



Heavy quark molecules and heavy quark symmetry

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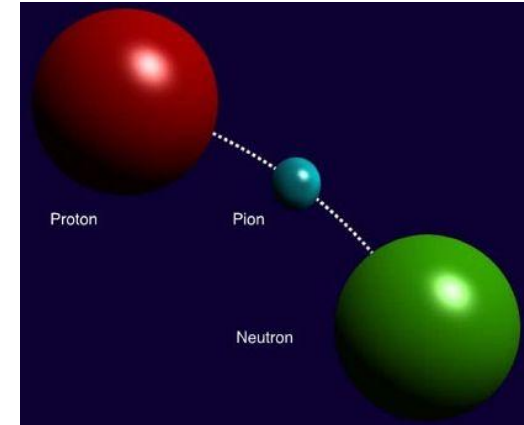
Constraints with heavy quark symmetry

- (I): $Q\bar{Q}$ spin in meson-antimeson molecules
- (II): Strong decays
- (III): Coupled channel effects for exotic JPC

Hadronic bound state: Deuteron

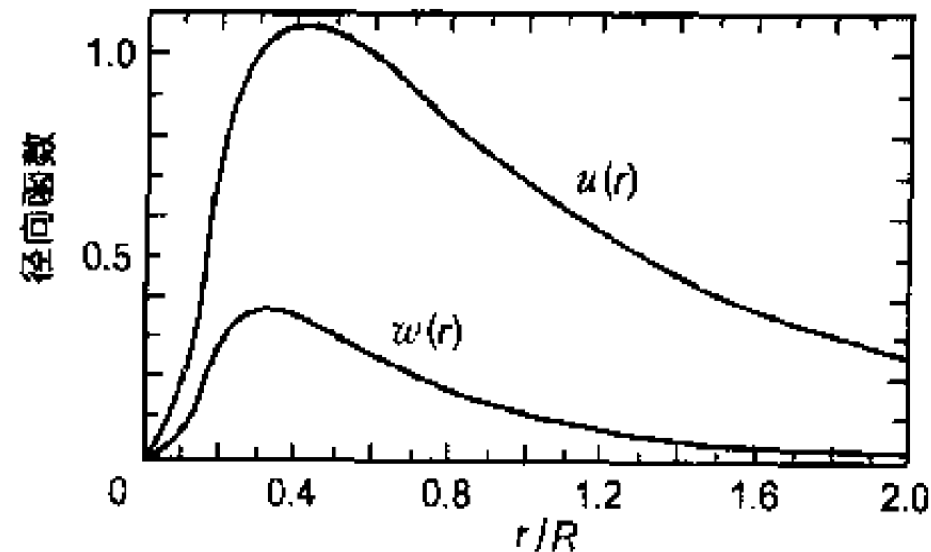
- Shallow NN bound state: $\sim(pn-np)$

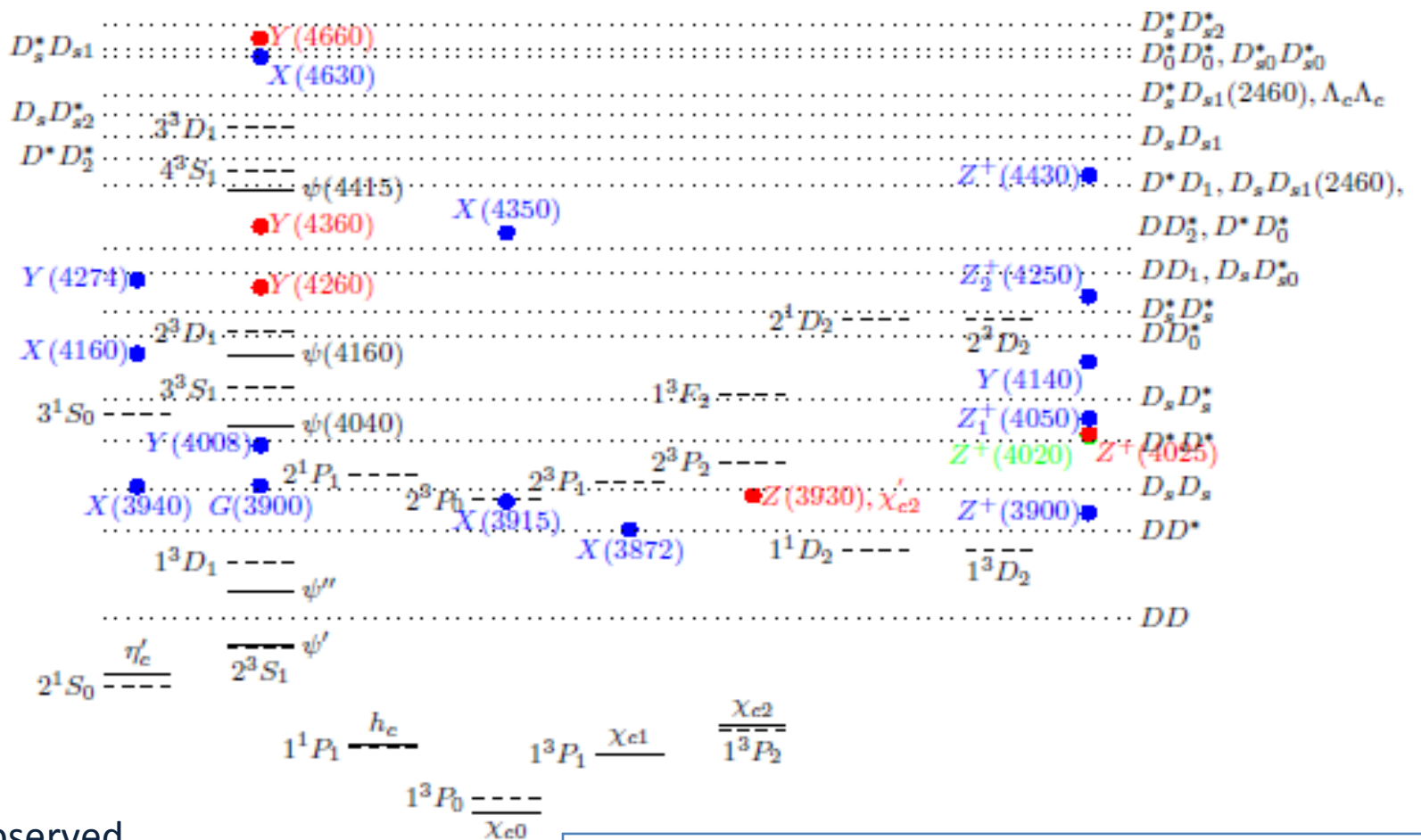
$$l=0, J=1 \quad \vec{J} = \vec{L} + \vec{S}$$



