QCD phase structures based on chiral effective models and the Ginzburg-Landau approach

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Outline

- Modeling the QCD phase diagram
- QCD phase structures based on the Ginzburg-Landau approach
  - The topological phase structures of massless two-flavor systems
  - Mapping with the Nambu–Jona-Lasinio model at the chiral and isospin chemical potential
- Summary and discussion

I. Modeling the QCD phase diagram

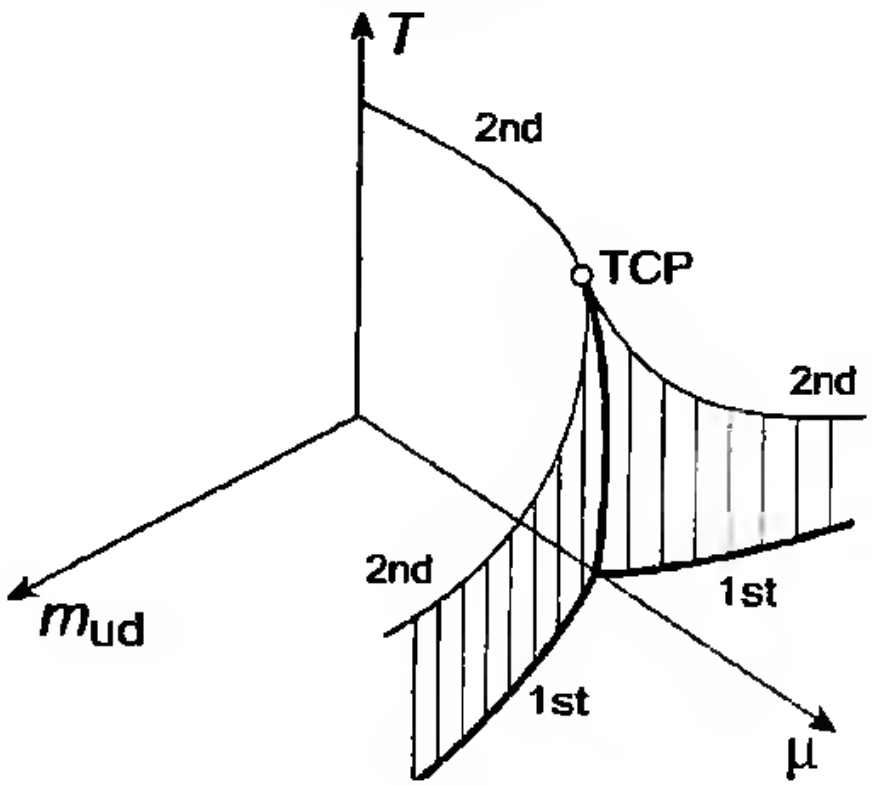
Two guiding principles for constructing models of the QCD phases:

a. The symmetry associated with the center Z(3) of the local $SU(3)_c$ color gauge group is exact in the limit of pure gauge QCD, realized for **infinitely heavy quarks**. In the high-T, deconfinement phase of is spontaneously broken, with the Polyakov loop acting as the order parameter.

b. Chiral $SU(N_f)_R \times SU(N_f)_L$ symmetry is an exact global symmetry of QCD with N_f **massless quark flavors**. In the low-T, this symmetry is spontaneously broken down to the flavor group $SU(N_f)_V$, As a consequence there exist $N_f^2 - 1$ pseudoscalar Nambu–Goldstone bosons and the QCD vacuum hosts a strong quark condensate:

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \rangle .$$

Chiral phase structures in two-flavor systems



II. QCD phase structures based on the Ginzburg-Landau approach

The Ginzburg–Landau–Wilson approach:

(1) If the phase transition is of second order or of weak first order, one may write down the free-energy functional in terms of the order parameter field as a power series of Φ/T_c.

(2) The large fluctuation of near the critical point is then taken into account by the renormalization group method.

2.1 The topological phase structures of massless two-flavor systems

➢ Chiral Symmetry

For massless two-flavor, a relevant order parameter for the chiral phase transition is the color-singlet chiral condensate

$$\Phi_{ij} \sim \left\langle \bar{q}_R^j q_L^i \right\rangle$$

The most general form of the **Ginzburg-Landau** free energy of the chiral field up to $\mathcal{O}(\Phi^4)$ with $SU_L(2)\times SU_R(2)$ symmetry reads

$$\Omega_\chi = \frac{a_0}{2}\mathrm{Tr}\Phi^\dagger\Phi + \frac{b_1}{4!}(\mathrm{Tr}\Phi^\dagger\Phi)^2 + \frac{b_2}{4!}\mathrm{Tr}(\Phi^\dagger\Phi)^2 - \frac{c_0}{2}(\det\Phi + \det\Phi^\dagger),$$

with $\Phi = \sigma + i\pi_a\tau_a$

H. M, N. Petropoulos, and W. Q. Zhao, J. Phys. G: Nucl.Part. Phys. 32, 2187 (2006).
T. D. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592 (2001)

In

v

$$\Omega_{2F}(\sigma, d) = \left(\frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 + \frac{f}{6}\sigma^6\right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4\right) + \lambda d^2\sigma^2.$$

Naoki Yamamoto,et al,PRD76, 074001 (2007)

Then the symmetry is breaking from $SU_L(2)\times SU_R(2)$ to $U_L(1)\times U_R(1)$

