

Next-to-Leading Order Corrections in $B \rightarrow \pi$ form factors

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Outline

1. Motivation of NLO calculation;
2. Factorization of $B \rightarrow \pi$ form factor
3. NLO corrections to $B \rightarrow \pi$ form factor (PRD85, 074004)
4. Resummation of Rapidity Logarithms (ArXiv:1210.2978)
5. Summary

Many thanks to Prof H. N. Li and Dr. Y. M. Wang

Motivation of NLO calculations

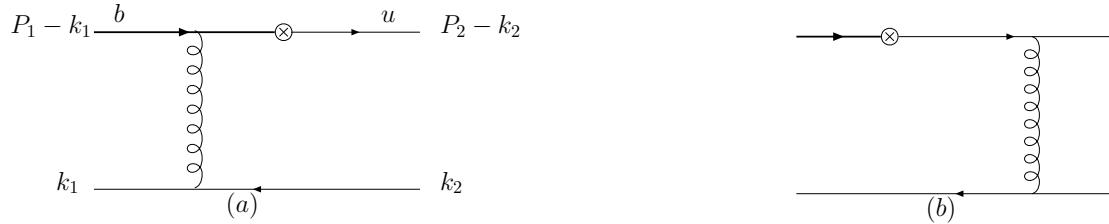
- As the accumulation of B decay data, theoretical predictions need to be updated.
- Exploring NLO contribution can help to fix factorization scale.
- The NLO corrections are also expected to alleviate the existed puzzles in B physics

k_T factorization of $B \rightarrow \pi$ form factors

- Breakdown of Collinear factorization: endpoint singularity
- Different understandings of the endpoint singularity
 - Breakdown of light-cone expansion in the endpoint region
 - double counting of soft and collinear modes
 - missing transverse field modes
- k_T factorization: introduce k_T to regularize the endpoint singularity

$B \rightarrow \pi$ form factor at leading order

- Leading order quark diagrams in $B \rightarrow \pi$ form factors



- LO hard kernel

$$H_a^{(0)}(x_1, k_{1T}, x_2, k_{2T}) = -4g^2 C_F \frac{[x_2 \eta \phi_B^{(+)}(x_1) + \phi_B^{(-)}(x_1)] P_2^\mu}{x_2 \eta (x_1 x_2 \eta m_B^2 + |\mathbf{k}_{1T} - \mathbf{k}_{2T}|^2)} \phi_\pi(x_2). \quad (1)$$

$$H_b^{(0)}(x_1, k_{1T}, x_2, k_{2T}) = -4g^2 C_F \frac{(\eta P_1^\mu - P_2^\mu) \phi_B^{(+)}(x_1) + P_2^\mu \phi_B^{(-)}(x_1)}{\eta (x_1 x_2 \eta m_B^2 + |\mathbf{k}_{1T} - \mathbf{k}_{2T}|^2)} \phi_\pi(x_2). \quad (2)$$

- The Power counting rule:

$$m_B^2, m_b^2 \gg x_2 m_B^2 \gg x_1 m_B^2 \gg x_1 x_2 m_B^2 \gg k_{1T}^2, k_{2T}^2, \quad (3)$$

- The leading power contribution is from the ϕ^- term of Fig(a).

The method for calculating $H^{(1)}$

- Expansion of the quark diagrams

$$\begin{aligned}
 G^{(0)}(x_1, k_{1T}; x_2, k_{2T}; \mu) &= \phi_{M_1}^{(0)} \otimes H^{(0)} \otimes \phi_{M_2}^{(0)} \\
 G^{(1)}(x_1, k_{1T}; x_2, k_{2T}; \mu) &= \phi_{M_1}^{(1)} \otimes H^{(0)} \otimes \phi_{M_2}^{(0)} + \phi_{M_1}^{(0)} \otimes H^{(1)} \otimes \phi_{M_2}^{(0)} + \phi_{M_1}^{(0)} \otimes H^{(0)} \otimes \phi_{M_2}^{(1)} \\
 &\dots
 \end{aligned} \tag{4}$$