$$(L=0,2; \quad S=1)$$

- B.E. ≈ 2.2 MeV
- S-wave: 96%;
- D-wave: 4%
- Tensor force important

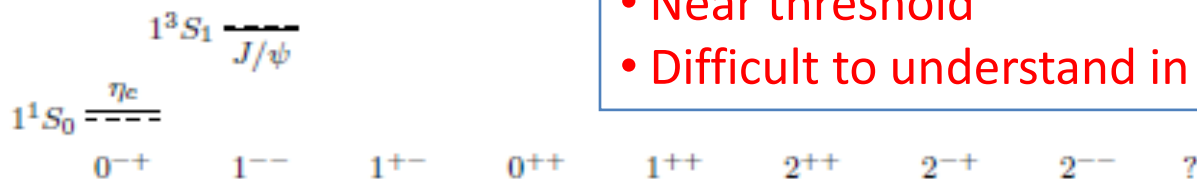




Solid: observed
 Dashed: QM
 Dots: unexpected

Unexpected states:

- Above DD threshold
- Near threshold
- Difficult to understand in QM



Charged exotic mesons

$Z^+(4430)$	4443_{-18}^{+24}	107_{-71}^{+113}	?	$B \rightarrow K(\pi^+\psi(2S))$	Belle[PRL100-14] BaBar[PRD79-1]
				$B \rightarrow K(\pi^+J/\psi)$	Belle, BaBar
				$\bar{B}^0 \rightarrow J/\psi K^- \pi^+$	Belle[1408.6457]
	$4475 \pm 7_{-25}^{+15}$	$172 \pm 13_{-34}^{+37}$	1^+	$B^0 \rightarrow \psi' K^+ \pi^-$	LHCb[1404.1903]
$Z_1^+(4050)$	4051_{-43}^{+24}	82_{-55}^{+51}	?	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	Belle[PRD78-07] BaBar[PRD85-0]
$Z_2^+(4250)$	4248_{-45}^{+185}	177_{-72}^{+821}	?	$B \rightarrow K(\pi^+\chi_{c1}(1P))$	Belle[PRD78-07] BaBar[PRD85-0]
$Z_c(3900)^+$	$3899.0 \pm 3.6 \pm 4.9$	$46 \pm 10 \pm 20$?	$e^+e^- \rightarrow \pi^-(\pi^+J/\psi)$	BES[PRL110,25]
	$3894.5 \pm 6.6 \pm 4.5$	$63 \pm 24 \pm 26$?	$e^+e^- \rightarrow \pi^-(\pi^+J/\psi)$	Belle[PRL110,25]
	$3885 \pm 5 \pm 1$	$34 \pm 12 \pm 4$?	$e^+e^- \rightarrow \pi^+\pi^-J/\psi$	Cleo-c data[1304.1896]
				$e^+e^- \rightarrow \pi^+\pi^-h_c$	BES[1309.1896]
$Z_c(3885)^+$	$3883.9 \pm 1.5 \pm 4.2$	$24.8 \pm 3.3 \pm 11.0$	1^+	$e^+e^- \rightarrow (D\bar{D}^*)^\pm \pi^\mp$	BES[1310.1163]
$Z_c(4020)^+$	$4022.9 \pm 0.8 \pm 2.7$	$7.9 \pm 2.7 \pm 2.6$?	$e^+e^- \rightarrow \pi^\mp(\pi^\pm h_c)$	BES[1309.1896]
	$4023.9 \pm 2.2 \pm 3.8$	Same	?	$e^+e^- \rightarrow \pi^0(\pi^0 h_c)$	BES[1409.6577]
$Z_c(4025)$	$4026.3 \pm 2.6 \pm 3.7$	$24.8 \pm 5.6 \pm 7.7$?	$e^+e^- \rightarrow (D^*\bar{D}^*)^\pm \pi^\mp$	BES[1308.2760]
$Z_c(4200)$	4196_{-29-13}^{+31+17}	$370_{-70-132}^{+70+70}$	$1^+(?)$	$\bar{B}^0 \rightarrow J/\psi K^- \pi^+$	Belle[1408.6457]

