

手征有效场论与核物理

- 对Weinberg power counting的修改及其在核子-核子散射中的应用

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FHNP'15, 北京, 01/2015

Propaganda

QCD



Chiral effective field theory



Nuclear physics
(Model-independent)

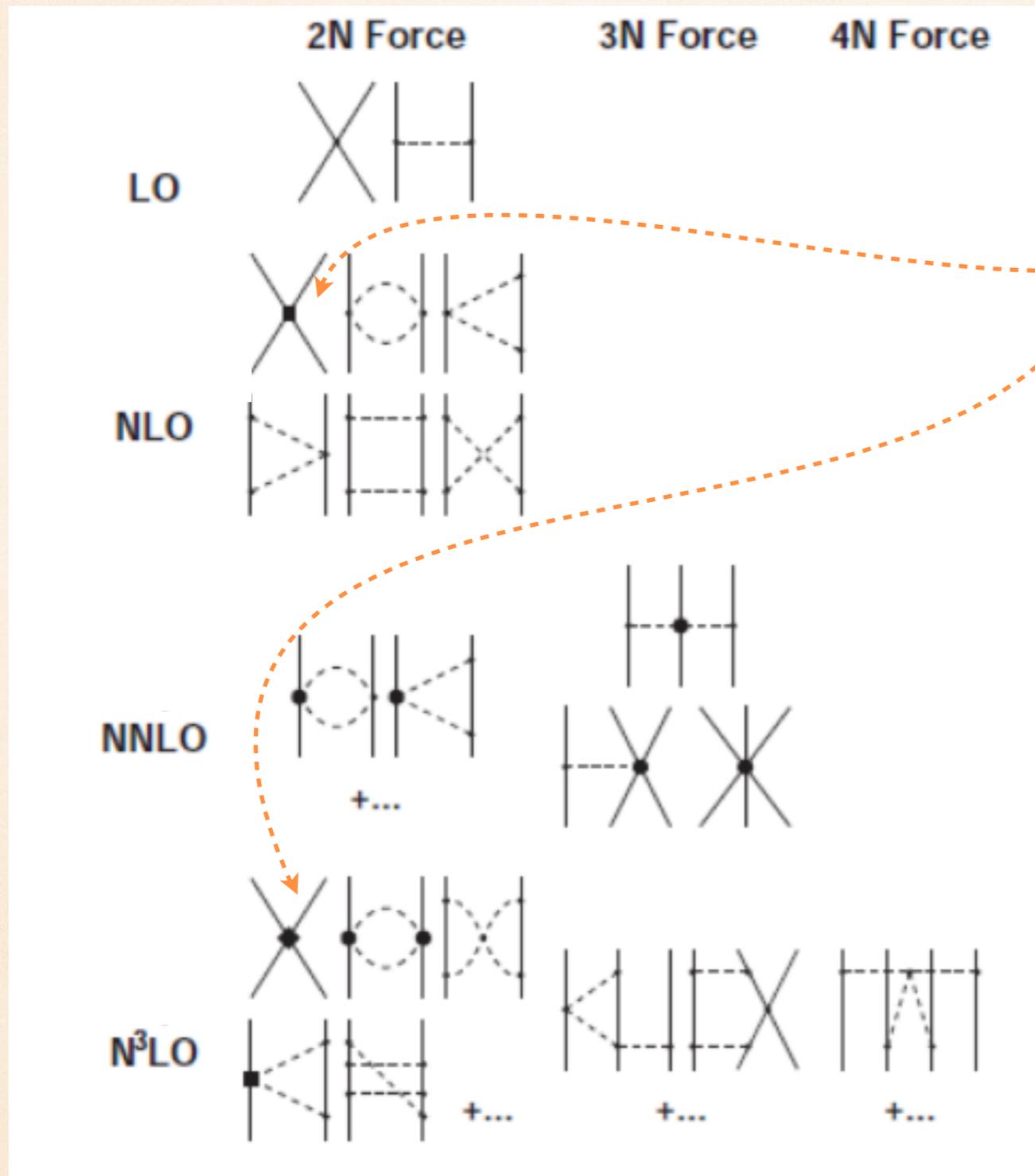


1. Nuclear reactions involving few-N

2. Electro-weak probes

3. Nuclear structure of light nuclei

What are we really doing?



