Law of double-β decay half-lives with two neutrinos

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Outline

- **A brief introduction of double-β decay**
- **Law of double-β decay half-lives including $\beta^-\beta^-$ and $\beta^+\beta^+$ transitions**
- **Comparison with the new Geiger-Nuttall law for α decay**
- **Summary**

Review on alpha decay, beta decay…

- Becquerel: discovery of natural radioactivity (1896)
- Rutherford: natural radioactivity, α, β, γ (1903)
- M. Curie: new elements Po, Ra by strong radioactivity
- Pauli: existence of a new particle, neutrino (1930s)
- Fermi: Fermi's theory for the description of β decay (1934)
- Lee, Yang: parity violation in weak processes (1956)
- Wu and collaborators: its confirmation by the β-decay experiment with polarized ^{60}Co (1957)
- Feynman, Gell-Mann, Sudarshan, Marshak: vector and axial-vector theory of weak interactions for four fermions (1958)
- Weinberg, Salam, Glashow: unified theory of weak and electromagnetic interactions (1960s)

Year	Physics	Chemistry	Physiology or Medicine
1901	W.C. Röntgen (G)	J.H. van't Hoff (NL)	E.A. von Behring (G)
1902	H.A. Lorentz (NL) P. Zeeman (NL)	H.E. Fischer (G)	R. Ross (GB)
1903	A.H. Becquerel (F) P. Curie (F) M. Curie (F)	S.A. Arrhenius (Swe)	N.R. Finsen (D)
1904	J.W.S. Rayleigh (GB)	W. Ramsey (GB)	I.P. Pavlov (R)
1905	P.E.A. von Lenard (G)	J.F.W.A. von Baeyer (G)	R. Koch (G)
1906	J.J. Thomson (GB)	H. Moissan (F)	C. Golgi (I) S. Ramón y Cajal (Sp)
1907	A.A. Michelson (US)	E. Buchner (G)	C.L.A. Laveran (F)
1908	G. Lippman (F)	E. Rutherford (GB)	I.I. Mechnikov (R) P. Ehrlich (G)
1909	G. Marconi (I) C.F. Braun (G)	W. Ostwald (G)	E.T. Kocher (Swi)
1910	J. D. van der Waals (NL)	O. Wallach (G)	A. Kossel (G)
1911	W. Wien (G)	M. Curie (F)	A. Gullstrand (Swe)
1912	N.G. Dalén (Swe)	V. Grignard (F) P. Sabatier (F)	A. Carrel (F)
1913	H. Kamerlingh-Onnes (NL)	A. Werner (Swi)	C.R. Richet (F)
1914	M. von Laue (G)	T.W. Richards (US)	R. Bárány (Au)
1915	W.H. Bragg (GB) L.W. Bragg (GB)	R.M. Willstätter (G)	Not awarded
1916	Not awarded	Not awarded	Not awarded
1917	C.G. Barkla (GB)	Not awarded	Not awarded

Alpha cluster and alpha decay: PRL 110, 262501 (2013)….

PRL **110,** 262501 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Nonlocalized Clustering: A New Concept in Nuclear Cluster Structure Physics

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(Received 5 April 2013; revised manuscript received 17 May 2013; published 24 June 2013)

We investigate the $\alpha + {}^{16}\mathrm{O}$ cluster structure in the inversion-doublet band ($K^\pi = 0_1^\pm$) states of $^{20}\mathrm{Ne}$ with an angular-momentum-projected version of the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function, which was successful "in its original form" for the description of, e.g., the famous Hoyle state. In contrast with the traditional view on clusters as localized objects, especially in inversion doublets, we find that these *single* THSR wave functions, which are based on the concept of nonlocalized clustering, can well describe the $K^\pi = 0_1^-$ band and the $K^\pi = 0_1^+$ band. For instance, they have 99.98% and 99.87% squared overlaps for 1^- and 3^- states (99.29%, 98.79%, and 97.75% for 0^+, 2^+, and 4^+ states), respectively, with the corresponding exact solution of the $\alpha + {}^{16}\mathrm{O}$ resonating group method. These astounding results shed a completely new light on the physics of low energy nuclear cluster states in nuclei: The clusters are nonlocalized and move around in the whole nuclear volume, only avoiding mutual overlap due to the Pauli blocking effect.

PRC2014：合成新核素^{205}Ac

PHYSICAL REVIEW C **89**, 014308 (2014)

α decay of the new neutron-deficient isotope ^{205}Ac

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(Received 1 December 2013; published 13 January 2014)

The new neutron-deficient isotope ^{205}Ac was synthesized in the complete-fusion reaction ^{169}Tm(^{40}Ca, 4n)^{205}Ac. The evaporation residues were separated in-flight by the gas-filled recoil separator SHANS in Lanzhou and subsequently identified by the α-α position and time correlation method. The α-decay energy and half-life of ^{205}Ac were determined to be 7.935(30) MeV and 20^{+97}_{-9} ms, respectively. Previously reported decay properties of the ground state in ^{206}Ac were confirmed.

PRC2014：合成新核素^{205}Ac

In Refs. [16,17], a new version of the Geiger-Nuttall law including the quantum numbers of α-core relative motion was proposed, which reproduces the α-decay half-lives of heavy nuclei with $N \leqslant 126$ very well. In Fig. 3(b), a calculation using this law is carried out for the favored α-decay transitions, and the results are compared with experimental values. The calculated 15-ms half-life of ^{205}Ac is in good agreement with the value measured in the present experiment.

The calculated half-life (15 ms) with the new Geiger-Nuttall law [16,17] agrees well with the measured data (20^{+97}_{-9}ms).

[16] Yuejiao Ren and Zhongzhou Ren, Phys. Rev. C **85**, 044608 (2012).
[17] Yuejiao Ren and Zhongzhou Ren, Nucl. Sci. Tech. **24**, 050518 (2013), http://www.j.sinap.ac.cn/nst/EN/Y2013/V24/I5/50518.

New isotope in China: 265Bh (Z=107)

Eur. Phys. J. A **20**, 385–387 (2004)

THE EUROPEAN PHYSICAL JOURNAL A

Letter

New isotope ^{265}Bh

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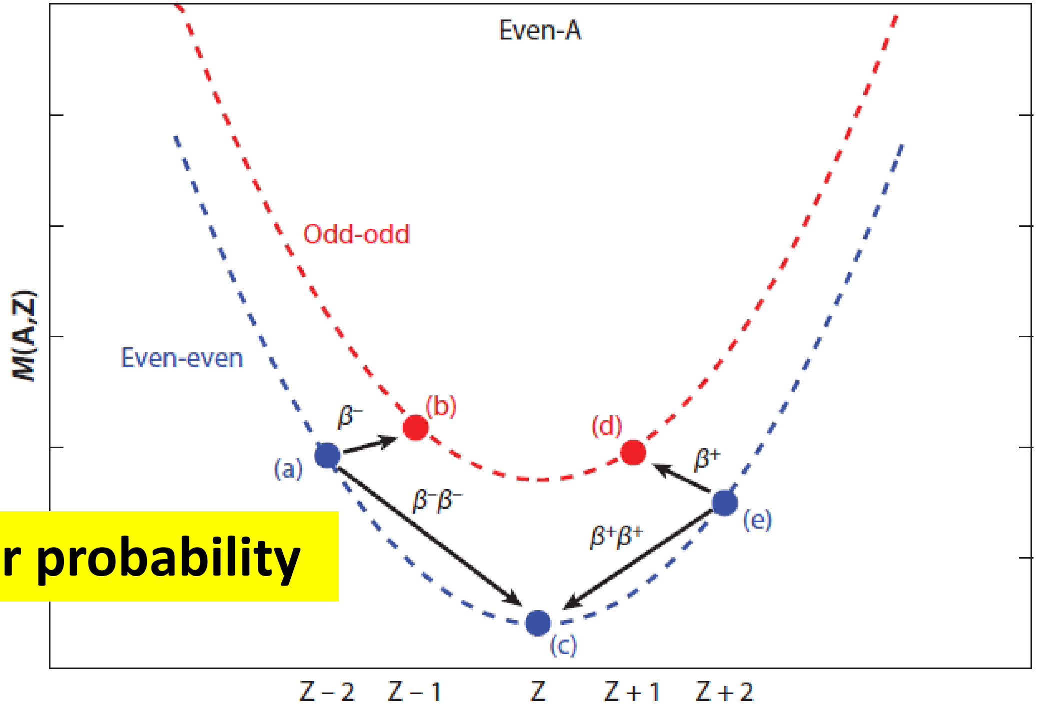
Data of 265Bh agree with theory [12,13]

