中国科学院大学
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胶球性质理论研究

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Reference

- **“Finding the 0^{--} Glueball”**
  Cong-Feng Qiao and Liang Tang,
  arXiv:1408.3995,
  Phys. Rev. Lett. 113, 221601 (2014).

Content

1. **Introduction of Glueballs**

2. **Current Status of Glueballs**

3. 0^{--} **Trigluon glueballs via QCDSR**

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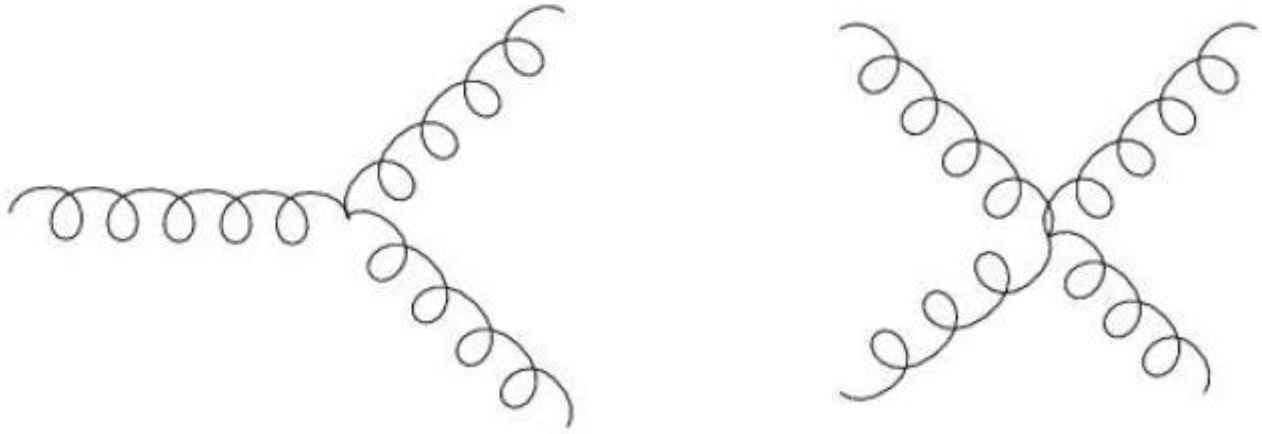
Introduction of Glueballs

- QCD = gauge theory with the color group $SU_c(3)$

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \sum_q \bar{\psi}_q(i\gamma^\mu D_\mu - m_q)\psi_q$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$

- Interactions of gluons



Introduction of Glueballs

- Color structure
  - Quark= fundamental representation **3**
  - Gluon= Adjoint representation **8**
  - Observable particles=color singlet **1**
    - Mesons $3 \bigotimes 3 = 1 \bigoplus 8$
    - Baryons $3 \bigotimes 3 \bigotimes 3 = 1 \bigoplus 8 \bigotimes 8 \bigoplus 10$
    - Glueballs $\begin{cases} 8 \bigotimes 8 = 1 \bigoplus 8 \bigoplus 8 \bigoplus 10 \bigoplus \overline{10} \bigoplus 27 \\ 8 \bigotimes \cdots \bigotimes 8 = 1 \bigoplus 8 \bigoplus \cdots \end{cases}$
- Glueballs: Predicted by QCD
- No definite observations in the experiment.
  - lack knowledge of their production&decay properties
  - mixing with quark states adds difficulty to isolate them.

Current Status of Glueballs

- Theoretical Approaches

  - Lattice QCD
  - Flux tube model
  - MIT bag model
  - Coulomb gauge model
  - QCD Sum Rules (QCDSR)

