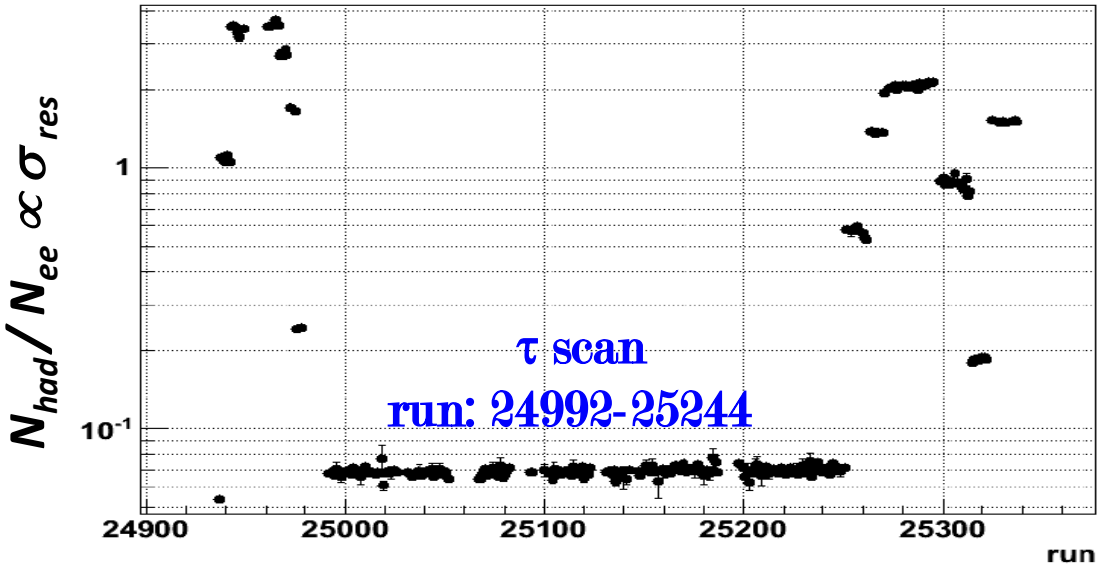


Pseudomass Method

$$m_\tau^2 = p_\tau^2 = \left(p_h + p_\nu \right)^2$$

$$= p_h^2 + p_\nu^2 + 2\, p_h \bullet p_\nu$$

$$= m_h^2 + 2\left(E_h E_\nu - \vec{p}_h \bullet \vec{p}_\nu \right)$$

$$\geq m_h^2 + 2\left(E_h E_\nu - \left| \vec{p}_h \right| \left| \vec{p}_\nu \right| \right)$$

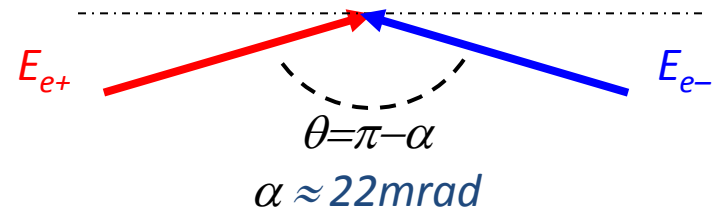
$$= m_h^2 + 2\left(E_h - \left| \vec{p}_h \right| \right)\left(E_\tau - E_h \right)$$