. The derived Q_α from the measured α energy for ^{265}Bh was 9.38 MeV, which was in agreement with the expected Q_α value by Zhongzhou Ren *et al.* [12,13]. The experimental half-life of ^{265}Bh also agrees with the calculations [13] $T_{1/2} = 2.6$

Double-β decay in nuclei



β⁻β⁻ with higher probability

$$\beta^-\beta^-: \ (\mathrm{A,\ Z}) \rightarrow (\mathrm{A,\ Z+2}) + 2e^- + 2\bar{\nu}_e, \ \ Q_{\beta\beta} = M(\mathrm{A,\ Z}) - M(\mathrm{A,\ Z{+}2})$$

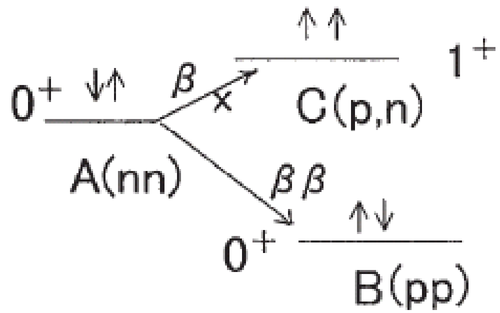
$$\beta^+\beta^+: \ (\mathrm{A,\ Z}) \rightarrow (\mathrm{A,\ Z-2}) + 2e^+ + 2\nu_e, \ \ Q_{\beta\beta} = M(\mathrm{A,\ Z}) - M(\mathrm{A,\ Z{-}2}) - 4m_e c^2$$

$$\mathrm{ECEC}: \ 2e^- + (\mathrm{A,\ Z}) \rightarrow (\mathrm{A,\ Z-2}) + 2\nu_e,$$

$$\mathrm{EC}\beta^+: \ e^- + (\mathrm{A,\ Z}) \rightarrow (\mathrm{A,\ Z-2}) + e^+ + 2\nu_e,$$

$\beta^- \beta^-$ transitions

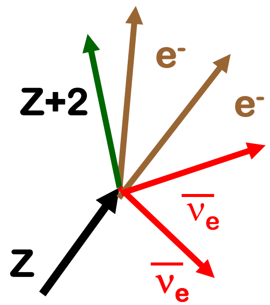
There are two different modes for $\beta^- \beta^-$ transitions



1) Two-Neutrino Double-β decay (2ν mode)

Nucleus (A, Z) $\rightarrow$ Nucleus (A, Z+2) + e^- + $\bar{\nu}_e$ + e^- + $\bar{\nu}_e$

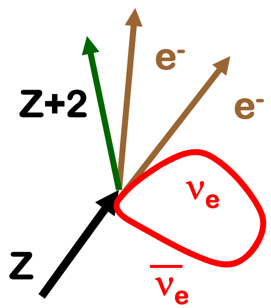
$^{76}Ge \rightarrow {}^{76}Se + e^- + \bar{\nu}_e + e^- + \bar{\nu}_e$



2) Neutrinoless Double-β decay (0ν mode):

Nucleus (A, Z) $\rightarrow$ Nucleus (A, Z+2) + e^- + e^-

$^{76}Ge \rightarrow {}^{76}Se + e^- + e^-$



Double β decay and neutrino physics

The 2ν mode is independent of a possible small neutrino mass.

$$(T_{1/2}^{2\nu})^{-1} = G_{2\nu}(Q_{2\beta}, Z)\left|M_{2\nu}\right|^2$$

The 0ν mode violates lepton number conservation and requires the existence of massive Majorana.

$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{2\beta}, Z)\left|M_{0\nu}\right|^2 \left\langle m_{\beta\beta} \right\rangle^2$$

Observation of 0ν double β decay could be used to determine the absolute scale of the neutrino masses, and can distinguish if neutrinos are Dirac or Majorana particles.

Experimental data for two-neutrino double-β decay to the ground state and excited states already exist for a group of nuclei. There are no confirmed experimental data so far for neutrinoless double-β decay.

First theoretical estimation for double β decay [2ν mode]: a half-life of over 10^{17} years.
M. Goeppert-Mayer, Phys. Rev. 48 (1935) 512

SEPTEMBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Double Beta-Disintegration

M. Goeppert-Mayer, *The Johns Hopkins University*
(Received May 20, 1935)

From the Fermi theory of β-disintegration the probability of simultaneous emission of two electrons (and two neutrinos) has been calculated. The result is that this process occurs sufficiently rarely to allow a half-life of over 10^{17} years for a nucleus, even if its isobar of atomic number different by 2 were more stable by 20 times the electron mass.

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1. Introduction

In a table showing the existing atomic nuclei it is observed that many groups of isobars occur, the term isobar referring to nuclei of the same atomic weight but different atomic number. It is unreasonable to assume that all isobars have exactly the same energy; one of them therefore

only isobars of even difference in atomic number occur.

A metastable isobar can, however, change into a more stable one by simultaneous emission of two electrons. It is generally assumed that the frequency of such a process is very small. In this paper the propability of a disintegration of that kind has been calculated.

Bardin, Gollon, Ullman, Wu, Phys. Lett. B 26 (1967) 112: ^{48}Ca

ELSEVIER

Physics Letters B

Volume 26, Issue 2, 25 December 1967, Pages 112–116

Double beta decay in ^{48}Ca and the conservation of leptons ☆

R.K. Bardin, D.J. Gollon, J.D. Ullman, C.S. Wu

Columbia University, New York, New York, USA

Received 18 November 1967, Available online 21 October 2002

Abstract

A new experimental investigation of double beta decay in ^{48}Ca using apparatus of unique sensitivity and discrimination is reported. We obtain a lower limit of 1.6×10^{21} years at a statistical confidence level of 80% for the lifetime of the lepton-nonconserving neutrinoless mode, and of 3×10^{19} years for the lepton-conserving two-neutrino mode.

☆ Work partially supported by the U.S. Atomic Energy Commission.

First direct laboratory detection of double β decay [2ν mode]: $^{82}Se \rightarrow ^{82}Kr + 2e^- + 2\bar{\nu}_e$. Phys. Rev. Lett. 59 (1987) 2020

VOLUME 59, NUMBER 18 PHYSICAL REVIEW LETTERS 2 NOVEMBER 1987

Direct Evidence for Two-Neutrino Double-Beta Decay in ^{82}Se

S. R. Elliott, A. A. Hahn, and M. K. Moe
Department of Physics, University of California, Irvine, Irvine, California 92717
(Received 31 August 1987)

The two-neutrino mode of double-beta decay in ^{82}Se has been observed in a time-projection chamber at a half-life of $(1.1^{+0.8}_{-0.3})\times 10^{20}$ yr (68% confidence level). This result from direct counting confirms the earlier geochemical measurements and helps provide a standard by which to test the double-beta-decay matrix elements of nuclear theory. It is the rarest natural decay process ever observed directly in the laboratory.

1987年核物理方法第一次观察到^{82}Se的双β衰变
Elliott, Hahn, Moe， PRL 59 (1987) 2020

Physics Reports

Volume 242, Issues 4–6, July 1994, Pages 403–422

Operator expansion method and nuclear ββ decay

M. Hirsch[a], X.R. Wu[a, 1], H.V. Klapdor-Kleingrothaus[a], Ching Cheng-rui[b, 2], Ho Tso-hsiu[b, 2]

Available online 30 September 2002

doi:10.1016/0370-1573(94)90174-0

Abstract

Recent developments in the theoretical description of double beta decay matrix elements are outlined. We present the formalism of the operator expansion method (OEM) and discuss its physical basis. The OEM has been combined with ground state wave functions obtained in a usual QRPA calculation and applied to the problem of evaluating nuclear matrix elements for double beta decay in the 2νββ as well as in the 0νββ decay mode. For 2νββ decay we have calculated that half-lives for all possible $\beta^-\beta^-$ decay candidates with $A \geq 70$. We confront our results with the existing experimental data and discuss the quality of the model. 0νββ decay matrix elements are then used to extract upper limits for the effective neutrino mass from published experimental lower half-life limits. The presently sharpest limits come from experiments using isotopically enriched ^{76}Ge. A short discussion is devoted to a recent measurement using ^{238}U in a radiochemical experiment. Our conclusion is that the experimental result should be interpreted as being due to 2νββ decay, and lepton number violation is not necessary to explain the experimental finding.

R. Saakyan, Annu. Rev. Nucl. Part. Sci. 2013, 63: 503

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Two-Neutrino Double-Beta Decay

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Annu. Rev. Nucl. Part. Sci. 2013. 63:503–29

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Keywords

nuclear matrix elements, low-background technologies, underground detectors

Abstract

Two-neutrino double-β decay is a radioactive process with the longest lifetime ever observed. It has been a subject of experimental research for more than 60 years and remains an important topic in modern nuclear and particle physics. This review examines the process in detail, covers its theoretical and experimental aspects, and describes the results obtained so far and future challenges.

