QCD Radiative Corrections to $B \to \pi, K$ Form Factors

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Outline

- $B \to \pi l \nu_l$ **form factor and the extraction of** $|V_{ub}|$ **.**
- $B \to K l^+ l^-$ **form factors.**
- **Summary**

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- Partial rate for $B \to \pi l \nu_l$

$$\Delta B(0,12\mathrm{Gev}^2) = \tau_{B^0} |V_{ub}|^2 \times \frac{G_F^2}{24\pi^3} \times \int_0^{12} dq^2 \; p_\pi^3 \; |f_+^{B\to\pi}(q^2)|^2$$

- Experimental observations

BABAR ,PRD 83 (2011) 032007

$$\Delta B(0,12\mathrm{Gev}^2) = (0.88 \pm 0.06) \times 10^{-4}$$

BABAR , PRD 83 (2011) 052011

$$\Delta B(0,12\mathrm{Gev}^2) = (0.84 \pm 0.03 \pm 0.04) \times 10^{-4}$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- BABAR, PRD 83 (2011) 052011

$$f_+^{B\to\pi}(0)|v_{ub}| = (9.4 \pm 0.3 \pm 0.3) \times 10^{-4}$$

- $B \to \pi l \nu_l$ transition is parameterized as

$$\left\langle \pi(p) \middle| \bar{u}\gamma_\mu b \middle| B(p+q) \right\rangle = 2 f_+^{B\to\pi}(q^2) p_\mu + (f_+^{B\to\pi}(q^2) + f_-^{B\to\pi}(q^2)) q_\mu$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- $$correlation\ function \sim \sum_n T_H^{(n)} \otimes \Psi^{(n)}$$

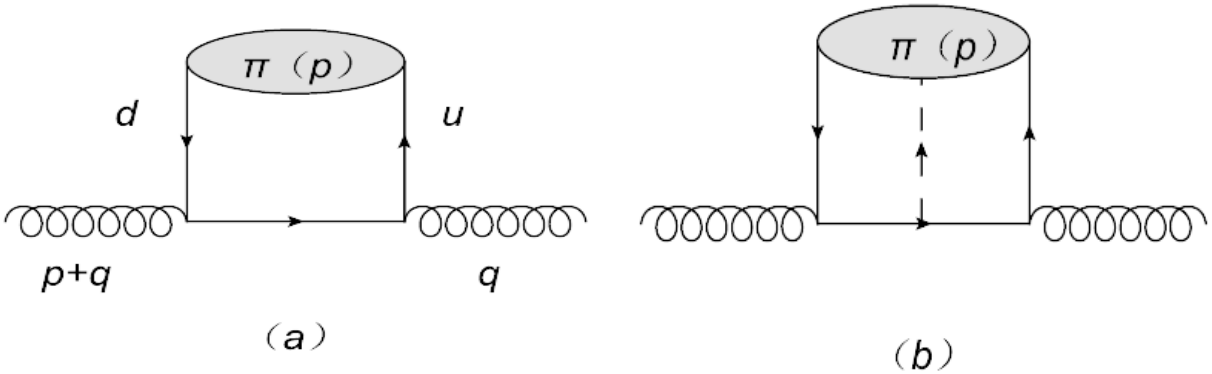
$$T_H^{(n)} = T_0^{(n)} + \frac{\alpha_s C_F}{4\pi} T_1^{(n)} + \cdots$$

- $$\begin{aligned}\Pi_\mu(p,q) &= i\int d^4x e^{iqx}\langle\pi(p)|T\{J_\mu^{V+A}(x), J_B^{P+S}(0)\}|0\rangle \\ &= F(q^2,(p+q)^2)p_\mu + \tilde{F}(q^2,(p+q)^2)q_\mu.\end{aligned}$$

$$J_\mu^{V+A}(x) = \bar{u}(x)\gamma_\mu(1+\gamma_5)b(x) \longleftarrow J_\mu^V(x) = \bar{u}(x)\gamma_\mu b(x)$$