CM energy setting

$$S = (E_{e^+} + E_{e^-})^2 - (p_{e^+} + p_{e^-})^2$$

$$E_{cm}^{AA} = (E_{e^+} + E_{e^-}) \cdot \cos\frac{\alpha}{2}$$

$$E_{cm}^{GA} = 2\sqrt{E_{e^+}E_{e^-}} \cdot \cos\frac{\alpha}{2}$$

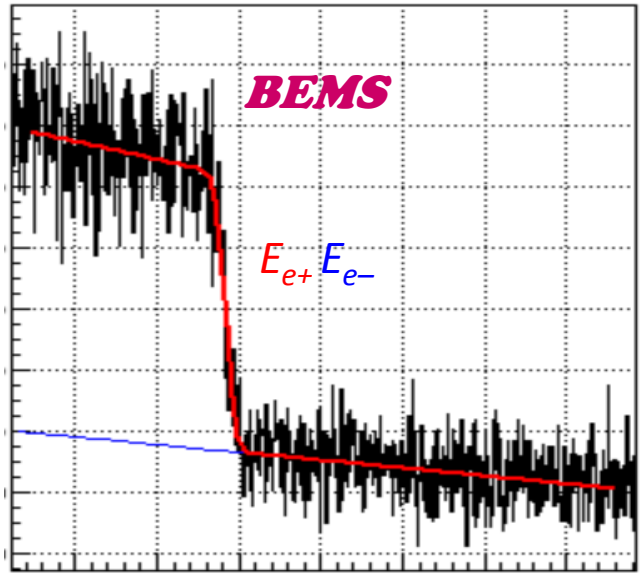


$$E_{cm}^{GA} \approx E_{cm}^{GA} \approx 2E_{beam}\left(1-\frac{\alpha^2}{8}\right)$$

$E_{cm}^{GA} \approx E_{cm}^{GA} \approx E_{cm}^{\sqrt{S}}$

α-effect $\approx 6\times10^{-5}$,

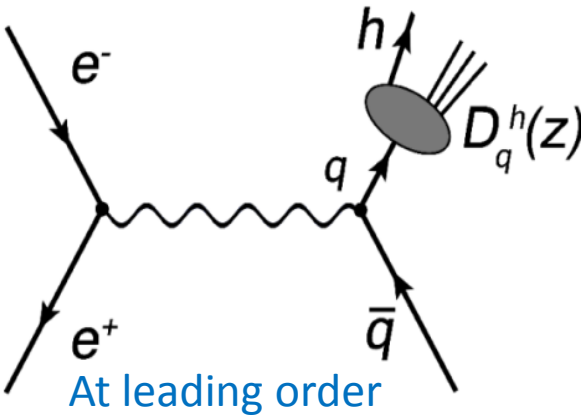
0.11MeV @ τ threshold



$$E_{cm}^{\sqrt{S}} \approx 2E_{beam}\left(1-\frac{\alpha^2}{8}\right)$$

$$E_{cm}^{\sqrt{S}} = \sqrt{2m_e^2 + 2E_{e^+}E_{e^-} - 2\sqrt{E_{e^+}^2 - m_e^2}\sqrt{E_{e^-}^2 - m_e^2} \cdot \cos(\pi - \alpha)}$$

Fragmentation Function



Fragmentation function $D^h_q(z)$: probability that hadron h is found in the debris of a parton (quark/gluon) carrying a fraction $z = 2E_h/\sqrt{s}$ of parton's energy.

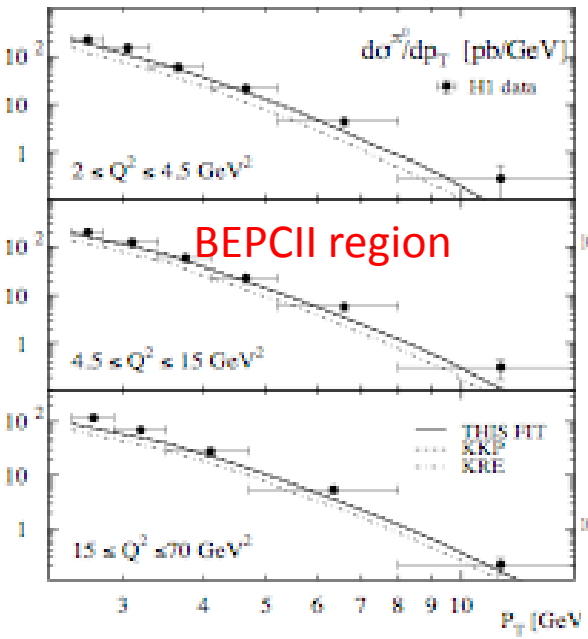
LO: $d\sigma(e^+e^- \rightarrow h+X)/dz=\sum_q \sigma(e + e- \rightarrow q\underline{q})(D^h_q(z) + D^h_{\underline{q}}(z))$

