# Puzzle on the Pion DA from the Pion-Photon TFF with the Belle and BaBar Data

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# MOTIVATION

- Pion distribution amplitude (DA)
  - **Physical meaning**:

    the momentun distribution of the valence quarks in the pion.
  - **Application**:

    the important component for the QCD LCSR and the QCD factorization theory in dealing with the exclusive processes: *pion electromagnetic form factor, pion-photon transition form factor, B to pion transition form factor*…….

# MOTIVATION

- Pion distribution amplitude (DA)
  - **Research methods**:

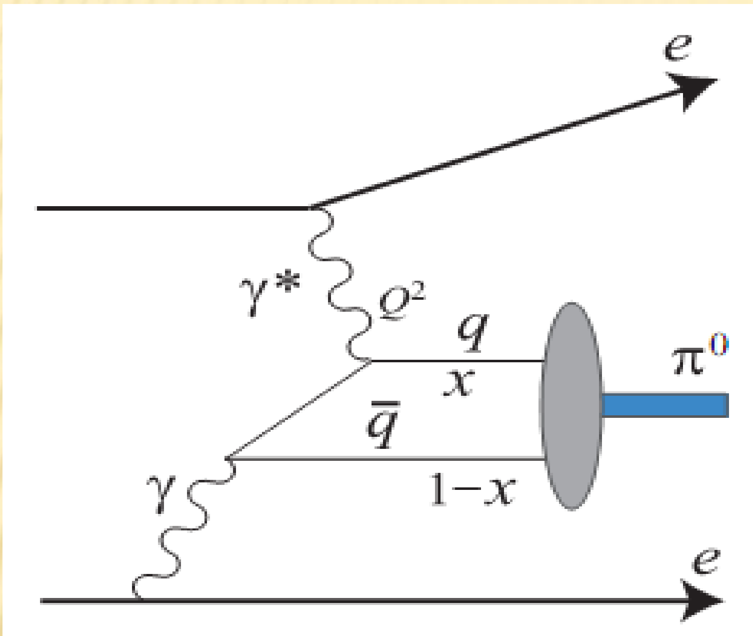
    solve DA's evolution equation(the asymptotic form);

    SVZ SRs and Lattice;

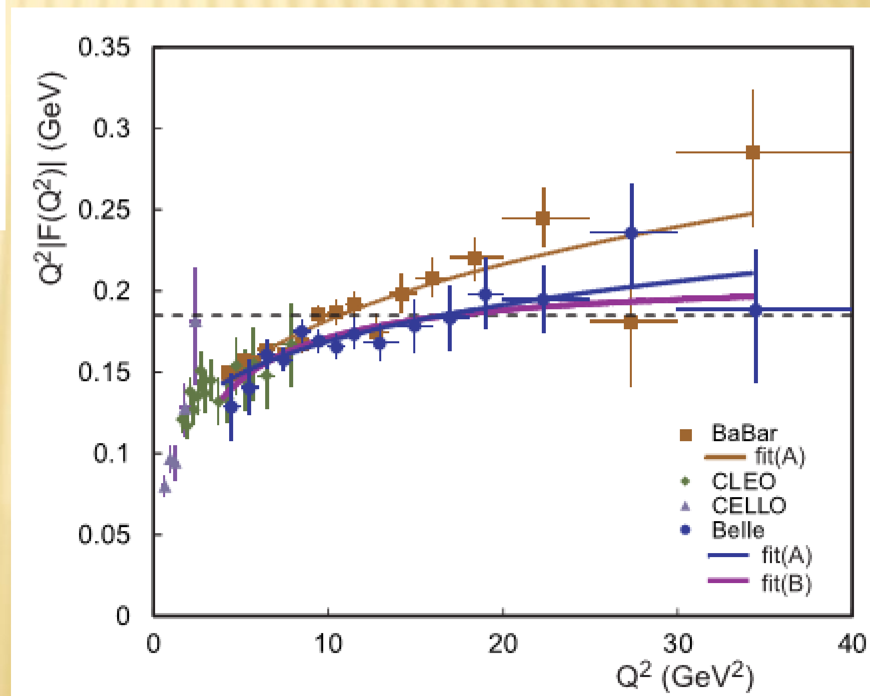
    models(***BHL LC harmonic oscillator model***, CZ(Chernyak-Zhitnitsky)-model, ……).
  - **Existent controversy**:

    Where the DA's asymptotic form is applicable? How about the DA's form in the low and middle region?
  - ***Information from the pion-photon transition form factor(TFF)***

# PION-PHOTON TFF




- **Experimentally**
  - CELLO
    (1991, $<3\text{GeV}^2$ )
  - CLEO
    (1998, $1.5\text{-}9.2\text{GeV}^2$)




- **Experimentally**
  - BABAR
    (2009, $4\text{-}40\text{GeV}^2$ )
  - Belle
    (2012, $4\text{-}40\text{GeV}^2$ )