- $Z_b(10610), Z_b(10650):$

$$N_{\text{quark}} \geq 4$$

$$\pi^\pm Y(nS) \quad \pi^\pm h_b(mP)$$

PRL 108, 122001 (2012) [Belle]

Heavy quark symmetry

- Heavy quark molecules (hadronic bound states)?
- **Heavy quark symmetry** (HQS) important in studying internal structures & interactions
- Discuss results constrained with HQS

Heavy quark symmetry

- For $Q\bar{q}$ mesons

(1) spin doublets

Degenerate doublets \Rightarrow

$$\begin{aligned}
 H_{j\ell=\frac{1}{2}}^{L=0} &= \begin{pmatrix} (\bar{q}Q)_{JP=0^-} \\ (\bar{q}Q)_{JP=1^-} \end{pmatrix}, & m_D &= m_{D^*}, \\
 H_{j\ell=\frac{1}{2}}^{L=1} &= \begin{pmatrix} (\bar{q}Q)_{JP=0^+} \\ (\bar{q}Q)_{JP=1^+} \end{pmatrix}, & m_{D_0} &= m_{D'_1}, \\
 H_{j\ell=\frac{3}{2}}^{L=1} &= \begin{pmatrix} (\bar{q}Q)_{JP=1^+} \\ (\bar{q}Q)_{JP=2^+} \end{pmatrix}, & m_{D_1} &= m_{D_2}, \\
 & & & \dots
 \end{aligned}$$

(2) chiral multiplets

Degenerate before chiral symmetry breaking \Rightarrow

$$\begin{pmatrix} 0^- \\ 1^- \end{pmatrix} \xleftrightarrow{\text{Chiral partner}} \begin{pmatrix} 0^+ \\ 1^+ \end{pmatrix}$$

Heavy quark symmetry

- For $Q\bar{Q}$ mesons, $m^{2S+1}L_J$. Only heavy quark spin symmetry

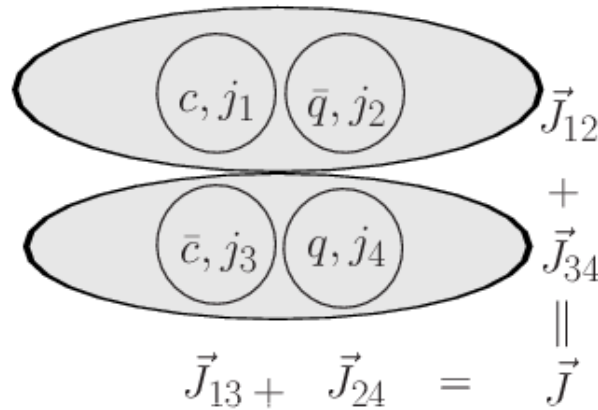
$$\begin{aligned}
 H_m^{L=0} &= \left(\begin{array}{l} (Q\bar{Q})_{S=0, J^{PC}=0^{-+}} \\ (Q\bar{Q})_{S=1, J^{PC}=1^{--}} \end{array} \right), & m_{\eta_c} &= m_{J/\psi}, \\
 H_m^{L=1} &= \left(\begin{array}{l} (Q\bar{Q})_{S=0, J^{PC}=1^{+-}} \\ (Q\bar{Q})_{S=1, J^{PC}=0^{++}} \\ (Q\bar{Q})_{S=1, J^{PC}=1^{++}} \\ (Q\bar{Q})_{S=1, J^{PC}=2^{++}} \end{array} \right), & m_{h_c} &= m_{\chi_{c0}} = m_{\chi_{c1}} = m_{\chi_{c2}}, \\
 & & & \dots
 \end{aligned}$$

$S_{Q\bar{Q}}$ conserved

Casalbuoni et al, PLB302,95 (1993)

(I): $Q\bar{Q}$ spin in meson-antimeson molecules

Recoupling:



- S-wave $A\bar{B} \pm B\bar{A}$: only $S_{c\bar{c}} = 1$

(1) $J = \max(J_\ell) + 1$ or $J = \min(J_\ell) - 1$

(2) $J^C = 1^+, 2^-, 3^+, \dots$ if (A,B) form a doublet

YRL, PRD.88,074008(2013)