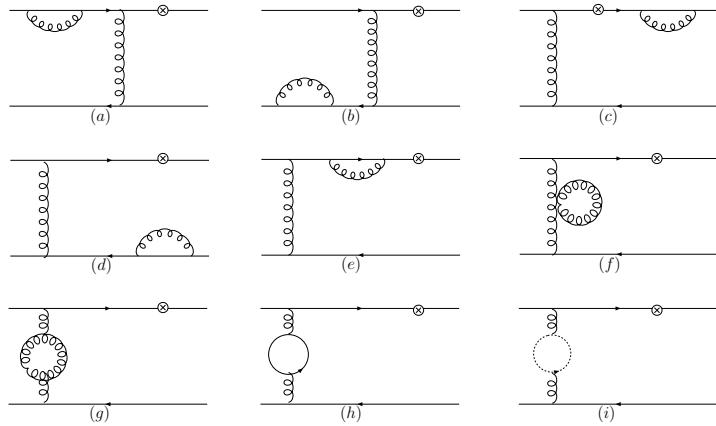
- The NLO hard kernel

$$\begin{aligned}
 H^{(1)}(x_1, \mathbf{k}_{1T}, x_2, \mathbf{k}_{2T}) &= G^{(1)}(x_1, \mathbf{k}_{1T}, x_2, \mathbf{k}_{2T}) \\
 &\quad - \int dx'_1 d^2\mathbf{k}'_{1T} \Phi_B^{(1)}(x_1, \mathbf{k}_{1T}; x'_1, \mathbf{k}'_{1T}) H^{(0)}(x'_1, \mathbf{k}'_{1T}, x_2, \mathbf{k}_{2T}) \\
 &\quad - \int dx'_2 d^2\mathbf{k}'_{2T} H^{(0)}(x_1, \mathbf{k}_{1T}, x'_2, \mathbf{k}'_{2T}) \Phi_\pi^{(1)}(x_2, \mathbf{k}_{2T}; x'_2, \mathbf{k}'_{2T}).
 \end{aligned} \tag{5}$$

- The hadronic wave functions serve as the infrared regulator, and also cancel the gauge dependence in the quark diagrams

The self energy corrections to the full diagram of $B \rightarrow \pi$

- the diagrams



- the results

$$G_{2a}^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left[\frac{6}{\delta_1} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} + \frac{5}{3} \right) + \frac{1}{2} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} - 2 \ln \frac{m_B^2}{m_g^2} - 1 \right) \right] H^{(0)}, \quad (6)$$

$$G_{2b}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{\delta_1 m_B^2 e^{\gamma_E}} + 2 \right) H^{(0)}, \quad (7)$$

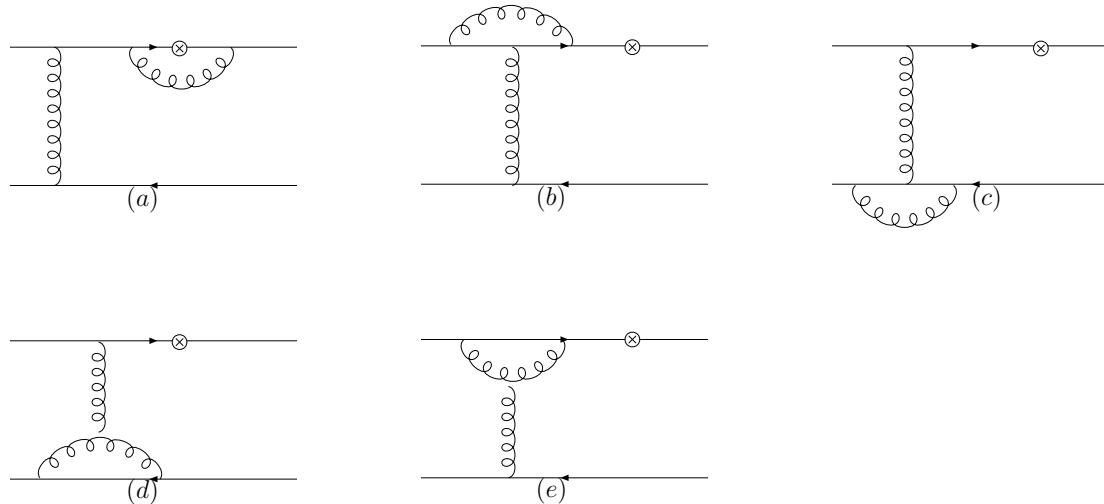
$$G_{2c,2d}^{(1)} = -\frac{\alpha_s C_F}{8\pi} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{\delta_2 m_B^2 e^{\gamma_E}} + 2 \right) H^{(0)}, \quad (8)$$