The GL potential becomes: $\pi = <\pi_1>, <\pi_2> = <\pi_3> = 0$

$$\Omega = \frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 + \frac{c}{6}\sigma^6 + \frac{\alpha}{2}\pi^2 + \frac{\beta}{4}\pi^4 + \frac{\gamma}{6}\pi^6 + \lambda\pi^2\sigma^2$$

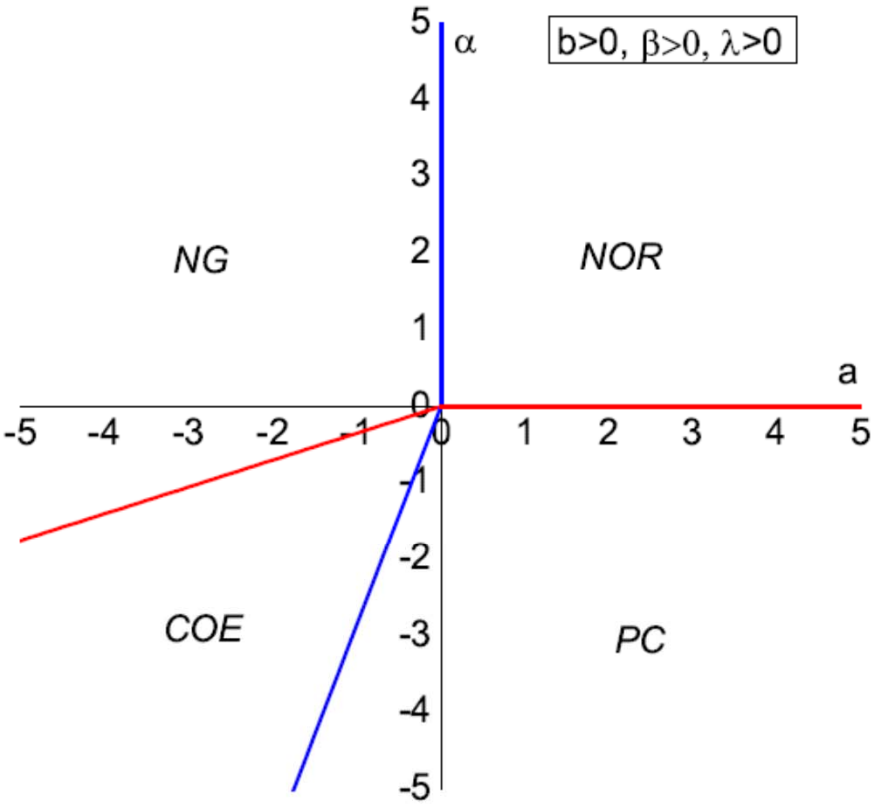
Normal (**NOR**) phase: $\sigma=0,\ \pi=0$

Pion condensate (**PC**) phase: $\sigma=0,\ \pi\neq0$

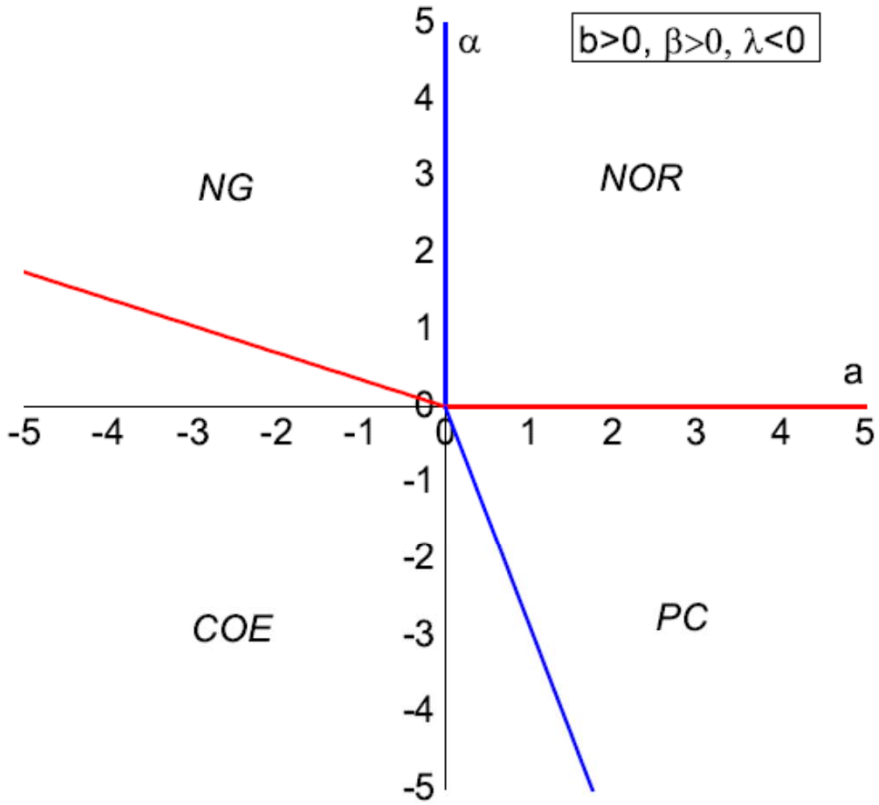
Nanbu-Goldstone (**NG**) phase: $\sigma\neq0,\ \pi=0$