Motivation

There are some simple and famous formulas for α-decay half-lives, such as the Geiger-Nuttall law, the Viola-Seaborg formula, and the new Geiger-Nuttall law. However, there is not a simple and accurate formula for double-β-decay half-lives in publications.

After decades of researches, accumulation of double-β-decay data provides a good opportunity to make a systematic analysis on the available double-β-decay data.

The resulting formula could be very helpful for its simplicity. It can be conveniently used to analyze double-β-decay data and to predict half-lives of double-β candidates.

REN: new law for double-β decay: PRC 89, 064603 (2014)

PHYSICAL REVIEW C **89**, 064603 (2014)

Systematic law for half-lives of double-β decay with two neutrinos

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(Received 5 March 2014; revised manuscript received 14 April 2014; published 6 June 2014)

Nuclear double-β decay with two neutrinos is a rare and important process for natural radioactivity of unstable nuclei. The experimental data of nuclear double-β^- decay with two neutrinos are analyzed and a systematic law to calculate the half-lives of this rare process is proposed. It is the first analytical and simple formula for double-β-decay half-lives where the leading effects from both the Coulomb potential and the nuclear structure are included. The systematic law shows that the logarithms of the half-lives are inversely proportional to the decay energies for the ground-state transitions between parent nuclei and daughter nuclei. The calculated half-lives are in agreement with the experimental data of ground-state transitions of all known 11 nuclei with an average factor of 3.06. The half-lives of other possible double-β-decay candidates with two neutrinos are predicted and these can be useful for future experiments. The law, without introducing any extra adjustment, is also generalized to the calculations of double-β-decay half-lives from the ground states of parent nuclei to the first 0^+ excited states of daughter nuclei, and the calculated half-lives agree very well with the available data. The calculated half-lives from the ground states of parent nuclei ^{48}Ca and ^{150}Nd to the first 0^+ excited states of daughter nuclei are the first theoretical results as far as we know. The similarity and difference between the law of α decay and that of double-β^- decay are also analyzed and discussed.

Tab. 1: Experimental data for $2\nu\beta^-\beta^-$ decay of 11 isotopes

Nucl.	$T_{1/2}$(expt) (Ey)	$\log_{10}T_{1/2}$(expt)	$Q_{2\beta}$ (MeV)	$[\log_{10}T_{1/2}(\text{expt})\text{x}Q_{2\beta}]^{1/2}$
^{48}Ca	44(+6 -5)	1.643	4.267	2.649
^{76}Ge	1840(+140 -100)	3.265	2.039	2.580
^{82}Se	92(7)	1.964	2.996	2.424
^{96}Zr	23.5(0.21)	1.371	3.349	2.143
^{100}Mo	7.1(0.4)	0.851	3.034	1.606
^{116}Cd	28(2)	1.447	2.813	2.017
^{128}Te	1.9(0.4)x10^6	6.279	0.8665	2.333
^{130}Te	700(140)	2.845	2.528	2.682
^{136}Xe	2300(120)	3.362	2.458	2.876
^{150}Nd	9.11(0.68)	0.960	3.371	1.799
^{238}U	2000(600)	3.301	1.144	1.944

$$\log_{10} T_{1/2}\,(\text{Ey}) = a/Q_{2\beta}\,(\text{MeV})$$

The effect from the Coulomb potential

$$\rho(Z,\varepsilon) = 2\pi\eta \big/ (1 - e^{-2\pi\eta}), \text{ with } \eta = (+Z/137)(\varepsilon/cp)$$

A correction to double-β[-]-decay half-lives $\log_{10} T_{1/2}$

$$-2\log_{10}\left[(2\pi Z)/137\right]$$

The leading effect of nuclear structure (shell effect) is simulated by introducing an addition quantity *S*:

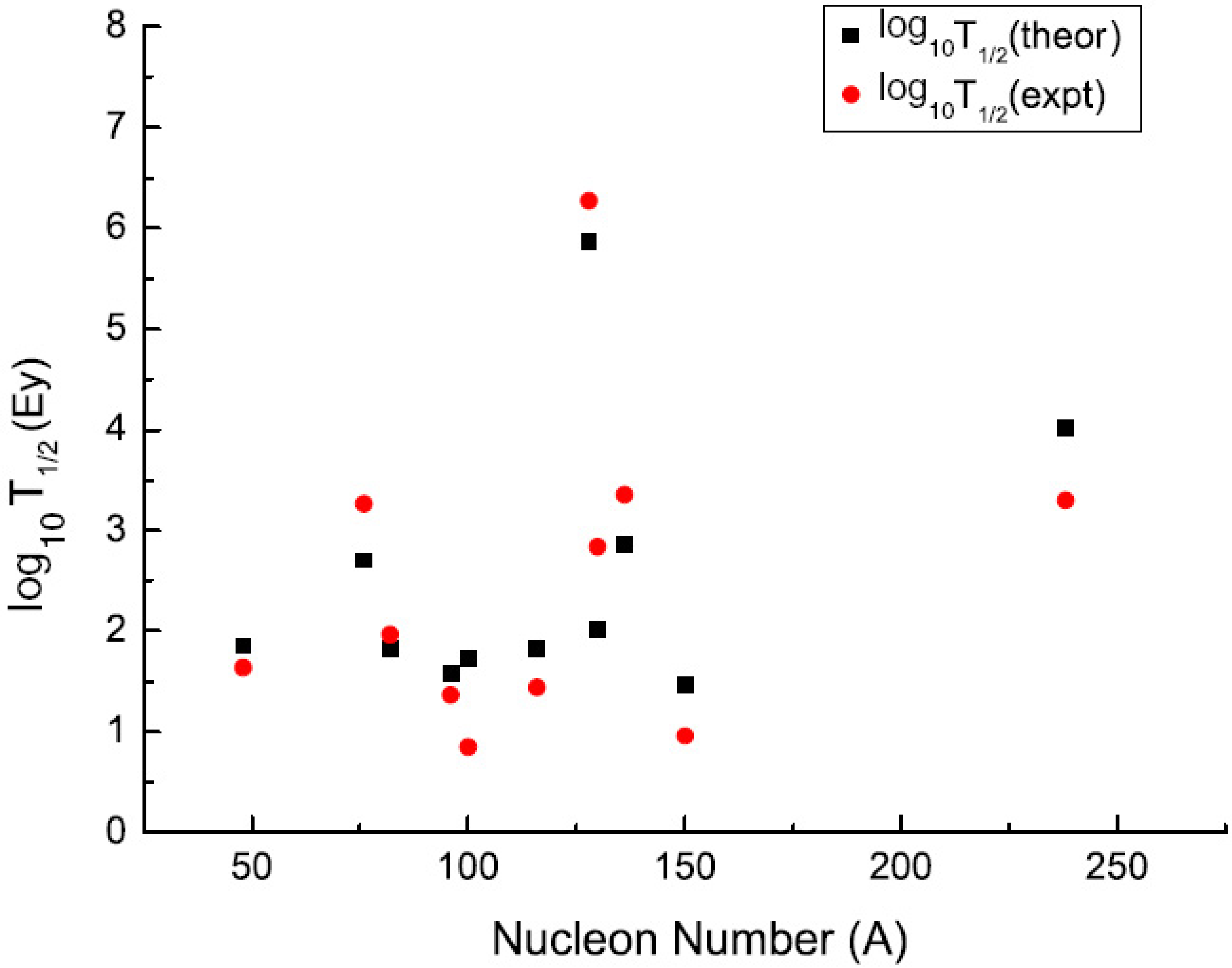
***S*=2 when neutron numbers of parent nuclei are magic,**

***S*=0 when neutron numbers are nonmagic.**

$$\log_{10} T_{1/2}(\mathrm{Ey}) = \left[a - 2\log_{10}(2\pi Z/137) + S\right] \big/ Q_{2\beta}(\mathrm{MeV}),$$

parameter a is obtained as 5.843 by fitting 11 ground-state data.

