# Glauber－gluon and $\eta_{C}$－mixing effects in $B \rightarrow \pi \pi$ decays 

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## Outlines

- Introduction
- Glauber gluons
- $\eta_{C}$ mixing
- Numerical results
- Summary


## Introduction

## A serious puzzle in $B$ decays

- $B\left(\pi^{0} \pi^{0}\right)$ is becoming a serious puzzle

6 times larger than PQCD prediction
- While
$\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \rho^{0}\right)=\left\{\begin{array}{lc}(0.92 \pm 0.32 \pm 0.14) \times 10^{-6} & (\mathrm{BABAR}) \\ \left(0.4 \pm 0.4_{-0.3}^{+0.2}\right) \times 10^{-6} & (\text { Belle }), \\ \left(0.73_{-0.28}^{+0.27}\right) \times 10^{-6} & \text { (HFAG) } .\end{array}\right.$
consistent with PQCD and QCDF predictions


## Color-suppressed tree

- $B\left(\pi^{0} \pi^{0}\right), B\left(\rho^{0} \rho^{0}\right)$ both depend on colorsuppressed tree amplitude C


Color-allowed tree $T$


Color-suppressed tree C

- $C$ is an important but least understood quantity in $B$ decays


## Smallness of $C$ at LO

- Factorizable contribution is proportional to Wilson coefficient $\mathrm{a}_{2}\left(\mathrm{~m}_{\mathrm{b}}\right) \sim 0$
- Spectator contribution cancels between

- Sensitive to subleading corrections or new QCD mechanism. Can C be large?
- Mechanism must differentiate $\pi$ from $\rho$


## NLO PQCD results

Hadronic uncertainty

| Mode | Data [1] | LO | $\mathrm{LO}_{\text {NLOWC }}$ | +VC | +QL | +MP | +NLO |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\overline{B^{ \pm} \rightarrow \pi^{ \pm} K^{0}}$ | $24.1 \pm 1.3$ | 17.3 | 32.9 | 31.6 | 34.9 | 24.5 | $24.9_{-8.2}^{+13.9}$ ( |  |
| $B^{ \pm} \rightarrow \pi^{0} K^{ \pm}$ | $12.1 \pm 0.8$ | 10.4 | 18.7 | 17.7 | 19.7 | 14.2 | $14.2_{-5.8}^{+10.2}$ |  |
| $B^{0} \rightarrow \pi^{\mp} K^{ \pm}$ | $18.9 \pm 0.7$ | 14.3 | 28.0 | 26.9 | 29.7 | 20.7 | $21.1_{-8.4}^{+15.7}$ |  |
| $B^{0} \rightarrow \pi^{0} K^{0}$ | $11.5 \pm 1.0$ | 5.7 | 12.2 | 11.9 | 13.0 | 8.8 | $9.2{ }_{-3.3}^{+5.6}$ ( + |  |
| $B^{0} \rightarrow \pi^{\mp} \pi^{ \pm}$ | $5.0 \pm 0.4$ | 7.1 | 6.8 | 6.6 | 6.9 | 6.7 | $6.6{ }_{-3.8}^{+6.7}$ |  |
| $B^{ \pm} \rightarrow \pi^{ \pm} \pi^{0}$ | $5.5 \pm 0.6$ | 3.5 | 4.2 | 4.1 | 4.2 | 4.2 | $4.11^{+3.5}$ ( + |  |
| $B^{0} \rightarrow \pi^{0} \pi^{0}$ | $1.45 \pm 0.29$ | 0.12 | 0.28 | 0.37 | 0.29 |  | 0.30 ${ }_{-0.2}$ | +0.12) |


| Mode | BABAR [1] | Belle [1] | LO | LO $_{\text {NLOWC }}+\mathrm{VC}$ | $+\mathrm{QL}+\mathrm{MP}$ | +NLO |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{0} \rightarrow \rho^{\mp} \rho^{ \pm}$ | $30 \pm 4 \pm 5$ | $22.8 \pm 3.8_{-2.6}^{+2.3}$ | 27.8 | 26.1 | 25.2 | 26.6 | 25.9 | $25.3_{-13.8(+12.1)}^{+25.3(+9)}$ |
| $B^{ \pm} \rightarrow \rho^{ \pm} \rho^{0}$ | $17.2 \pm 2.5 \pm 2.8$ | $31.7 \pm 7.1_{-6.7}^{+3.8}$ | 13.7 | 16.2 | 16.0 | 16.2 | 16.2 | $16.0^{+15.0(+7.8)}$ |
| $B^{0} \rightarrow \rho^{0} \rho^{0}$ | $<1.1$ | - | 0.33 | 0.56 | 1.02 | 0.62 | 0.45 | $0.92_{-0.56(-0.40)}^{+1.10(+0.64)}$ |

