# Glauber-gluon and $\eta_c$ -mixing effects in $B \rightarrow \pi \pi$ decays

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Academia Sinica, Taipei presented at 10<sup>th</sup> HFCPV, Qingdao Oct. 28, 2012 collaborated with 劉新肖振軍

# Outlines

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#### Introduction

# A serious puzzle in B decays

•  $B(\pi^0\pi^0)$  is becoming a serious puzzle

$$\mathcal{B}(B^{0} \to \pi^{0} \pi^{0}) = \begin{cases} (1.83 \pm 0.21 \pm 0.13) \times 10^{-6} & \text{(BABAR)} \\ (2.3^{+0.4+0.2}_{-0.5-0.3}) \times 10^{-6} & \text{(Belle)} \\ (1.91^{+0.22}_{-0.23}) \times 10^{-6} & \text{(HFAG)} ; \end{cases}$$

- 6 times larger than PQCD prediction
- While

$$\begin{split} \mathcal{B}(B^0 \to \rho^0 \rho^0) \;\; = \; \begin{cases} \; (0.92 \pm 0.32 \pm 0.14) \times 10^{-6} & (\text{BABAR}) \\ \; (0.4 \pm 0.4^{+0.2}_{-0.3}) \times 10^{-6} & (\text{Belle}) \;, \\ \; (0.73^{+0.27}_{-0.28}) \times 10^{-6} & (\text{HFAG}) \;. \end{cases} \end{split}$$

consistent with PQCD and QCDF predictions

#### Color-suppressed tree

•  $B(\pi^0\pi^0), B(\rho^0\rho^0)$  both depend on colorsuppressed tree amplitude C



 C is an important but least understood quantity in B decays

#### Smallness of C at LO

- Factorizable contribution is proportional to Wilson coefficient a<sub>2</sub>(m<sub>b</sub>) ~ 0
- Spectator contribution cancels between



- Sensitive to subleading corrections or new QCD mechanism. Can C be large?
- Mechanism must differentiate  $\pi$  from  $\rho$

# NLO PQCD results

Hadronic

uncertainty

Mode	Data [1]	$_{\rm LO}$	$\rm LO_{NLOWC}$	$+\mathrm{VC}$	$+\mathrm{QL}$	$+\mathrm{MP}$	+NLO
$B^{\pm} \rightarrow \pi^{\pm} K^0$	$24.1\pm1.3$	17.3	32.9	31.6	34.9	24.5	$24.9^{+13.9(+13.2)}_{-8.2(-8.2)}$
$B^\pm \to \pi^0 K^\pm$	$12.1\pm0.8$	10.4	18.7	17.7	19.7	14.2	$14.2^{+10.2}_{-5.8}(-4.3)$
$B^0 \to \pi^\mp K^\pm$	$18.9\pm0.7$	14.3	28.0	26.9	29.7	20.7	$21.1^{+15.7}_{-8.4}(-6.6)$
$B^{0} \to \pi^{0} K^{0}$	$11.5\pm1.0$	5.7	12.2	11.9	13.0	8.8	$9.2^{+5.6}_{-3.3}(-3.0)$
$B^0 \to \pi^{\mp} \pi^{\pm}$	$5.0\pm0.4$	7.1	6.8	6.6	6.9	6.7	$6.6^{+6.7(+2.7)}_{-3.8(-1.8)}$
$B^\pm \to \pi^\pm \pi^0$	$5.5\pm0.6$	3.5	4.2	4.1	4.2	4.2	$4.1^{+3.5(+1.7)}_{-2.0(-1.2)}$
$B^0  ightarrow \pi^0 \pi^0$	$1.45\pm0.29$	0.12	0.28	0.37	0.29	0.21	$0.30^{+0.49(+0.12)}_{-0.21(-0.09)}$

Mode	BABAR [1]	Belle [1]	LO	LO <sub>NLOWC</sub>	+VC	$+\mathrm{QL}$	+MP	+NLO
$B^0 \to \rho^{\mp} \rho^{\pm}$	$30\pm4\pm5$	$22.8\pm3.8^{+2.3}_{-2.6}$	27.8	26.1	25.2	26.6	25.9	$25.3^{+25.3}_{-13.8}(-7.9)$
$B^\pm \to \rho^\pm \rho^0$	$17.2\pm2.5\pm2.8$	$31.7\pm7.1^{+3.8}_{-6.7}$	13.7	16.2	16.0	16.2	16.2	$16.0^{+15.0(+7.8)}_{-8.1(-5.3)}$
$B^0 \to \rho^0 \rho^0$	< 1.1		0.33	0.56	1.02	0.62	0.45	$0.92^{+1.10}_{-0.56}(^{+0.64}_{-0.40})$