Coexistence (**COE**) phase: $\sigma\neq0,\ \pi\neq0$

➢Numerical results(1) with b>0 and β>0

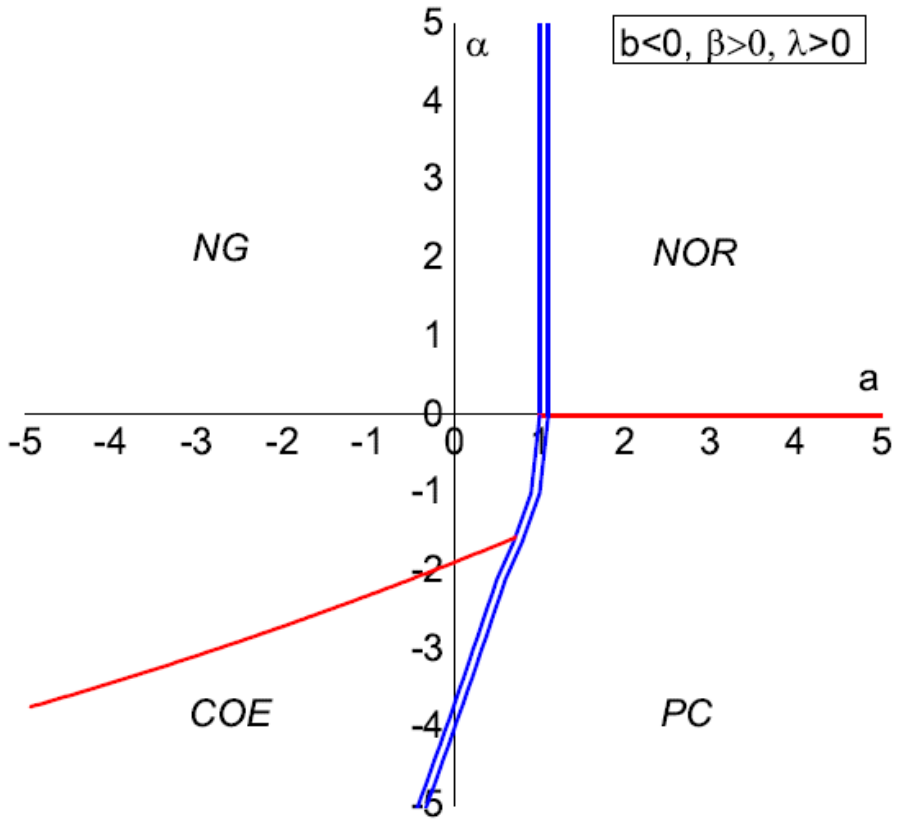


1.1 λ>0

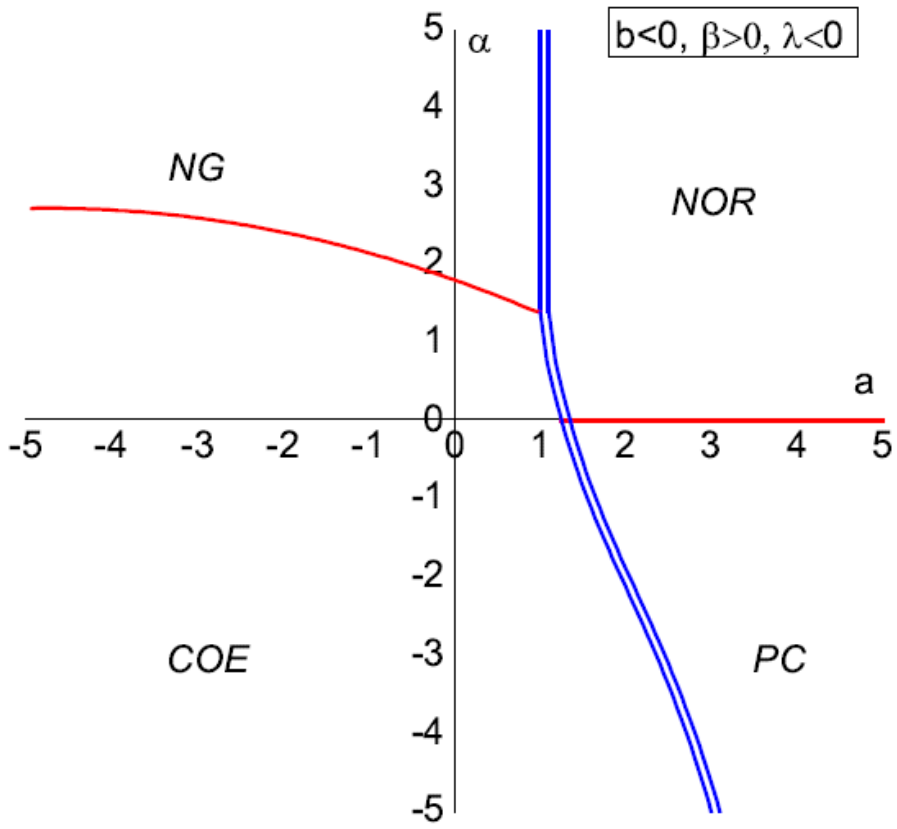


1.2 λ<0

➢Numerical results(2) with b<0 and β>0 or b>0 and β<0

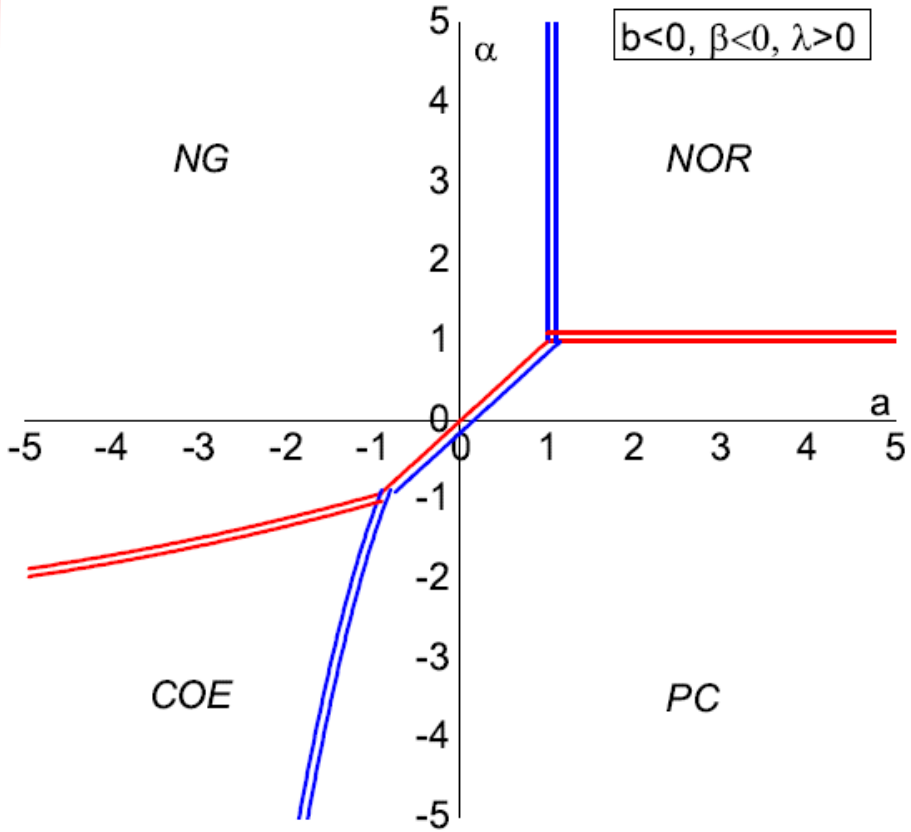


2.1 λ>0

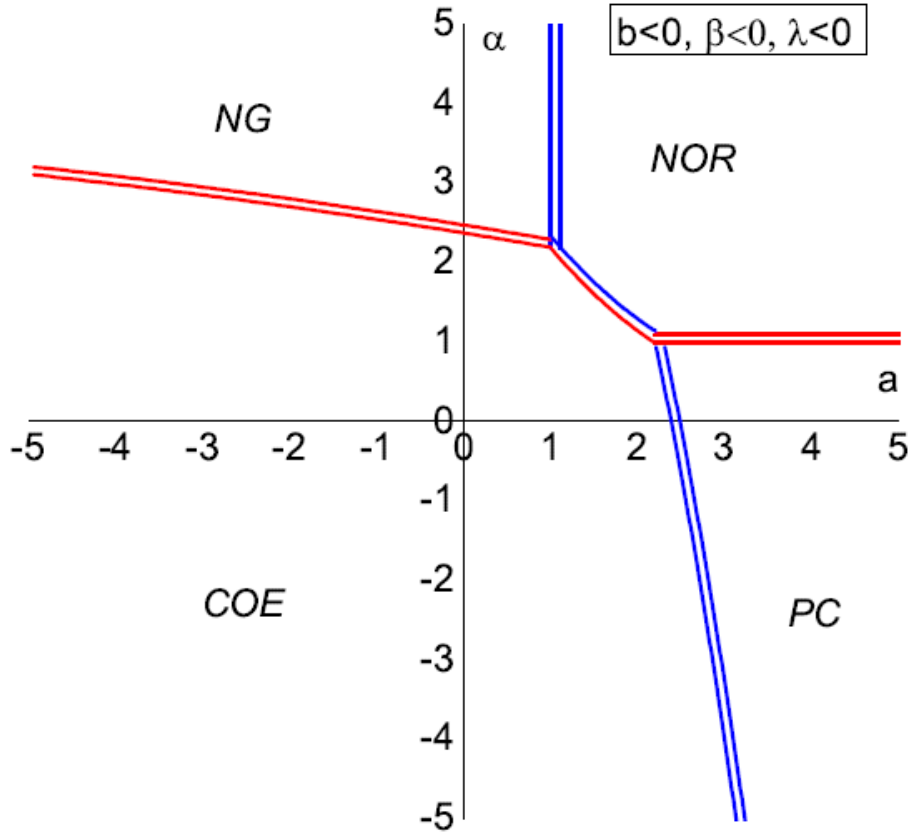


2.2 λ<0

➢Numerical results(3) with b<0 and β<0



3.1 λ>0



3.2 λ<0

2.2 Mapping with the Nambu–Jona-Lasinio model at the chiral and isospin chemical potential

➢ NJL model

The two-flavor NJL Lagrangian

$$\mathcal{L} = \bar{q}(\gamma_\nu \partial_\nu + m_0 - \hat{\mu}\gamma_4)q - G_s[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2]$$

with $\hat{\mu} = \mu_5\gamma_5 + \mu_I\tau_3$

The thermodynamic potential is obtained in the mean-field approximation a

$$\Omega = -N_c \sum_\alpha \int \frac{d^3\mathbf{p}}{(2\pi)^3}\left[E_\alpha + \frac{2}{\beta}\ln\left(1 + e^{-\beta E_\alpha}\right)\right] + U_M$$