# PION-PHOTON TFF

- **Definition**

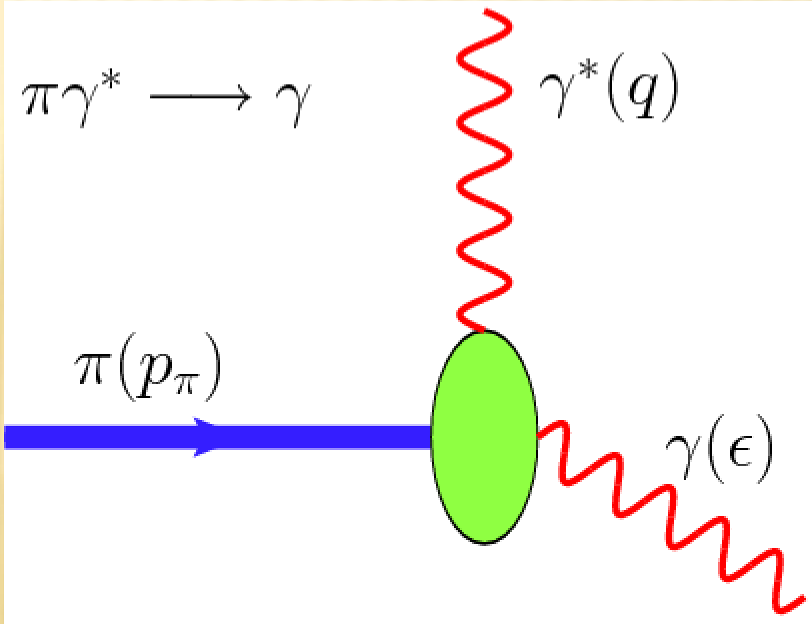
Pion-photon TFF

$$\Gamma_\mu = -ie^2 F_{\pi\gamma} \varepsilon_{\mu\nu\alpha\beta} p_\pi^\nu \varepsilon^\alpha q^\beta$$

The amplitude of $e\pi \rightarrow e\gamma$

- **Theoretical approach**
  - Light-Cone Sum Rules(LCSRs)
  - ***Light-cone pQCD approach***

  ……

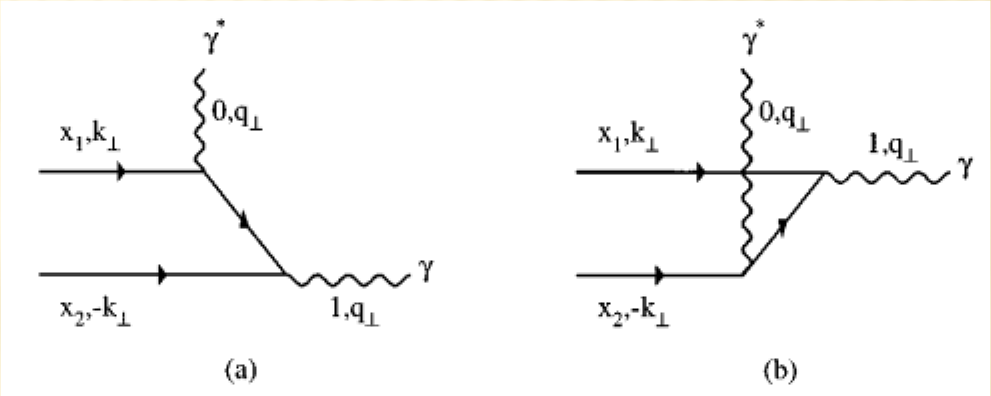


# PION-PHOTON TFF(*LC PQCD*)

- The pion-photon TFF can be written as the convolution of the hard-scattering amplitude and the pion wave function:

$$F_{\pi\gamma}(Q^2) = \int_0^1 dx_1 dx_2 \delta(1 - x_1 - x_2) T_H(x_i, Q) \Psi(x_i, Q)$$

- The hard-scattering amplitude can be calculated perturbativel:



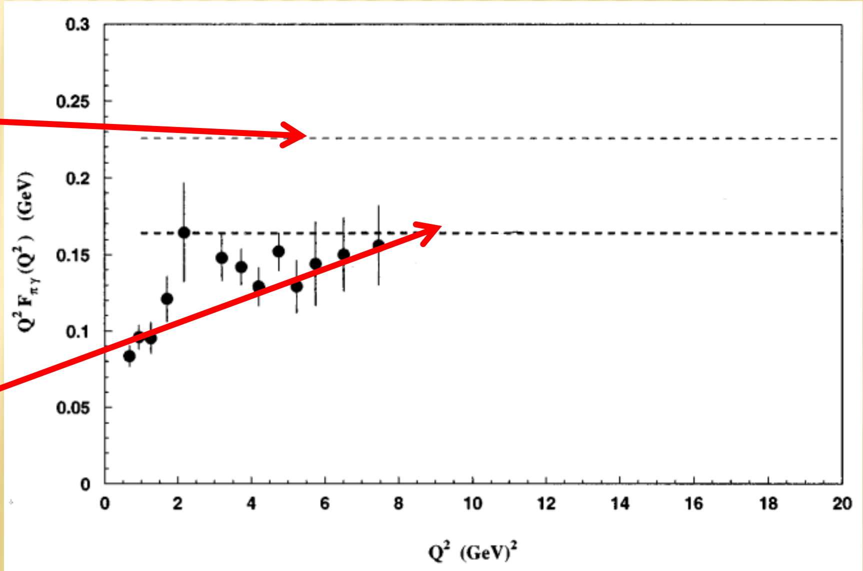
# PION-PHOTON TFF(*LC PQCD*)