(I): $Q\bar{Q}$ spin in meson-antimeson molecules

- Hosaka et al in [PRD 86, 117502 \(2012\)](#) :
P-wave $B\bar{B}^*$ with $J^{PC} = J^{--}$, only $S_{c\bar{c}} = 1$
- L-wave case:

$$\chi_J = \frac{1}{\sqrt{2}} \left\{ (j_1 \bar{j}_2)_{J_{12}} (\bar{j}_3 j_4)_{J_{34}} + C_X (-1)^{J - J_{12} - J_{34}} (j_3 \bar{j}_4)_{J_{34}} (\bar{j}_1 j_2)_{J_{12}} \right\}$$



$$\chi_J = \frac{1}{\sqrt{2}} \left\{ (j_1 \bar{j}_2)_{J_{12}} (\bar{j}_3 j_4)_{J_{34}} + C_X (-1)^{L+S - J_{12} - J_{34}} (j_3 \bar{j}_4)_{J_{34}} (\bar{j}_1 j_2)_{J_{12}} \right\}$$

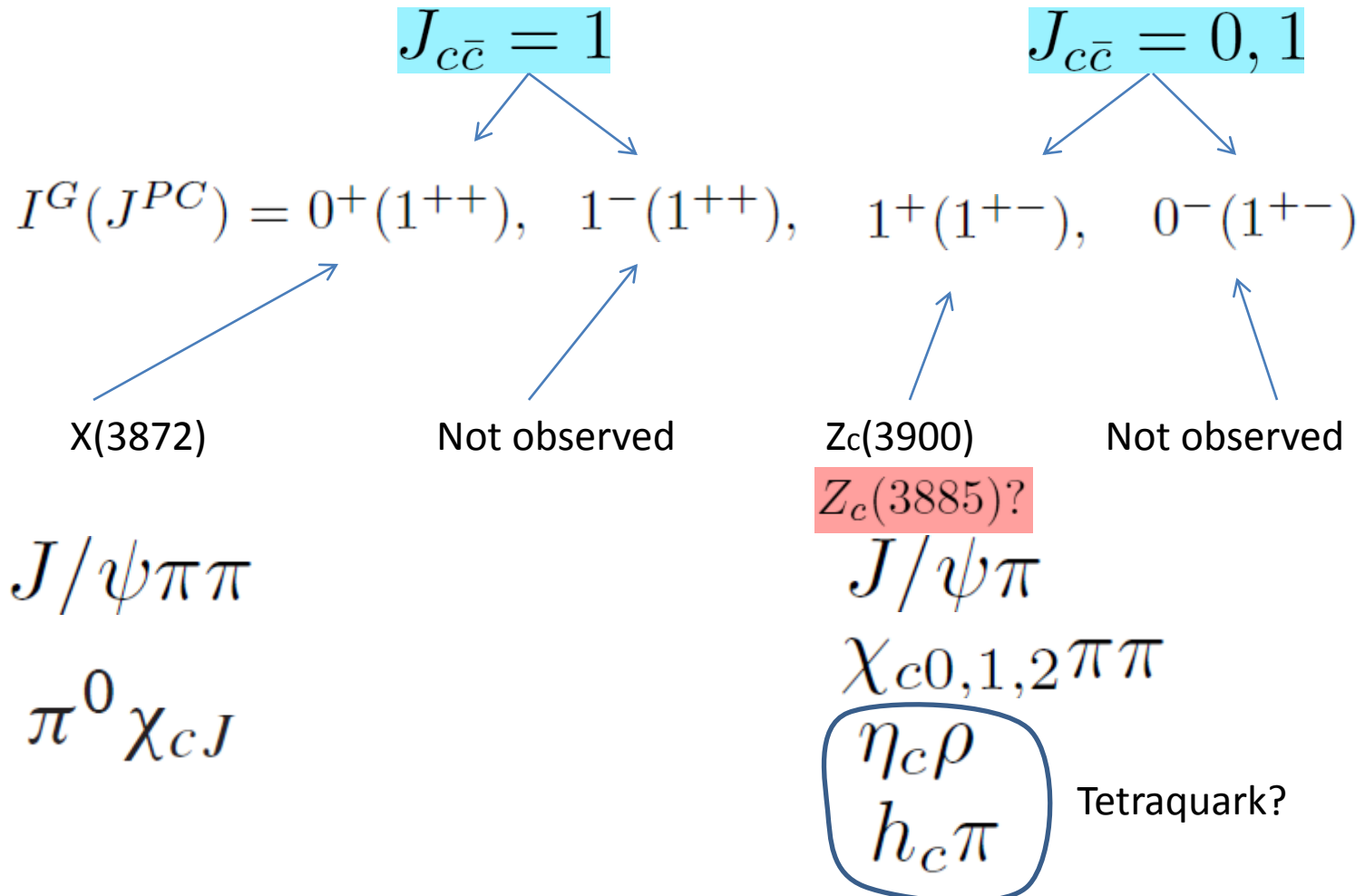
\Rightarrow For $B\bar{B}^*$, P-wave $J^{PC} = J^{--}$ and S-wave $J^{PC} = 1^{++}$:
the same spin selection rule

Consistent results!