M.Huang, P.F.Zhuang and W.Q.Zhao,PRD65, 076012(2002)

$$\alpha = \{s = \pm, f = \pm\}$$

where $$E_\alpha = \sqrt{(\sqrt{(p - s\mu_5)^2 + M^2} - f\mu_I)^2 + \pi^2}$$

$$M = m_0 - \sigma,\ \sigma = 2G_s\langle\bar{q}q\rangle \text{ and } \pi = 2G_s\langle\bar{q}i\gamma_5\tau_1 q\rangle$$

$$U_M = (\sigma^2 + \pi^2)/4G_s$$

The mean-field values are determined by the stationary conditions

$$\frac{\partial\Omega}{\partial\sigma} = \frac{\partial\Omega}{\partial\pi} = 0$$

few remarks on chiral chemical potential

(1) μ_5 **CAN NOT** be considered as a true chemical potential

$$\partial_\mu V^\mu = 0, \quad \Longleftarrow \quad U(1)_V$$

$$\partial_\mu A^\mu = 2i\bar{q}\gamma_5 \mathscr{M} q + \frac{3g_3^2}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\mathscr{G}_a^{\mu\nu}\mathscr{G}_a^{\rho\sigma}, \qquad \varepsilon_{0123} = 1,$$

$U(1)_A$

(2) It is well known that QCD with three colors suffers the sign problem: namely, the fermion determinant of QCD with three colors is complex at finite quark chemical potential, making the usual Monte Carlo sampling of configurations in the lattice simulations not possible when the quark chemical potential is larger than the temperature**. However, the theory with $\mu_5 \neq 0$ is a sign free theory.**

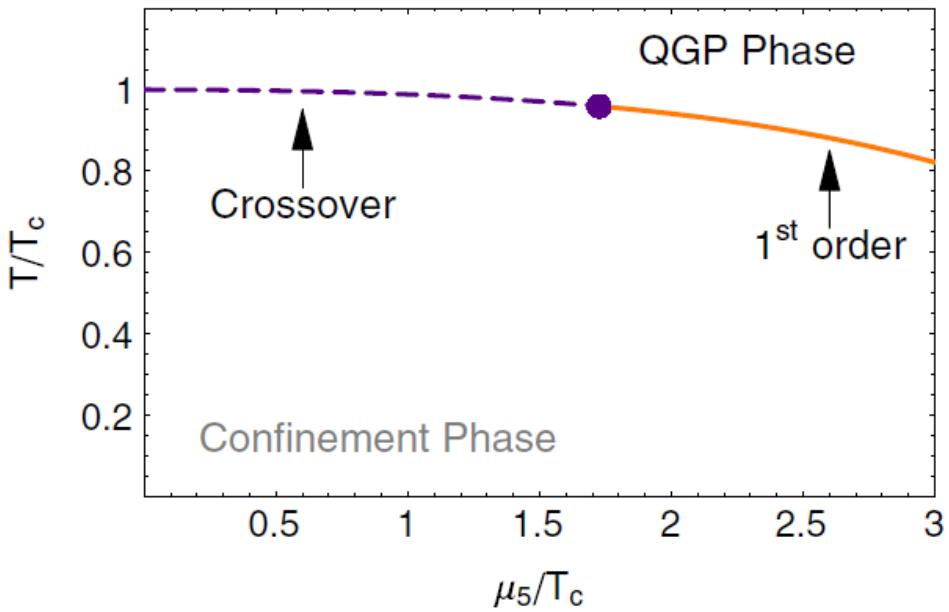
$$\gamma_5 D(\mu_5)\gamma_5 = D^\dagger(\mu_5)$$

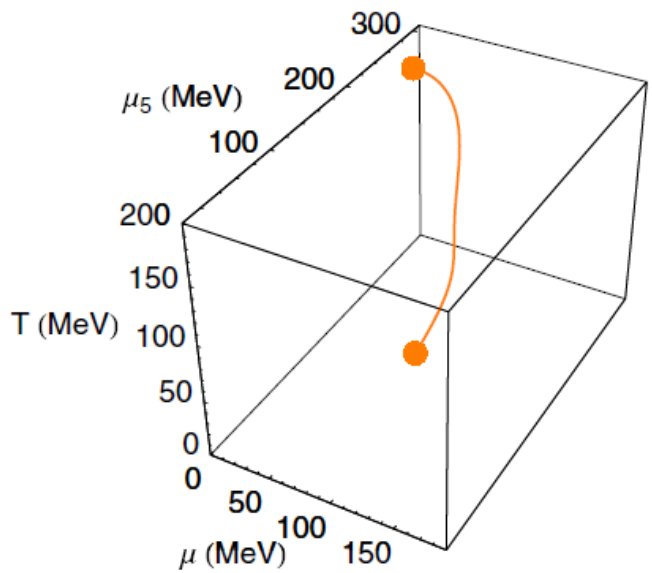
the fermion determinant is real and positive at $\mu_5 \neq 0$, and grand canonical ensembles with finite μ_5 can be simulated on the lattice.

