Order-v^4 Relativistic Corrections to Υ Inclusive Decay into a Charm pair

桑文龙

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Outline

- Motivations
- NRQCD factorization formula
- Treatment in detail
- Summaries

Motivations

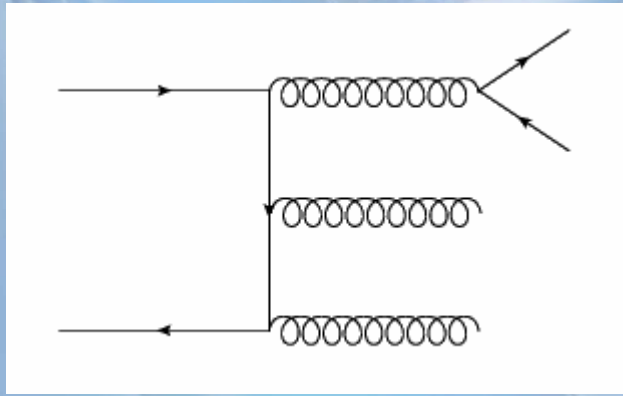
(1) NLO relativistic corrections are extremely large

	r	$\Gamma^{(0)}(\mathrm{keV})$	$\Gamma^{(2)}(\mathrm{keV})$	$\Gamma^{(2)}/\Gamma^{(0)}$	$G_1(^3S_1)/F_1(^3S_1)$
$J/\psi \to e^+e^-gg$	1.08×10^{-7}	4.73×10^{-1}	-5.91×10^{-1}	-125%	-5.56
$J/\psi \to \mu^+\mu^-gg$	4.69×10^{-3}	1.08×10^{-1}	-1.57×10^{-1}	-145%	-6.49
$\psi(2S) \to e^+e^-gg$	7.66×10^{-8}	2.43×10^{-1}	-8.56×10^{-1}	-352%	-5.55
$\psi(2S) \to \mu^+\mu^-gg$	3.31×10^{-3}	5.98×10^{-2}	-2.41×10^{-1}	-403%	-6.37
$\Upsilon(1S) \to e^+e^-gg$	1.16×10^{-8}	3.68×10^{-2}	-1.16×10^{-2}	-31.5%	-5.53
$\Upsilon(1S) \to \mu^+\mu^-gg$	5.02×10^{-4}	1.22×10^{-2}	-4.16×10^{-3}	-34.0%	-5.97
$\Upsilon(1S) \to \tau^+\tau^-gg$	1.41×10^{-1}	1.05×10^{-3}	-7.06×10^{-4}	-67.3%	-11.8
$\Upsilon(1S) \to c\bar{c}gg$	1.56×10^{-1}	1.44	-1.01	-70.4%	-12.4
$\Upsilon(2S) \to c\bar{c}gg$	1.39×10^{-1}	7.99×10^{-1}	-1.68	-210%	-11.7
$\Upsilon(3S) \to c\bar{c}gg$	1.30×10^{-1}	6.63×10^{-1}	-1.90	-287%	-11.4

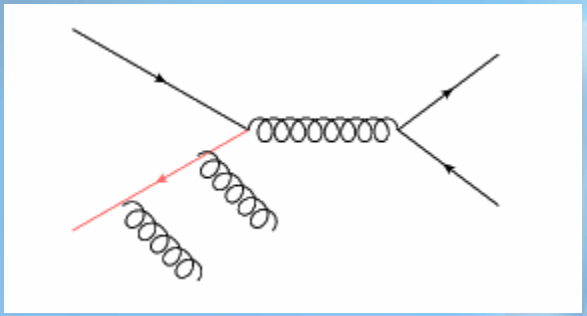
It needs to check the convergence of relativistic expansion

Motivations

(2) There exists double IR divergence.