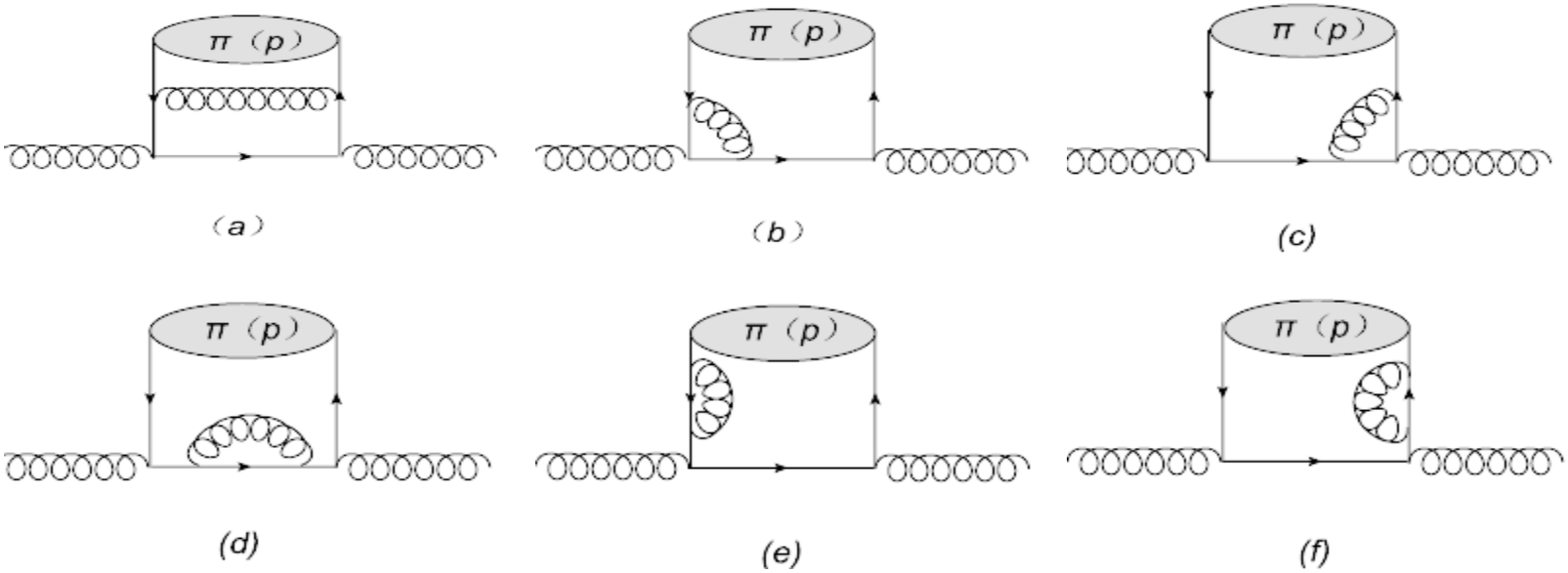
$$J_B^{P+S} = m_b\bar{b}(0)i(1+\gamma_5)d(0) \longleftarrow J_B = m_b\bar{b}(0)i\gamma_5 d(0)$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.



$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

One-loop Feynman diagrams contributing to the correction function



$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

$$f_+^{B\to\pi}(q^2) = \frac{e^{\frac{m_B^2}{M^2}}}{2m_B^2 f_B}\left[F_0(q^2, M^2, s_0^B) + \frac{\alpha_s C_F}{4\pi} F_1(q^2, M^2, s_0^B)\right]$$

$$F_0(q^2, M^2, s_0^B) = 2m_b^2 f_\pi \int_{u_0}^{1} du\, e^{-\frac{m_b^2 - q^2\bar{u}}{uM^2}} \left\{ \frac{\varphi_\pi(u)}{u} + \frac{1}{m_b^2 - q^2} \left(-\frac{m_b^2 u}{4(m_b^2 - q^2)} \frac{d^2\phi_{4\pi}(u)}{du^2} + u\psi_{4\pi}(u) + \int_0^u dv\, \psi_{4\pi}(v) - I_{4\pi} \right) \right\}$$