Constituent Models

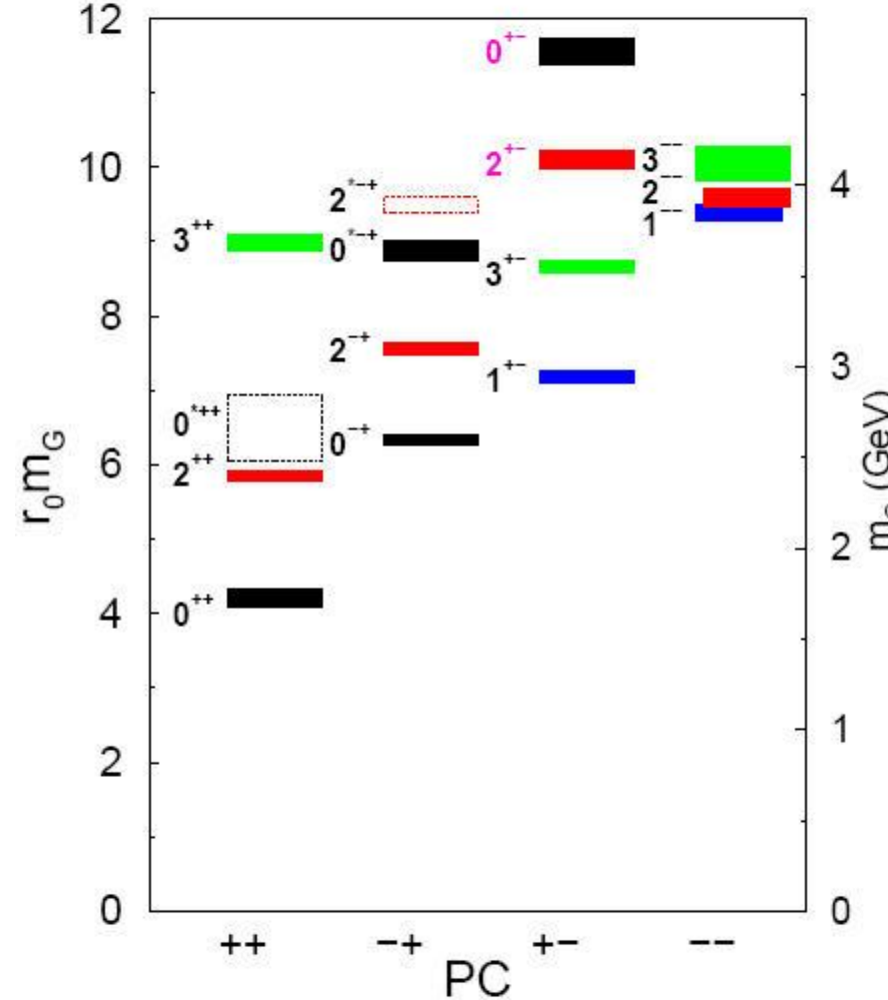
V.Mathieu, N.Kochelev&V.Vento, Int.J.Mod.Phys. E18,1(2009)

Current Status of Glueballs

- Results of Lattice QCD

$$r_0^{-1} = 410 \pm 20 \text{ MeV}$$

J^{PC}	Other J	$r_0 m_G$	m_G (MeV)
0^{++}		4.21 (11)(4)	1730 (50)(80)
2^{++}		5.85 (2)(6)	2400 (25)(120)
0^{-+}		6.33 (7)(6)	2590 (40)(130)
0^{*++}		6.50 (44)(7)†	2670 (180)(130)
1^{+-}		7.18 (4)(7)	2940 (30)(140)
2^{-+}		7.55 (3)(8)	3100 (30)(150)
3^{+-}		8.66 (4)(9)	3550 (40)(170)
0^{*-+}		8.88 (11)(9)	3640 (60)(180)
3^{++}	$6, 7, 9, \ldots$	8.99 (4)(9)	3690 (40)(180)
1^{--}	$3, 5, 7, \ldots$	9.40 (6)(9)	3850 (50)(190)
2^{*-+}	$4, 5, 8, \ldots$	9.50 (4)(9)†	3890 (40)(190)
2^{--}	$3, 5, 7, \ldots$	9.59 (4)(10)	3930 (40)(190)
3^{--}	$6, 7, 9, \ldots$	10.06 (21)(10)	4130 (90)(200)
2^{+-}	$5, 7, 11, \ldots$	10.10 (7)(10)	4140 (50)(200)
0^{+-}	$4, 6, 8, \ldots$	11.57 (12)(12)	4740 (70)(230)



Morningstar & Peardon, PRD60(1999)034509.

Current Status of Glueballs

- Results of Lattice QCD

Chen *et al.*,PRD73(2006)014516.

Morningstar & Peardon, PRD60(1999)034509.

R^{PC}	Possible J^{PC}	$r_0 M_G$	$r_0 M_G$
A_1^{++}	0^{++}	4.16(11)	4.21(11)
E^{++}	2^{++}	5.82(5)	5.85(2)
T_2^{++}	2^{++}	5.83(4)	5.85(2)
A_2^{++}	3^{++}	9.00(8)	8.99(4)
T_1^{++}	3^{++}	8.87(8)	8.99(4)
A_1^{-+}	0^{-+}	6.25(6)	6.33(7)
T_1^{+-}	1^{+-}	7.27(4)	7.18(3)
E^{-+}	2^{-+}	7.49(7)	7.55(3)
T_2^{-+}	2^{-+}	7.34(11)	7.55(3)
T_2^{+-}	3^{+-}	8.80(3)	8.66(4)
A_2^{+-}	3^{+-}	8.78(5)	8.66(3)
T_1^{--}	1^{--}	9.34(4)	9.50(4)
E^{--}	2^{--}	9.71(3)	9.59(4)
T_2^{--}	2^{--}	9.83(8)	9.59(4)
A_2^{--}	3^{--}	10.25(4)	10.06(21)
E^{+-}	2^{+-}	10.32(7)	10.10(7)
A_1^{+-}	0^{+-}	11.66(7)	11.57(12)

Mass(0^{--})=(5166±1000) MeV (Unquenched)

Gregory, *et al.*, JHEP1210(2012)170.

Current Status of Glueballs

- Production of glueballs via Lattice QCD.

  - Scalar glueball in radiative J/ψ decay on lattice.
    Long-Cheng Gui, *et al*., (CLQCD Collaboration),
    Phys. Rev. Lett. 110, 021601 (2013).