The two gluons become soft



It is interesting and technically challenging to cancel the IR divergence and obtain a IR-independent decay rate

NRQCD factorization formula

Scales in quarkonium. v signifies the typical velocity in quarkonium

"soft" → ← "Hard"

Mv^2, Λ_{QCD} $\ll$ Mv $\ll$ M

重夸克势能 重夸克动量 重夸克质量

$c\bar{c} : v^2 \approx 0.3 \qquad b\bar{b} : v^2 \approx 0.1$

NRQCD factorization formula

NRQCD factorization

长距离矩阵元

a.描述正反夸克对演变成色单态强子过程

b.按v展开的。

c.通过非微扰途径得到，比如势模型、格点QCD、或者通过实验匹配

短距离系数

a.描述正反重夸克对产生或湮灭的过程

b.按照耦合常数展开

c.是微扰可算的，

NRQCD因子化公式是按α_s和v双重展开

The factorization formula for $\Upsilon \to c\bar{c} + X$

$$
\begin{aligned}
&d\Gamma[\Upsilon \to c\bar{c} + X] \\
&= \frac{dF_1(^3S_1^{[1]})}{m^2}\langle \mathcal{O}(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF_2(^3S_1^{[1]})}{m^4}\langle \mathcal{P}(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF(^1S_0^{[8]})}{m^2}\langle \mathcal{O}(^1S_0^{[8]})\rangle_\Upsilon \\
&+ \frac{dF_3(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_1(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF_4(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_2(^3S_1^{[1]})\rangle_\Upsilon \\
&+ \frac{dF(^3S_1^{[8]})}{m^2}\langle \mathcal{O}(^3S_1^{[8]})\rangle_\Upsilon + \sum_{J=0,1,2} \frac{dF(^3P_J^{[8]})}{m^4}\langle \mathcal{O}(^3P_J^{[8]})\rangle_\Upsilon,
\end{aligned}
$$

The factorization formula for $\Upsilon \to c\bar{c} + X$

relative order v^0

v^2

v^3

$$d\Gamma[\Upsilon \to c\bar{c} + X]$$

$$= \frac{dF_1(^3S_1^{[1]})}{m^2}\langle \mathcal{O}(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF_2(^3S_1^{[1]})}{m^4}\langle \mathcal{P}(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF(^1S_0^{[8]})}{m^2}\langle \mathcal{O}(^1S_0^{[8]})\rangle_\Upsilon$$

$$+ \frac{dF_3(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_1(^3S_1^{[1]})\rangle_\Upsilon + \frac{dF_4(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_2(^3S_1^{[1]})\rangle_\Upsilon$$

$$+ \frac{dF(^3S_1^{[8]})}{m^2}\langle \mathcal{O}(^3S_1^{[8]})\rangle_\Upsilon + \sum_{J=0,1,2} \frac{dF(^3P_J^{[8]})}{m^4}\langle \mathcal{O}(^3P_J^{[8]})\rangle_\Upsilon,$$

v^4

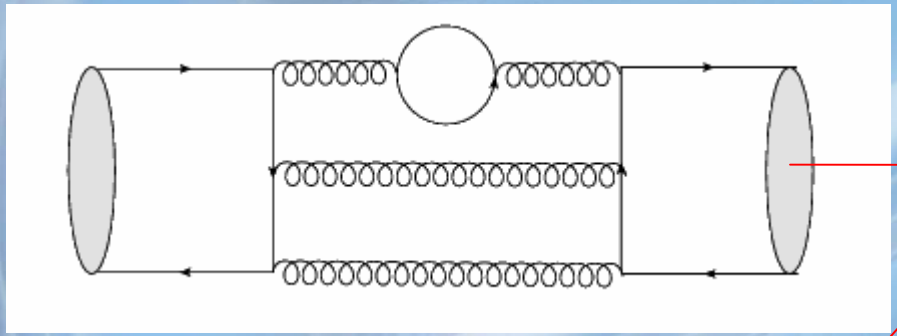
Matching precedure

Through perturbative process $b\bar{b}(^3S_1^{[1]}) \to c\bar{c}gg$

$$d\Gamma(^3S_1^{[1]}) = \frac{dF_3(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_1(^3S_1^{[1]})\rangle_H + \frac{dF_4(^3S_1^{[1]})}{m^6}\langle \mathcal{Q}_2(^3S_1^{[1]})\rangle_H + \frac{dF(^3S_1^{[8]})}{m^2}\langle \mathcal{O}(^3S_1^{[8]})\rangle_H + \sum_J \frac{dF(^3P_J^{[8]})}{m^4}\langle \mathcal{O}(^3P_J^{[8]})\rangle_H,$$