Arata Yamamoto, PRL 107, 031601 (2011)

(3) Phase diagrams at μ_5 and μ

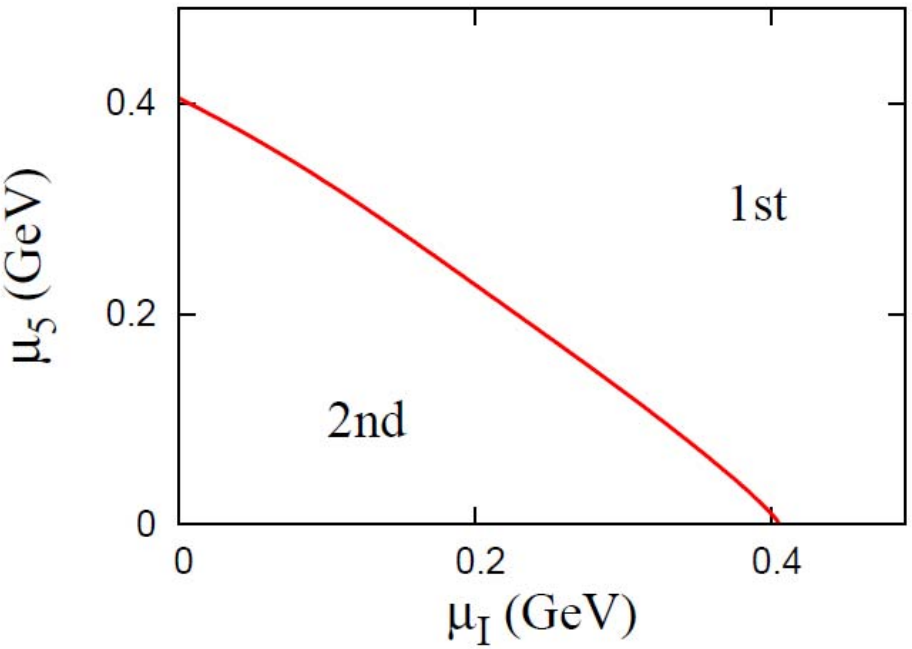
Marco Ruggieri, PRD84, 014011 (2011)



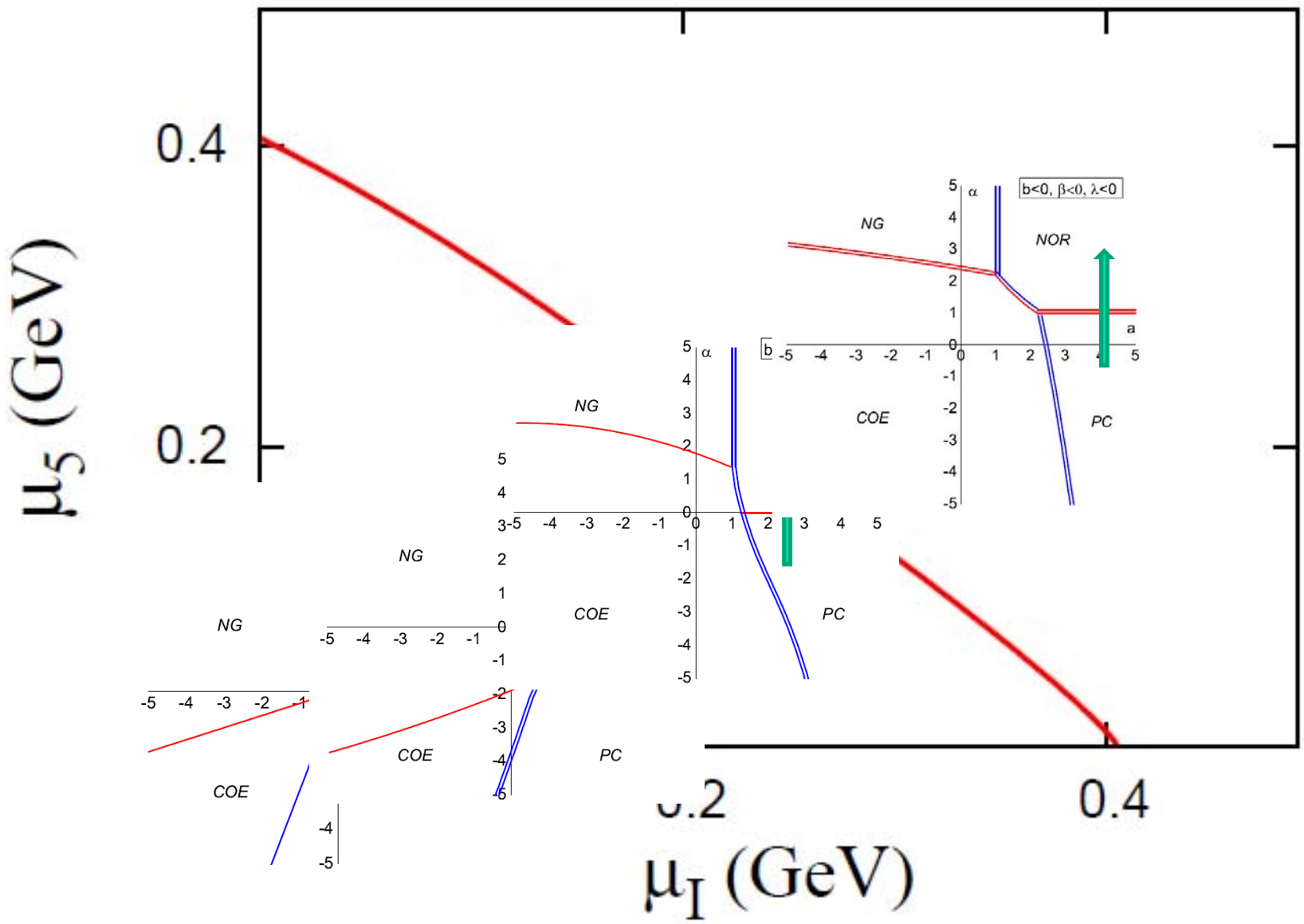


➢ Phase diagrams in NJL model at the chiral and isospin chemical potential :