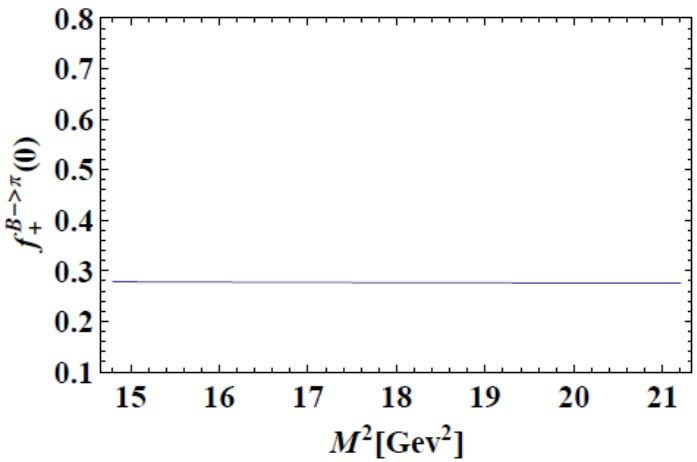
$$I_{4\pi}(u) = \int_0^u d\alpha_1 \int_{(u-\alpha_1)/1-\alpha_1}^{1} \frac{dv}{v} [2\Psi_{4\pi}(\alpha_i) + 2\widetilde{\Psi}_{4\pi}(\alpha_i) - \Phi_{4\pi}(\alpha_i) - \widetilde{\Phi}_{4\pi}(\alpha_i)] | \alpha_2 = 1 - \alpha_1 - \alpha_3, \alpha_3 = \frac{u - \alpha_1}{v}$$

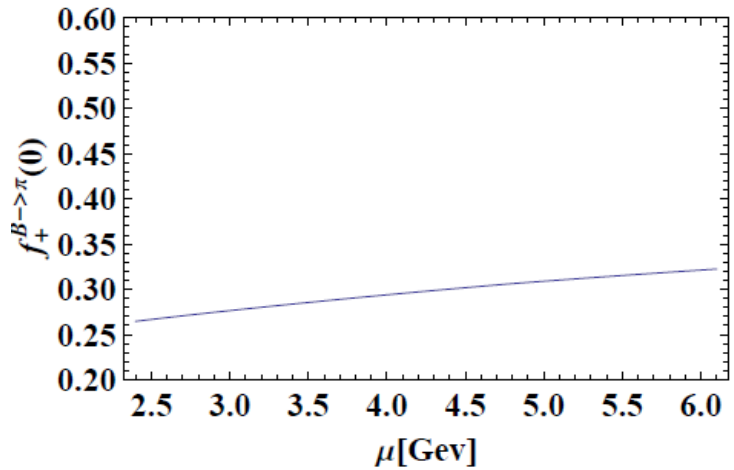
$$F_1(q^2, M^2, s_0^B) = \frac{f_\pi}{\pi} \int_{m_b^2}^{s_0^B} ds\, e^{-\frac{s}{M^2}} \int_0^1 du\, \mathrm{Im}_s\, T_1(q^2, s, u)\, \varphi_\pi(u)$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

$$T_1 = 4\left(\frac{1}{1-\rho} + \frac{r_2 - 1}{u(r_2 - r_1)^2}\right)G(r_1) + 4\left(\frac{1}{1-\rho} - \frac{1 - r_1}{\bar{u}(r_2 - r_1)^2}\right)G(r_2)$$
$$-4\left(\frac{2}{1-\rho} + \frac{r_2 - 1}{u(r_2 - r_1)^2} - \frac{1 - r_1}{\bar{u}(r_2 - r_1)^2}\right)G(\rho) - \frac{2(\rho+1)}{(\rho-1)^2}\left(3\ln\frac{m_b^2}{\mu^2}\right.$$
$$\left.-\frac{3\rho+1}{\rho}\right) - \frac{4}{r_2}\left(\frac{r_2 - 1}{\rho - 1} - \frac{r_2 - 1}{\bar{u}(r_2 - r_1)}\ln(1 - r_2)\right) - \frac{2}{r_2}$$
$$\left(\frac{r_2 - 2}{\rho} - \frac{r_2}{\rho^2} + \frac{2(r_2 - 1)}{\bar{u}(r_2 - r_1)}\right)\ln(1 - \rho)$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.