$$G_{2e}^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left[\frac{6}{x_2\eta} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} + \frac{5}{3} \right) + \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} + 4 \ln(x_2\eta) - 5 \right) \right] H^{(0)}, \quad (9)$$

$$G_{2f+2g+2h+2i}^{(1)} = \frac{\alpha_s}{4\pi} \left(\frac{5}{3} N_c - \frac{2}{3} N_f \right) \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{\delta_{12} m_B^2 e^{\gamma_E}} \right) H^{(0)}. \quad (10)$$

The Vertex Corrections

- the diagrams



- Fig(3a)

$$G_{3a}^{(1)} = \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} - 2 \ln \left(\frac{\delta_2}{\eta} \right) (1 + \ln x_2) + \ln^2 x_2 - \frac{\pi^2 - 3}{2} \right] H^{(0)} \quad (11)$$

- the IR divergence only depend on $\delta_2 = k_{2T}^2/m_B^2$
 - the double logarithm $2 \ln \delta_2 \ln x_2$ leads to the known Sudakov logarithm $\ln^2 \delta_2$ and the known threshold logarithm $\ln^2 x_2$

$$2 \ln \delta_2 \ln x_2 = \ln^2 \delta_2 + \ln^2 x_2 - \ln^2 \frac{\delta_2}{x_2}. \quad (12)$$

- Fig3(b,e): No infrared divergence

$$G_{3b}^{(1)} = -\frac{\alpha_s}{8\pi N_c} \left[\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} + 4 \ln(x_2\eta) \right] H^{(0)}, \quad (13)$$

$$G_{3e}^{(1)} = \frac{\alpha_s N_c}{8\pi} \left[\frac{3}{\epsilon} + 3 \ln \frac{4\pi\mu^2}{m_B^2 e^{\gamma_E}} - \frac{1}{2} \ln^2 \left(\frac{\delta_{12}}{\eta^2} \right) + 2(\ln x_2 - 1) \ln \left(\frac{x_1}{\eta} \right) - \frac{\pi^2}{2} + 3 \right] H^{(0)}, \quad (14)$$

- Fig3(c,d)

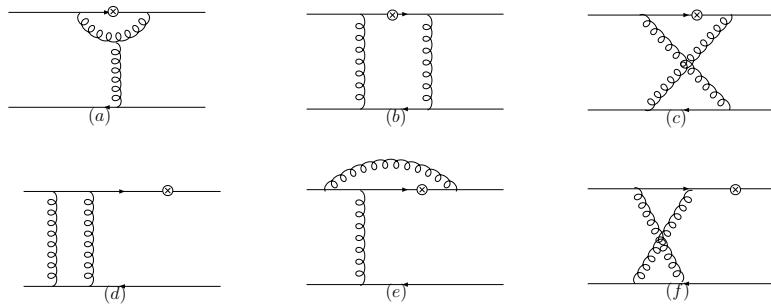
- the double log $\ln \delta_1 \ln \delta_2$ is generated in 3(c)
- only single logs appear in 3(d)