  - Lattice study of radiative J/ψ decay to a tensor glueball.
    Yi-Bong Yang, *et al*., (CLQCD Collaboration),
    Phys. Rev. Lett. 111 (2013) 9, 091601.

Current Status of Glueballs

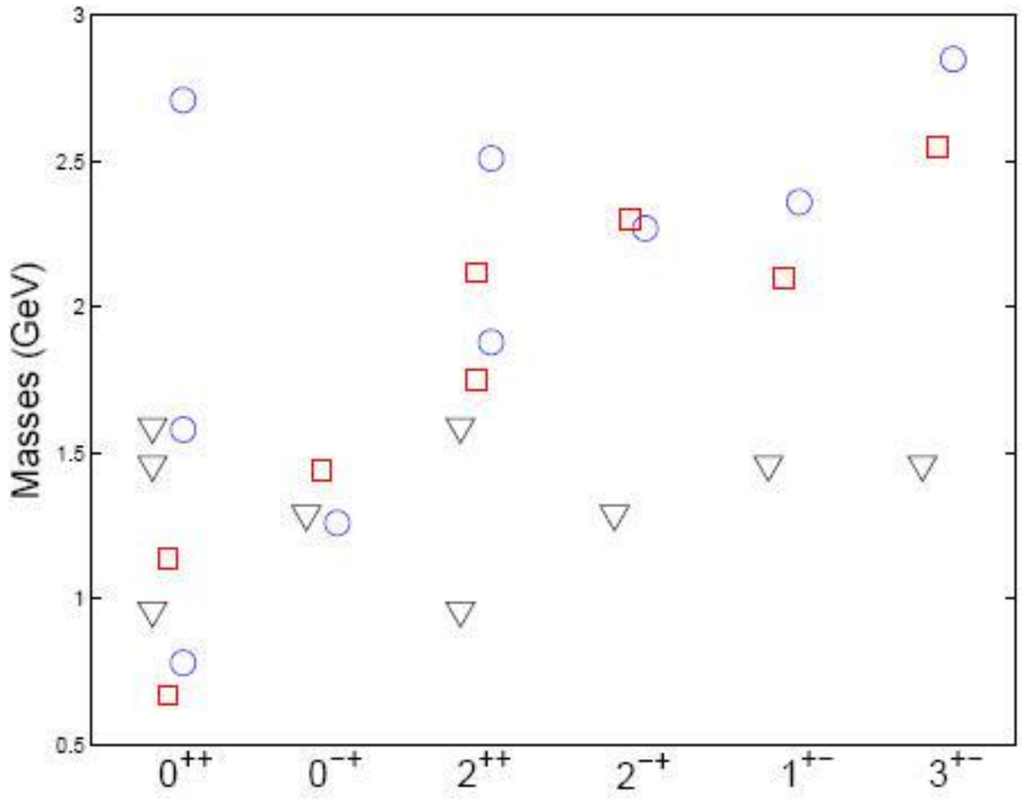
- Flux tube model

J^{PC}	Mass (GeV)
0^{++}	1.52
1^{+-}	2.25
0^{++}	2.75
$0^{++}, 0^{+-}, \boxed{0^{-+}, 0^{--}}$	$\boxed{2.79}$
2^{++}	2.84
$2^{++}, 2^{++}, 2^{++}, 2^{++}$	2.84
1^{+-}	3.25
3^{+-}	3.35

Isgur & Parton, PRD31(1985)2910.

Current Status of Glueballs

- MIT bag model



▽ =Jaffe &Johnson, PLB60,201(1976).

◯=Carlson *et al.*, PRD27 (1983)1556.

□ =Chanowitz &Sharpe, NPB222(1983)211.

Current Status of Glueballs

- Coulomb Gauge model

Model	J^{PC}	0^{-+}	1^{--}	2^{--}	3^{--}	5^{--}	7^{--}
	color	f	d	d	d	d	d
	S	0	1	2	3	3	3
	L	0	0	0	0	2	4
H^g_{eff} (this work)		3900	3950	4150	4150	5050	5900
H_M (this work)		3400	3490	3660	3920	5150	6140

Llances-Estrada, Bicudo &Cotanch, PRL96(2006)081601

Current Status of Glueballs

- QCD Sum Rules

Two-gluon glueballs in QCDSR

	Novikov *et. al.*	Forkel	Bagan *et. al.*	Huang *et. al*
0^{++}	0.7-0.9 GeV	1.25 GeV	1.7 GeV	1.66 GeV
0^{-+}	-	2.2 GeV	-	-

Novikov et al., NPB165(1980)67.
Bagan&Steele, PLB243(1990)43.
Forkel, PRD64(2001)034015.
Huang, Jin&Zhang, PRD59(1999)034026.