- 1980, Lepage and Brodsky neglect the transverse momentum of quark to predict:

$$F_{\pi\gamma}(Q^2) = \frac{2}{\sqrt{3}Q^2}\int_0^1 \frac{[dx]}{x_1 x_2}\phi_\pi(x)\left[1 + O\left(\alpha_s, \frac{m^2}{Q^2}\right)\right]$$

CZ-Model
$\phi^{CZ} = 15/2(2x-1)^2(1-(2x-1)^2)$

AS wave function
$\phi^{as} = 6x(1-x)$



# PION-PHOTON TFF(*LC PQCD*)

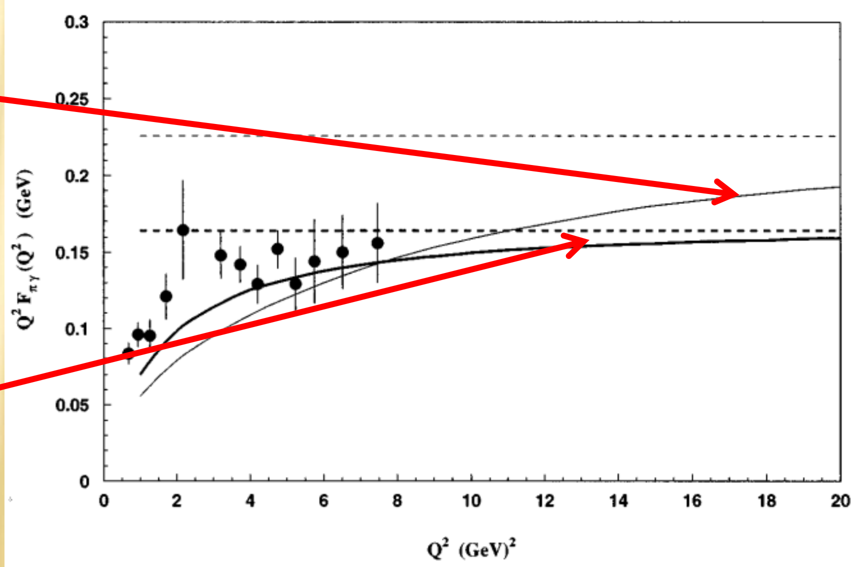
- 1996, Fu-Guang Cao, Tao Huang and Bo-Qiang Ma, with the transverse momentum of quark dependence:

$$F_{\pi\gamma}(Q^2) = 2\sqrt{n_c}(e_u^2 - e_d^2)\int_0^1 [dx]\int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\Psi(x_i,\mathbf{k}_\perp)T_H(x_1,x_2,\mathbf{k}_\perp)$$

CZ-Model

BHL model

$$\Psi^{BHL}(x,\mathbf{k}_\perp) = A\exp\left[-\frac{\mathbf{k}_\perp^2 + m_q^2}{8\beta^2 x(1-x)}\right]$$

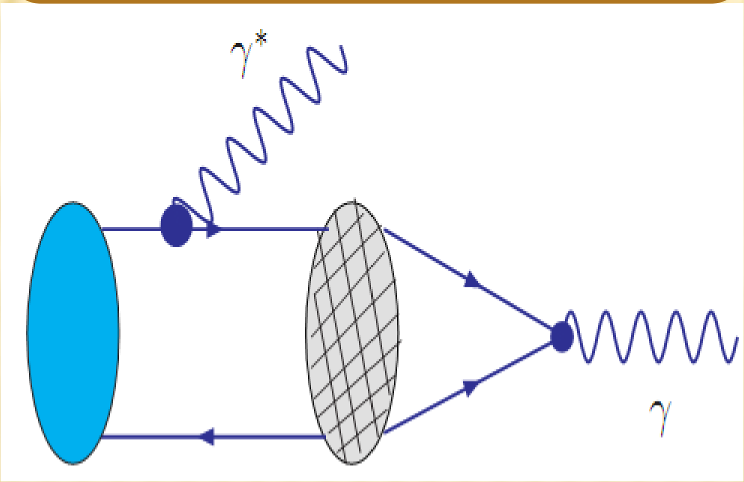


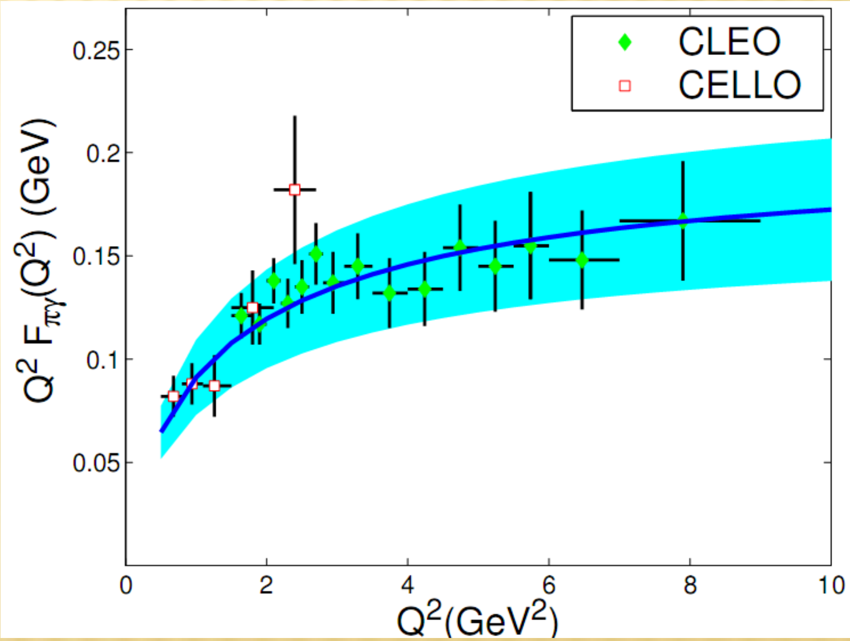
# PION-PHOTON TFF(*LC PQCD*)