Comparison of the experimental and theoretical double-β-decay half-lives for ground-state transitions of 11 even-even nuclei



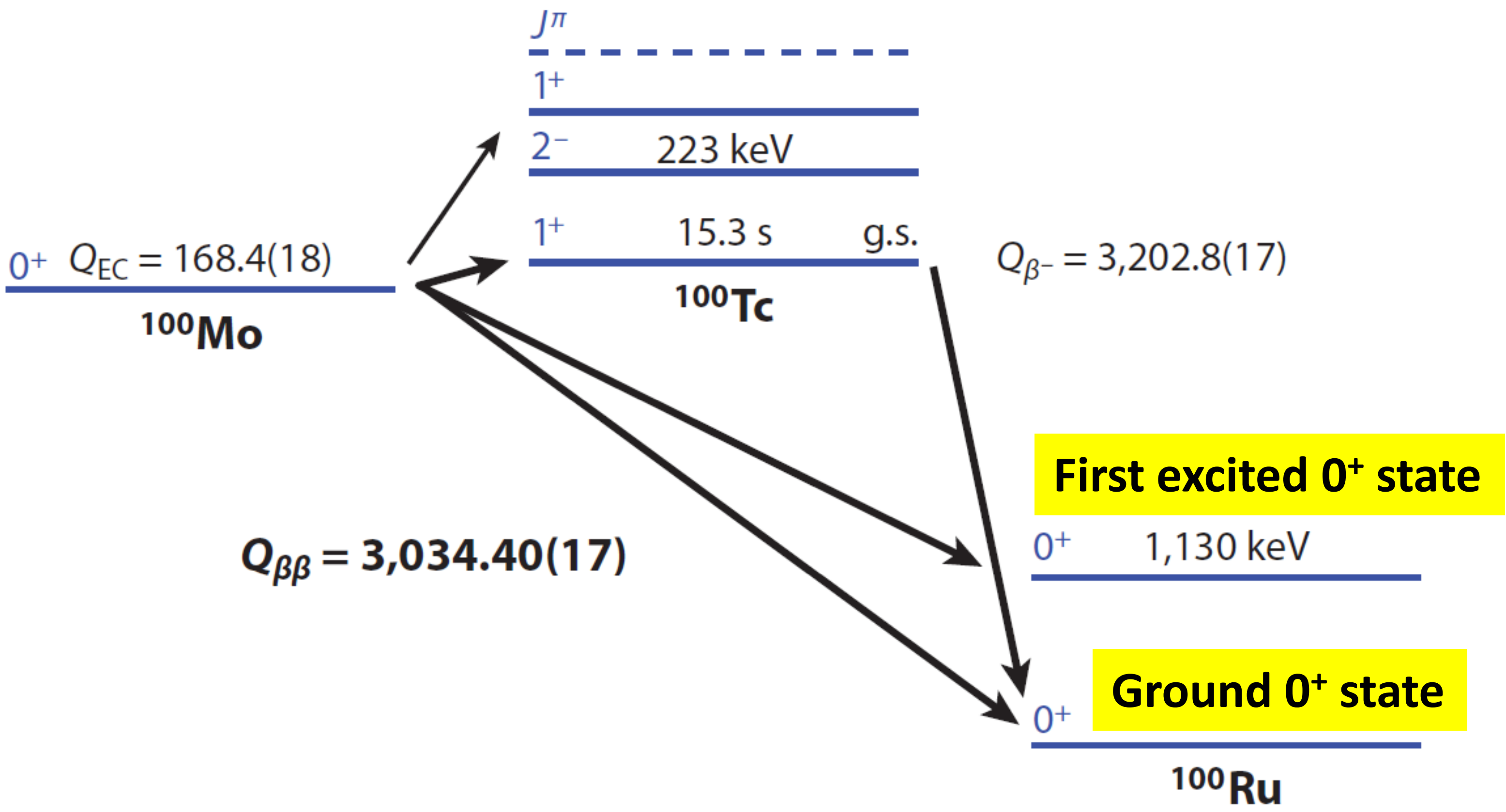
Tab. 2: Logarithms of double-β-decay half-lives calculated with the new law and the experimental data (with a factor of 3.06)

Nucl.	$T_{1/2}$(expt) (Ey)	$\log_{10}T_{1/2}$(expt)	$Q_{2\beta}$ (MeV)	$\log_{10}T_{1/2}$(theor)
^{48}Ca	44(+6 -5)	1.643	4.267	1.856
^{76}Ge	1840(+140 -100)	3.265	2.039	2.702
^{82}Se	92(7)	1.964	2.996	1.822
^{96}Zr	23.5(0.21)	1.371	3.349	1.587
^{100}Mo	7.1(0.4)	0.851	3.034	1.738
^{116}Cd	28(2)	1.447	2.813	1.834
^{128}Te	1.9(0.4)x10^{6}	6.279	0.8665	5.872
^{130}Te	700(140)	2.845	2.528	2.013
^{136}Xe	2300(120)	3.362	2.458	2.871
^{150}Nd	9.11(0.68)	0.960	3.371	1.473
^{238}U	2000(600)	3.301	1.144	4.015

Tab. 3: Predicted double-β-decay half-lives with two neutrinos for ground-state transitions of 11 even-even isotopes

Nucl.	$Q_{2\beta}$ (MeV)	$\log_{10}T_{1/2}$(theor)	$T_{1/2}$(theor) (Ey)
^{46}Ca	0.989	5.984	9.64×10^5
^{86}Kr	1.258	5.889	7.74×10^5
^{94}Zr	1.142	4.655	4.52×10^4
^{104}Ru	1.301	4.023	1.05×10^4
^{110}Pd	2.017	2.576	3.77×10^2
^{148}Nd	1.928	2.575	3.76×10^2
^{154}Sm	1.251	3.945	8.81×10^3
^{160}Gd	1.731	2.835	6.84×10^2
^{198}Pt	1.049	4.515	3.27×10^4
^{124}Sn	2.291	2.236	1.72×10^2
^{244}Pu	1.35	3.388	2.44×10^3

Nuclear-level diagram for the $\beta^- \beta^-$ transition of ^{100}Mo to ground and first excited 0^+ states



Best current results for 2νββ decay to first excited 0^+ and 2^+ states

Isotope	Excited state, $Q_{\beta\beta}{}^X$	Experimental half-life (years)	Theoretical half-life (years)
^{48}Ca	0_1^+, 1,275 keV 2_1^+, 3,289 keV	$>1.5 \times 10^{20}$ (63) $>1.8 \times 10^{20}$ (63)	N/A 1.7×10^{24} (68)
^{76}Ge	0_1^+, 917 keV 2_1^+, 1,480 keV	$>6.2 \times 10^{21}$ (64) $>1.1 \times 10^{21}$ (65)	$(7.5–310) \times 10^{21}$ (41, 66) 4.5×10^{21} (67) 5.8×10^{28} (68) $(7.8–10) \times 10^{25}$ (41, 66)
^{82}Se	0_1^+, 1,506 keV 2_1^+, 2,219 keV	$>3.0 \times 10^{21}$ (69) $>1.4 \times 10^{21}$ (69)	$(1.5–3.3) \times 10^{21}$ (41, 66) $(2.8–3,300) \times 10^{23}$ (41, 66)
^{96}Zr	0_1^+, 2,203 keV 2_1^+, 2,572 keV	$>6.8 \times 10^{19}$ (70) $>7.9 \times 10^{19}$ (70)	$(2.4–2.7) \times 10^{21}$ (41, 66) 3.8×10^{21} (67) 2.3×10^{25} (68) $(3.8–4.8) \times 10^{21}$ (41, 66)
^{100}Mo	0_1^+, 1,904 keV 2_1^+, 2,495 keV	$6.1^{+1.8}_{-1.1} \times 10^{20}$ (43) $(9.3^{+2.8}_{-1.7} \pm 1.4) \times 10^{20}$ (71) $(6.0^{+1.9}_{-1.1} \pm 0.6) \times 10^{20}$ (72, 73) $(5.7^{+1.3}_{-0.9} \pm 0.8) \times 10^{20}$ (44) $>1.6 \times 10^{21}$ (43)	1.6×10^{21} (74) 2.1×10^{21} (67) 1.2×10^{25}(68) 3.4×10^{22}(67)
^{116}Cd	0_1^+, 1,048 keV 2_1^+, 1,512 keV	$>2.0 \times 10^{21}$ (75) $>2.3 \times 10^{21}$ (75)	1.1×10^{22} (41, 66) 1.1×10^{21} (67) 3.4×10^{26} (68) 1.1×10^{24} (41, 66)
^{130}Te	0_1^+, 735 keV 2_1^+, 1,993 keV	$>2.3 \times 10^{21}$ (76) $>2.8 \times 10^{21}$ (77)	$(5.1–14) \times 10^{22}$ (41, 66, 76) 6.9×10^{26} (68) $(3–27) \times 10^{22}$ (41, 66)
^{150}Nd	0_1^+, 2,627 keV 2_1^+, 3,034 keV	$[1.33^{+0.36}_{-0.23}\,(\mathrm{stat})^{+0.27}_{-0.13}\,(\mathrm{syst})] \times 10^{20}$ (78) $>2.2 \times 10^{20}$ (78)	N/A N/A

Two positive measurements

R. Saakyan, Annu. Rev. Nucl. Part. Sci. 63, 503 (2013)

Tab. 4: Calculations for double-β decays from the ground state of parent nuclei to the first 0^+ excited states of daughter nuclei