Tri-gluon glueballs in QCDSR

	0^{++}	0^{-+}	1^{-+}	1^{--}	2^{++}
Latorre *et. al.*	3.1 GeV	-	-	-	-
Liu *et. al.*	1.45 GeV	-	1.87 GeV	2.4 GeV	2.0 GeV
Hao *et. al.*	-	1.9-2.7 GeV	-	-	-

Latorre *et al*., PLB191(1987)437. Liu, CPL15(1998)784. Hao et al., PLB642(2006)53.

0^{--} Trigluon glueballs: Why important?

- Oddballs

Oddballs: glueballs with exotic quantum numbers

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+} \text{ and so on}$$

Physics at BESIII, Editors Kuang-Ta Chao & Yifang Wang, Int. JMPA24,1,(2009).

- $C = -1$ →Trigluon glueballs.

- Exotic quantum numbers ➡ Do not mix with $q\bar{q}$

- 0^{--} is the lowest spin state of oddballs. (i.e. the simplest Lorentz structure).

0^{--} Trigluon glueballs: Why important?

➢ It can be produced in the decay of heavy vector quarkonium or quarkoniumlike states easier.

= exotic state (not easy to be detected, only $\pi_1(1400)$ with 1^{-+} in PDG)

= unfavorable production channel

Like in football + =

$$1^{--} \to G_{0^{--}} + 1^{++}$$

$$1^{--} \to G_{2^{+-}} + 1^{-+}/2^{-+}/3^{-+}$$

$$1^{--} \to G_{0^{+-}} + 1^{-+}$$

$$1^{--} \to G_{3^{-+}} + 2^{+-}$$

$$1^{--} \to G_{1^{-+}} + 0^{--}$$

$$1^{++} = f_1(1285)/\chi_{c1}(3511)/\chi_{b1}(10255)$$

0^{--} Trigluon glueballs:QCDSR

➢QCDSR

- The two-point correlation function

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T\Big\{j_{0^{--}}(x), j_{0^{--}}(0)\Big\}|0\rangle \ ,$$

- The QCD side of the correlation function

$$\Pi^{\rm QCD}(Q^2) = a_0 Q^{12}\ln\frac{Q^2}{\mu^2} + b_0 Q^8\langle\alpha_s G^2\rangle + \left(c_0 + c_1\ln\frac{Q^2}{\mu^2}\right) Q^6\langle g_s G^3\rangle + d_0 Q^4\langle\alpha_s G^2\rangle^2 \ .$$

- The phenomenological side of the correlation function

$$\frac{1}{\pi}{\rm Im}\Pi^{\rm phe}(s) = f_G^2 M_{0^{--}}^{12}\delta(s - M_{0^{--}}^2) + \rho(s)\theta(s - s_0) \ .$$

0^{--} Trigluon glueballs:QCDSR

- The dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s+Q^2} + \left(\Pi(0) - Q^2\Pi'(0) + \frac{1}{2}Q^4\Pi''(0) - \frac{1}{6}Q^6\Pi'''(0)\right) ,$$

- The Borel tansformation

$$\hat{B}_\tau \equiv \lim_{\substack{Q^2\to\infty, n\to\infty \\ \frac{Q^2}{n}=\frac{1}{\tau}}} \frac{(-Q^2)^n}{(n-1)!}\left(\frac{d}{dQ^2}\right)^n ,$$

- The quark-hadron duality approximation

$$\frac{1}{\pi}\int_{s_0}^\infty e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds \simeq \int_{s_0}^\infty \rho(s)e^{-s\tau}ds ,$$

0^{--} Trigluon glueballs:QCDSR

- The moments

$$L_0(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$
$$L_1(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} se^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds\ ,$$

- The mass function

$$M^i_{0^{--}}(\tau, s_0) = \sqrt{\frac{L_1(\tau, s_0)}{L_0(\tau, s_0)}}$$

- Ratios to constrain the windows of τ

$$R_i^{\mathrm{OPE}} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\langle \mathrm{g_s G^3}\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{QCD}}(s)ds}$$

$$R_i^{\mathrm{PC}} = \frac{L_0(\tau, s_0)}{L_0(\tau, \infty)}\ .$$

0^{--} Trigluon glueballs

- Interpolating currents of 0^{--} trigluon glueballs
  - Constraints: quantum number, gauge invariance, Lorentz invariance and $SU_c(3)$ symmetry