- 2007, Tao Huang and Xing-Gang Wu, with the non-valence-quark part contribution:

$$F_{\pi\gamma}(Q^2) = F_{\pi\gamma}^{(V)}(Q^2) + F_{\pi\gamma}^{(NV)}(Q^2)$$

The non-valence-quark contribution



# Pion-Photon TFF(*LC pQCD*)

- Considering the $k_\perp$-correction, non-valence-quark contribution and the NLO correction to the valence-quark contribution, the pion-photon TFF would be:

$$F_{\pi\gamma}(Q^2) = F_{\pi\gamma}^{(V)}(Q^2) + F_{\pi\gamma}^{(NV)}(Q^2),$$

$$F_{\pi\gamma}^{(V)}(Q^2) = \frac{1}{4\sqrt{3}\pi^2}\int_0^1\int_0^{x^2Q^2}\frac{dx}{xQ^2}\left[1-\frac{\alpha_s(Q^2)}{3\pi}\left(\ln\frac{Q^2}{xQ^2+k_\perp^2}\right.\right.$$

$$\left.\left.+2\ln x+3-\frac{\pi^2}{3}\right)\right]\Psi_{q\bar{q}}(x,k_\perp^2)dk_\perp^2,$$

$$F_{\pi\gamma}^{(NV)}(Q^2) = \frac{\alpha}{(1+Q^2/\kappa^2)}.$$

# PION DA

- The model of the pion WF can be written as:

$$\Psi_{q\bar{q}}(x,\mathbf{k}_\perp) = \sum_{\lambda_1\lambda_2} \chi^{\lambda_1\lambda_2}(x,\mathbf{k}_\perp)\Psi^R_{q\bar{q}}(x,\mathbf{k}_\perp),$$

$$\Psi^R_{q\bar{q}}(x,\mathbf{k}_\perp) =$$

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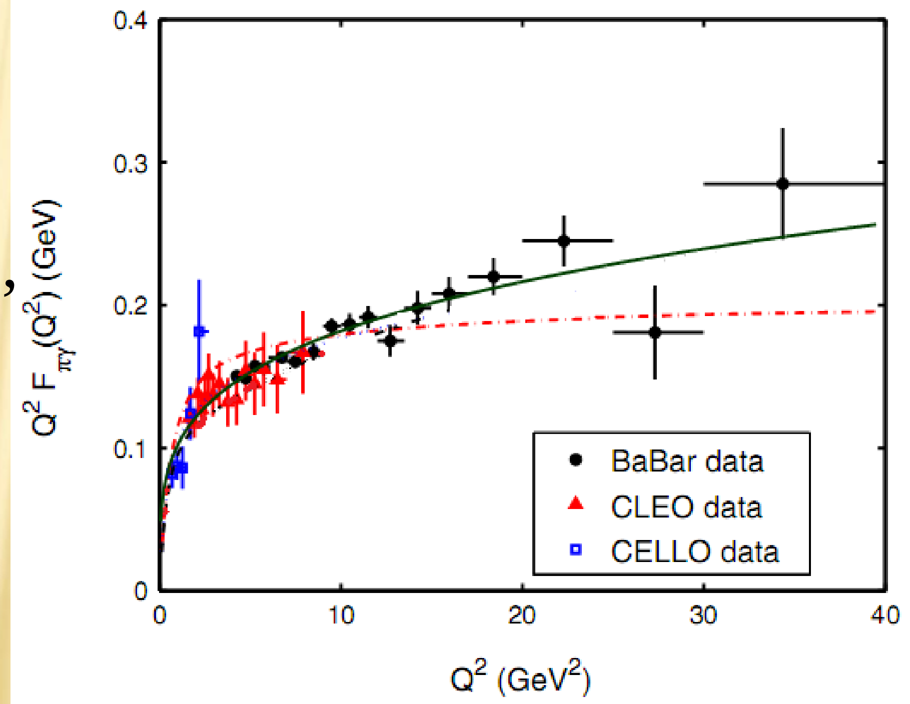