$$G_{3c}^{(1)} = -\frac{\alpha_s}{8\pi N_c} \left[\frac{1}{\epsilon} + \ln \frac{4\pi\mu^2}{\delta_{12} m_B^2 e^{\gamma_E}} - 2 \ln \left(\frac{\delta_1}{\delta_{12}} \right) \ln \left(\frac{\delta_2}{\delta_{12}} \right) - 2 \ln \left(\frac{\delta_1 \delta_2}{\delta_{12}^2} \right) - \frac{2\pi^2}{3} + \frac{9}{2} \right] H^{(0)}, \quad (15)$$

$$G_{3d}^{(1)} = \frac{\alpha_s N_c}{8\pi} \left[\frac{3}{\epsilon} - 3\gamma_E + 3 \ln \frac{4\pi\mu^2}{\delta_{12} m_B^2 e^{\gamma_E}} + 2 \ln \left(\frac{\delta_{12}^2}{\delta_1 \delta_2} \right) + 7 \right] H^{(0)}, \quad (16)$$

The box and pentagon diagrams

- the diagrams



$$G_{4a}^{(1)} = -\frac{\alpha_s N_c}{4\pi} \left[\ln \left(\frac{x_2 \eta^2}{\delta_2} \right) + 1 \right] x_2 H^{(0)}, \quad (17)$$

$$G_{4c}^{(1)} = -\frac{\alpha_s}{4\pi N_c} \left[\ln \left(\frac{x_1 \eta}{\delta_1} \right) \ln \left(\frac{\delta_{12}}{\delta_2} \right) + \frac{\pi^2}{6} \right] H^{(0)}, \quad (18)$$

$$G_{4d}^{(1)} = -\frac{\alpha_s C_F}{4\pi} \left[\ln^2 \left(\frac{\delta_1}{x_1^2} \right) - \ln^2 x_1 - \frac{7\pi^2}{3} \right] H^{(0)}. \quad (19)$$

$$G_{4e}^{(1)} = \frac{\alpha_s}{8\pi N_c} \left[\ln^2 \left(\frac{x_2 \eta^2}{\delta_2} \right) + \pi^2 \right] H^{(0)}, \quad (20)$$

$$G_{4f}^{(1)} = \frac{\alpha_s}{8\pi N_c} \left[\ln \left(\frac{\delta_{12}}{\delta_2} \right) \left(\ln(\delta_{12}\delta_2) - 4 \ln(x_2 \eta) \right) \right] H^{(0)}. \quad (21)$$

Notes:

- The result indicates fig4(a) is power suppressed
- fig4(c) cancels the $\ln \delta_1 \ln \delta_2$ term in fig3(c)

Ward Identity: Summary of the IR divergence

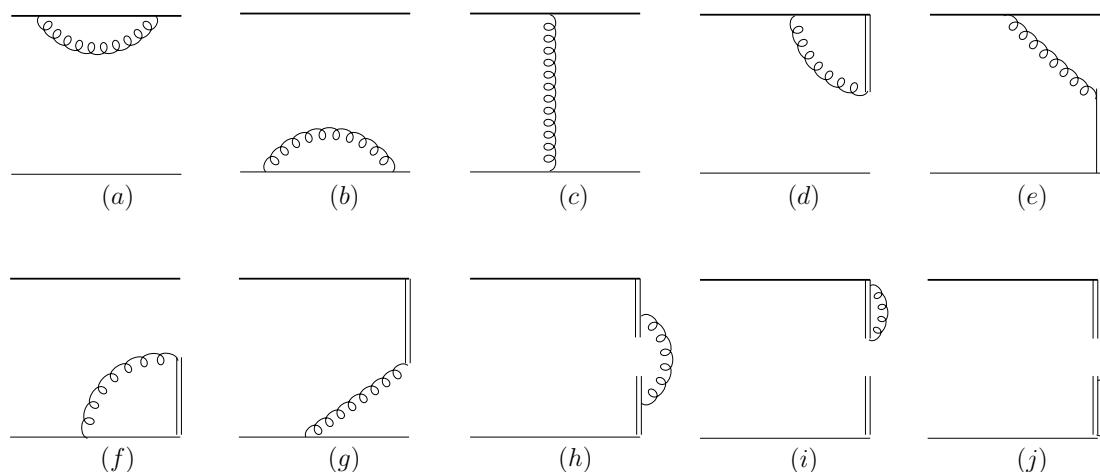
diagrams	Coefficient of $\ln\delta_1$	diagrams	Coefficient of $\ln\delta_2$
2b	C_F	2c	C_F
3c	$\frac{2}{N_c}(1 - \ln\delta_{12})$	5a	$4(1 + \ln x_2)C_F$
3d	$-2N_c$	4a	$2N_c x_2$ (suppressed)
4c	$\frac{2}{N_c}\ln\delta_{12}$	4c	$\frac{2}{N_c}(\ln\delta_{12} - \ln\delta_2)$
4f	0	4e	$-\frac{2}{N_c}(2\ln\eta + \ln x_2)$
2e	0	2d	C_F
3b	0	3c	$\frac{2}{N_c}(1 - \ln\delta_{12})$
3e	0	3d	$-2N_c$
4f	0	4f	$\frac{4}{N_c}\ln(x_2\eta)$
4e	0	-	-