Selection rules & strong decays

for $D\bar{D}^*$ $S_{Q\bar{Q}}$ conserved

State	J^C	J_{24}	Selection rule for $J_{c\bar{c}} \neq 0$
$D\bar{D}^*/D_0^*\bar{D}'_1$	1^\pm	0, 1	$J^C = 1^+$
$D^*\bar{D}^*/D'_1\bar{D}'_1$	$0^+, 1^-, 2^+$	0, 1	$J = 2$
$D^*\bar{D}'_1$	$0^\pm, 1^\pm, 2^\pm$	0, 1	$J = 2$
$D^*\bar{D}_1/D'_1\bar{D}_1$	$0^\pm, 1^\pm, 2^\pm$	1, 2	$J = 0$
$D^*\bar{D}_2^*/D'_1\bar{D}_2^*$	$1^\pm, 2^\pm, 3^\pm$	1, 2	$J = 3$
$D_1\bar{D}_2^*$	$1^\pm, 2^\pm, 3^\pm$	0 ~ 3	$J^C = 1^+, 2^-, 3^+$
$D_2^*\bar{D}_2^*$	$0^+, 1^-, 2^+, 3^-, 4^+$	0 ~ 3	$J = 4$



(II):

Strong decays

- Hidden charm decay: (1406.6879)
- Open charm decay:

$$\psi(nS) \rightarrow DD: DD^*: D^*D^* \Rightarrow 1:4:7$$

[Rujula, Georgi, Glashow, PRL 37,398 (76)]

$$\psi(nD) \rightarrow DD: DD^*: D^*D^* \Rightarrow 5:5:2(?)$$

$$\psi(nS) \rightarrow D_1D_1: D_1D_2: D_2D_2 \Rightarrow 1:7:4 (?)$$

...

(III):

HQSS constraints

- For interactions: EFT

$$\begin{aligned}\mathcal{L}_{PP^*} = & D_\mu P D^\mu P^\dagger - M_P^2 P P^\dagger + f_Q (P A^\mu P_\mu^{*\dagger} + P_\mu^* A^\mu P^\dagger) \\ & - \frac{1}{2} P^{*\mu\nu} P_{\mu\nu}^{*\dagger} + M_{P^*}^2 P^{*\mu} P_\mu^{*\dagger} \\ & + \frac{1}{2} g_Q \varepsilon_{\mu\nu\lambda\kappa} (P^{*\mu\nu} A^\lambda P^{*\kappa\dagger} + P^{*\kappa} A^\lambda P^{*\mu\nu\dagger}), \quad (2.20)\end{aligned}$$

T.M. Yan et al., PRD46,1148 (1992)

$$\text{HQSS} \Rightarrow g = \frac{1}{2} f$$

- With doublets

$$\begin{aligned}\mathcal{L} = & g \text{Tr}[H \not{A} \gamma_5 \bar{H}] + g' \text{Tr}[S \not{A} \gamma_5 \bar{S}] + g'' \text{Tr}[T_\mu \not{A} \gamma_5 \bar{T}^\mu] + [h \text{Tr}[S \not{A} \gamma_5 \bar{H}] + h.c.] \\ & + [h' \text{Tr}[T^\mu A_\mu \gamma_5 \bar{S}] + h.c.] + \frac{h_1}{\Lambda_\chi} \text{Tr}[T^\mu (D_\mu \not{A}) \gamma_5 \bar{H}] + h.c. \\ & + \frac{h_2}{\Lambda_\chi} \text{Tr}[T^\mu (\not{D} A_\mu) \gamma_5 \bar{H}] + h.c.\end{aligned}$$

$$\begin{aligned}H &= \frac{1+\phi}{2} [P^{*\mu} \gamma_\mu + \delta_H P \gamma_5], \quad S = \frac{1+\phi}{2} [P_1^{*\mu} \gamma_\mu + \delta_S P_0^*], \\ T^\mu &= \frac{1+\phi}{2} P_2^{*\mu\nu} \gamma_\nu + \delta_T \sqrt{\frac{3}{2}} P_1^\nu \gamma_5 [g_\nu^\mu - \frac{1}{3} \gamma_\nu (\gamma^\mu - v^\mu)]\end{aligned}$$

(III):

Meson-antimeson states: exotic J^{PC}

0^{--}	$D\bar{D}_0^*, D^*\bar{D}_1, D^*\bar{D}'_1$
0^{+-}	$D_1\bar{D}'_1$
1^{-+}	$D\bar{D}_1, D\bar{D}'_1, D^*\bar{D}_0, D^*\bar{D}_1, D^*\bar{D}'_1, D^*\bar{D}_2^*$
2^{+-}	$D_0^*\bar{D}_2^*, D_1\bar{D}'_1, D_1\bar{D}_2^*, D'_1\bar{D}_2^*$
3^{-+}	$D^*\bar{D}_2^*$

$$S_{c\bar{c}} = 1$$

TABLE I: The meson-antimeson states with exotic J^{PC} .