$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- Central values and respective uncertainties due to each of inputs

Central value	M^2	s_0^B	μ	m_b	f_B	a_2^π	a_4^π
$f_+^{B\to\pi}(0)$	+0.002	+0.007	+0.05	+0.008	+0.007	+0.008	+0.01
0.277	-0.001	-0.008	-0.01	-0.008	-0.009	-0.008	-0.01

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- Comparison of theoretical predictions

Approach[Ref.]	$f_+^{B\to\pi}(0)$
LCSR Phys.Rev.D83(2011)094031	0.281 ± 0.05
Phys.Rev.D71(2005)014015	$0.26^{+0.04}_{-0.03}$
JHEP 04(2008)014	0.258 ± 0.331
This work	$0.28^{+0.05}_{-0.02}$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

$$|V_{ub}| = (3.4^{+0.2}_{-0.6} \pm 0.1 \pm 0.1) \times 10^{-3}$$

Zuo.Hong.Li, Nan Zhu,Xiaojiao Fan,Tao Huang,JHEP 1205(2012)160.

$$|V_{ub}| = (3.4^{+0.24}_{-0.48}|\text{th.} \pm 0.16|\text{exp.}) \times 10^{-3}$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

- Accessing the large q^2 region with z-parameterization

$$z(q^2, t_0) = \frac{\sqrt{(m_B + m_\pi)^2 - q^2} - \sqrt{(m_B + m_\pi)^2 - t_0}}{\sqrt{(m_B + m_\pi)^2 - q^2} + \sqrt{(m_B + m_\pi)^2 - t_0}}$$

$$t_0 = (m_B + m_\pi)^2 - 2\sqrt{m_B m_\pi}\sqrt{(m_B + m_\pi)^2 - q^2_{min}}$$

$$f_+^{B\to\pi}(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{N} \tilde{b}_k [Z(q^2, t_0)]^k$$

This parameterization ensures general analytic properties of the form factor

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

To obey the expected near –threshod behavior ,the following relation Is introduced

$$\tilde{b}_N = -\frac{(-1)^N}{N}\sum_{k=0}^{N-1}(-1)^k k\tilde{b}_k$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.

$$\tilde{b}_k = f_+^{B\to\pi}(0) b_k$$

So that

$$b_0 = 1 - \sum_{k=1}^{N=1} b_k [z(0,t_0)^k - (-1)^{k-N} \frac{k}{N} z(0,t_0)^N]$$