## Glauber gluons

## Spectator diagram

- Have checked $\mathrm{k}_{\mathrm{T}}$ factorization of spectator diagram
- Considered factorization of $M_{2}$ wave function from this diagram
- Found existence

of Glauber gluons, which give additional phase factors (Li, Mishima, 11)
- Interference between two LO diagrams becomes constructive


## Relevant diagrams for $\mathrm{M}_{2}$



## Soft effect

- Resum Glauber divergence to all orders, like summing collinear divergence into meson wave function
$k_{2} \quad P_{2}-k_{2}$

delta function


minus sign due to radiation from anti-quark
- like $1-1 \Rightarrow e^{i S_{e}}-e^{-i S_{e}}$, large imaginary C


## Glauber gluons for $\mathrm{M}_{1}$ ?

Yes, they couple $M_{1}$ and $B-M_{2}$ system, and have the same strength as
$\mathrm{M}_{2}$ Glauber effect.
Li, Mishima, in preparation

## Relevant diagrams for $\mathrm{M}_{1}$



## Nambu-Goldstone boson

- Pion as a $q \bar{q}$ bound state and as a massless Nambu-Goldstone boson?
- Massless boson => huge spacetime => large separation between $q \bar{q}=>$ high mass under confinement $=>$ contradiction!
- Reconciliation: leading $q \bar{q}$ state is tight, higher Fock state gives soft cloud (Lepage, Brodsky 79; Nussinov, Shrock 08; Duraisamy, Kagan 08) Pion is unique.
- Our soft factor corresponds to this soft cloud: 3 partons in $\mathrm{k}_{\mathrm{T}}$ space


## C-sensitive quantities

- Se dependence

for Se $\sim-p i / 2$
$C / T=0.53 e^{-2.2 i}$
Acp(pi- K+) and S(rho0 KS) are almost not affected Li, Mishima, 2011



## Two questions

- Can Glauber phase Se be so large?
- Se is factorized and universal. How is it different between $\pi, \rho$ ?
- Though Glauber gluons are factorized, loop momentum $\mathrm{I}_{\mathrm{T}}$ flows through mesons
- If mesons have different intrinsic $\mathrm{k}_{\mathrm{T}}$ dependence, Glauber effects are different
- Can any available models of intrinsic $\mathrm{k}_{\mathrm{T}}$ dependence answer the above two questions?


## Convolution in $\mathrm{k}_{\mathrm{T}}$ space

- Consider intrinsic $\mathrm{k}_{\mathrm{T}}$ dependence

$$
\int \frac{d^{2} k_{T}}{(2 \pi)^{2}} \frac{d^{2} k_{1 T}}{(2 \pi)^{2}} \frac{d^{2} k_{2 T}}{(2 \pi)^{2}} \int \frac{d^{2} l_{T}}{(2 \pi)^{2}} \phi_{B}\left(k_{T}\right) \phi_{1}\left(k_{1 T}\right)
$$

$$
\bar{\phi}_{2}\left(k_{2 T},-k_{2 T}+l_{T}\right) G_{2}\left(l_{T}\right) H_{a}\left(k_{T}, k_{1 T}, k_{2 T}, l_{T}\right)
$$