Tensor force
may also be
important

States	Angular momentum	Channels							
		1	2	3	4	5	6	7	8
$D\bar{D}_0^*$	J=0	1S_0							
$D^*\bar{D}_1, D^*\bar{D}'_1, D_1\bar{D}'_1$	J=0	1S_0	5D_0						
	J=1	3S_1	3D_1	5D_1					
	J=2	5S_2	1D_2	3D_2	5D_2	5G_2			
$D^*\bar{D}_2^*, D_1\bar{D}_2^*, D'_1\bar{D}_2^*$	J=1	3S_1	3D_1	5D_1	7D_1	7G_1			
	J=2	5S_2	3D_2	5D_2	7D_2	5G_2	7G_2		
	J=3	7S_3	3D_3	5D_3	7D_3	3G_3	5G_3	7G_3	7I_3

TABLE II: Coupled channels ($^{2S+1}L_J$) for various meson-antimeson systems. 14

(III):

Reformulated Lagrangian

$$V(NN \rightarrow NN) : V(r) = V_C(r) + S_{ten} V_T(r)$$

$$V(NN \rightarrow N\Delta) :$$

- Spin-3/2:

$$\Delta_\mu \equiv S_\mu \chi$$

transition spin

$$\chi_{s_z=3/2} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_{s_z=1/2} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \chi_{s_z=-1/2} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \chi_{s_z=-3/2} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$V(r) = V_C(r) + S_{ten} V_T(r)$$

- Now: **mesonic case**

(III):

Reformulated Lagrangian

$$\mathcal{L}_g = -\frac{2g}{f} \text{Tr} \left\{ \Sigma_{V\mu} [\Phi_V \partial^\mu \mathcal{M} \Phi_V^\dagger] - \delta_H^\dagger S_V^\mu [\Phi_V \partial_\mu \mathcal{M} P^\dagger] - \delta_H S_V^{\dagger\mu} [P \partial_\mu \mathcal{M} \Phi_V^\dagger] \right\},$$

$$\mathcal{L}_{g'} = -\frac{2g'}{f} \text{Tr} \left\{ \Sigma_{A,\mu} [\Phi'_A \partial^\mu \mathcal{M} \Phi'_A{}^\dagger] - \delta_S^\dagger S_A^\mu [\Phi'_A \partial_\mu \mathcal{M} P_0^{*\dagger}] - \delta_S S_A^{\dagger\mu} [P_0^* \partial_\mu \mathcal{M} \Phi'_A{}^\dagger] \right\},$$

$$\mathcal{L}_{g''} = -\frac{2g''}{f} \text{Tr} \left\{ \Sigma_{T,\alpha} [\Phi_T (\partial^\alpha \mathcal{M}) \Phi_T^\dagger] - \frac{5}{6} \Sigma_{A,\alpha} [\Phi_A (\partial^\alpha \mathcal{M}) \Phi_A^\dagger] \right. \\ \left. - \delta_T^\dagger \sqrt{\frac{1}{6}} \Sigma_{TA,\alpha} [\Phi_T (\partial^\alpha \mathcal{M}) \Phi_A^\dagger] - \delta_T \sqrt{\frac{1}{6}} \Sigma_{TA,\alpha}^\dagger [\Phi_A (\partial^\alpha \mathcal{M}) \Phi_T^\dagger] \right\},$$

$[\Sigma_{V\mu}, \Sigma_{V\nu}] = i\epsilon_{\mu\nu\alpha\beta} v^\alpha \Sigma_V^\beta$
 $[\Sigma_{T\mu}, \Sigma_{T\nu}] = -\frac{i}{2} \epsilon_{\mu\nu\alpha\beta} v^\alpha \Sigma_T^\beta$

$$\mathcal{L}_h = -\frac{2h}{f} \text{Tr} \left\{ v_\mu \Sigma_{AV} [\Phi'_A (\partial^\mu \mathcal{M}) \Phi_V^\dagger] - \delta_S \delta_H^\dagger v^\mu [P_0^* (\partial_\mu \mathcal{M}) P^\dagger] \right\} + h.c.,$$

$$\mathcal{L}_{h'} = -\frac{h'}{f} \text{Tr} \left\{ -2 \Sigma_{TA}^\mu [\Phi_T (\partial_\mu \mathcal{M}) \Phi_A^\dagger] + \delta_T \sqrt{\frac{2}{3}} \Sigma_A^\mu [\Phi_A (\partial_\mu \mathcal{M}) \Phi_A^\dagger] + \delta_T \delta_S^\dagger \sqrt{\frac{8}{3}} S_A^\mu [\Phi_A (\partial_\mu \mathcal{M}) P_0^{*\dagger}] \right\} + h.c.,$$

$$\mathcal{L}_{h_\chi} = -\frac{2h_\chi}{f} \text{Tr} \left\{ \Sigma_{TV}^{\mu\alpha} [\Phi_T (\partial_\mu \partial_\alpha \mathcal{M}) \Phi_V^\dagger] - \delta_H^\dagger S_T^{\mu\nu} [\Phi_T (\partial_\mu \partial_\nu \mathcal{M}) P^\dagger] - \delta_T \sqrt{\frac{1}{6}} \Sigma_{AV}^{\mu\alpha} [\Phi_A (\partial_\mu \partial_\alpha \mathcal{M}) \Phi_V^\dagger] \right\} + h.c.$$