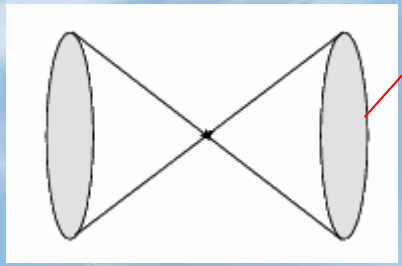
$b\bar{b}(^3S_1^{[1]})$

Matching precedure

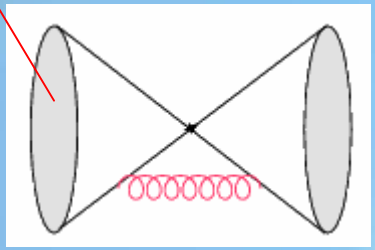


$b\bar{b}(^3S_1^{[1]})$

$= \; dF(^3S_1^{[1]}) \times$



$+ \; dF(^3P_J^{[8]}) \times$



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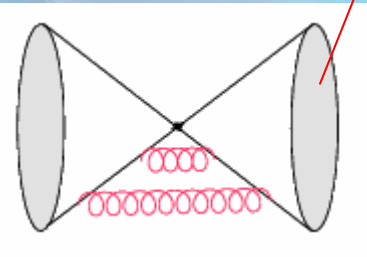

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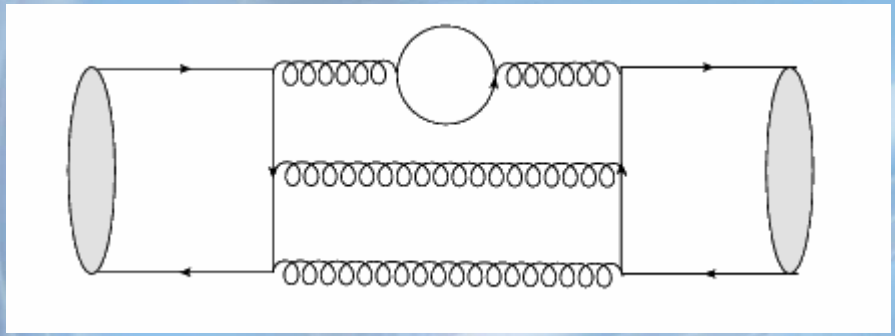
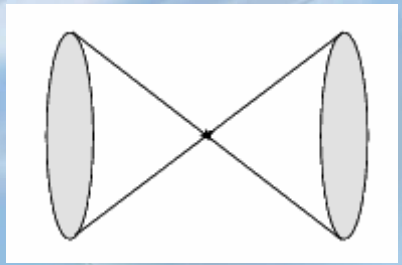
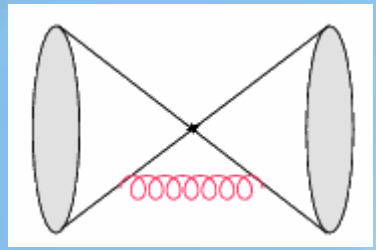
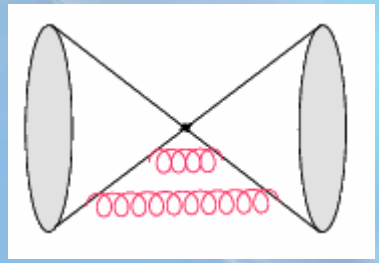
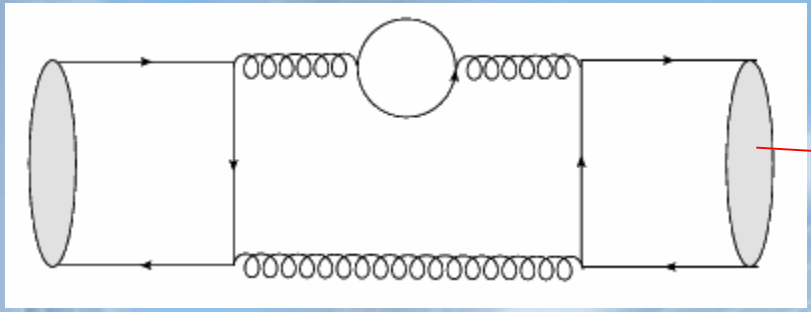