- $\text{fig}(3c) + \text{fig}(3d) + \text{fig}(4c) = -4C_F$ for the coefficient of $\ln\delta_1$
- $\text{fig}(4c) + \text{fig}(4e) + \text{fig}(3c) + \text{fig}(3d) + \text{fig}(4f) = -4C_F$ for the coefficient of $\ln\delta_2$

The effective diagram of B meson

- Operator definition of B wave function

$$\Phi_{\pm}^B(x_1, k_{1T}; x'_1, k'_{1T}) = \int \frac{dz^-}{2\pi} \frac{d^2 z_T}{(2\pi)^2} e^{-ix'_1 P_1^+ z^- + i\mathbf{k}'_{1T} \cdot \mathbf{z}_T} \langle 0 | \bar{q}(z) W_z(n_1)^\dagger I_{n_1; z, 0} W_0(n_1) \not{h}_{\mp} \Gamma h(0) | h_v \bar{q}(k_1) \rangle,$$



$$\Phi_{5a}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{m_g^2 e^{\gamma_E}} \right) H^{(0)}, \quad (22)$$

$$\Phi_{5b}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{8\pi} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{\delta_1 m_B^2 e^{\gamma_E}} + 2 \right) H^{(0)}, \quad (23)$$

$$\Phi_{5c}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} \left(\ln^2 \frac{\delta_1}{x_1^2} \right) H^{(0)}, \quad (24)$$

$$\Phi_{5d}^{(1)} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} \ln \frac{\zeta_1^2}{m_B^2} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{m_g^2 e^{\gamma_E}} \right) H^{(0)}, \quad (25)$$

$$\Phi_{5e}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \ln \frac{\zeta_1^2}{m_B^2} \left(\ln \frac{\zeta_1^2}{m_g^2} - \frac{1}{2} \ln \frac{\zeta_1^2}{m_B^2} + 2 \ln x_1 \right) H^{(0)}, \quad (26)$$

$$\Phi_{5f}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{4\pi\mu_f^2}{x_1^2 \zeta_1^2 e^{\gamma_E}} - \ln^2 \frac{\delta_1 m_B^2}{x_1^2 \zeta_1^2} - 2 \ln \frac{\delta_1 m_B^2}{x_1^2 \zeta_1^2} + \frac{\pi^2}{3} \right) H^{(0)}, \quad (27)$$

$$\Phi_{5g}^{(1)} \otimes H^{(0)} = \frac{\alpha_s C_F}{4\pi} \left(\ln^2 \frac{\delta_1 m_B^2}{x_1^2 \zeta_1^2} - \frac{2\pi^2}{3} \right) H^{(0)}, \quad (28)$$

Notes:

- The soft divergence in fig (d) and (e) cancel each other because of color transparency.
- The double log terms in fig (f) and (g) cancel each other, while single collinear log is left.
- The double log $\ln^2 \zeta^2$ which arises from light-cone singularity regulator leads to scheme dependence, should be resummed.