(III):

Formalism: OPEP

$$(D^* \bar{D}_2^*)_J$$

Chiral quark model: $g'' = -g$

$$V = I_f G^\pi \frac{gg'' m^3}{24\pi f^2} \left\{ D_S V_e^{dir}(r) - D_T V_t^{dir}(r) \right\} \\ + I_f G^\pi \frac{|h_\chi|^2 \mu^5}{8\pi f^2} c (-1)^J \left\{ \frac{2}{15} C_S V_e^{cro}(r) + \frac{4}{21} C_{T1} V_{t1}^{cro}(r) + \frac{1}{7} C_{T2} V_{t2}^{cro}(r) \right\}$$

- c : C-parity of the state;
- $I_f = 1$ (-3) for isovector (isoscalar);
- $G^\pi = -1$: G-parity of π meson

$$D_S = \frac{1}{2} [(\vec{\Sigma}_T \cdot \vec{\Sigma}_V) + (V \leftrightarrow T)], \quad D_T = \frac{1}{2} [(\vec{\Sigma}_T \cdot \vec{\Sigma}_V - 3\vec{\Sigma}_T \cdot \hat{r} \vec{\Sigma}_V \cdot \hat{r}) + (V \leftrightarrow T)], \\ C_S = \frac{1}{2} [(\Sigma_{TV}^{ij} \Sigma_{VT}^{ij}) + (V \leftrightarrow T)], \quad C_{T1} = \frac{1}{2} [(\Sigma_{TV}^{ij} \Sigma_{VT}^{ij} - 3\Sigma_{TV}^{ij} \hat{r}_j \Sigma_{VT}^{ij'} \hat{r}_{j'}) + (V \leftrightarrow T)], \\ C_{T2} = \frac{1}{9} \{ [2(\Sigma_{TV}^{ij} \Sigma_{VT}^{ij}) - 20(\Sigma_{TV}^{ij} \hat{r}_j \Sigma_{VT}^{ij'} \hat{r}_{j'}) + 35(\Sigma_{TV}^{ij} \hat{r}_i \hat{r}_j)(T_{VT}^{i'j'} \hat{r}_{i'} \hat{r}_{j'})] + (V \leftrightarrow T) \},$$

Examples: Coupled channel effects

- | state | $I^G(J^{PC})$ | $\Lambda(\text{GeV})$ | B.E.(MeV) | $r_{rms}(\text{fm})$ |
|----------------|-----------------------|-----------------------|-----------|----------------------|
| $D\bar{D}_0^*$ | $0^-(0^{--})^\dagger$ | 1.4 ~ 4 | 30 ~ 24 | 1.3 ~ 1.4 |

binding solution always exists (EPJC70,183)