$= dF(^3S_1^{[1]}) \times$  $+ \quad dF(^3P_J^{[8]}) \times$ 

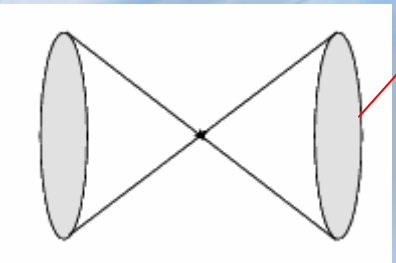
$+ \quad dF(^3S_1^{[8]}) \times$ 

Matching precedure



$$b\bar{b}(^3P_J^{[8]})$$

$$= \quad dF(^3P_J^{[8]}) \times$$



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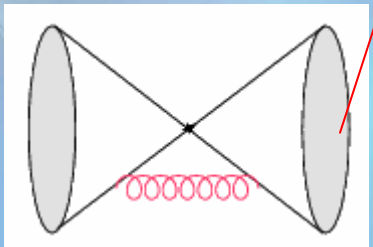

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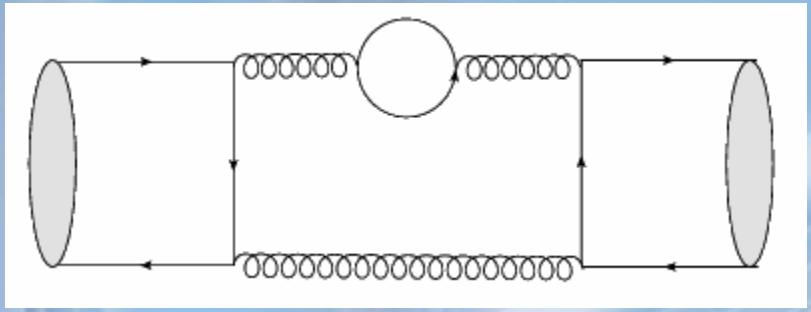
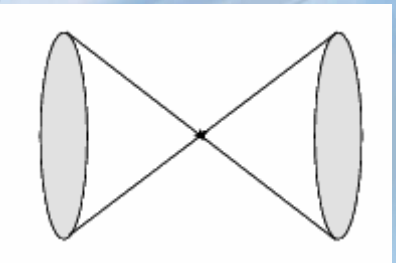


$$= dF(^3P_J^{[8]}) \times$$



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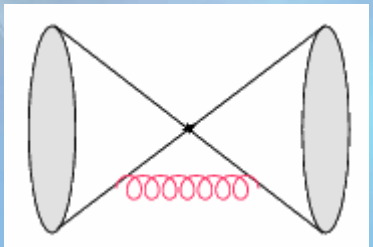