Nucl.	$T_{1/2}$(expt) (Ey)	$Q_{2\beta}$ (MeV)	$T_{1/2}$(theor) (Ey)	$T_{1/2}$(other1) (Ey) [39,40,42,43]	$T_{1/2}$(other2) (Ey) [41]
^{48}Ca		1.275	1.63×10^6		
^{76}Ge		0.917	1.02×10^6	$(7.5\text{-}310) \times 10^3$	4.5×10^3
^{82}Se		1.506	4.21×10^3	$(1.5\text{-}3.3) \times 10^3$	
^{96}Zr		2.203	2.59×10^2	$(24\text{-}27) \times 10^2$	38×10^2
^{100}Mo	590(80)	1.904	589	16×10^2	21×10^2
^{116}Cd		1.048	8.36×10^4	1.1×10^4	0.11×10^4
^{130}Te		0.735	8.38×10^6	$(5.1\text{-}14) \times 10^4$	
^{150}Nd	133(45)	2.627	77.6		

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Our result agrees with new data of ^{150}Nd（2014）

PHYSICAL REVIEW C **90**, 055501 (2014)

Two-neutrino double-β decay of ^{150}Nd to excited final states in ^{150}Sm

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(Received 29 May 2014; revised manuscript received 9 September 2014; published 19 November 2014)

Background: Double-β decay is a rare nuclear process in which two neutrons in the nucleus are converted to two protons with the emission of two electrons and two electron antineutrinos.
Purpose: We measured the half-life of the two-neutrino double-β decay of ^{150}Nd to excited final states of ^{150}Sm by detecting the deexcitation γ rays of the daughter nucleus.
Method: This study yields the first detection of the coincidence γ rays from the 0_1^+ excited state of ^{150}Sm. These γ rays have energies of 333.97 and 406.52 keV and are emitted in coincidence through a $0_1^+ \to 2_1^+ \to 0_{gs}^+$ transition.
Results: The enriched Nd_2O_3 sample consisted of 40.13 g ^{150}Nd and was observed for 642.8 days at the Kimballton Underground Research Facility, producing 21.6 net events in the region of interest. This count rate gives a half-life of $T_{1/2} = [1.07^{+0.45}_{-0.25}(\text{stat}) \pm 0.07(\text{syst})] \times 10^{20}$ yr. The effective nuclear matrix element was found

PHYSICAL REVIEW C **90**, 055501 (2014)

Two-neutrino double-β decay of ^{150}Nd to excited final states in ^{150}Sm

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(Received 29 May 2014; revised manuscript received 9 September 2014; published 19 November 2014)

gives a half-life of $T_{1/2} = [1.07^{+0.45}_{-0.25}(\mathrm{stat}) \pm 0.07(\mathrm{syst})] \times 10^{20}$ yr.

The recent systematic analysis of Ren and Ren [21] gives the value $T_{1/2} = 0.776 \times 10^{20}$ yr, in closer agreement with our result than that of Ref. [5]. The observation of double-β decay to an excited final state has only been previously observed in one other nucleus, ^{100}Mo (Ref. [14] and references therein).

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New Geiger-Nuttall law for α decay: PRC 85, 044608 (2012)

PHYSICAL REVIEW C **85**, 044608 (2012)

New Geiger-Nuttall law for α decay of heavy nuclei

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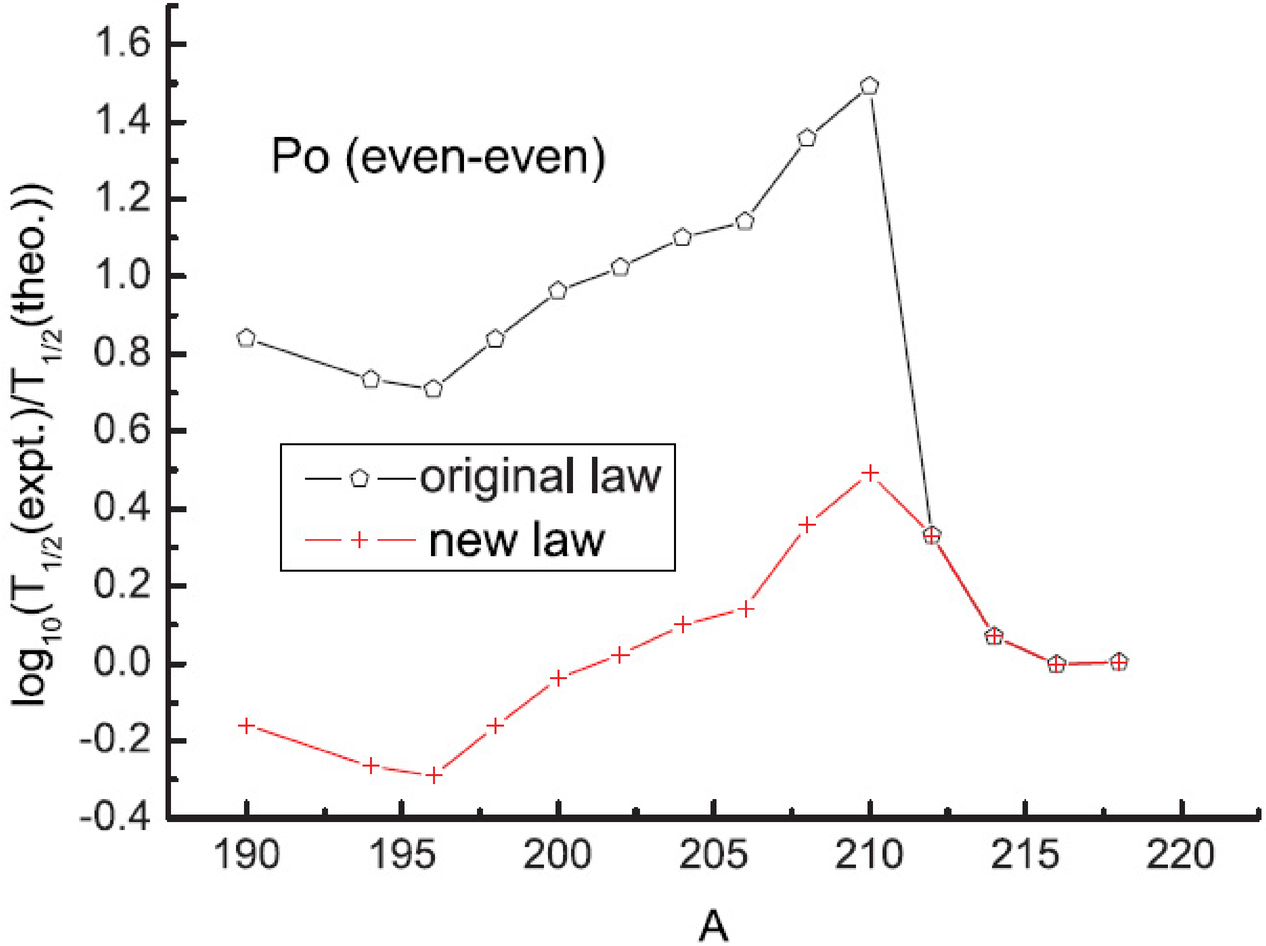
Recent α-decay data of heavy nuclei are collected and systematic analysis shows that there is a sudden change between the logarithm of decay half-life and the reciprocal of the square root of decay energy across the $N = 126$ shell closure. In order to reproduce this sudden change, the new Geiger-Nuttall law is proposed where the effects of the quantum numbers of α-core relative motion are naturally embedded in the law. The remedy achieved by a very simple parametrization of these effects is remarkable. By adding terms to the Geiger-Nuttall law, the parameters in the formula of decay half-lives need not be changed, except for some odd nuclei. This is an important development to the original Geiger-Nuttall law, which is valid for the ground-state transitions of even-even nuclei with $N \geqslant 128$. The law is generalized to the favored and hindered transitions of the $N \leqslant 128$ nuclei and of high-spin isomers. The results of this article point to the simplicity of the underlying mechanism of the decay.

$$\log_{10} T_{1/2} = a\sqrt{\mu} Z_c Z_d \big/ \sqrt{Q} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S + P\ell(\ell+1)$$

Effects of quantum numbers on α-decay data (shell effects):
S=0 for N>126 and S=1 for N<=126

Ratios between experiment and theory for even-even Po nuclei with the original law and with the new law, showing the reduction of a sudden change across the N=126 shell closure.



Comparison between α decay and double-β decay

Common points:

1. Both of them are natural phenomena and they also obey the same exponential decay law.
2. They obey the laws of quantum mechanics and quantum filed theory .
3. They often occur for ground-state transitions of even-even nuclei. The changes of quantum numbers between parent and daughter nuclei are very similar, 0^+.