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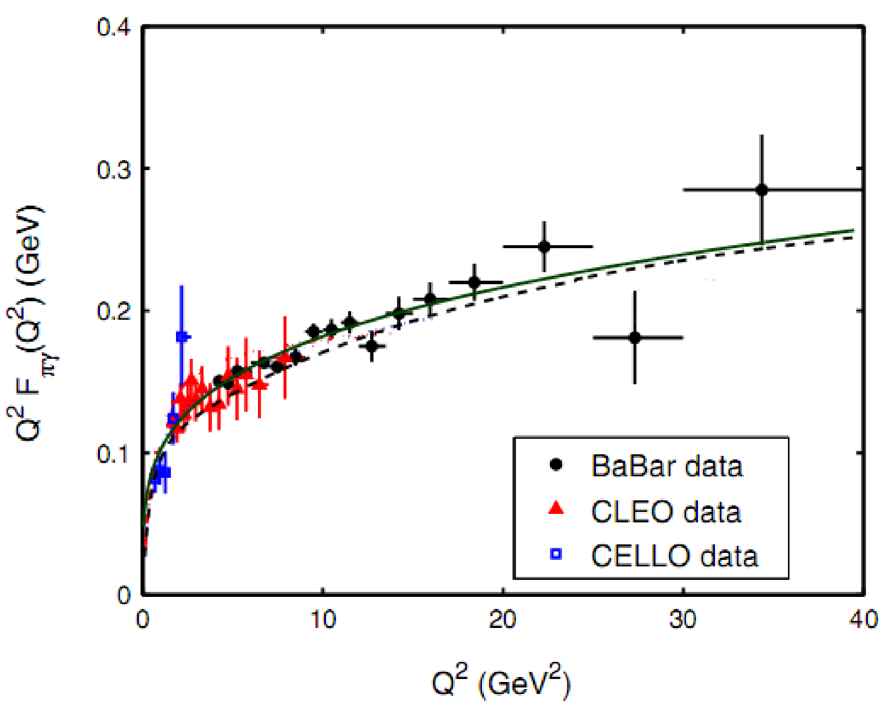
$$\Psi^R_{q\bar{q}}(x,\mathbf{k}_\perp) =$$

$$A\exp\left[-\frac{\mathbf{k}_\perp^2 + m_q^2}{8\beta^2 x(1-x)}\right],$$

$$\varphi_\pi(x) =$$

$$1 + B \times C_2^{3/2}(2x-1).$$



# PION DA

- How to determine the parameters of the WF:

The wave-function normalization condition

$$\int_0^1 dx \int_{|\mathbf{k}_\perp|^2<\mu_0^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_{q\bar{q}}(x,\mathbf{k}_\perp) = \frac{f_\pi}{2\sqrt{3}}$$

The constraint derived from $\pi^0 \to \gamma\gamma$ decay amplitude

$$\int_0^1 dx \Psi_{q\bar{q}}(x,\mathbf{k}_\perp = 0) = \frac{\sqrt{3}}{f_\pi}$$

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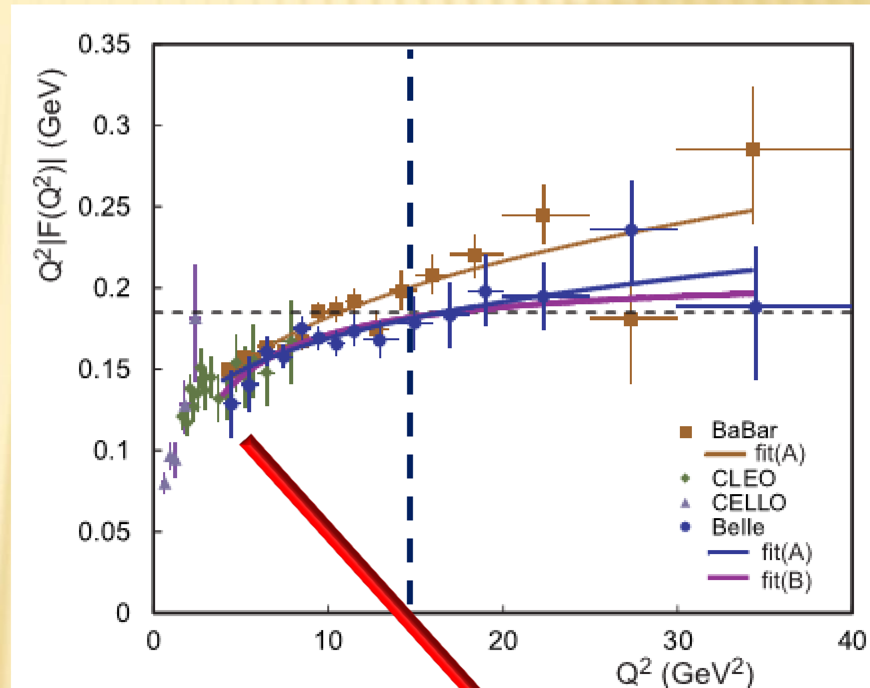



$$B = 0.00 \sim 0.60$$

# PION DA

- The pion DA can be obtained as following:

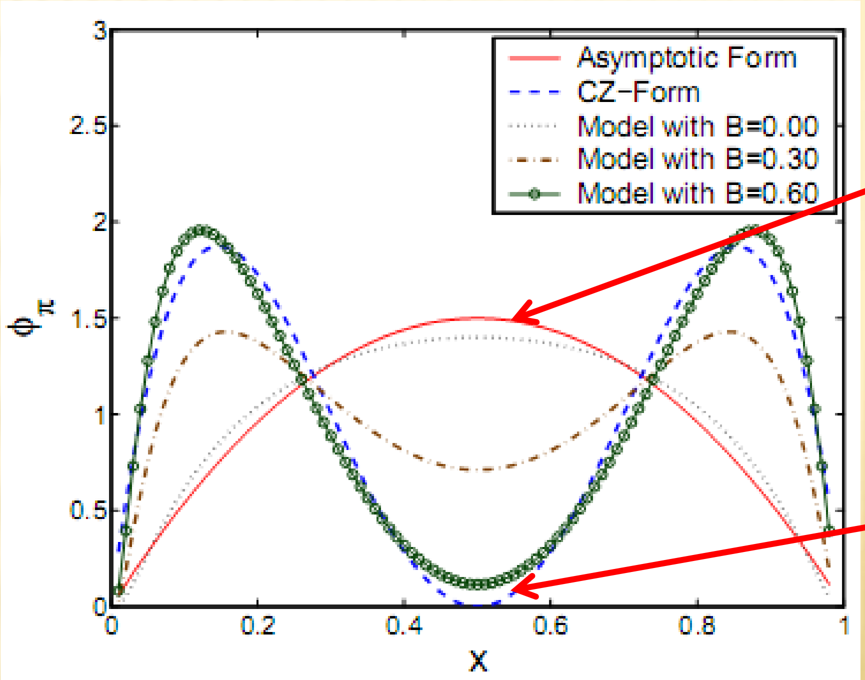
$$\phi_\pi(x,\mu_0^2) = \frac{2\sqrt{3}}{f_\pi}\int_{|\mathbf{k}_\perp|^2\le\mu_0^2}\frac{d^2\mathbf{k}_\perp}{16\pi^3}\Psi_{q\bar{q}}(x,\mathbf{k}_\perp)$$