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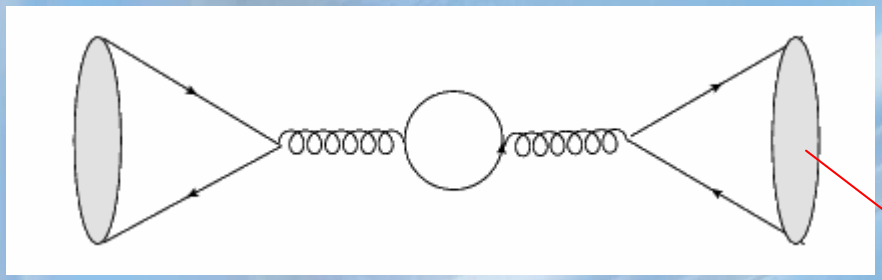
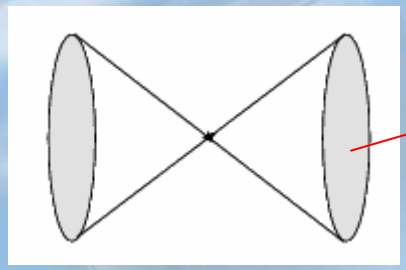
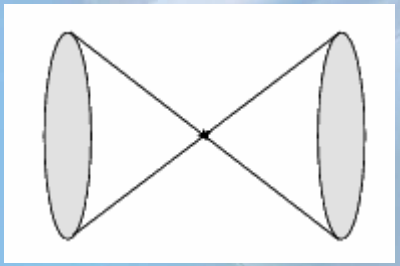


$$= \quad dF(^3S_1^{[8]}) \times$$



$$b\bar{b}(^3S_1^{[8]})$$

Perturbative NRQCD matrix elements



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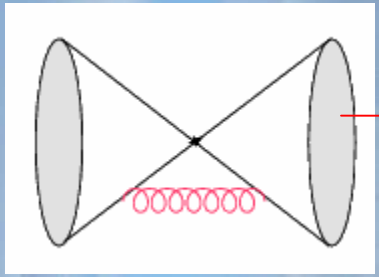
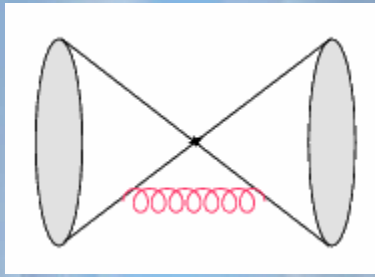
Perturbative NRQCD matrix elements



$$b\bar{b}(^3S_1^{[8]})$$

$$
\begin{aligned}
I_a &= \frac{(ig_s)^2}{4m^2}\frac{-1}{N_c}T^a\otimes T^a\int\frac{d^dk}{(2\pi)^d}\frac{i}{p^0-k^0-\frac{(\vec{p}-\vec{k})^2}{2m}+i\epsilon}\frac{i}{p'^0-k^0-\frac{(\vec{p}'-\vec{k})^2}{2m}+i\epsilon}\\
&\quad\times\frac{4i(\boldsymbol{p}\cdot\boldsymbol{p}'-\boldsymbol{p}\cdot\boldsymbol{k}\boldsymbol{p}'\cdot\boldsymbol{k}/\boldsymbol{k}^2)}{k^2+i\epsilon}\boldsymbol{p}\cdot\boldsymbol{p}'\\
&= -\frac{g_s^2}{2m^2}\frac{T^a\otimes T^a}{N_c}\boldsymbol{p}\cdot\boldsymbol{p}'\,\boldsymbol{p}\cdot\boldsymbol{p}'\frac{d-2}{d-1}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{|\boldsymbol{k}|^3}\\
&= -\frac{g_s^2}{2m^2}\frac{T^a\otimes T^a}{N_c}\frac{\boldsymbol{p}^2\boldsymbol{p}'^2}{d-1}\frac{d-2}{d-1}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{|\boldsymbol{k}|^3}\\
&= -\frac{\alpha_s}{3\pi m^2}\frac{T^a\otimes T^a}{N_c}\left(\frac{\tilde{f}_\epsilon}{\epsilon_{\rm UV}}\frac{\boldsymbol{p}^4}{d-1}-\frac{\tilde{f}_\epsilon}{\epsilon_{\rm IR}}\frac{\boldsymbol{p}^4}{d-1}\right)
\end{aligned}
$$