Different points:

The long-range Coulomb repulsive potential among protons leads to the appearance of α decay, while the very short-range weak interactions among nucleons leads to double-β decay.

α decay law VS double-β decay law

$$\log_{10} T_{1/2}^{\alpha}\ (\mathrm{s}) = a\sqrt{\mu} Z_c Z_d \Big/ \sqrt{Q_\alpha} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S + P\ell(\ell+1)$$

$$\log_{10} T_{1/2}^{2\beta}(\mathrm{Ey}) = \left[a - 2\log_{10}(2\pi Z / 137) + S\right] \Big/ Q_{2\beta}$$

A common point is that their half-lives are sensitive to the decay energies and affected by the shell effect.

The first term in the new Geiger-Nuttall law is dependent on both charge numbers and decay energies because the total effect from the Coulomb potential is related to charge numbers. The first term in the systematic law for double-β-decay half-lives is only dependent on decay energies because the weak interaction is universal for natural decay processes and the total effect from the weak interaction is not very sensitive to the change of nucleon numbers.

α decay law VS double-β decay law

Another important difference is from the difference of the perturbation approximation in quantum mechanics.

For the new Geiger-Nuttall law, α decay is a first-order process of the electromagnetic interaction and there are significant influences from the strong interactions.

Double-β decay is a second-order process of the weak interaction with the V-A four-fermion theory where a single β decay is forbidden in many double-β emitters.

Double-β^+ decay： Science China 58 (2015) 012002

$\beta^+\beta^+$ transitions are suppressed compared with $\beta^-\beta^-$ decay due to their smaller phase-space factors. So far, β^+ β^+ processes have NOT been observed in a direct experiment.

Theoretically, there are many successful calculations on double-β^- decay. But calculations on double-β^+ decay are much less.

The 2012 nuclear mass table suggests 40 possible double-β^+ emitters, ranging from A=36 (^{36}Ar) to A=252 (^{252}Fm). Our evaluation of $Q_{\beta\beta}$ shows that many of them have negative decay energy or approximately zero decay energy.
Only eight of them have significantly positive energies for double-β^+ decay.

Tab. 5: Decay energy, isotopic abundance (IS) of parent nuclei (or their α-decay half-lives T^{α}) for eight double-β^{+} candidates

Parent	Daughter	M(A,Z) (MeV)	M(A,Z-2) (MeV)	$Q_{2\beta}$ (MeV)	IS or T^{α}
^{78}Kr	^{78}Se	-74.180	-77.026	0.802	IS=0.355%
^{96}Ru	^{96}Mo	-86.079	-88.794	0.671	IS=5.54%
^{106}Cd	^{106}Pd	-87.132	-89.907	0.731	IS=1.25%
^{124}Xe	^{124}Te	-87.661	-90.525	0.820	IS=0.095%
^{130}Ba	^{130}Xe	-87.262	-89.880	0.574	IS=0.106%
^{136}Ce	^{136}Ba	-86.509	-88.887	0.334	IS=0.185%
^{148}Gd	^{148}Sm	-76.269	-79.336	1.023	T^{α}=70.9 y
^{154}Dy	^{154}Gd	-70.394	-73.705	1.267	T^{α}=3.0 My

The effect from the Coulomb potential

$$\rho(Z,\varepsilon) = 2\pi\eta \big/ (1-e^{-2\pi\eta}),\ \text{with}\ \eta = (-Z/137)(\varepsilon/cp)$$

A correction to double-β⁺-decay half-lives $\log_{10}T_{1/2}$

$$+2\log_{10}\left[(2\pi Z)/137\right]$$

The leading effect of nuclear structure (shell effect) is simulated by introducing an addition quantity *S*:

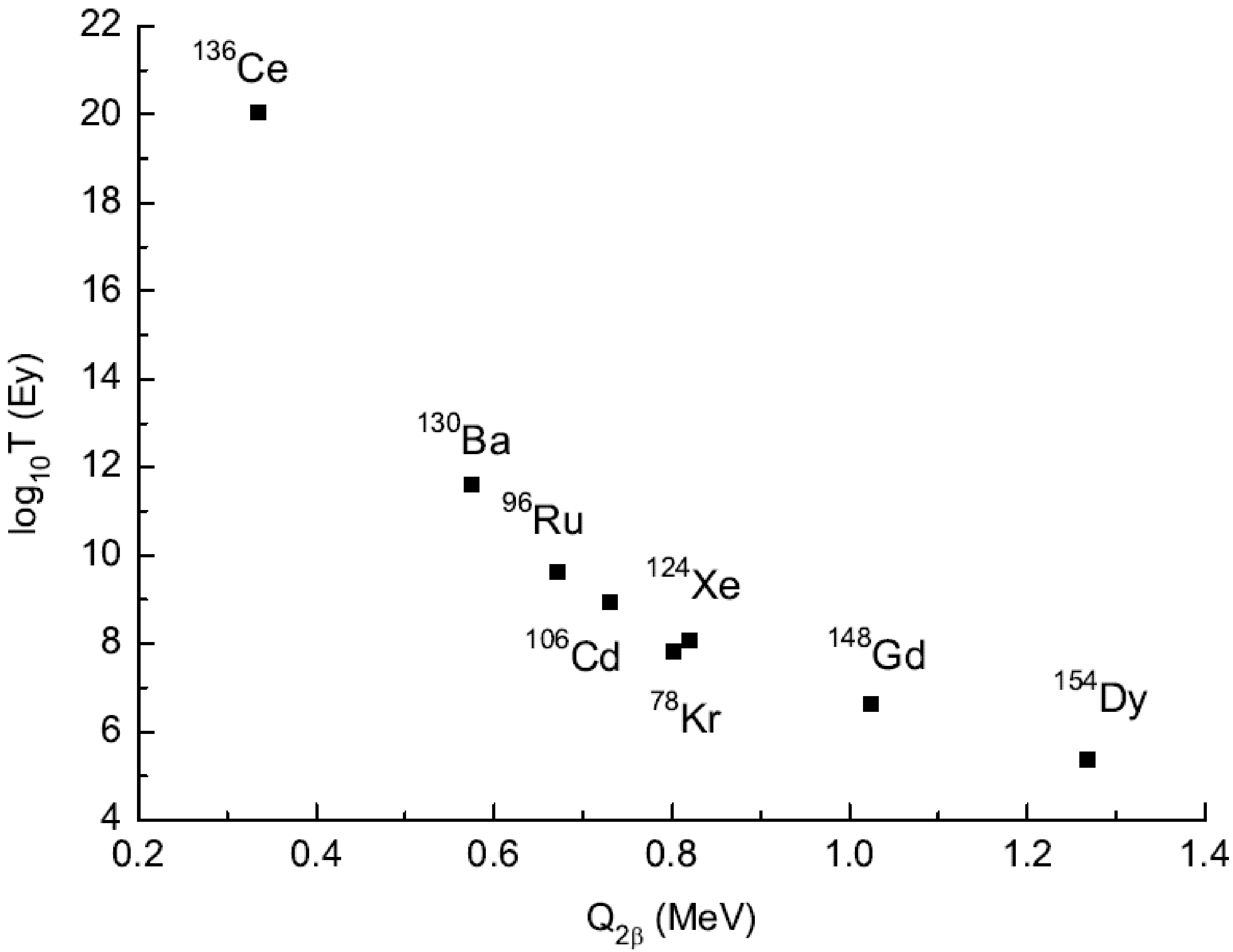
The proton number of the eight candidates are all not magic.

Therefore *S*=0 is adopted for double-β⁺ decay.

$$\log_{10} T_{1/2}(\text{Ey}) = \left[a + 2\log_{10}(2\pi Z/137)\right] \big/ Q_{2\beta}\ (\text{MeV}),$$

the parameter a is kept the same value as double-β^- decay.

Logarithms of the theoretical double-β^+-decay half-lives for ground-state transitions of eight even-even nuclei



Tab. 6: Calculated half-lives for double-β^+ decay of even-even isotopes, compared with the other theoretical results [A,B,C]

Nucl.	$Q_{2\beta}$ (MeV)	$T_{1/2}$(theor) (Ey)	$T_{1/2}$(other1) (Ey)	$T_{1/2}$(other2) (Ey)
^{78}Kr	0.802	6.73×10^{7}	$(4.94\text{-}15.8) \times 10^{7}$ [A]	1.93×10^{8} [C]
^{96}Ru	0.671	4.13×10^{9}	$(1.2\text{-}10) \times 10^{8}$ [B]	5.31×10^{8} [C]
^{106}Cd	0.731	8.51×10^{8}		4.94×10^{7} [C]
^{124}Xe	0.820	1.22×10^{9}		8.17×10^{7} [C]
^{130}Ba	0.574	4.04×10^{11}		1.37×10^{11} [C]
^{136}Ce	0.334	1.09×10^{20}		4.51×10^{13} [C]
^{148}Gd	1.023	4.22×10^{6}		5.81×10^{8} [C]
^{154}Dy	1.267	2.35×10^{5}		

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Summary

- A systematic law for double-β decay is proposed including the effects of the decay energy, Coulomb potential and shell structure (the first formula for double-β decay).