#### Glauber gluons

# Spectator diagram

• Have checked  $k_T$  factorization of spectator diagram  $q(0) M_2 \bar{q}(z_2)$ 

 $-k_2$ 

 $P_1 - k_1$ 

 $M_1$ 

 $k_{2}$ 

 $P_B - k$ 

B

- Considered factorization of M<sub>2</sub> wave function from this diagram
- Found existence k  $k_1$  of Glauber gluons, which give additional phase factors (Li, Mishima, 11)
- Interference between two LO diagrams becomes constructive

#### Relevant diagrams for M<sub>2</sub>



#### Soft effect

 Resum Glauber divergence to all orders, like summing collinear divergence into meson wave function

 $P_2 - k_2$ 

 $P_1 - k_1$ 

 $k_2$ 

 $P_B - k$ 

b



minus sign due to radiation from anti-quark

 $\begin{array}{c} \bar{q} & \bigcirc & & \text{anti-quark} \\ \hline k & k_1 & & \\ \hline I_a \approx e^{iS_e} \mathcal{M}_a^{(0)} & & \\ \hline I_b \approx e^{-iS_e} \mathcal{M}_b^{(0)} \\ \hline \end{array} \\ \hline \bullet & \text{like } 1 - 1 \Longrightarrow e^{iS_e} - e^{-iS_e} \text{ , large imaginary C} \end{array}$ 

# Glauber gluons for $M_1$ ?

Yes, they couple  $M_1$  and  $B-M_2$  system, and have the same strength as  $M_2$  Glauber effect. Li, Mishima, in preparation

#### Relevant diagrams for M<sub>1</sub>



# Nambu-Goldstone boson

- Pion as a qq bound state and as a massless Nambu-Goldstone boson?
- Massless boson => huge spacetime => large separation between qq => high mass under confinement => contradiction!
- Reconciliation: leading  $q\overline{q}$  state is tight, higher Fock state gives soft cloud (Lepage, Brodsky 79; Nussinov, Shrock 08; Duraisamy, Kagan 08) Pion is unique.
- Our soft factor corresponds to this soft cloud: 3 partons in k<sub>T</sub> space

#### **C-sensitive quantities**

• Se dependence



for Se ~ -pi/2  $C/T = 0.53e^{-2.2i}$ Acp(pi- K+) and S(rho0 KS) are almost not affected Li, Mishima, 2011



#### Two questions

- Can Glauber phase Se be so large?
- Se is factorized and universal. How is it different between  $\pi$  ,  $\rho$ ?
- Though Glauber gluons are factorized, loop momentum  $I_{\rm T}$  flows through mesons
- If mesons have different intrinsic  $k_T$  dependence, Glauber effects are different
- Can any available models of intrinsic k<sub>T</sub> dependence answer the above two questions?

# Convolution in $k_T$ space

• Consider intrinsic  $k_T$  dependence

$$\int \frac{d^2 k_T}{(2\pi)^2} \frac{d^2 k_{1T}}{(2\pi)^2} \frac{d^2 k_{2T}}{(2\pi)^2} \int \frac{d^2 l_T}{(2\pi)^2} \phi_B(k_T) \phi_1(k_{1T})$$



# **Factorization formulas**

• Performing Fourier transformation, we



(a)

 $\int d^2b_1 d^2b_2 d^2b' \phi_B(b_1) \phi_1(b_1) \bar{\phi}_2(b_1 + b', b_2 + b_1 + b')$  $\exp[iS(b')] H_a(b_1, b_2).$  $\int d^2b_1 d^2b_2 d^2b' \phi_B(b_1) \phi_1(b_1) \bar{\phi}_2(b_2 + b_1 + b', b_1 + b')$  $\exp[-iS(b')] H_b(b_1, b_2)$ 

(b)