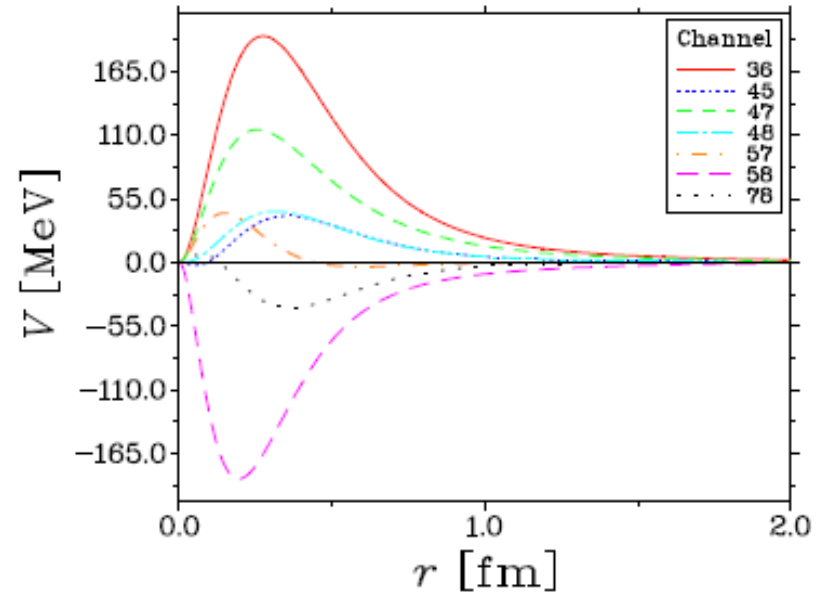
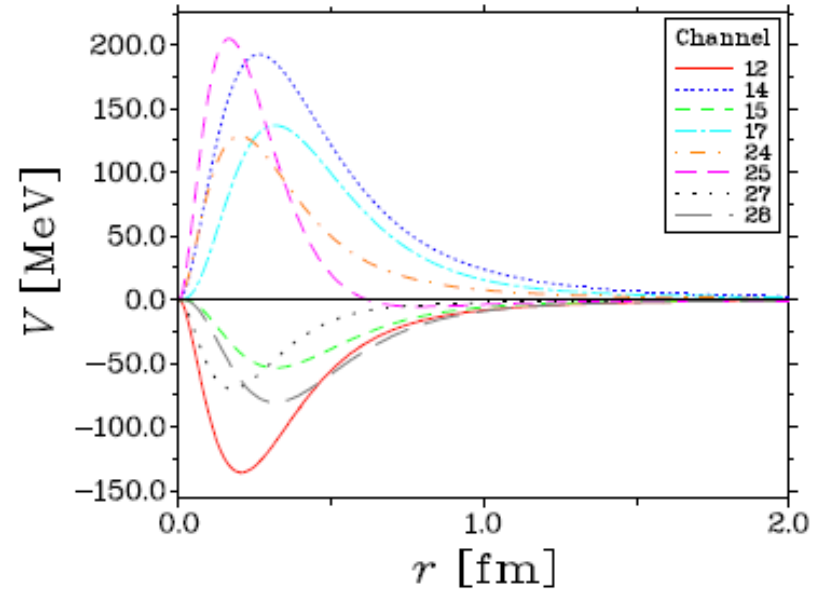
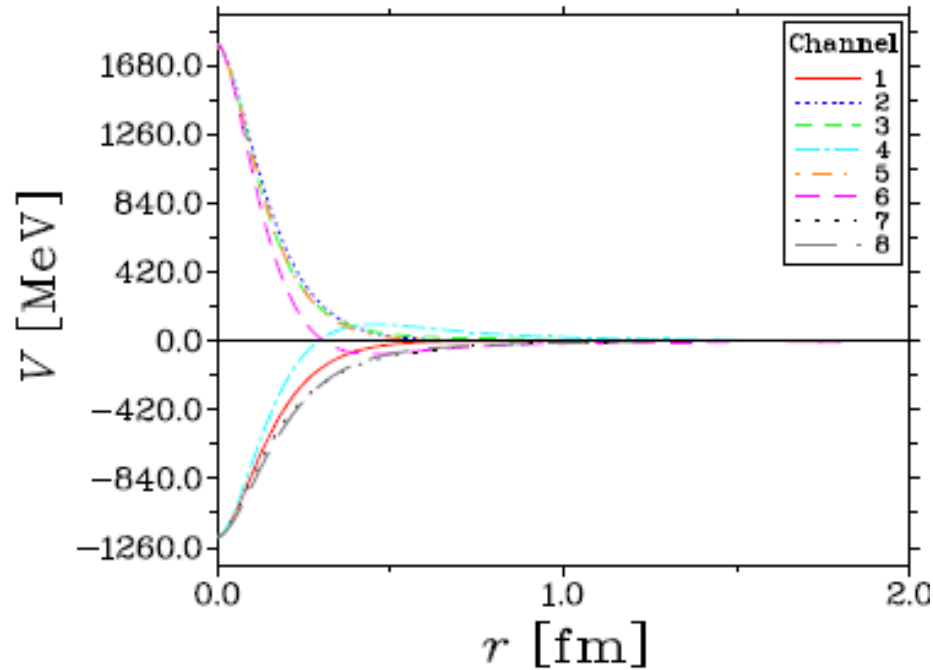
- | state | $I^G(J^{PC})$ | $\Lambda(\text{GeV})$ | B.E.(MeV) | $r_{rms}(\text{fm})$ | |
|------------------|-----------------------|-----------------------|------------|----------------------|---------|
| $D^*\bar{D}_2^*$ | $1^-(1^{--})^\dagger$ | 3.6 ~ 3.7 | 1.9 ~ 8.3 | 2.2 ~ 1.1 | ←S-wave |
| | $0^+(2^{-+})$ | 3.13 ~ 3.15 | 0.94 ~ 6.8 | 3.0 ~ 1.1 | |
| | $0^+(3^{-+})^\dagger$ | 2.3 ~ 2.5 | 0.58 ~ 9.6 | 3.9 ~ 1.0 | |

state	$I^G(J^{PC})$	$\Lambda(\text{GeV})$	B.E.(MeV)	$r_{rms}(\text{fm})$	
$D^*\bar{D}_2^*$	$1^-(1^{--})^\dagger$	3.4 ~ 3.5	3.0 ~ 11	1.8 ~ 0.97	←C.C.
	$0^+(1^{-+})^\dagger$	3.9 ~ 4	2.2 ~ 10	2.3 ~ 1.2	
	$0^+(2^{-+})$	3.02 ~ 3.09	0.94 ~ 15	3.2 ~ 0.85	
	$0^+(3^{-+})^\dagger$	2.0 ~ 2.3	0.91 ~ 16	3.3 ~ 0.94	

B.E. shift: 15~20 MeV

3^{-+} most attractive

C.C. effects ($\Lambda=2.3$ GeV)



- Reasons: V small or 0

Summary

- HQS constraints $Q\bar{Q}$ spin structure of hadrons
- HQS constraints strong decays
- Coupled channel effects are not very large for meson-antimeson states with exotic J^{PC} in OPEP model

Thank you for your attention!