- The law is generalized to $\beta^-\beta^-$ transitions from ground states to first 0^+ excited states. The results show good agreement with the experimental data.

- The law is extended to predict half-lives for $\beta^+\beta^+$ transitions between ground states (the first prediction for ^{154}Dy).

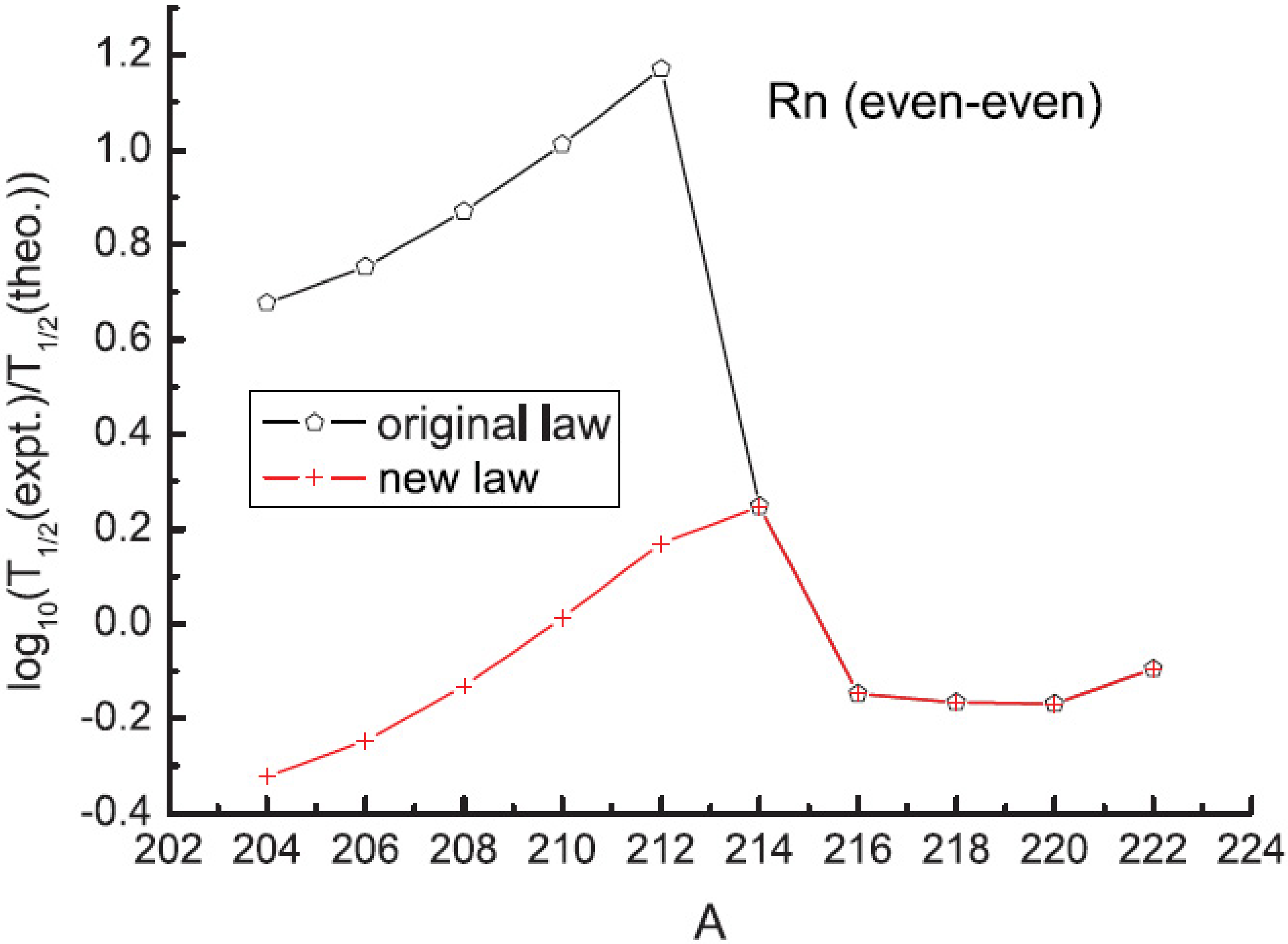
- There are common and different points between the systematic law for double-β-decay half-lives and the new Geiger-Nuttall law for α-decay half-lives.

谢谢 各位老师和同行多年来对我们工作的支持！

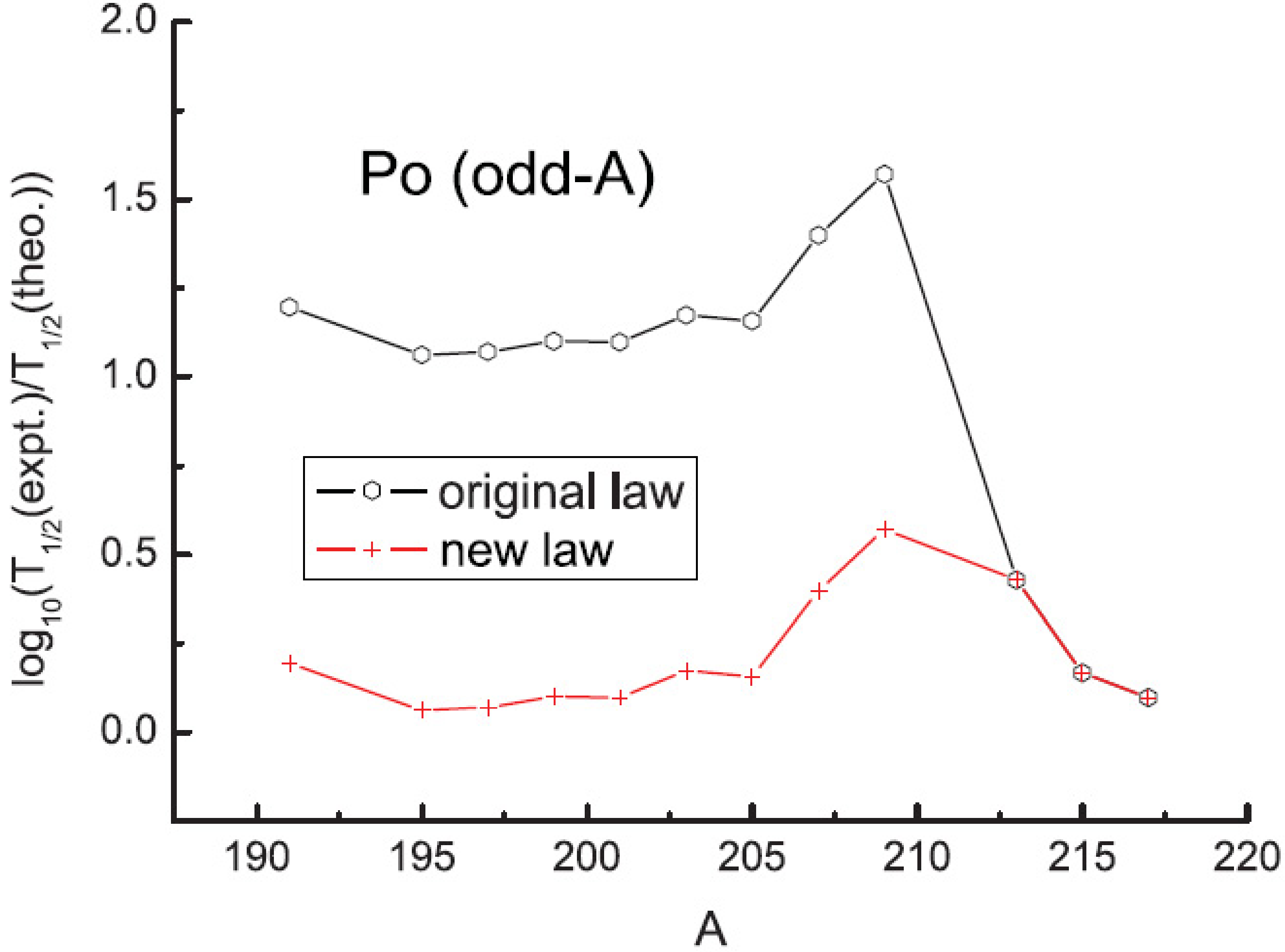
我们全家祝愿老师和长辈健康长寿！ 给大家拜年！

Thank you!