No good data at $\sqrt{s} < 10$ GeV

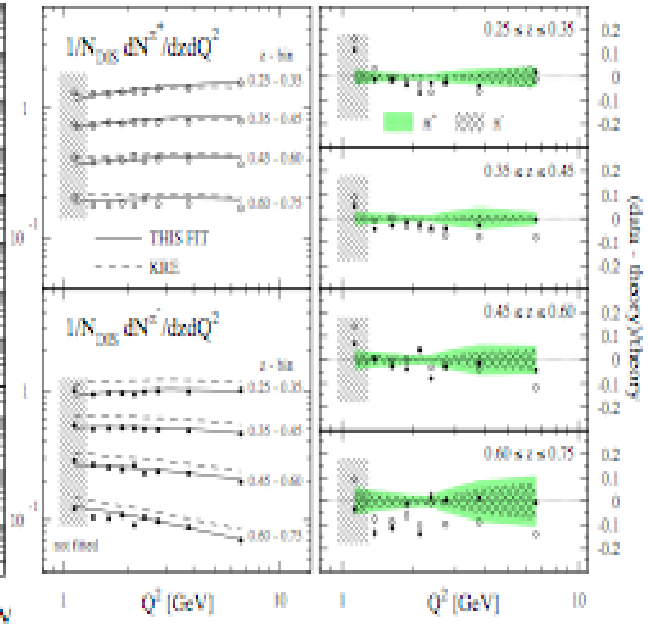
- DASP: $\pi^\pm$ at 3.6GeV; average stat. uncertainty 18%
- DASP: $k^\pm$ at 3.6GeV; average stat. uncertainty 55%

Frangmenation Function

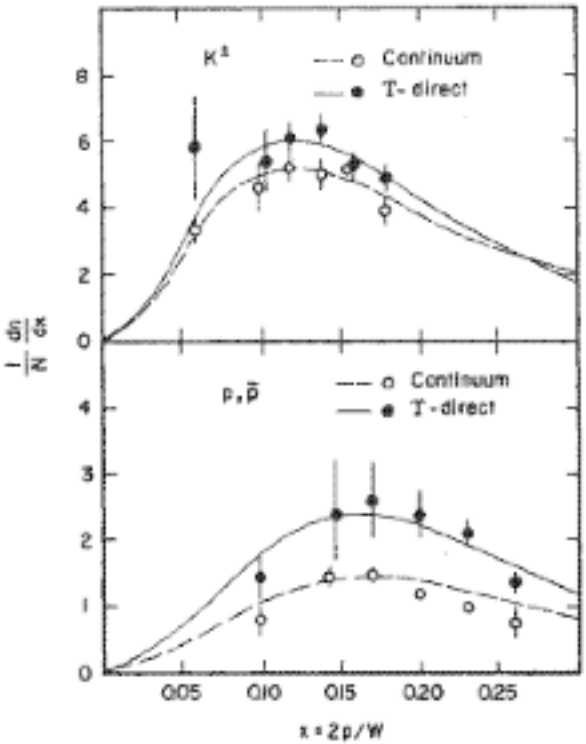
ep collision, H1 data



Fix target experiment HEMES



CLEO



BESIII can provide e^+e^- data in 2-5 GeV

R values and QCD

- R, α_s and charm quark mass
- Quark fragmentation functions
- Form factor of baryon (p, Λ, …)
- MLLA/LPHD predictions

  - ξ distribution (ξ=-ln(2p/$\sqrt{s}$), parameter Λ & KLPHD

  - Multiplicity, 2nd binomial moment R_2

R & QCD: analyses

- $$R = \frac{1}{\sigma_{\mu+\mu-}} \cdot \frac{N_{had} - N_{bg}}{L \cdot \varepsilon_{had} \cdot (1+\delta)}$$

N_{had}, N_{bg} : observed hadronic events, backgrounds

L: integrated luminosity

ε_{had}: detection efficiency for N_{had}

δ: radiative correction factor

- J/ψ line shape fitting
- QCD studies (so far):
  - ppbar cross section and form factor
  - $\pi^+\pi^-$ cross section and form factor
  - Fragmentation function

More on Form Factor

Puzzles related to proton timelike FF:

- Proton FF factor 2 higher in Timelike region compared to Spacelike Region (pQCD precicts them to be equal)

- Neutron FF ~ factor 2 higher than proton FF → measurement of Neutron FF at BES-III very important

- Steps at 2.2 and 2.9 GeV
- Threshold enhancement

} (see slide 26)