$$= \frac{\sqrt{3}Am\beta}{2\sqrt{2}\pi^{3/2}f_\pi}\sqrt{x(1-x)}\times(1+BC_2^{3/2}(2x-1))$$

$$\times\left(erf\left[\sqrt{\frac{m^2+\mu_0^2}{8\beta^2x(1-x)}}\right]-erf\left[\sqrt{\frac{m^2}{8\beta^2x(1-x)}}\right]\right)$$

# PION DA

- Comparison of the pion DA model with the asymptotic-DA and the CZ-DA:




$$\phi^{as} = 6x(1-x)$$

$$\phi^{CZ} = \frac{15}{2}(2x-1)^2(1-(2x-1)^2)$$

# PION DA

- The pion DA Gegenbauer moments can be calculated by the following way:

$$a_n(\mu_0^2) = \frac{\int_0^1 dx\phi_\pi(x,\mu_0^2)C_n^{3/2}(2x-1)}{\int_0^1 dx6x(1-x)\left[C_n^{3/2}(2x-1)\right]^2}$$

- The soft part contribution of the pion electromagnetic form factor can be written as:

$$F_{\pi^+}^s(Q^2) = \int \frac{dxd^2\mathbf{k}_\perp}{16\pi^3} \sum_{\lambda_1\lambda_2} \Psi_{q\bar{q}}^*(x,\mathbf{k}_\perp,\lambda_1)\Psi_{q\bar{q}}(x,\mathbf{k}'_\perp,\lambda_2)$$

- The probability for finding the lowest valence quark state:

$$P_{q\bar{q}} = F_{\pi^+}^s(Q^2)|_{Q^2=0}$$

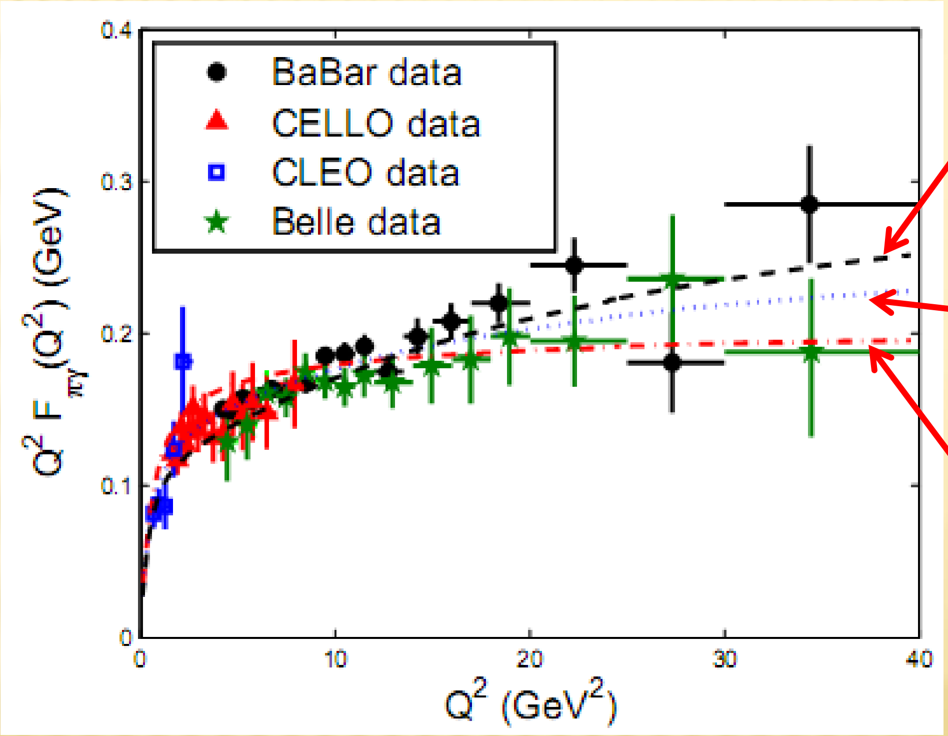
- The charged mean square radius:

$$\langle r_{\pi^+}^2 \rangle^{q\bar{q}} \approx -6\frac{\partial F_{\pi^+}^s(Q^2)}{\partial Q^2}|_{Q^2=0}$$

| $B$ | $A(\mathrm{GeV}^{-1})$ | $\beta(\mathrm{GeV})$ | $P_{q\bar{q}}$ | $\sqrt{\langle r_{\pi^+}^2\rangle^{q\bar{q}}}$ | $a_2(\mu_0^2)$ |
|---|---|---|---|---|---|
| 0.00 | 25.06 | 0.586 | 63.5% | 0.341 | 0.03 |
| 0.30 | 20.26 | 0.668 | 62.0% | 0.378 | 0.36 |
| 0.60 | 16.62 | 0.745 | 79.9% | 0.451 | 0.68 |