## Factorization formulas

- Performing Fourier transformation, we have

(a)
(b)

$$
\begin{gathered}
\int d^{2} b_{1} d^{2} b_{2} d^{2} b^{\prime} \phi_{B}\left(b_{1}\right) \phi_{1}\left(b_{1}\right) \bar{\phi}_{2}\left(b_{1}+b^{\prime}, b_{2}+b_{1}+b^{\prime}\right) \\
\exp \left[i S\left(b^{\prime}\right)\right] H_{a}\left(b_{1}, b_{2}\right) . \\
\int d^{2} b_{1} d^{2} b_{\bullet} d^{2} b^{\prime} \phi_{B}\left(b_{1}\right) \phi_{1}\left(b_{1}\right) \bar{o}_{\boldsymbol{\prime}}\left(b_{2}+b_{1}+b^{\prime}, b_{1}+b^{\prime}\right) \\
\exp \left[-i S\left(b^{\prime}\right)\right] H_{b}\left(b_{1}, b_{2}\right)
\end{gathered}
$$

## Include $\mathrm{M}_{1}$ Glauber gluons

$$
\begin{aligned}
& \int \frac{d^{2} k_{T}}{(2 \pi)^{2}} \frac{d^{2} k_{1 T}}{(2 \pi)^{2}} \frac{d^{2} k_{2 T}}{(2 \pi)^{2}} \int \frac{d^{2} l_{1 T}}{(2 \pi)^{2}} \frac{d^{2} l_{2 T}}{(2 \pi)^{2}} \phi_{B}\left(k_{T}\right) \bar{\phi}_{1}\left(k_{1 T}+l_{2 T},-k_{1 T}-l_{1 T}-l_{2 T}\right) \\
& \times \bar{\phi}_{2}\left(k_{2 T}+l_{1 T}+l_{2 T},-k_{2 T}-l_{1 T}\right) G_{1}\left(l_{1 T}\right) G_{2}\left(l_{2 T}\right) H_{a}\left(k_{T}, k_{1 T}, k_{2 T}\right) .
\end{aligned}
$$

$$
\int d^{2} b_{1} d^{2} b_{2} \int d^{2} b_{s 1} d^{2} b_{s 2} \bar{\phi}_{B}\left(b_{1}\right) \bar{\phi}_{1}\left(b_{s 1}+b_{1}+b_{2}, b_{s 1}+b_{2}\right)
$$

$$
\times \bar{\phi}_{2}\left(b_{s 2}+b_{1}, b_{s 2}+b_{1}+b_{2}\right) \operatorname{ex}\left[-i S\left(b_{s 1}\right)+i S\left(b_{s 2}\right)\right] H_{a}\left(b_{1}, b_{2}\right)
$$

$$
\int d^{2} b_{1} d^{2} b_{2} \int d^{2} b_{s 1} d^{2} b_{s 2} \bar{\phi}_{B}\left(b_{1}\right) \bar{\phi}_{1}\left(b_{s 1}+b_{1}+b_{2}, b_{s 1}+b_{2}\right)
$$

$$
\times \bar{\phi}_{2}\left(b_{s 2}+b_{1}+b_{2}, b_{s 2}+b_{1}\right) \operatorname{exr} \underbrace{\left.-i S\left(b_{s 1}\right)-i S\left(b_{s 2}\right)\right] H_{b}\left(b_{1}, b_{2}\right) .}
$$

## Glauber gluon effects

- $\mathrm{M}_{2}$ Glauber gluons modify interference between the two LO spectator diagrams into constructive one
- Increase of $C$ enhances $B\left(\pi^{0} \pi^{0}\right)$
- $\mathrm{M}_{1}$ Glauber gluons rotate C , and modify interference between T and C
- Increase $B\left(\pi^{+} \pi^{0}\right)$
- Expect to improve consistency for all modes


## $\eta_{C}$ mixing

## Tetramixing

- Have studied $\eta-\eta^{\prime}-G$ (see $\times$ Liu's talk)
- Have studied tetramixing $\eta-\eta^{\prime}-G-\eta_{C}$ (Tsai, Li, Zhao, 2012)
- Found charm content of $\eta^{\prime}$ important for understanding large observed $\operatorname{Br}\left(B \rightarrow \eta^{\prime} K\right) \approx 70 \times 10^{-6}$
- Peng and Ma (2011) have studied $\pi-\eta-\eta^{\prime}-\eta_{C}$ and $\omega-\rho-\phi-J / \psi$
- Determined charm content in $\pi$ and $\rho$