Motivations

- Hadronic contribution to
  - QED running coupling constant $\alpha_{QED}(M_Z)$

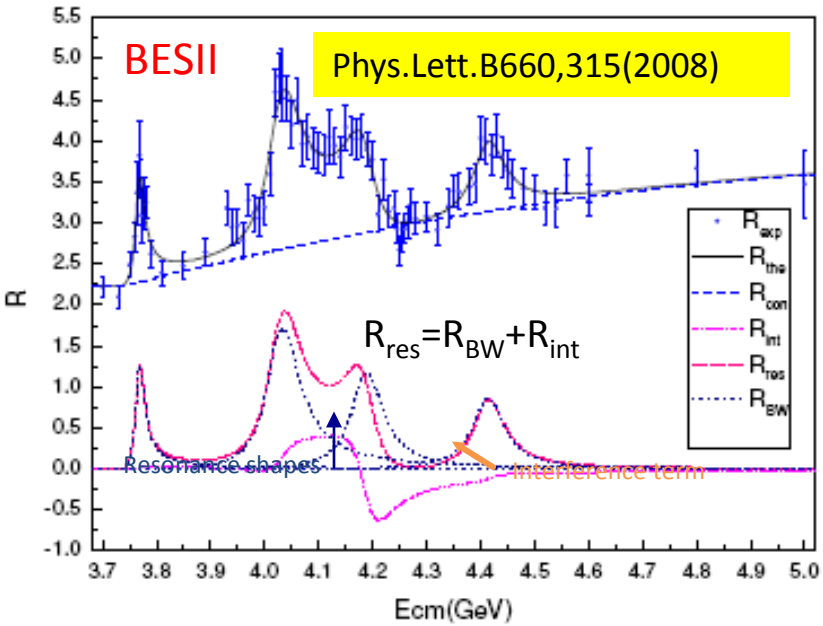
$$\Delta\alpha_{had}^{(5)}(s) = -\frac{\alpha s}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s' - s - i\varepsilon}$$

  - Anomalous magenet moment of the muon a_μ, or $(g_\mu - 2)$

$$a_\mu^{had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{K}(s')}{s'^2} R(s')$$

- Resonance structure and component in open charm region;
- Strong coupling constant α_s determination;
- Proton form factor;
- Charm quark mass m_c determination;
- X, Y, Z particles and other possible new resonances
- Physics with D_s, Charmed baryons, ……

Resonances in the Open Charm Region



- All possible two-body decays of ψ(3770), ψ(4040), ψ(4160), ψ(4415) are included in the fit.
- Interference, phase and energy-dependent width must be taken into account in the fit.

$$
\begin{aligned}
\psi(3770) &\Rightarrow D\bar{D};\\
\psi(4040) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s;\\
\psi(4160) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*;\\
\psi(4415) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, \bar{D}D^*, D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s^*.
\end{aligned}
$$

We need high statistic data taken at each peak position to measure the resonance parameters by knowing the cross section of their exclusive decay channels.