(I) in chiral limit

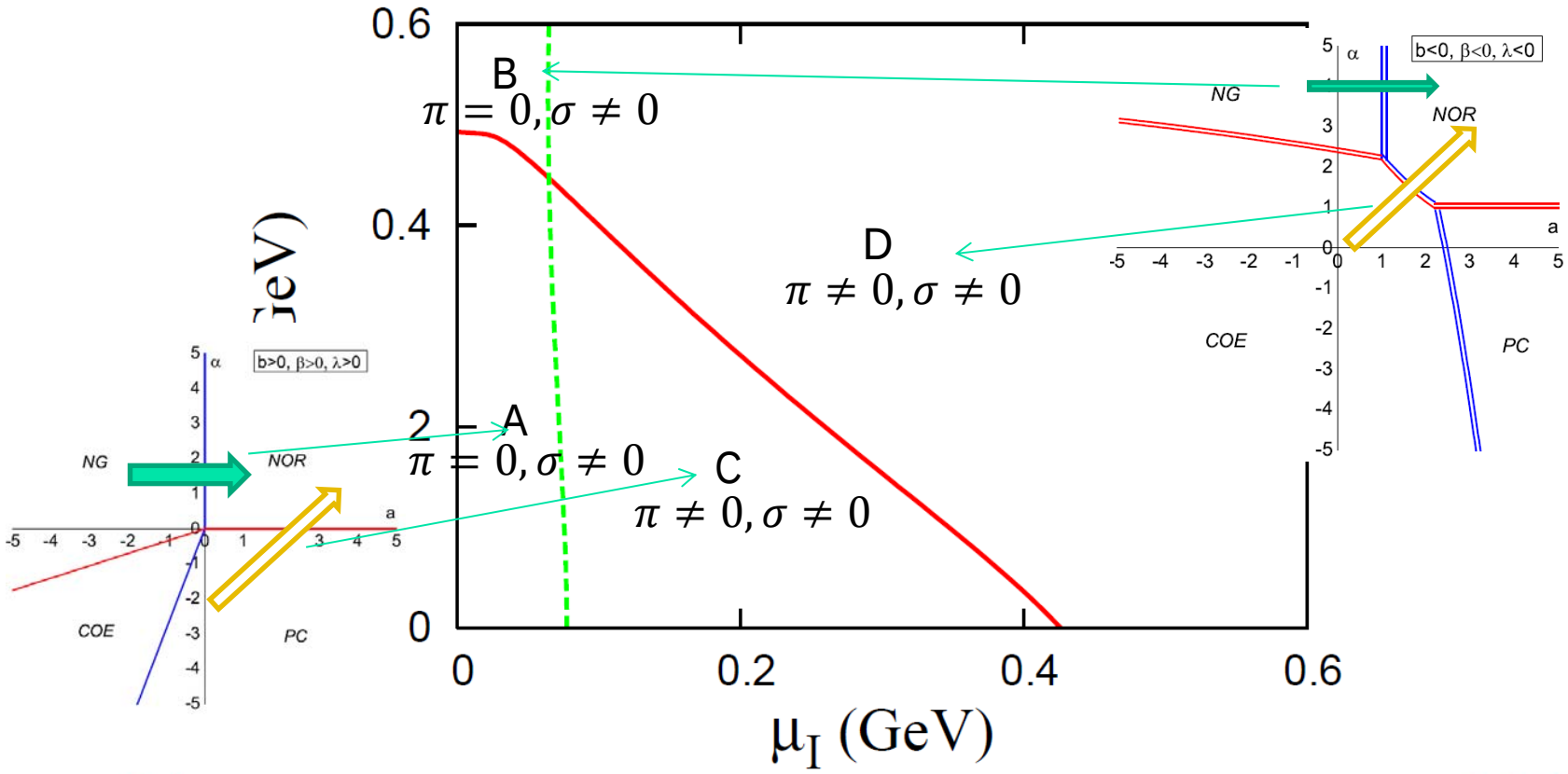


Oder of the finite temperature transition in the μ_5-μ_I plane for m_0 =0.


0.4
0.2
μ_5 (GeV)
0.2
0.4
μ_I (GeV)
NG
COE
PC
NOR
b<0, β<0, λ<0
α
a
b

(II) in real world

order param.	A	B	C	D
σ	crossover	1st order	crossover	crossover
π	none	none	2nd order	1st order



Oder of the finite temperature transition in the μ_5-μ_I plane for m_0 =5.5 MeV.

III. Summary and Discussion

- Based on the Ginzburg-Landau approach, we have studied the general phase structures of two-flavor system, and mapping with the NJL model calculations at the chiral and isospin chemical potentials.