## Intrinsic charm content

- charmonium mixing effects different by an order of magnitude also differentiate $\pi, \rho$

$$
\begin{aligned}
U_{\pi \eta \eta^{\prime} \eta_{c}} & =\left(\begin{array}{cccc}
0.9895 & 0.0552 & -0.1119 & \widehat{0.0342} \\
-0.1082 & 0.8175 & -0.5614 & -0.0259 \\
0.0590 & 0.5696 & 0.8160 & 0.0452 \\
-0.0395 & -0.0065 & -0.0478 & 0.9960
\end{array}\right) \\
U_{\omega \rho \phi J / \psi} & =\left(\begin{array}{cccc}
0.9886 & -0.0122 & -0.1429 & 0.0076 \\
0.0299 & 0.9910 & 0.1221 & -0.0025 \\
0.1400 & -0.1250 & 0.9808 & 0.0258 \\
-0.0111 & 0.0058 & -0.0239 & 0.9986
\end{array}\right)
\end{aligned}
$$

## Add $B \rightarrow \eta_{C} \pi$ amplitudes

- NLO PQCD give

$$
\begin{aligned}
B R\left(B^{0} \rightarrow \eta_{c} \pi^{0}\right)_{\mathrm{NLO}} & =1.24_{-0.28}^{+0.36}\left(\omega_{B}\right)_{-0.12}^{+0.12}\left(a_{2}^{\pi}\right) \times 10^{-5} \\
B R\left(B^{+} \rightarrow \eta_{c} \pi^{+}\right)_{\mathrm{NLO}} & =2.66_{-0.60}^{+0.77}\left(\omega_{B}\right)_{-0.26}^{+0.26}\left(a_{2}^{\pi}\right) \times 10^{-5}
\end{aligned}
$$

- Same formalism explained $B \rightarrow \eta_{C} K$ data
- Add the above two amplitudes into hard kernels of corresponding $B \rightarrow \pi \pi$ decays
- Adopt the mixing matrix elements

$$
U_{\pi \eta \eta^{\prime} \eta_{c}}^{14} \approx 0.1 \quad U_{\omega \rho \phi J / \psi}^{24} \approx 0
$$

- The former comes from normalization

Numerical results

## Model of Brodsky et al

- Adopt model of Brodsky et al (1980) for intrinsic $\mathrm{k}_{\mathrm{T}}$ dependence

$$
\begin{aligned}
\phi_{p}\left(x, k_{T}\right) & =\frac{f_{p}}{2 \sqrt{2 N_{c}}} \frac{24 \pi m_{p}^{2}}{\left[m_{p}^{2}+k_{T}^{2} / x+k_{T}^{2} /(1-x)\right]^{2}} \\
\phi_{p}(x, b) & \equiv \int \frac{d^{2} k_{T}}{(2 \pi)^{2}} \exp \left(-i k_{T} \cdot b\right) \phi_{p}\left(x, k_{T}\right)
\end{aligned}
$$

$\xrightarrow[\text { Gegenbauer }]{\text { Contain higher }} \Longrightarrow \phi_{p}(x) \sqrt{x(1-x)} m_{p} b K_{1}\left(\sqrt{x(1-x)} m_{p} b\right)$

- Expect to differentiate $\pi$ and $\rho$, due to very different meson masses


## Two-argument TMD

- Include Glauber gluon transverse momentum

$$
\begin{aligned}
& \bar{\phi}_{p}\left(x, b^{\prime}, b\right) \\
& =\phi_{p}(x) \frac{m_{p}^{2}}{2 \pi} \int \frac{K_{1}\left(\sqrt{x m_{p}^{2} b^{\prime 2}+t^{2}}\right)}{\sqrt{x m_{p}^{2} b^{\prime 2}+t^{2}}} J_{0}\left(\frac{\sqrt{(1-x)} b}{\sqrt{x} b^{\prime}} t\right) t d t
\end{aligned}
$$

- Parameterization of Glauber factor

$$
S(b)=-\alpha b^{2}
$$

- Normalization from RHIC physics (H Liu et al. 2006)