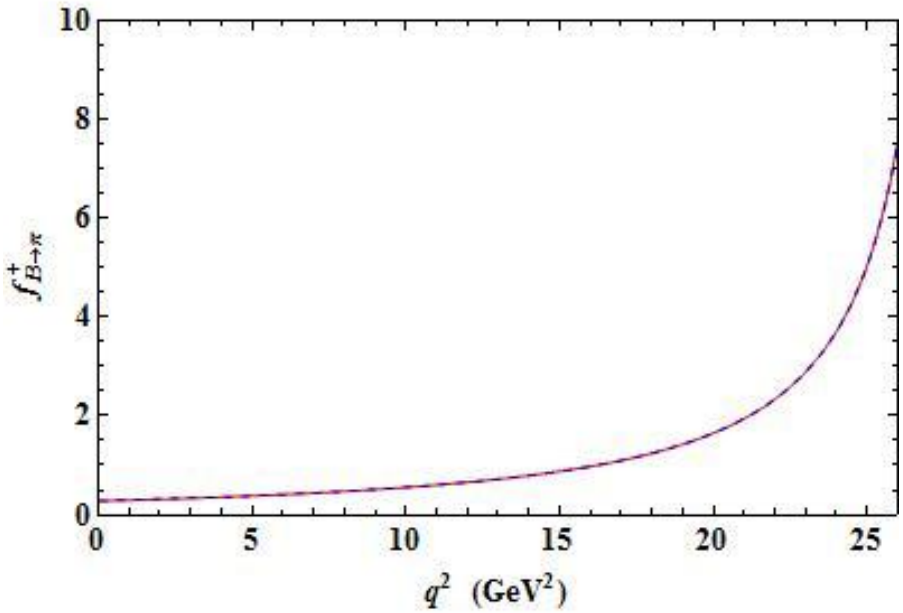
Finally, we have

$$f_+^{B\to\pi}(q^2) = \frac{f_+^{B\to\pi}(0)}{1 - q^2/m_{B^*}^2} \left\{ 1 + \sum_{k=0}^{N-1} b_k \left(z(q^2,t_0)^k - z(0,t_0)^k - (-1)^{N-k} \frac{k}{N} [z(q^2,t_0)^N - z(0,t_0)^N] \right) \right\}$$

- We fitted the numerical LCSR prediction for the form factor with N=2

$$b_1 = -2.39^{+1.29}_{-1.00}$$

$B \to \pi l \nu_l$ form factor $f_+^{B\to\pi}(q^2)$ and the extraction of $|V_{ub}|$.



$B \to K$ form factors $f_+^{B\to K}(q^2)$, $f_0^{B\to K}(q^2)$, $f_T^{B\to K}(q^2)$

$$\Pi_\mu(p,q) = i\int d^4x\, e^{iq.x}\, \langle K(p)|T\{\bar{s}(x)\Gamma_\mu b(x), m_b \bar{b}_0 i\gamma_5 d(0)\}|0\rangle$$

$$= \begin{cases} F(q^2,(p+q)^2)p_\mu + \tilde{F}(q^2,(p+q)^2)q_\mu, & \Gamma_\mu = \gamma_\mu \\ F^T(q^2,(p+q)^2)[p_\mu q^2 - q_\mu(qp)], & \Gamma_\mu = -i\sigma_{\mu\nu}q^\nu \end{cases}$$

$$\left\langle K(p)\left|\bar{s}\gamma_\mu b\right|B(p+q)\right\rangle = 2f_+^{B\to K}(q^2)p_\mu + (f_+^{B\to K}(q^2) + f_-^{B\to K}(q^2))q_\mu$$

$$\left\langle K(p)\left|\bar{s}\sigma_{\mu\nu}q^\nu b\right|B(p+q)\right\rangle = [q^2(2p_\mu + q_\mu) - (m_B^2 - m_K^2)q_\mu]\frac{if_T^{B\to K}(q^2)}{m_B + m_K}$$

$B \to K$ form factors $f_+^{B\to K}(q^2)$, $f_0^{B\to K}(q^2)$, $f_T^{B\to K}(q^2)$

$$f_+^{B\to K}(0) = \frac{e^{m_B^2/M^2}}{2 f_B m_B^2}\left(F_0^{K,2}(q^2, M^2, s_0^B) + F_0^{K,4}(q^2, M^2, s_0^B) + \frac{\alpha_s C_F}{4\pi} F_1^{K,2}(q^2, M^2, s_0^B) \right)$$

$$F_0^{K,2}(q^2, M^2, s_0^B) = 2 m_b^2 f_K \int_{u_0}^{1} \frac{du}{u} e^{-\frac{m_b^2 - q^2 \bar{u} + m_K^2 u \bar{u}}{u M^2}} \phi_K(u)$$

$$F_0^{K,4}(q^2, M^2, s_0^D) = 2 m_b^2 f_K \int_{u_0}^{1} \frac{du}{u} e^{-\frac{m_b^2 - q^2 \bar{u} + m_K^2 u \bar{u}}{u M^2}} \Bigg\{$$