Further studies:

- **dense QCD Simulating in Ultracold atomic systems**

High density QCD matter and ultracold atomic systems, although differing by some twenty orders of magnitude in energy scales, share analogous physical aspects:

Kenji Maeda, Gordon Baym, and Tetsuo Hatsuda, PRL 103, 085301 (2009)

Cold atoms	Dense QCD
b (bosonic atom)	D (spin-0 diquark)
$f_{\uparrow,\downarrow}$ (fermionic atom)	$q_{\uparrow,\downarrow}$ (unpaired quark)
$N_{\uparrow,\downarrow}$ (b–f molecule)	$\mathcal{N}_{\uparrow,\downarrow}$ (D–q bound state = nucleon)
b–f attraction	gluonic D–q attraction
b–BEC	2SC
N–BCS	nucleon superfluidity

Meson condensation analogs in ultracold atomic and molecular dipolar gases, **Kenji Maeda, Gordon Baym, and Tetsuo Hatsuda, arXiv:1205.1086v1**

➢ Hadron-quark phase transition in a unified model

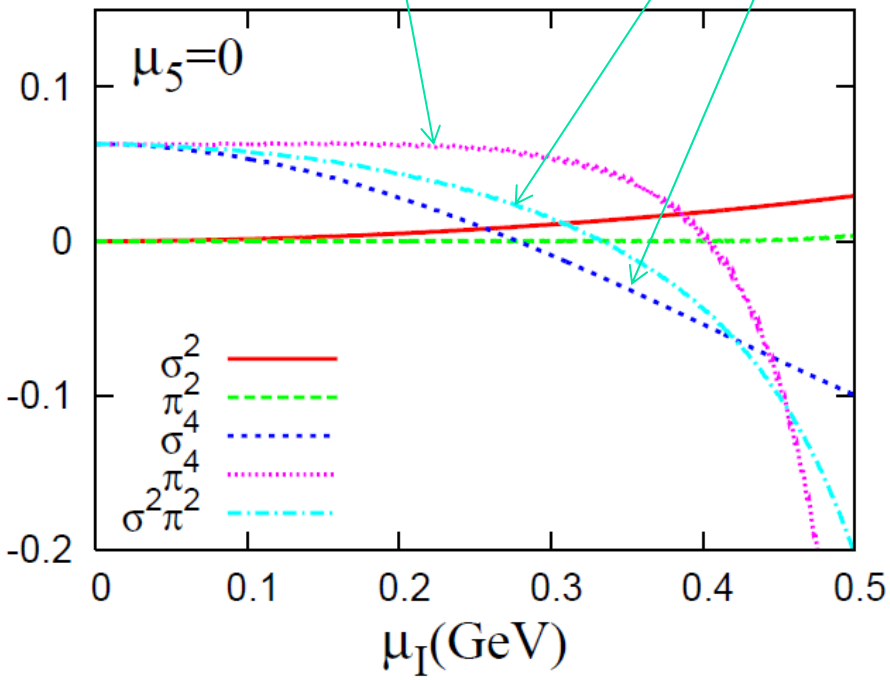
In the framework of the NJL model, we are working on description the hadron quark phase transition self-consistently by instead of RMF-PNJL or RMF-MIT methods.

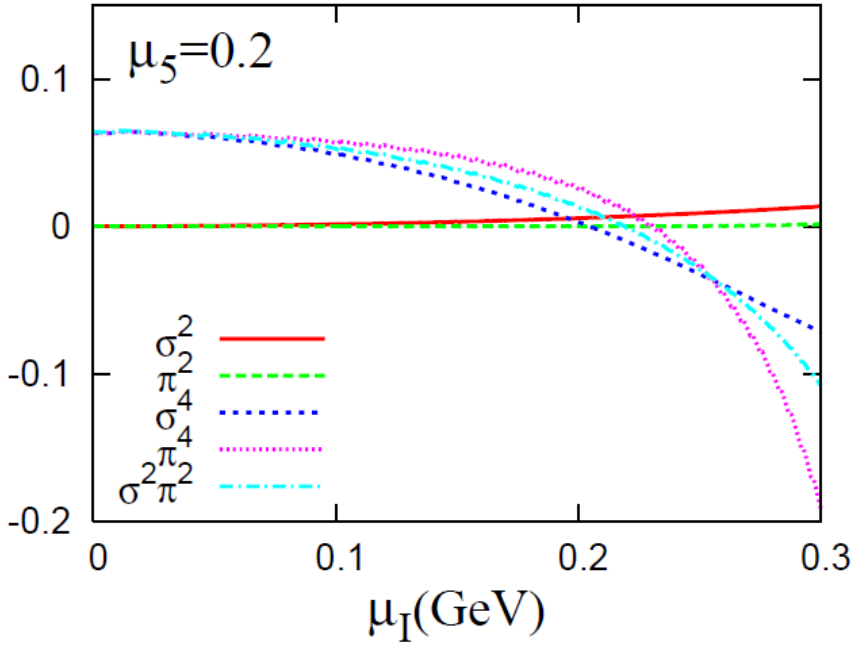
Thanks!

- **The coefficients at the critical point are calculated in the NJL model as**

$$\frac{a}{2} = \frac{\partial \Omega}{\partial \sigma^2}, \quad \frac{\alpha}{2} = \frac{\partial \Omega}{\partial \pi^2}, \quad \frac{b}{2} = \frac{\partial^2 \Omega}{\partial (\sigma^2)^2},$$

$$\frac{\beta}{2} = \frac{\partial^2 \Omega}{\partial (\pi^2)^2}, \quad \lambda = \frac{\partial^2 \Omega}{\partial \sigma^2 \partial \pi^2},$$





Comments

a. Case (1) and (2) can be solved analytically, but case (3) **CAN NOT**. So we can only numerically show the most general phase structures in all possibilities.

b. Although, the phase boundaries and the curves are parameter-dependent, the topological structures should not be essentially changed due to different choosing parameters.

c. For the massive quarks, we need to add the explicit symmetry breaking term $\boldsymbol{h\sigma}$. *(1) The effect of a finite external field, h, generally smears the second order phase transition and changes it to a smooth crossover. But, (2) The first order phase transition is stable against a small external field (small h). However, it will be washed out eventually for large h.*