NLO Hard Kernel

- The NLO hard kernel reads

$$\begin{aligned}
 H^{(1)} = & \frac{\alpha_s(\mu_f)C_F}{4\pi} \left[\frac{21}{4} \ln \frac{\mu^2}{m_B^2} - \left(\ln \frac{m_B^2}{\zeta_1^2} + \frac{13}{2} \right) \ln \frac{\mu_f^2}{m_B^2} + \frac{9}{16} (\ln^2 x_1 + 2 \ln x_1 \ln x_2 - \ln x_2^2) \right. \\
 & + \left(2 \ln \frac{m_B^2}{\zeta_1^2} + \frac{7}{8} \ln \eta - \frac{1}{4} \right) \ln x_1 + \left(2 \ln \frac{m_B^2}{\zeta_2^2} + \frac{7}{8} \ln \eta - \frac{5}{2} \right) \ln x_2 + 2 \ln \frac{m_B^2}{\zeta_2^2} + \left(\frac{15}{4} - \frac{7}{16} \ln \eta \right) \ln \eta \\
 & \left. - \frac{1}{2} \ln \frac{m_B^2}{\zeta_1^2} \left(3 \ln \frac{m_B^2}{\zeta_1^2} + 2 \right) + \frac{85}{48} \pi^2 + \frac{219}{16} \right] H^{(0)}
 \end{aligned} \tag{29}$$

- The hard kernel is infrared safe
- The hard kernel is scale and scheme dependence.
- The double logarithms $\ln^2 x_2$ which is enhanced in the small x region can be absorbed into the jet function $J(x_2)$

$$J^{(1)} H^{(0)} = -\frac{\alpha_s}{4\pi} C_F \left(\ln^2 x_2 + \ln x_2 + \frac{\pi^2}{3} \right) H^{(0)}, \tag{30}$$

- Our choice of scheme: $\zeta_2 = 1$ which is consistent with pion EM form factor and $\zeta_1 = 25m_B$ which serves to cancel the double log term and constant term

The renormalization scale and factorization scale

- Scenario I: factorization scale

$$t^a = \max(\sqrt{x_2 \eta} m_B, 1/b_1, 1/b_2), \quad t^b = \max(\sqrt{x_1 \eta} m_B, 1/b_1, 1/b_2), \quad (31)$$

The renormalization scale: Diminish all single logarithmic and constant terms

$$t_s(\mu_f) = \left\{ \text{Exp} \left[c_1 + \left(\ln \frac{m_B^2}{\zeta_1^2} + \frac{5}{4} \right) \ln \frac{\mu_f^2}{m_B^2} \right] x_1^{c_2} x_2^{c_3} \right\}^{2/21} \mu_f, \quad (32)$$

- Scenario II: $\mu = \mu_f = t^a(t^b)$
- Scenario III: $\mu_f = m_B, \mu = t_s(\mu)$
- NLO corrections in these three scenarios are about 30%, 40% and 15% respectively

Resummation of Rapidity Logarithms

- the scheme dependence term $\ln \zeta^2 (\zeta^2 = (P \cdot n)^2/n^2)$ is similar to the rapidity logs in SCET
- the evolution equation

$$\begin{aligned}\zeta^2 \frac{d}{d\zeta^2} \Phi_B^{(r)}(x, k_T, \zeta^2, \mu_f) &= K^{(b,1)} \otimes \Phi_B^{(r)}(x, k_T, \zeta^2, \mu_f) \\ &\quad - \frac{1}{Z_\Phi} \left(\zeta^2 \frac{d}{d\zeta^2} Z_\Phi \right) \Phi_B^{(r)}(x, k_T, \zeta^2, \mu_f),\end{aligned}\tag{33}$$

- transform to (N, b) space

$$\zeta^2 \frac{d}{d\zeta^2} \tilde{\Phi}_B(N, b, \zeta^2, \mu_f) = \tilde{K}^{(1)}(N, b, \zeta^2, \mu_f) \tilde{\Phi}_B(N, b, \zeta^2, \mu_f),\tag{34}$$