Perturbative NRQCD matrix elements



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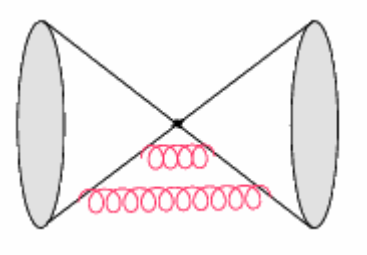
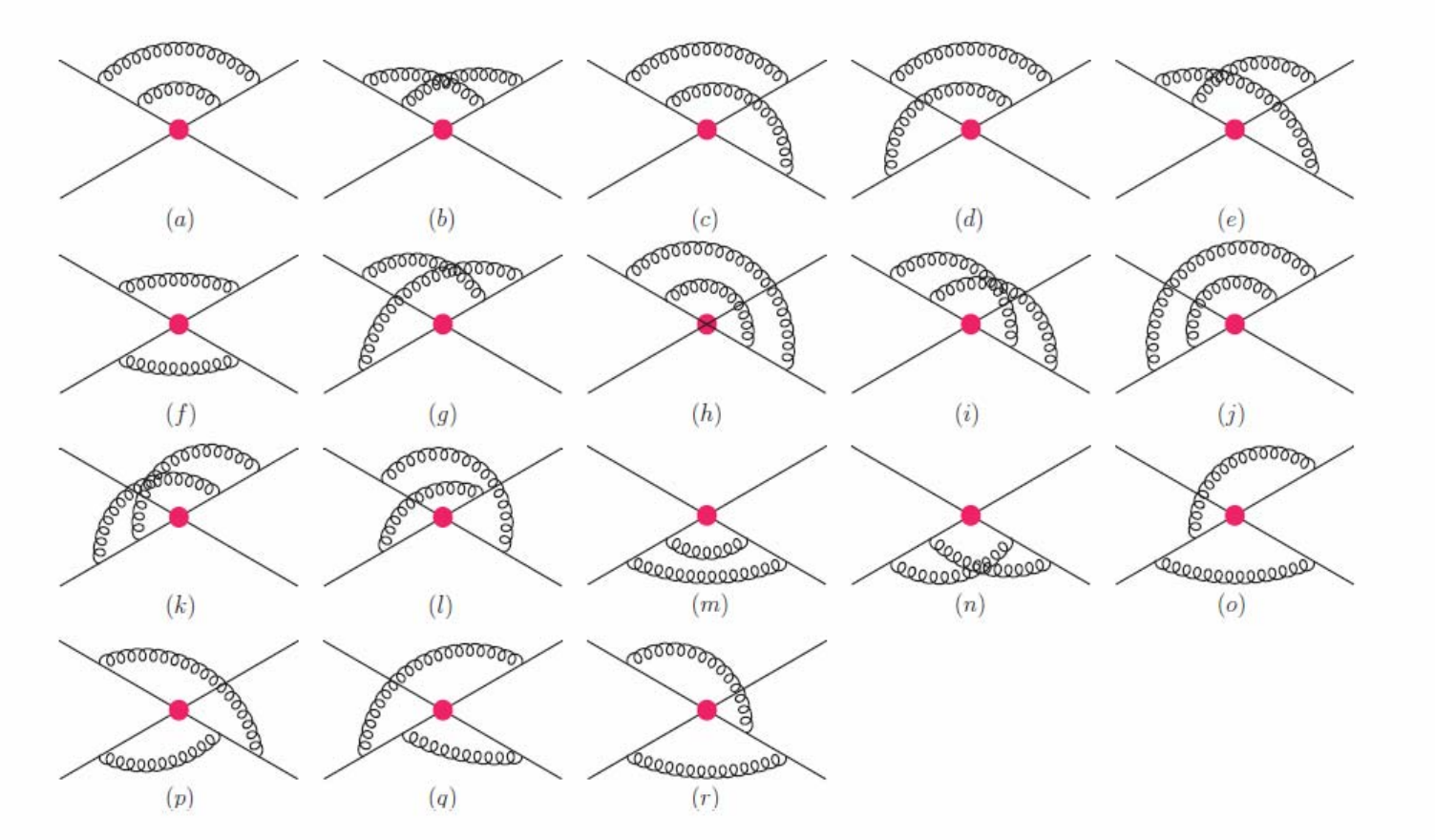
&= -\frac{g_s^2}{2m^2}\frac{T^a\otimes T^a}{N_c}\boldsymbol{p}\cdot\boldsymbol{p}'\boldsymbol{p}\cdot\boldsymbol{p}'\boxed{\frac{d-2}{d-1}}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{|\boldsymbol{k}|^3}\\
&= -\frac{g_s^2}{2m^2}\frac{T^a\otimes T^a}{N_c}\frac{\boldsymbol{p}^2\boldsymbol{p}'^2}{\boxed{d-1}}\frac{d-2}{d-1}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{|\boldsymbol{k}|^3}\\
&= -\frac{\alpha_s}{3\pi m^2}\frac{T^a\otimes T^a}{N_c}\left(\frac{\tilde{f}_\epsilon}{\epsilon_{\rm UV}}\frac{\boldsymbol{p}^4}{d-1}-\frac{\tilde{f}_\epsilon}{\epsilon_{\rm IR}}\frac{\boldsymbol{p}^4}{d-1}\right)
\end{aligned}
$$