#### Include M<sub>1</sub> Glauber gluons

 $\int \frac{d^2 k_T}{(2\pi)^2} \frac{d^2 k_{1T}}{(2\pi)^2} \frac{d^2 k_{2T}}{(2\pi)^2} \int \frac{d^2 l_{1T}}{(2\pi)^2} \frac{d^2 l_{2T}}{(2\pi)^2} \phi_B(k_T) \bar{\phi}_1(k_{1T} + l_{2T}, -k_{1T} - l_{1T} - l_{2T}) \\ \times \bar{\phi}_2(k_{2T} + l_{1T} + l_{2T}, -k_{2T} - l_{1T}) G_1(l_{1T}) G_2(l_{2T}) H_a(k_T, k_{1T}, k_{2T}).$ 

$$\begin{split} &\int d^2 b_1 d^2 b_2 \int d^2 b_{s1} d^2 b_{s2} \bar{\phi}_B(b_1) \bar{\phi}_1(b_{s1} + b_1 + b_2, b_{s1} + b_2) \\ &\times \bar{\phi}_2(b_{s2} + b_1, b_{s2} + b_1 + b_2) \exp\left[-iS(b_{s1}) + iS(b_{s2})\right] H_a(b_1, b_2) \\ &\int d^2 b_1 d^2 b_2 \int d^2 b_{s1} d^2 b_{s2} \bar{\phi}_B(b_1) \bar{\phi}_1(b_{s1} + b_1 + b_2, b_{s1} + b_2) \\ &\times \bar{\phi}_2(b_{s2} + b_1 + b_2, b_{s2} + b_1) \exp\left[-iS(b_{s1}) - iS(b_{s2})\right] H_b(b_1, b_2) \end{split}$$

# Glauber gluon effects

- M<sub>2</sub> Glauber gluons modify interference between the two LO spectator diagrams into constructive one
- Increase of C enhances  $B(\pi^0\pi^0)$
- M<sub>1</sub> Glauber gluons rotate C, and modify interference between T and C
- Increase  $B(\pi^+\pi^0)$
- Expect to improve consistency for all modes

# $\eta_C$ mixing

# Tetramixing

- Have studied  $\eta \eta' G$  (see X Liu's talk)
- Have studied tetramixing  $\eta \eta' G \eta_C$ (Tsai, Li, Zhao, 2012)
- Found charm content of  $\eta'$  important for understanding large observed  $Br(B \rightarrow \eta' K) \approx 70 \times 10^{-6}$
- Peng and Ma (2011) have studied  $\pi \eta \eta' \eta_c$  and  $\omega \rho \phi J/\psi$
- Determined charm content in  $\pi$  and  $\rho$

#### Intrinsic charm content

• charmonium mixing effects different by an order of magnitude also differentiate  $\pi$ ,  $\rho$ 

$$U_{\pi\eta\eta'\eta_c} = \begin{pmatrix} 0.9895 & 0.0552 & -0.1119 & 0.0342 \\ -0.1082 & 0.8175 & -0.5614 & -0.0259 \\ 0.0590 & 0.5696 & 0.8160 & 0.0452 \\ -0.0395 & -0.0065 & -0.0478 & 0.9960 \end{pmatrix}$$

$$U_{\omega\rho\phi J/\psi} = \begin{pmatrix} 0.9886 & -0.0122 & -0.1429 & 0.0076 \\ 0.0299 & 0.9910 & 0.1221 & -0.0025 \\ 0.1400 & -0.1250 & 0.9808 & 0.0258 \\ 0.0111 & 0.0058 & 0.0230 & 0.9986 \end{pmatrix}$$

#### Add $B \rightarrow \eta_c \pi$ amplitudes

#### • NLO PQCD give $BR(B^0 \to \eta_c \pi^0)_{\text{NLO}} = 1.24^{+0.36}_{-0.28} (\omega_B)^{+0.12}_{-0.12} (a_2^{\pi}) \times 10^{-5}$ $BR(B^+ \to \eta_c \pi^+)_{\text{NLO}} = 2.66^{+0.77}_{-0.60} (\omega_B)^{+0.26}_{-0.26} (a_2^{\pi}) \times 10^{-5}$

- Same formalism explained  $B \rightarrow \eta_C K$  data
- Add the above two amplitudes into hard kernels of corresponding  $B \rightarrow \pi \pi$  decays
- Adopt the mixing matrix elements  $U^{14}_{\pi\eta\eta'\eta_c} \approx 0.1 \quad U^{24}_{\omega\rho\phi J/\psi} \approx 0$
- The former comes from normalization

#### Numerical results

# Model of Brodsky et al

 Adopt model of Brodsky et al (1980) for intrinsic k<sub>T</sub> dependence

$$\phi_p(x, k_T) = \frac{f_p}{2\sqrt{2N_c}} \frac{24\pi m_p^2}{[m_p^2 + k_T^2/x + k_T^2/(1-x)]^2}$$

$$\phi_p(x,b) \equiv \int \frac{d^2k_T}{(2\pi)^2} \exp(-ik_T \cdot b) \phi_p(x,k_T)$$

Contain higher Gegenbauer  $\longrightarrow \phi_p(x)\sqrt{x(1-x)}m_pbK_1\left(\sqrt{x(1-x)}m_pb\right)$ 