- RG evolution

$$\mu_f \frac{d}{d\mu_f} \Phi_B = -\frac{1}{Z_\Phi} \mu_f \frac{dZ_\Phi}{d\mu_f} \Phi_B \equiv -\hat{\gamma}_B \Phi_B,\tag{35}$$

$$\mu_f \frac{d}{d\mu_f} K^{(1)} = -\lambda_K,\tag{36}$$

- Solution in (N, b) space

$$\begin{aligned}\tilde{\Phi}_B(N, b) &= \exp \left[\int_{\zeta_0^2}^{N^2 \zeta_0^2} \frac{d\tilde{\zeta}^2}{\tilde{\zeta}^2} \mathcal{K}^{(1)}(N, b, \tilde{\zeta}^2, \mu_f) - \int_{\mu_0}^{\mu_f} \frac{d\mu}{\mu} \frac{\alpha_s(\mu)}{2\pi} C_F (\ln \zeta_0^2 - 2) \right] \\ &\quad \times \tilde{\Phi}_B(N, b, \zeta_0^2, \mu_0),\end{aligned}\tag{37}$$

- inverse Mellin transformation

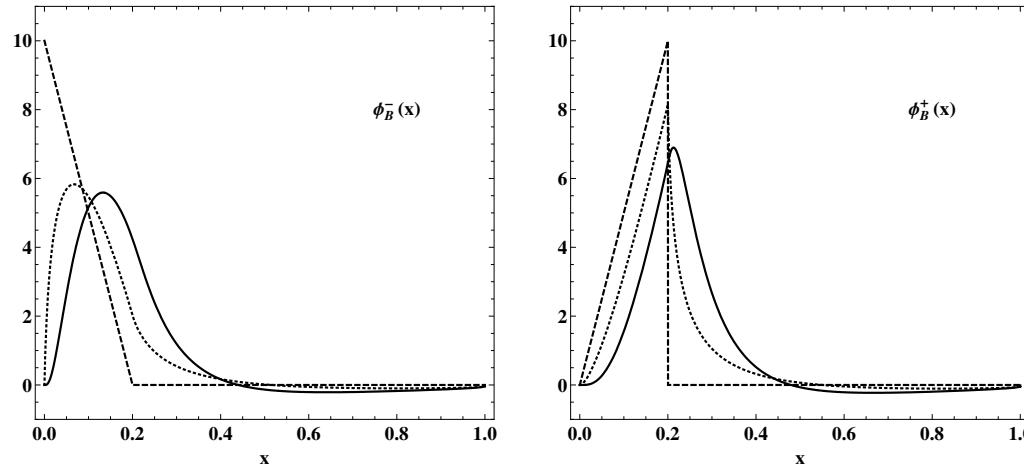
$$\Phi_B^\pm(x, k_T) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (1-x)^{-N} \tilde{\Phi}_B^\pm(N, k_T),\tag{38}$$