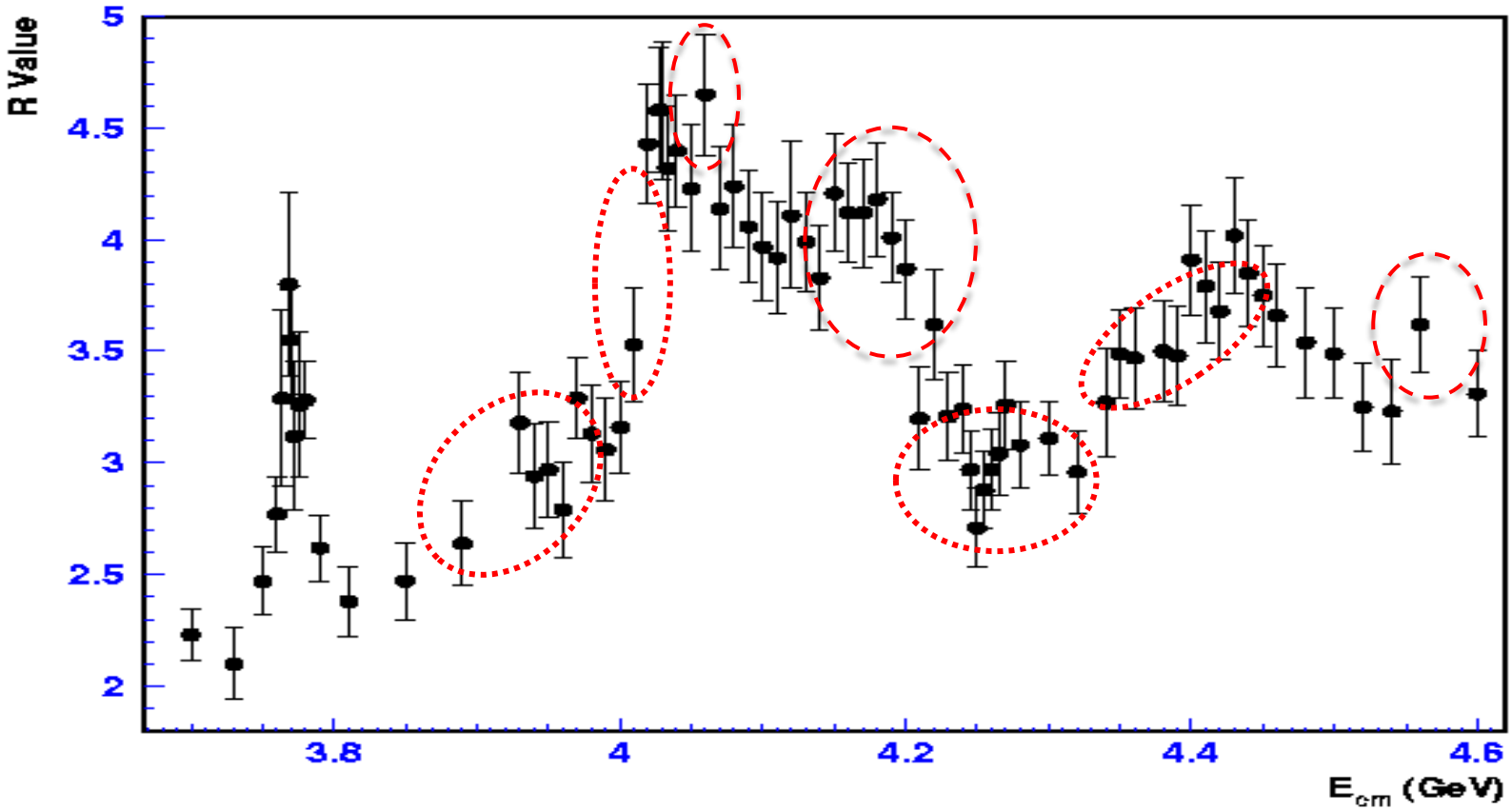
- Non-resonant contribution
- Open charm threshold

Parameters of the Broad Resonances

Parameters (M, Γ_{tot}, Γ_{ee}) of the $J^{PC} = 1^{--}$ conventional charmonia $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ remain quite uncertain and model dependent:

	M, MeV	Γ_{tot}, MeV	Γ_{ee}, keV	δ, deg	
ψ(3770)	3772.92 ± 0.35	27.3 ± 1.0	0.265 ± 0.018		PDG09
	3772.0 ± 1.9	30.4 ± 8.5	0.22 ± 0.05	0	BES08
ψ(4040)	4039 ± 1	80 ± 10	0.86 ± 0.07		PDG09
	4039.6 ± 4.3	84.5 ± 12.3	0.83 ± 0.20	130 ± 46	BES08
ψ(4160)	4153 ± 3	103 ± 8	0.83 ± 0.07		PDG09
	4191.7 ± 6.5	71.8 ± 12.3	0.48 ± 0.22	293 ± 57	BES08
ψ(4415)	4421 ± 4	62 ± 20	0.58 ± 0.07		PDG09
	4415.1 ± 7.9	71.5 ± 19.0	0.35 ± 0.12	234 ± 88	BES08