• Expect to differentiate  $\pi$  and  $\rho$ , due to very different meson masses

# Two-argument TMD

Include Glauber gluon transverse
 momentum

$$\begin{split} \bar{\phi}_p(x,b',b) \\ &= \phi_p(x) \frac{m_p^2}{2\pi} \int \frac{K_1(\sqrt{xm_p^2 b'^2 + t^2})}{\sqrt{xm_p^2 b'^2 + t^2}} J_0\left(\frac{\sqrt{(1-x)b}}{\sqrt{xb'}}t\right) t dt. \end{split}$$

• Parameterization of Glauber factor

$$S(b) = -\alpha b^2$$

Normalization from RHIC physics (H Liu et al. 2006)

# Numerical results

• With  $\alpha = -0.42$  for all following

Mode	Data [1]	NLO $[4]$	NLO (this work)
$B^0 \to \pi^+ \pi^-$	$5.11\pm0.22$	$6.5^{+}_{-3.8(-1.8)}$	$6.58^{+2.21}_{-1.62}(\omega_B)^{+0.24}_{-0.19}(a_2^{\pi})$
$B^+ \to \pi^+ \pi^0$	$5.48^{+0.35}_{-0.34}$	$4.0^{+}_{-1.9(-1.2)}$	$5.60^{+0.00}_{-1.90}(\omega_B)^{+0.00}_{-2.39}(a_2^{\pi})$
$B^0  o \pi^0 \pi^0$	$1.91^{+0.22}_{-0.23}$	$0.29^{+0.50(+0.13)}_{-0.20(-0.08)}$	$1.10^{+0.00}_{-0.88}(\omega_B)^{+0.00}_{-0.65}(a_2^{\pi})$
$B^0 \to \rho^0 \rho^0$	$0.73\substack{+0.27 \\ -0.28}$	$0.92^{+1.10(+0.64)}_{-0.56(-0.40)}$	$0.61^{+1.02}_{-0.00}(\omega_B)^{+0.15}_{-0.06}(a_2^{\rho})$
$Br(B^0$	$\rightarrow \pi^0 \pi^0$ ) -	$\int 4.11 \times 10^{-1}$	vith Glauber
DI(D	~~~) -	$\overline{}$ 3.38 × 10 <sup>-</sup>	$\eta_{c}$ with $\eta_{c}$

• Glauber alone is not enough. Glauber and  $\eta_c$  mixing effects are equally important

#### Model of Huang, Ma et al

• If adopting model of Huang et al, Ma et al.

$$\phi_{\pi}(x,k_T) = A_{\pi} \frac{m}{\sqrt{x(1-x)}\mathcal{M}} \exp\left(-\frac{\mathcal{M}^2}{8\beta_{\pi}^2}\right),$$
  

$$\phi_{\rho}(x,k_T) = A_{\rho} \frac{2x(1-x)\mathcal{M}+m}{\sqrt{x(1-x)}(\mathcal{M}+2m)} \exp\left(-\frac{\mathcal{M}^2}{8\beta_{\rho}^2}\right)$$
  

$$\mathcal{M}^2 = \frac{k_T^2 + m^2}{x} + \frac{k_T^2 + m^2}{1-x}$$

 $\pi$  and  $\rho$  do not differ much

#### Numerical results

• For  $\alpha = -0.12$ , which gives the best fit, and stretching parameter difference

 $\beta_{\pi} \sim 0.4 \quad \beta_{\rho} \sim 0.7$ 

$$BR(B^{0} \to \pi^{0} \pi^{0}) = 0.65 \times 10^{-6}$$
  

$$BR(B^{0} \to \pi^{+} \pi^{-}) = 5.52 \times 10^{-6}$$
  

$$BR(B^{+} \to \pi^{+} \pi^{0}) = 4.40 \times 10^{-6}$$
  

$$BR(B^{0} \to \rho^{0} \rho^{0}) = 1.10 \times 10^{-6}$$

• Cannot explain the data

#### Summary

- Imaginary Glauber divergences exist in spectator B decay amplitudes
- Glauber effects change interference between two LO spectator diagrams and between T,C
- Glauber effects from pion and rho differ through convolution of universal Glauber factor with different transverse momentum dependence
- 1% charmonium in pion further differentiates pion and rho. Its effect is equally important
- Using Brodsky's model, consistency between theory and data for all  $B \rightarrow \pi\pi$ ,  $\rho\rho$  is improved but  $B(\pi^0\pi^0)$  still not resolved