- the KKQT model and the endpoint behavior after rapidity resummation

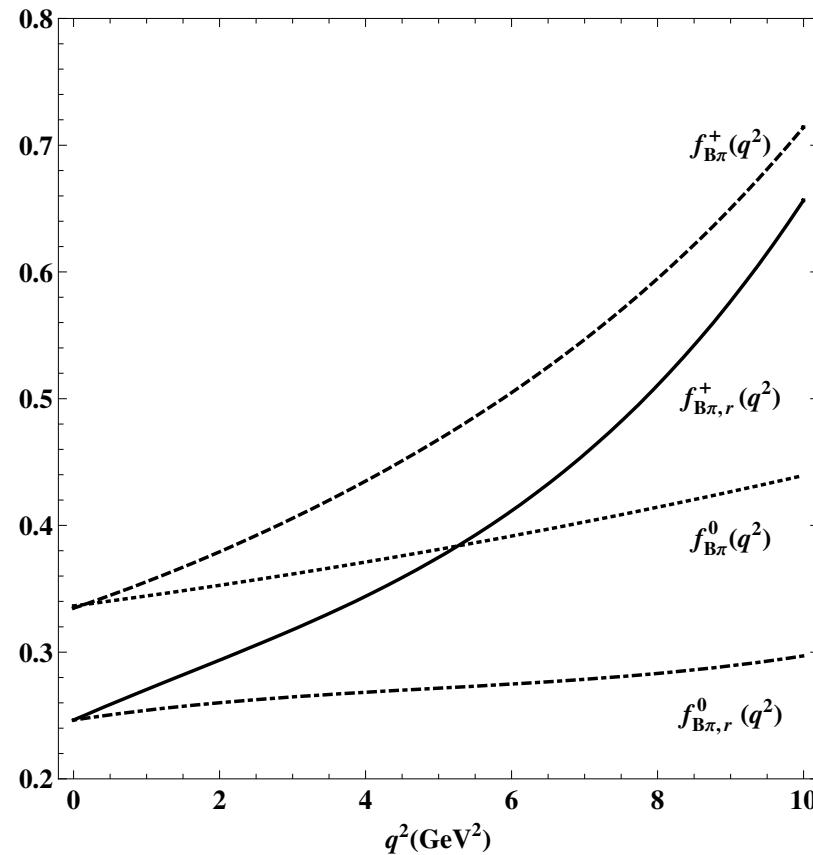
$$\Phi_B^\pm(x, k_T, \zeta_0^2) = \phi_B^\pm(x, \zeta_0^2) \phi(k_T),\tag{39}$$

$$\phi_B^-(x, \zeta_0^2) = \frac{2x_0 - x}{2x_0^2} \theta(2x_0 - x),\tag{40}$$

$$\phi_B^+(x, \zeta_0^2) = \frac{x}{2x_0^2} \theta(2x_0 - x).\tag{41}$$



- resummation effect on $B \rightarrow \pi$ form factor



Summary

- We computed the next-to-leading corrections to $B \rightarrow \pi$ form factors in k_T factorization, and obtain a infrared safe and gauge invariant hard kernel.
- The NLO corrections in various scenarios is about 15 % to 40%.
- We performed the rapidity resummation on the B meson wave function, which can improve the end point behavior of the form factor.

Thanks for your patience!

B meson projector

$$M_{\beta\alpha}^B = -\frac{if_B M}{4} \left[\frac{1+\not{v}}{2} \left\{ \phi_+^B(\omega) \not{n}_+ + \phi_-^B(\omega) \not{n}_- \right. \right. \\ \left. \left. - \int_0^{l_+} d\eta (\phi_-^B(\eta) - \phi_+^B(\eta)) \gamma^\mu \frac{\partial}{\partial l_{\perp\mu}} \right\} \gamma_5 \right]_{\alpha\beta}. \quad (42)$$

The equation of motion under WW approximation

$$\frac{\partial \tilde{\phi}_-^B}{\partial t} + \frac{1}{t} (\tilde{\phi}_-^B - \tilde{\phi}_+^B) |_{z^2=0} = 0, \quad (43)$$

$$\frac{\partial \tilde{\phi}_+^B}{\partial z^2} + \frac{1}{4} \frac{\partial^2 \tilde{\phi}_-^B}{\partial t^2} |_{z^2=0} = 0. \quad (44)$$

In momentum space

$$\int_0^{l_+} d\eta (\phi_-^B(\eta) - \phi_+^B(\eta)) = l_+ \phi_-^B(l_+) \quad \text{or} \quad \phi_+^B(l_+) = -l_+ \phi'_-^B(l_+), \quad (45)$$

The projector then leads to

$$\frac{1}{2\sqrt{N_c}} (P_1 + m_B) \gamma_5 \left[\not{n}_+ \phi_B^{(+)}(x_1) + \left(\not{n}_- - k_1^+ \frac{\partial}{\partial \mathbf{k}_{1T}} \right) \phi_B^{(-)}(x_1) \right] \quad (46)$$