# NUMERICAL ANALYSIS

- $Q^2 F_{\pi\gamma}(Q^2)$ with the model WF by taking $m_q$=0.30 GeV and by varying $B$ within the region of [0.00,0.60]:



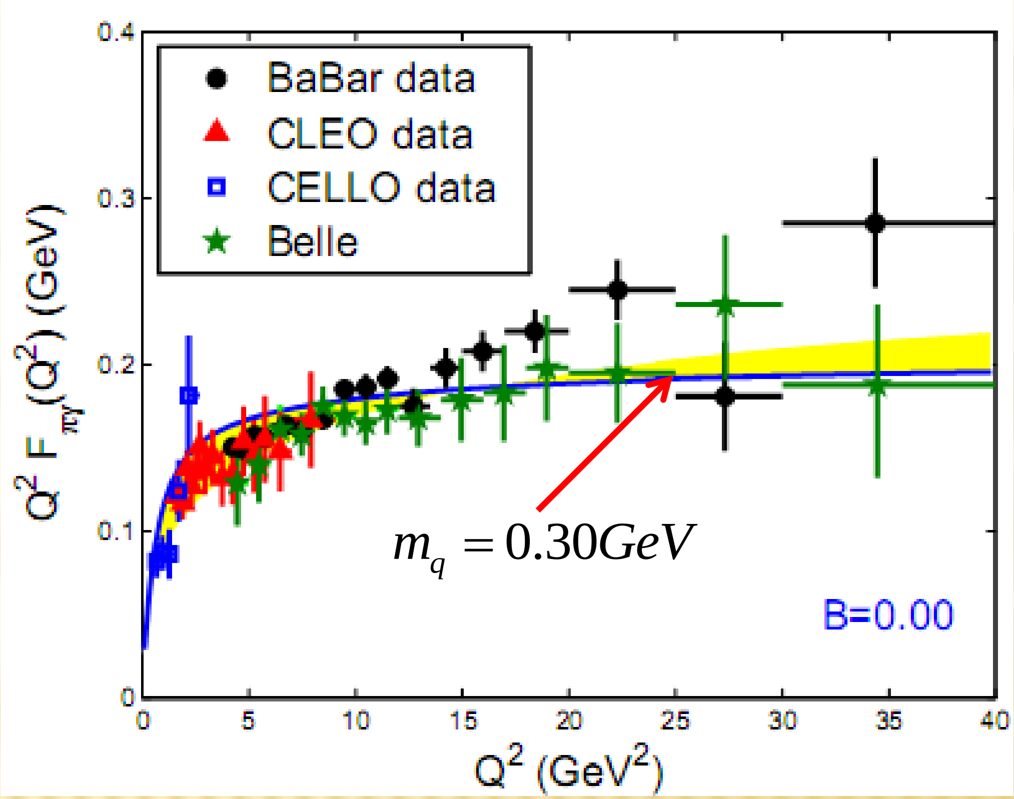

$B$=0.60

$B$=0.30

$B$=0.00

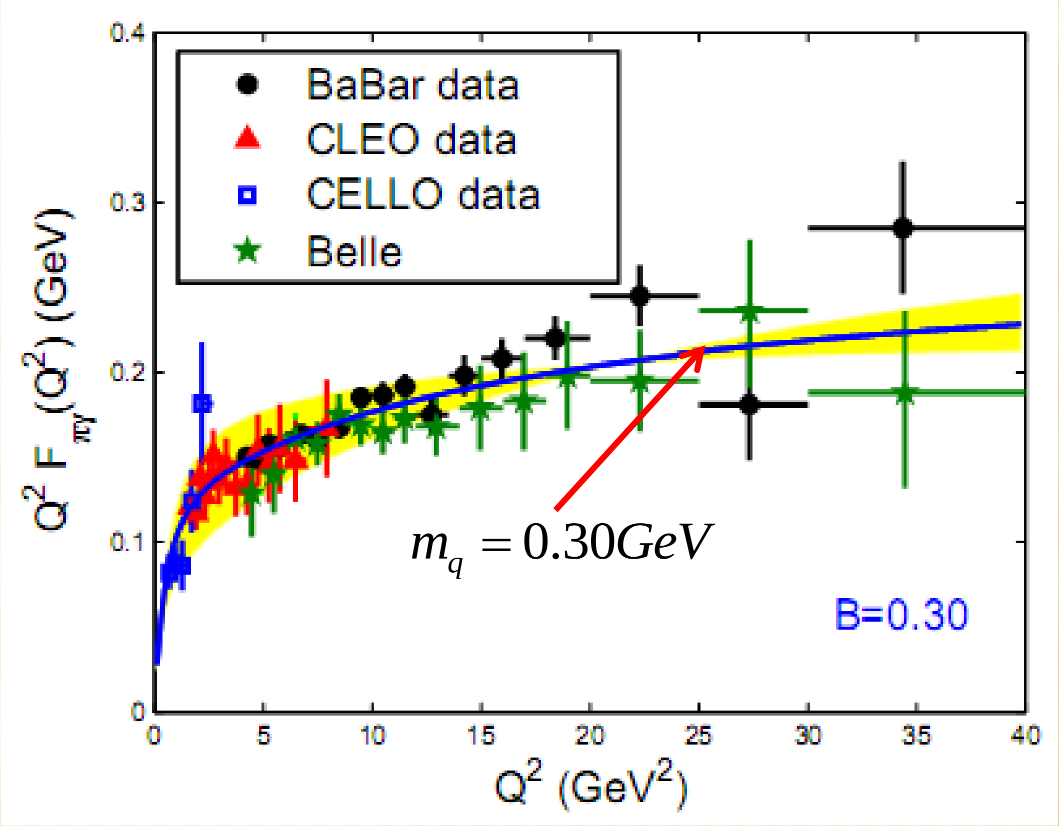
# NUMERICAL ANALYSIS

- $Q^2 F_{\pi\gamma}(Q^2)$ with the model WF by fixing $B$=0.00 (Asymptotic-like DA) and by varying $m_q$ within the region [0.20,0.30] GeV:



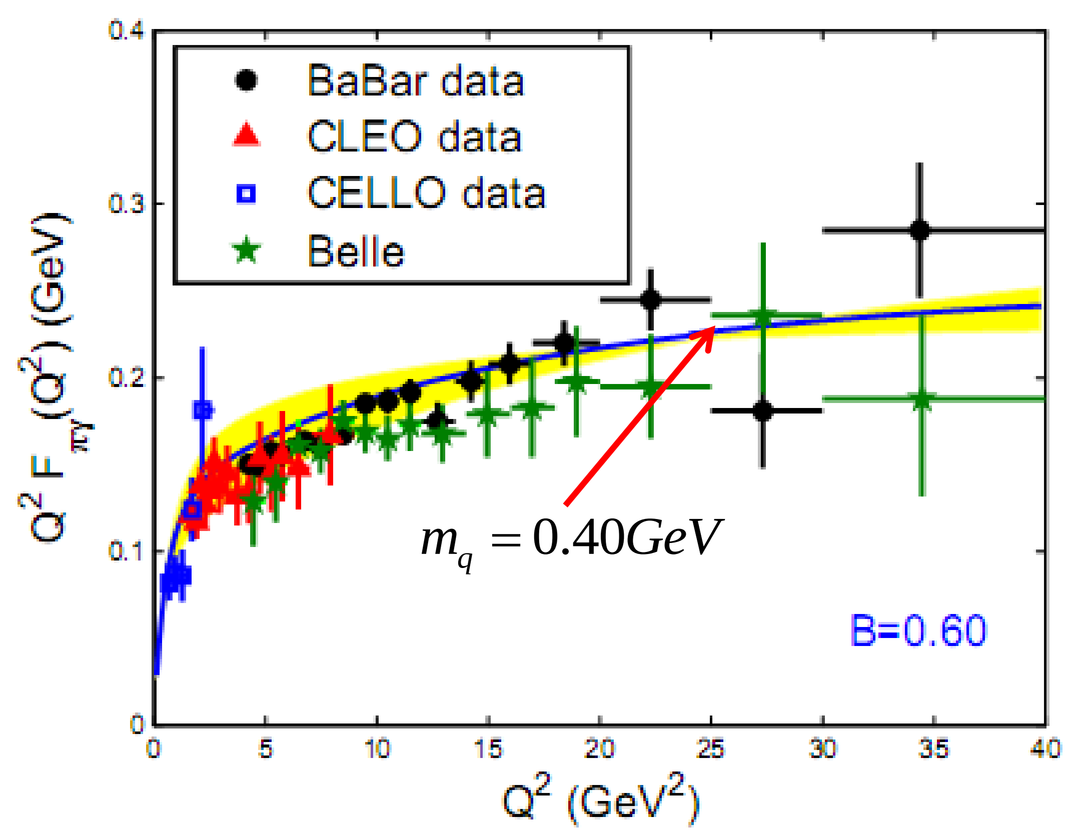
# NUMERICAL ANALYSIS

- $Q^2 F_{\pi\gamma}(Q^2)$ with the model WF by fixing $B$=0.30 and by varying $m_q$ within the region of [0.20,0.40] GeV:



# NUMERICAL ANALYSIS

- $Q^2 F_{\pi\gamma}(Q^2)$ with the model WF by fixing $B$=0.60 (CZ-like DA) and by varying $m_q$ within the region [0.30,0.50] GeV:



# SUMMARY

- 1. If we know the pion-photon TFF well, we can conveniently ***derive the pion DA's correct behavior***.
- 2. In large $Q^2$ region, the ***new Belle data agrees with the asymptotic DA estimation***, while to be consistent with the BABAR data,we need a much broader DA.
- 3. If the BABAR collaboration still insists on their neasurements, then there may indicate ***new physics*** in these form factors from pQCD.
- 4. Because of the ***effective end-point suppression*** due to the BHL-transverse-momentum dependence, our present model of the pion WF/DA will present a basis for deriving more reliable pQCD estimates.

***Thank you!***