From loop integration

Extracting S wave

Perturbative NRQCD matrix elements





Perturbative NRQCD matrix elements

Summing all the contributions

$$
\begin{aligned}
I &= \frac{5}{54}\frac{2g_s^4}{m^4}\frac{(d-2)^2}{(d-1)^2}\,1\otimes 1\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\int\frac{d^{d-1}l}{(2\pi)^{d-1}}\frac{(\boldsymbol{p}\cdot\boldsymbol{p}')^2}{|\boldsymbol{k}|^3|\boldsymbol{l}|^3} \\
&= \frac{20\alpha_s^2}{243\pi^2(d-1)}\frac{\boldsymbol{p}^4}{m^4}\left(\frac{\tilde{f}_\epsilon}{\epsilon_{\rm UV}}-\frac{\tilde{f}_\epsilon}{\epsilon_{\rm IR}}\right)^2 1\otimes 1
\end{aligned}
$$

Renormalization to cancel the UV divergence

$$
\mathcal{O}({}^3S_1^{[8]})_{\overline{\rm MS}} = \mathcal{O}({}^3S_1^{[8]}) + \delta_1\mathcal{O} + \delta_2\mathcal{O} + \mathcal{O}(\alpha_s^3)
$$

Determining short-distance coefficients

$$\frac{dF(^3S_1^{[8]})}{dz} = \frac{\pi\alpha_s L}{2}\delta(1-z)$$

$$\frac{dF(^3P_J^{[8]})}{dz} = \frac{-5\alpha_s^2 L}{18}\left\{\left[\ln\frac{\mu^2}{M^2}+\frac{5}{3}-2\ln(1-r)\right]\delta(1-z)-2\left(\frac{1}{1-z}\right)_+ + A_J\right\}$$

$$\frac{dF(^3S_1^{[1]})}{dz} = \frac{dF_d(^3S_1^{[1]})}{dz}+\frac{dF_s(^3S_1^{[1]})}{dz}+\frac{dF_r(^3S_1^{[1]})}{dz}$$

$$\frac{dF_d(^3S_1^{[1]})}{dz} = \frac{10\alpha_s^3 L}{729\pi}\left\{\left[\ln^2\frac{\mu^2}{M^2}+\frac{10-12\ln(1-r)}{3}\ln\frac{\mu^2}{M^2}+4\ln^2(1-r)-\frac{20}{3}\ln(1-r)+\frac{25}{9}\right.\right.$$
$$\left.\left.-\frac{2\pi^2}{3}\right]\delta(1-z)-\left(\frac{1}{1-z}\right)_+\left[4\ln\frac{\mu^2}{M^2}+2\ln z+\frac{20}{3}\right]+8\left(\frac{\ln(1-z)}{1-z}\right)_+\right\}$$