Ratios between experimental data and theoretical results for Rn nuclei with the original law and with the new law (PRC, 2012)



Ratios between experimental data and theoretical results for odd-A Po nuclei with original law and with new law (PRC, 2012)



Main positive results for 2νββ decay to ground states

Isotope	Experiment (type)	$T_{1/2}(2\nu)$ (years)
^{48}Ca	Hoover Dam (TPC) TGV (planar HPGe) NEMO3 (track calorimeter)	$[4.3^{+2.4}_{-1.1}\,(\text{stat.}) \pm 1.4\,(\text{syst.})] \times 10^{19}$ $(4.2^{+3.3}_{-1.3}) \times 10^{19}$ $(4.4 \pm 0.64) \times 10^{19}$
^{76}Ge	IGEX (HPGe) Heidelberg–Moscow (HPGe) GERDA (HPGe)	$(1.45 \pm 0.15) \times 10^{21}$ $[1.74 \pm 0.01\,(\text{stat.})^{+0.18}_{-0.16}\,(\text{syst.})] \times 10^{21}$ $1.84^{+0.14}_{-0.10} \times 10^{21}$
^{82}Se	Geochemistry NEMO3 (track calorimeter)	$(1.3 \pm 0.05) \times 10^{20}$ $[0.96 \pm 0.03\,(\text{stat.}) \pm 0.1\,(\text{syst.})] \times 10^{20}$
^{96}Zr	Geochemistry Geochemistry NEMO3 (track calorimeter)	$(3.9 \pm 0.9) \times 10^{19}$ $(0.94 \pm 0.32) \times 10^{19}$ $[2.35 \pm 0.14\,(\text{stat.}) \pm 0.16\,(\text{syst.})] \times 10^{19}$
^{100}Mo	Geochemistry Hoover Dam (TPC) DBA (liquid argon TPC) NEMO3 (track calorimeter)	$(2.1 \pm 0.3) \times 10^{18}$ $[6.82^{+0.38}_{-0.53}\,(\text{stat.}) \pm 0.68\,(\text{syst.})] \times 10^{18}$ $[7.2 \pm 1.1\,(\text{stat.}) \pm 1.8\,(\text{syst.})] \times 10^{18}$ $[7.17 \pm 0.01\,(\text{stat.}) \pm 0.54\,(\text{syst.})] \times 10^{18}$
^{116}Cd	Solotvina (scintillator) NEMO3 (track calorimeter)	$[2.9 \pm 0.06\,(\text{stat.})^{+0.4}_{-0.3}\,(\text{syst.})] \times 10^{19}$ $(2.88 \pm 0.17) \times 10^{19}$
^{128}Te	Geochemistry	$\sim 2.2 \times 10^{24}$, $(7.7 \pm 0.4) \times 10^{24}$
^{130}Te	Geochemistry MiBETA (bolometer) NEMO3 (track calorimeter)	$\sim 0.8 \times 10^{21}$, $(2.7 \pm 0.1) \times 10^{21}$ $[6.1 \pm 1.4\,(\text{stat.})^{+2.9}_{-3.5}\,(\text{syst.})] \times 10^{20}$ $[7.0 \pm 0.9\,(\text{stat.}) \pm 1.1\,(\text{syst.})] \times 10^{20}$
^{136}Xe	EXO-200 (LXe TPC) KamLAND–Zen	$[2.11 \pm 0.04\,(\text{stat.}) \pm 0.21\,(\text{syst.})] \times 10^{21}$ $[2.38 \pm 0.02\,(\text{stat.}) \pm 0.14\,(\text{syst.})] \times 10^{21}$
^{150}Nd	Hoover Dam (TPC) NEMO3 (track calorimeter)	$[6.75^{+0.37}_{-0.42}\,(\text{stat.}) \pm 0.68\,(\text{syst.})] \times 10^{18}$ $[9.11^{+0.25}_{-0.22}\,(\text{stat.}) \pm 0.63\,(\text{syst.})] \times 10^{18}$
^{238}U	Radiochemistry	$(2.0 \pm 0.6) \times 10^{21}$
^{130}BaECEC (2ν)	Geochemistry	$(2.2 \pm 0.5) \times 10^{21}$

R. Saakyan, Annu. Rev. Nucl. Part. Sci. 63, 503 (2013)

Shell model with an effective interaction for double-β decay

PRL **110**, 222502 (2013) PHYSICAL REVIEW LETTERS week ending 31 MAY 2013

Shell-Model Analysis of the ^{136}Xe Double Beta Decay Nuclear Matrix Elements

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(Received 24 December 2012; revised manuscript received 18 March 2013; published 30 May 2013)

Neutrinoless double beta decay, if observed, could distinguish whether the neutrino is a Dirac or a Majorana particle, and it could be used to determine the absolute scale of the neutrino masses. ^{136}Xe is one of the most promising candidates for observing this rare event. However, until recently there were no positive results for the allowed and less rare two-neutrino double beta decay mode. The small nuclear matrix element associated with the long half-life represents a challenge for nuclear structure models used for its calculation. We report a new shell-model analysis of the two-neutrino double beta decay of ^{136}Xe, which takes into account all relevant nuclear orbitals necessary to fully describe the associated Gamow-Teller strength. We further use the new model to analyze the main contributions to the neutrinoless double beta decay matrix element, and show that they are also diminished.

Quasiparticle random-phase approximation for double-β decay

PHYSICAL REVIEW C **87**, 064302 (2013)

Large-scale calculations of the double-β decay of ^{76}Ge, ^{130}Te, ^{136}Xe, and ^{150}Nd in the deformed self-consistent Skyrme quasiparticle random-phase approximation

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(Received 29 January 2013; revised manuscript received 8 May 2013; published 5 June 2013)

We use the axially deformed Skyrme quasiparticle random-phase approximation (QRPA) together with the SkM* energy-density functional, both as originally presented and with the time-odd part adjusted to reproduce the Gamow-Teller resonance energy in ^{208}Pb, to calculate the matrix elements that govern the neutrinoless double-β decay of ^{76}Ge, ^{130}Te, ^{136}Xe, and ^{150}Nd. Our matrix elements in ^{130}Te and ^{136}Xe are significantly smaller than those of previous QRPA calculations, primarily because of the difference in pairing or deformation between the initial and the final nuclei. In ^{76}Ge and ^{150}Nd, our results are similar to those of less computationally intensive QRPA calculations. We suspect the ^{76}Ge result, however, because we are forced to use a spherical ground state, even though our mean-field theory indicates a deformed minimum.

2013 Tom W. Bonner Prize of APS was awarded to M. K. Moe for his leadership in the first observation of rare process of two-neutrino double beta decay.

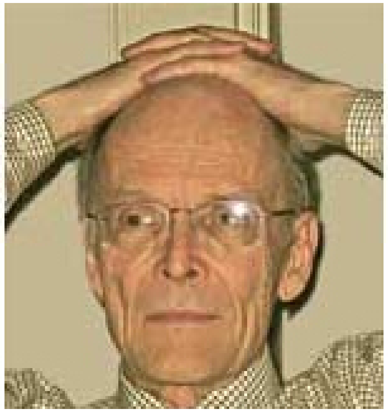
2013 Tom W. Bonner Prize in Nuclear Physics Recipient

Michael K. Moe
University of California, Irvine

Citation:

"For his leadership in the first observation of the rare process of two neutrino double beta decay, where his creative contributions were instrumental to its successful detection and transformed the field."

Background:



In 1966, a preprint sent by C. S. Wu sparked his interest in double beta decay. Moe designed a time projection chamber (TPC) for double beta decay, and developed it with Steve Elliott and Alan Hahn to finally see the first solid evidence of two-neutrino decay in ^{82}Se in 1987.

QRPA calculations: Phys. Rev. C 87 (2013) 045501

PHYSICAL REVIEW C **87**, 045501 (2013)

$0\nu\beta\beta$ and $2\nu\beta\beta$ nuclear matrix elements, quasiparticle random-phase approximation, and isospin symmetry restoration

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Within the quasiparticle random-phase approximation (QRPA) we achieve partial restoration of the isospin symmetry and hence fulfillment of the requirement that the $2\nu\beta\beta$ Fermi matrix element $M_F^{2\nu}$ vanishes, as it should, unlike in the previous version of the method. This is accomplished by separating the renormalization parameter g_{pp} of the particle-particle proton-neutron interaction into isovector and isoscalar parts. The isovector parameter $g_{pp}^{T=1}$ needs to be chosen to be essentially equal to the pairing constant g_{pair}, so no new parameter is needed. For the $0\nu\beta\beta$ decay the Fermi matrix element $M_F^{0\nu}$ is substantially reduced, while the full matrix element $M^{0\nu}$ is reduced by $\approx$10%. We argue that this more consistent approach should be used from now on in the proton-neutron QRPA and in analogous methods.