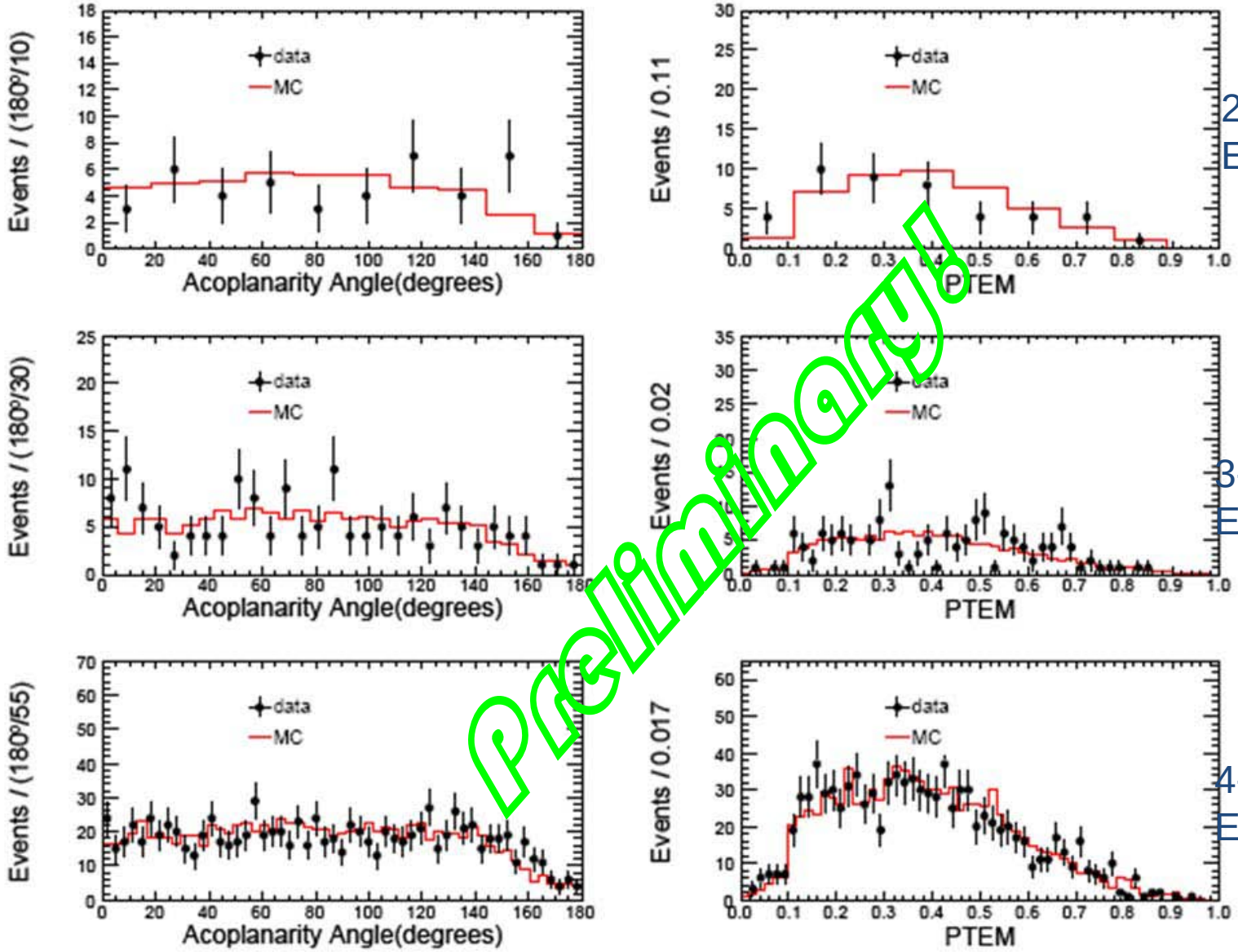
Broad Resonances Beyond Open Charm



- What are these broad resonances?
- Mass resgion where some X, Y particles are found.
- Possible new resonance that not yet discoveried?