Numerical results

Ratios of short distance coefficients

	r	$F_2(^3S_1^{[1]})/F_1(^3S_1^{[1]})$	$F(^3S_1^{[1]})/F_1(^3S_1^{[1]})$
$\Upsilon(1S)$	1.56×10^{-1}	-12.4	$19.4 + 0.6\ln^2(\frac{\mu^2}{M^2}) + 0.9\ln(\frac{\mu^2}{M^2})$
$\Upsilon(2S)$	1.39×10^{-1}	-11.7	$18.5 + 0.5\ln^2(\frac{\mu^2}{M^2}) + 0.6\ln(\frac{\mu^2}{M^2})$
$\Upsilon(3S)$	1.30×10^{-1}	-11.4	$18.0 + 0.5\ln^2(\frac{\mu^2}{M^2}) + 0.4\ln(\frac{\mu^2}{M^2})$

Numerical results

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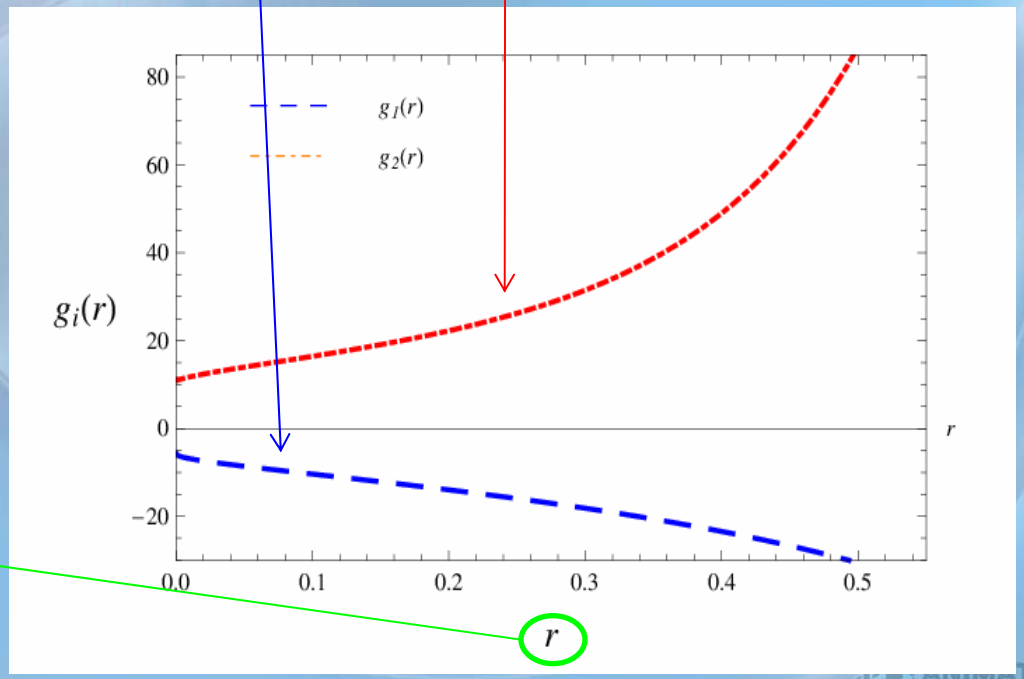
$\Upsilon(3S)$	1.30×10^{-1}	-11.4	$18.0+0.5\ln^2(\frac{\mu^2}{M^2})+0.4\ln(\frac{\mu^2}{M^2})$

The relativistic corrections rise rapidly with increase of r



$$\frac{m_c^2}{m_b^2}$$

Summaries

(1) The IR divergence appeared in matching precedure can be canceled through color-octet mechanism. It needs to carefully carry out the renormalization.

(2) The relativistic expansion for color-singlet contribution converges very well

(3) Through extrapolating the mass ratio of charm quark to bottom quark, we find the relativistic corrections rise rapidly with increase of the ratio.

谢 谢！