## Numerical results

- With $\alpha=-0.42$ for all followng

| Mode | Data [1] | NLO [4] | NLO (this work) |
| :---: | :---: | :---: | :---: |
| $B^{0} \rightarrow \pi^{+} \pi^{-}$ | $5.11 \pm 0.22$ | $6.5_{-3.8(-1.7)}^{+6.7(+2.8)}$ | $6.58_{-1.62}^{+2.21}\left(\omega_{B}\right)_{-0.19}^{+0.24}\left(a_{2}^{\pi}\right)$ |
| $B^{+} \rightarrow \pi^{+} \pi^{0}$ | $5.48_{-0.34}^{+0.35}$ | $4.0_{-1.4(+1.7)}^{+1.9(-1.2)}$ | $5.60_{-1.90}^{+0.00}\left(\omega_{B}\right)_{-2.39}^{+0.00}\left(a_{2}^{\pi}\right)$ |
| $B^{0} \rightarrow \pi^{0} \pi^{0}$ | $1.91_{-0.23}^{+0.22}$ | $0.29_{-0.50(+13)}^{+0.00(+0.08)}$ | $1.10_{-0.88}^{+0.00}\left(\omega_{B}\right)_{-0.65}^{+0.00}\left(a_{2}^{\pi}\right)$ |
| $B^{0} \rightarrow \rho^{0} \rho^{0}$ | $0.73_{-0.28}^{+0.27}$ | $0.92_{-0.56(-0.40)}^{+1.10(+0.64)}$ | $0.61_{-0.00}^{+1.02}\left(\omega_{B}\right)_{-0.06}^{+0.15}\left(a_{2}^{\rho}\right)$ |

$$
\operatorname{Br}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)=\left\{\begin{array}{l}
4.11 \times 10^{-7} \text { with Glauber } \\
3.38 \times 10^{-7} \text { with } \eta_{C}
\end{array}\right.
$$

- Glauber alone is not enough. Glauber and $\eta_{C}$ mixing effects are equally important


## Model of Huang, Ma et al

- If adopting model of Huang et al, Ma et al.

$$
\begin{aligned}
\phi_{\pi}\left(x, k_{T}\right) & =A_{\pi} \frac{m}{\sqrt{x(1-x)} \mathcal{M}} \exp \left(-\frac{\mathcal{M}^{2}}{8 \beta_{\pi}^{2}}\right), \\
\phi_{\rho}\left(x, k_{T}\right) & =A_{\rho} \frac{2 x(1-x) \mathcal{M}+m}{\sqrt{x(1-x)(\mathcal{M}+2 m)}} \exp \left(-\frac{\mathcal{M}^{2}}{8 \beta_{\rho}^{2}}\right) \\
\mathcal{M}^{2} & =\frac{k_{T}^{2}+m^{2}}{x}+\frac{k_{T}^{2}+m^{2}}{1-x}
\end{aligned}
$$

- $\pi$ and $\rho$ do not differ much


## Numerical results

- For $\alpha=-0.12$, which gives the best fit, and stretching parameter difference

$$
\beta_{\pi} \sim 0.4 \quad \beta_{\rho} \sim 0.7
$$

$$
\begin{aligned}
\operatorname{BR}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right) & =0.65 \times 10^{-6} \\
\operatorname{BR}\left(B^{0} \rightarrow \pi^{+} \pi^{-}\right) & =5.52 \times 10^{-6} \\
\operatorname{BR}\left(B^{+} \rightarrow \pi^{+} \pi^{0}\right) & =4.40 \times 10^{-6} \\
\operatorname{BR}\left(B^{0} \rightarrow \rho^{0} \rho^{0}\right) & =1.10 \times 10^{-6}
\end{aligned}
$$

- Cannot explain the data


## Summary

- Imaginary Glauber divergences exist in spectator B decay amplitudes
- Glauber effects change interference between two LO spectator diagrams and between T,C
- Glauber effects from pion and rho differ through convolution of universal Glauber factor with different transverse momentum dependence
- 1\% charmonium in pion further differentiates pion and rho. Its effect is equally important
- Using Brodsky's model, consistency between theory and data for all $B \rightarrow \pi \pi, \rho \rho$ is improved but $B\left(\pi^{0} \pi^{0}\right)$ still not resolved