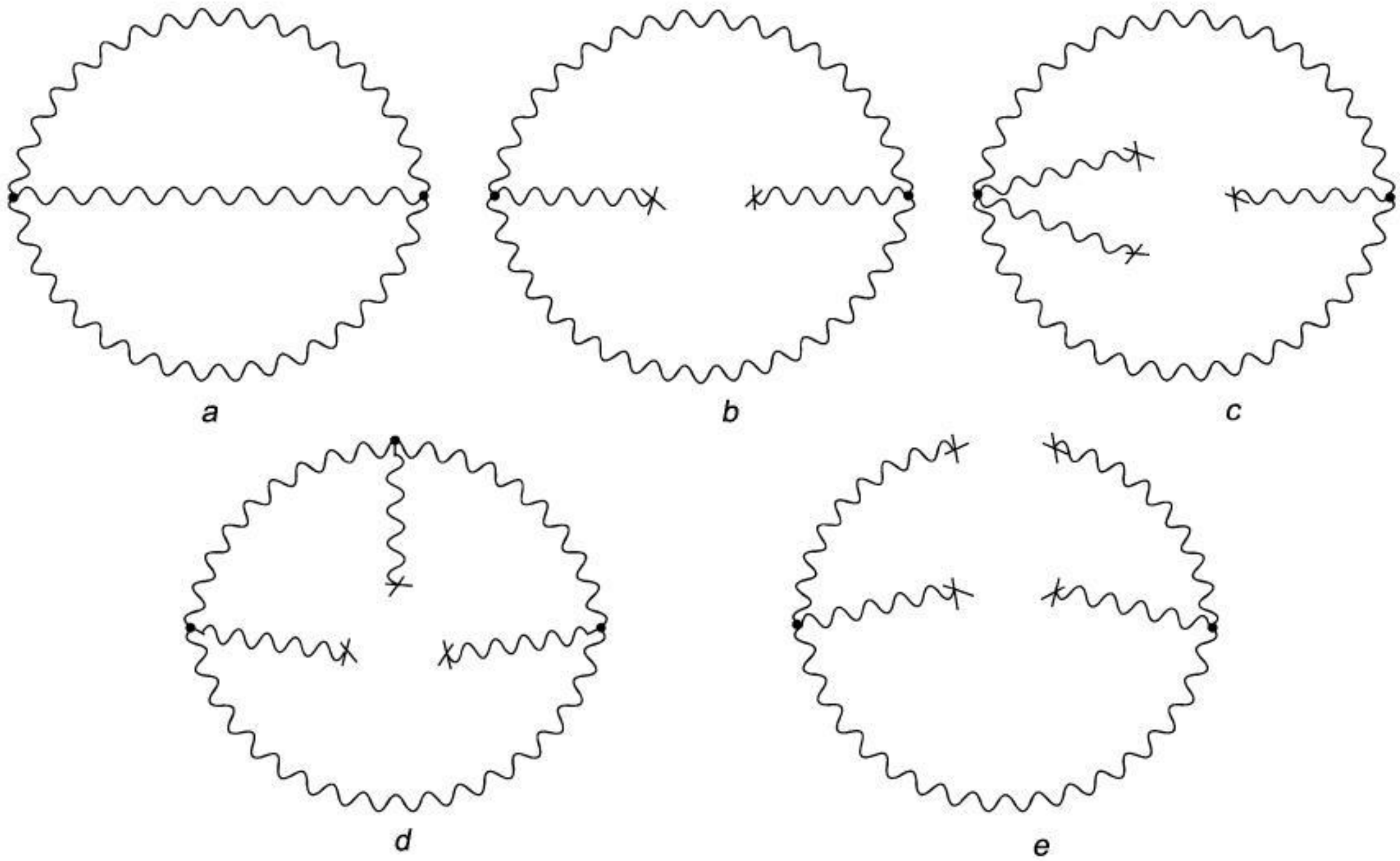
$$j^A_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^B_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][G^c_{\rho\mu}(x)]\,,$$
$$j^C_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}G^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta G^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$
$$j^D_{0^{--}}(x) = g_s^3 d^{abc}[g^t_{\alpha\beta}\tilde{G}^a_{\mu\nu}(x)][\partial_\alpha\partial_\beta \tilde{G}^b_{\nu\rho}(x)][\tilde{G}^c_{\rho\mu}(x)]\,,$$

where $g^t_{\alpha\beta} = g_{\alpha\beta} - \partial_\alpha\partial_\beta/\partial^2 \quad \tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\kappa\tau}G^a_{\kappa\tau}$

Cong-Feng Qiao & Liang Tang, arXiv:1408.3995, PRL113,221601(2014)

0^{--} Trigluon glueballs

- Typical Feynman diagrams of trigluon glueballs



0^{--} Trigluon glueballs

➢ Wilson coefficients in the QCD-side

$$a_0^i = \frac{487\alpha_s^3}{143 \times 2^6 \times 3^3\pi}\ ,\ b_0^i = -\frac{5\pi}{36}\alpha_s^2\ , c_0^A = -\frac{205}{12}\pi\alpha_s^3\ ,$$
$$c_1^A = -\frac{775}{144}\pi\alpha_s^3\ ,\ c_0^B = -\frac{2065}{48}\pi\alpha_s^3\ , c_1^B = -\frac{1075}{96}\pi\alpha_s^3\ ,$$
$$c_0^C = \frac{2275}{72}\pi\alpha_s^3\ ,\ c_1^C = \frac{2125}{144}\pi\alpha_s^3\ ,\ c_0^D = -\frac{1045}{144}\pi\alpha_s^3\ ,$$
$$c_1^D = -\frac{25}{32}\pi\alpha_s^3\ ,\ d_0^j = 0\ ,\ d_0^D = -\frac{5}{9}\pi^3\alpha_s\ ,$$

where, i=A, B, C, D; j=A, B, C; with A, B, C and D corresponding to the above four currents.