Acoplanarity angle

PTEM



2-point
Ecm=3553.8MeV

3-point
Ecm=3561.1MeV

4-point
Ecm=3600.2MeV

	Scan point	Run number	$E_{e^-}(MeV)$	E_{e^+} (MeV)	E_{cm} (MeV)
τ	1	24983-25015	1771.558 ± 0.067	1771.069 ± 0.053	3542.413 ± 0.085
	2	25016-25094	1777.307 ± 0.047	1776.730 ± 0.046	3553.822 ± 0.075
	3	25100-25141	1780.926 ± 0.055	1780.431 ± 0.065	3561.142 ± 0.085
	4	25143-25243	1800.526 ± 0.044	1799.878 ± 0.044	3600.186 ± 0.062
J/ψ	1	24937-24937	1544.542 ± 0.135	1544.312 ± 0.217	3088.667 ± 0.256
	2	24938-24942	1547.917 ± 0.099	1547.548 ± 0.106	3095.278 ± 0.145
	3	24943-24949	1548.692 ± 0.103	1548.171 ± 0.086	3096.676 ± 0.135
	4	24959-24966	1549.079 ± 0.109	1548.714 ± 0.075	3097.606 ± 0.133
	5	24967-24971	1549.451 ± 0.081	1549.014 ± 0.114	3098.278 ± 0.140
	6	24972-24975	1549.566 ± 0.101	1549.438 ± 0.083	3098.817 ± 0.131
	7	24976-24978	1552.186 ± 0.088	1551.936 ± 0.107	3103.934 ± 0.139
ψ'	1	25245-25251	1838.183 ± 0.256	1837.940 ± 0.157	3675.901 ± 0.300
	2	25252-25262	1842.177 ± 0.090	1841.279 ± 0.220	3683.653 ± 0.303
	3	25264-25270	1842.755 ± 0.153	1842.489 ± 0.087	3685.113 ± 0.230
	4	25271-25295	1843.402 ± 0.075	1842.893 ± 0.110	3686.337 ± 0.189
	5	25299-25314	1844.787 ± 0.125	1844.137 ± 0.107	3688.819 ± 0.226
	6	25315-25322	1846.832 ± 0.138	1846.487 ± 0.108	3693.515 ± 0.245
	7	25325-25337	1844.130 ± 0.091	1843.396 ± 0.088	3687.573 ± 0.158

$$E_{cm}^{\sqrt{s}} \approx 2E_{beam}\left(1-\frac{\alpha^2}{8}\right)$$

Bhabha

$e^+e^- \rightarrow e^+e^-$

Di-gamma

$e^+e^- \rightarrow \gamma\gamma$

	Scan point	N_2^{obs}	N_1^{obs}	L_{bhabha} (nb^{-1})	N^{obs}	$L_{digamma}$ (nb^{-1})
τ	1	1575827	58018	4502.89	74240	4252.17
	2	2043538	75371	5877.14	96570	5566.82
	3	1413321	52432	4082.30	67192	3889.29
	4	3411037	126081	10068.29	161482	9553.18
J/ψ	1	38143	1393	81.79	1804	78.52
	2	114205	7191	239.19	5016	219.26
	3	137995	21744	260.07	5557	243.13
	4	109972	17947	206.00	4718	206.55
	5	116221	15593	225.34	5104	223.53
	6	106130	10079	215.17	4950	216.87
	7	150860	6618	324.23	7218	317.31
ψ'	1	269201	9878	830.58	12763	787.04
	2	284362	10995	879.30	13291	823.10
	3	285762	12775	878.75	13432	832.47
	4	414291	20998	1266.84	19097	1184.34
	5	565681	27641	1734.27	26761	1660.77
	6	265322	11889	817.48	12366	767.97
	7	501530	19215	1559.59	23624	1470.75

Motivation of high accurate τ mass measurement

Elementary parameter in SM (PDG2012)

- M_e=0.510998910±0.000000013 (2.6×10^{-8})
- M_μ=105.658367±0.000004 (3.8×10^{-8})
- [illegible] (9.0×10^{-5})

Lepton universality testing

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \frac{B(\tau \to e\nu_e\nu_\tau)}{B(\mu \to e\nu_e\nu_\mu)}(1+\Delta_e)$$

g_τ and g_μ : coupling constants; τ_τ and τ_μ :life time of τ and μ ; $B(\tau \to e\nu_e\nu_\tau)$ and $B(\mu \to e\nu_e\nu_\mu)$: decay branching ratio; Δ_e:correct factor (phase factor, radiative correction factor of QED, correct factor of propagator of W-meson etc.)

Yoshio Koideo (1981) equality testing

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2$$

$$\Delta f_m = \sqrt{\sum_{i=e,\mu,\tau}\left(m_i - \frac{2}{3}\sum_{k=e,\mu,\tau}\sqrt{m_i m_k}\right)^2 \cdot \left(\frac{\delta m_i}{m_i}\right)^2}$$

→ $\Delta f_m \cong 1/3\ \delta m_\tau$