$$\frac{1}{m_b^2 - q^2 + m_K^2 u^2}\Bigg[u\psi_{4K}(u) + \left(1 - \frac{2u^2 m_K^2}{m_b^2 - q^2 + m_K^2 u^2}\right) \int_0^u dv \psi_{4K}(v)$$

$$- \frac{m_b^2 u}{4(m_b^2 - q^2 + m_K^2 u^2)} \left(\frac{d^2}{du^2} - \frac{6 u m_K^2}{m_b^2 - q^2 + m_K^2 u^2} \frac{d}{du} + \frac{12 u m_K^4}{(m_b^2 - q^2 + m_K^2 u^2)^2} \right) \varphi_{4K}(u)$$

$B \to K$ form factors $f_+^{B\to K}(q^2)$, $f_0^{B\to K}(q^2)$, $f_T^{B\to K}(q^2)$

$$
\begin{aligned}
&-\frac{m_b^2 u}{4(m_b^2-q^2+m_K^2u^2)}\left(\frac{d^2}{du^2}-\frac{6um_K^2}{m_b^2-q^2+m_K^2u^2}\frac{d}{du}+\frac{12um_K^4}{(m_b^2-q^2+m_K^2u^2)^2}\right)\varphi_{4K}(u)\\
&\quad-\left(\frac{d}{du}-\frac{2um_K^2}{m_b^2-q^2+m_K^2u^2}\right)\left(\mathrm{I}_{4K}(u)-\frac{d\mathrm{I}_{4K}^{\Xi}(u)}{du}\right)\\
&\quad-\frac{2um_K^2}{m_b^2-q^2+u^2m_K^2}\left(u\frac{d}{du}+\left(1-\frac{4u^2m_K^2}{m_b^2-q^2+u^2m_K^2}\right)\right)\bar{\mathrm{I}}_{4K}(u)\\
&\quad+\frac{2um_K^2(m_b^2-q^2+u^2m_K^2)}{(m_b^2-q^2+u^2m_K^2)^2}\left(\frac{d}{du}-\frac{6um_K^2}{m_b^2-q^2+u^2m_K^2}\right)\int_u^1 d\xi\,\bar{\mathrm{I}}_{4K}(\xi)\Bigg]\Bigg\}\\
&\quad+\frac{m_b^4 f_K e^{-\frac{m_c^2}{M^2}}}{4(m_b^2-q^2+u^2m_K^2)^2}\left(\frac{d\varphi_{4K}^{WW}(u)}{du}\right)_{u\to 1}
\end{aligned}
$$

$B \to K$ form factors $f_+^{B\to K}(q^2)$, $f_0^{B\to K}(q^2)$, $f_T^{B\to K}(q^2)$

$$F_1^{K,2}(q^2, M^2, s_0^B) = \frac{f_\pi}{\pi} \int_{m_b^2}^{s_0^B} ds e^{-s/M^2} \int_0^1 du \, \mathrm{Im}_s \, T\phi_K(u)$$

$$f_0^{B\to K}(q^2) = f_+^{B\to K}(q^2) + \frac{q^2}{m_B^2 - m_K^2} f_-^{B\to K}(q^2)$$

$B \to K$ form factors $f_+^{B\to K}(q^2)$, $f_0^{B\to K}(q^2)$, $f_T^{B\to K}(q^2)$

Form factors at $q^2 = 0$

$$f_+^{B\to K}(0) = f_0^{B\to K}(0) = 0.33^{+0.05}_{-0.04}$$

$$f_T^{B\to K}(0) = 0.41 \pm 0.05$$

Summary

- Form factor $f_+^{B\to\pi}(q^2)$ for $B \to \pi l \nu_l$ is calculated in a revised LCSR to twist-5 accuracy.
- Providing two methods to extract $|V_{ub}|$
- Giving the $B \to K\, l^+ l^-$ form factors.