There are symmetries within Wilson coefficients a_0^i , b_0^i and d_0^j . The position and number of $\tilde{G}$ do not influence the perturbative and $\langle\alpha_s G^2\rangle$ contributions, whereas they influence $\langle g_s G^3\rangle$ term. Since $\langle\alpha_s G^2\rangle^2$ involves no loop contribution, d_0^j are governed by the number of $\tilde{G}$.

0^{--} Trigluon glueballs

- Figures for case-A

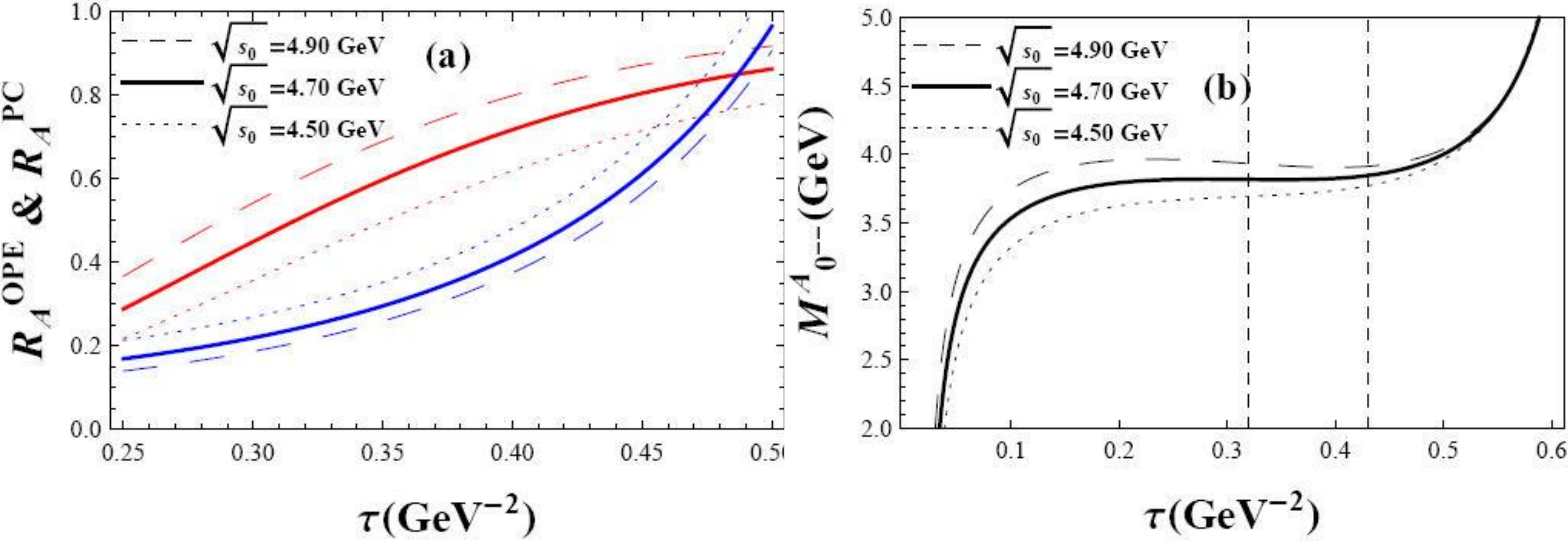


FIG. 1: (a) The ratios R_A^{OPE} and R_A^{PC} in case-A as functions of Borel parameter τ for different values of $\sqrt{s_0}$, where blue lines represent R_A^{OPE} and red lines denote R_A^{PC}. (b) The mass M^A_{0--} as function of the Borel parameter τ for different values of $\sqrt{s_0}$, where the two vertical lines indicate the upper and lower limits of the valid Borel window.

0^{--} Trigluon glueballs

➢ Figures for case-B

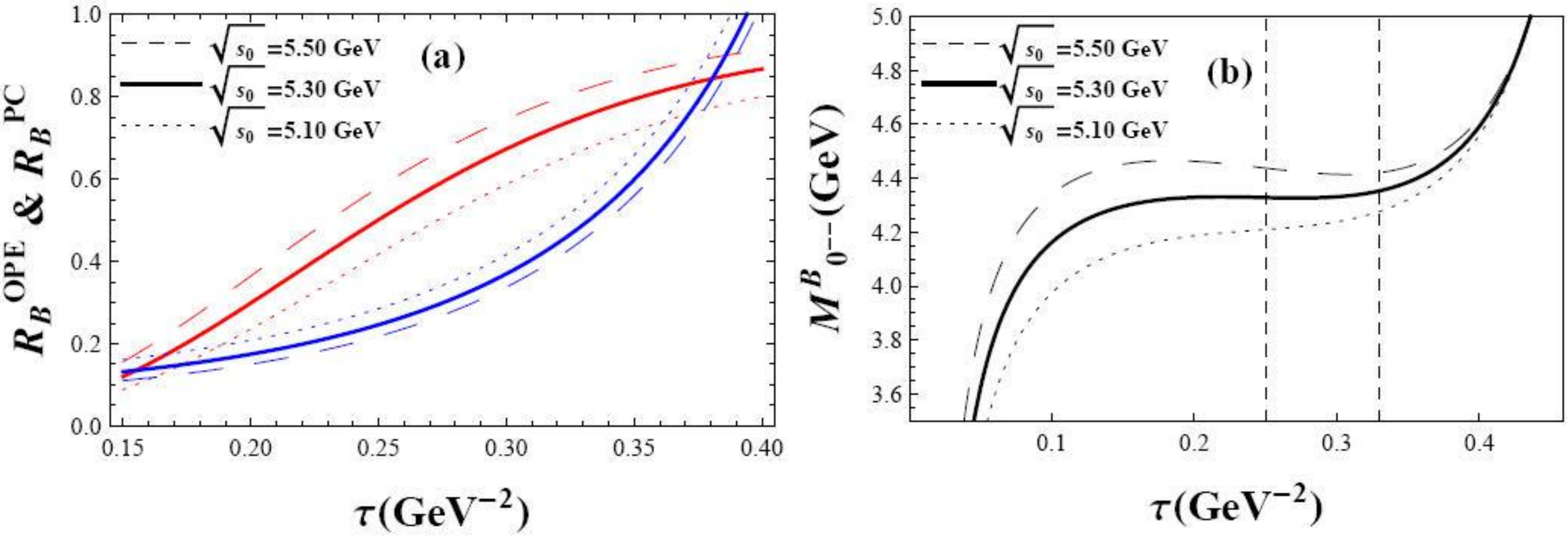


FIG. 2: The same caption as in Figure 1, but for case-B.

0^{--} Trigluon glueballs

➢ Figures for case-C

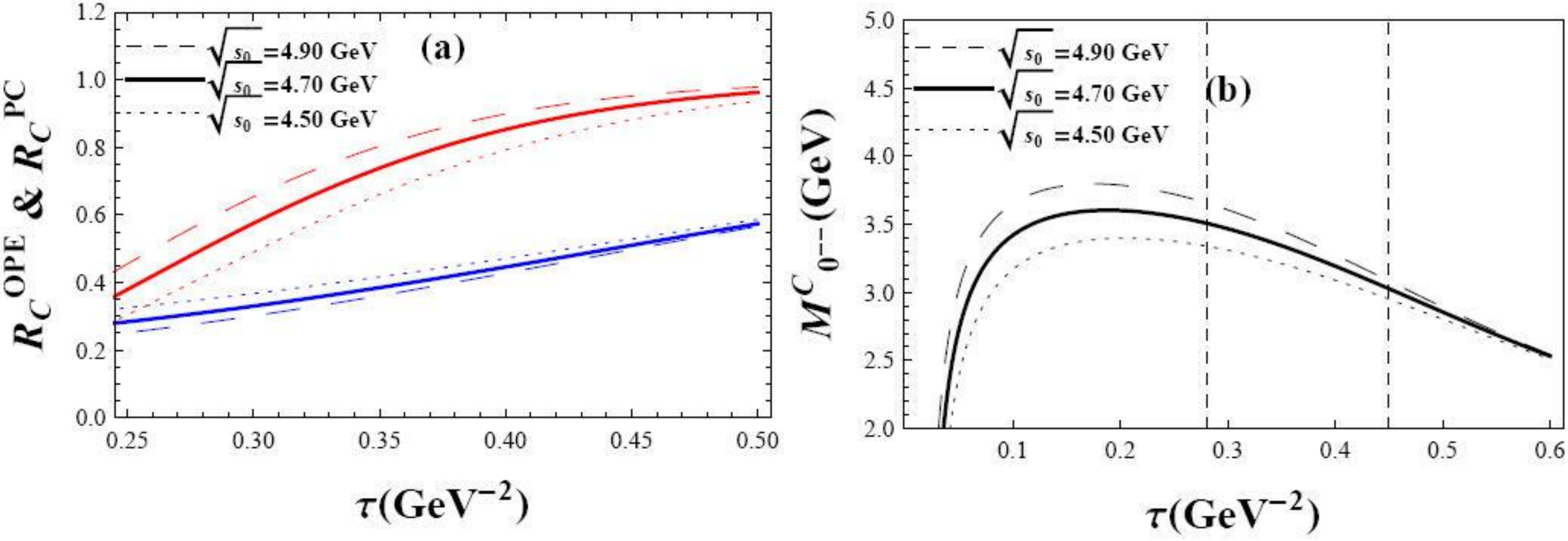


FIG. 3: The same caption as in Figure 1, but for case-C.

0^{--} Trigluon glueballs

- Figures for case-D

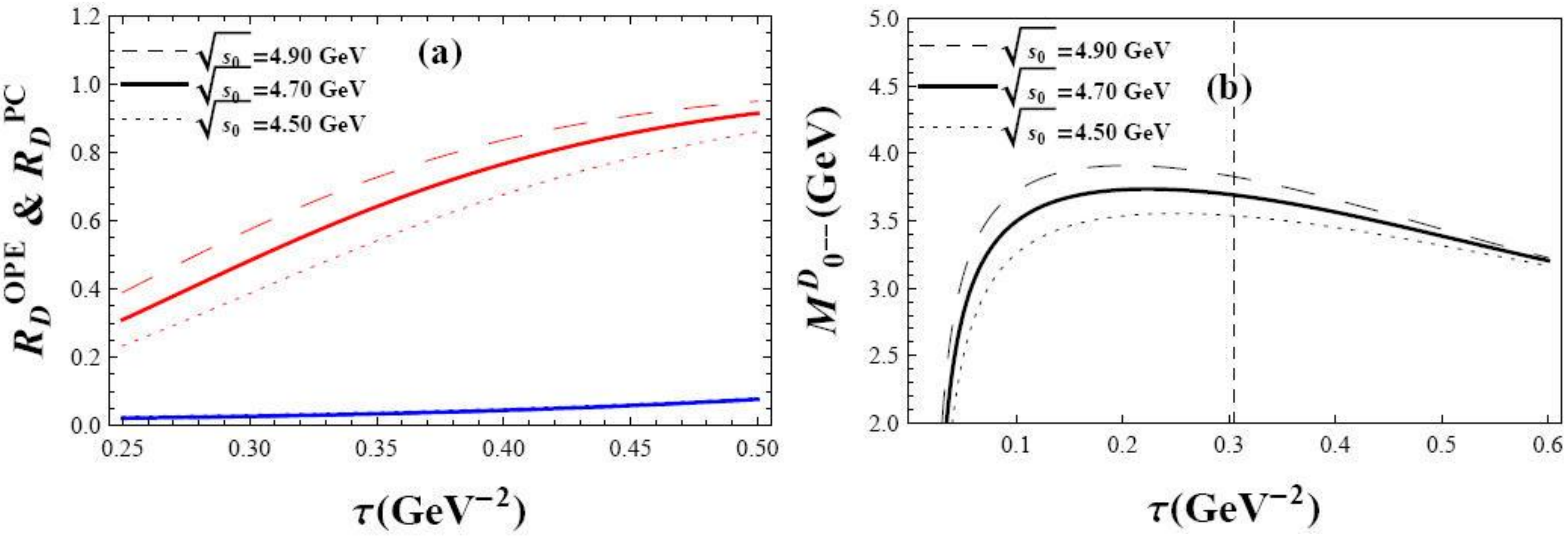


FIG. 4: The same caption as in Figure 1, but for case-D. Here the single vertical line indicates the lower limit of the valid Borel window while the upper limit is out of the region.

0^{--} Trigluon glueballs

- Masses of 0^{--} oddballs

$$M^A_{0^{--}} = 3.81 \pm 0.12\,\mathrm{GeV},$$

$$M^B_{0^{--}} = 4.33 \pm 0.13\,\mathrm{GeV},$$

- Nominate the above two 0^{--} oddballs as follows

$$M^A_{0^{--}} \Rightarrow G_{0^{--}}(3810)$$

$$M^B_{0^{--}} \Rightarrow G_{0^{--}}(4330)$$

0^{--} Trigluon glueballs

- Compare our 0^{--} oddball with Flux tube model

$$G_{0^{--}}(3810) > 2.79\,\mathrm{GeV}$$

Isgur & Parton, PRD31(1985)2910.

- Compare our 0^{--} oddball with Lattice QCD

$$G_{0^{--}}(4330) < (5166 \pm 1000)\,\mathrm{MeV}$$

Gregory, *et al.*, JHEP1210(2012)170.

0^{--} Trigluon glueballs

- Proposed production channels (Taking the light one as an example)

$$X(3872) \to \gamma + G_{0^{--}}(3810), \qquad \Upsilon(1S) \to f_1(1285) + G_{0^{--}}(3810),$$

$$\Upsilon(1S) \to \chi_{c_1} + G_{0^{--}}(3810), \qquad \chi_{b_1} \to J/\psi + G_{0^{--}}(3810),$$

$$\chi_{b_1} \to \omega + G_{0^{--}}(3810).$$

- Proposed decay channels

$$G_{0^{--}}(3810) \to \gamma + f_1(1285),$$

$$G_{0^{--}}(3810) \to \omega + f_1(1285).$$

$$G_{0^{--}}(3810) \to \gamma + \chi_{c_1},$$

Summary

- We obtained two stable 0^{--} oddballs with masses about 3.81 and 4.33 GeV.

- Oddballs can in principle mix with hybrids and tetraquark states, though naively the OZI suppression may hinder the mixing in certain degree.

- We briefly analyzed the 0^{--} oddball optimal production and decay mechanism. They are expected to be measured in BESIII, BELLEII, Super-B, PANDA, and LHCb experiments.

Thank you！