Modify power counting of NN contact interactions, so as

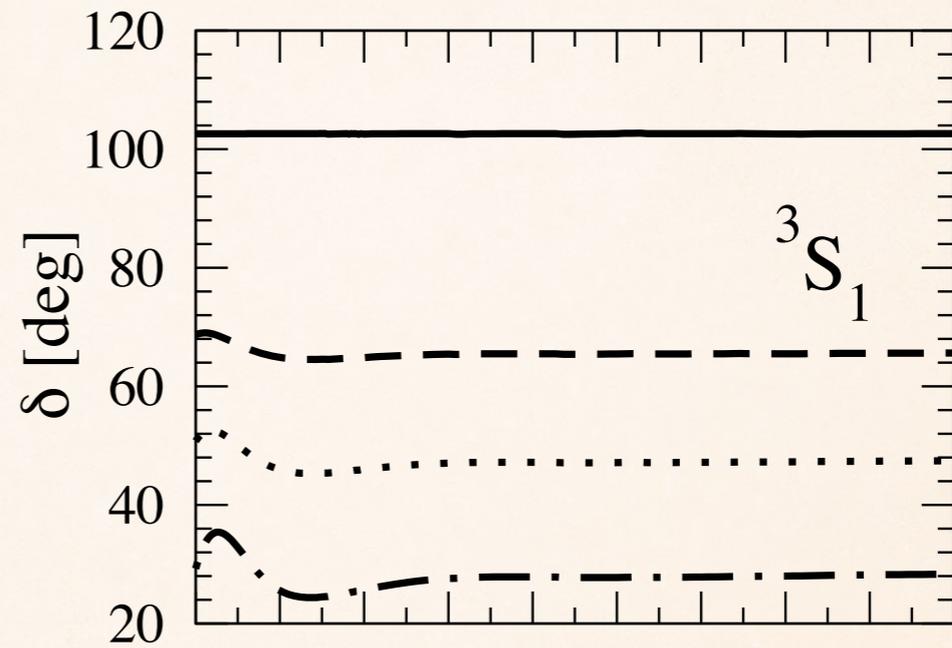
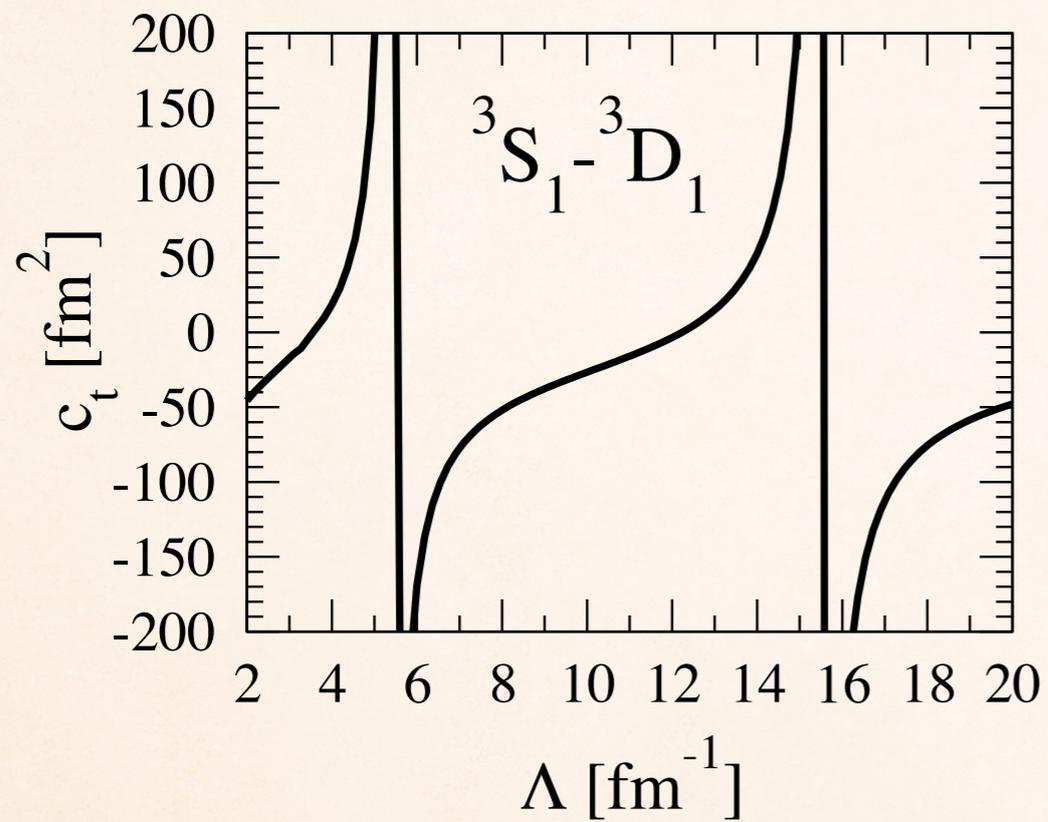
(1) to satisfy renormalization group invariance;

(2) to better understand how much of nuclear physics is decided by short-range interactions as opposed to chiral symmetry.

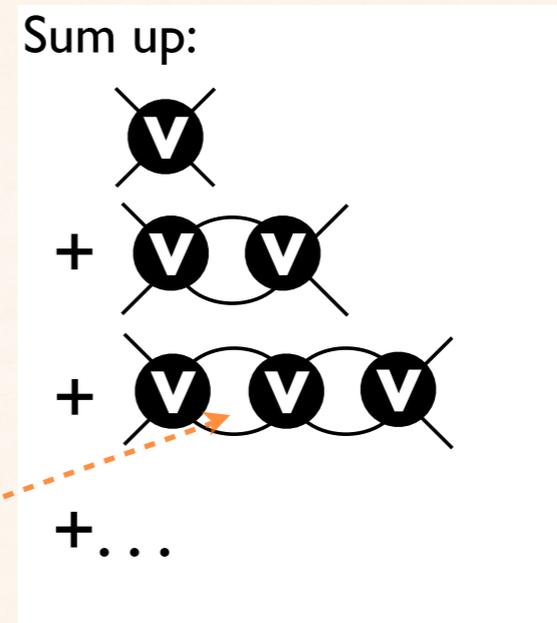
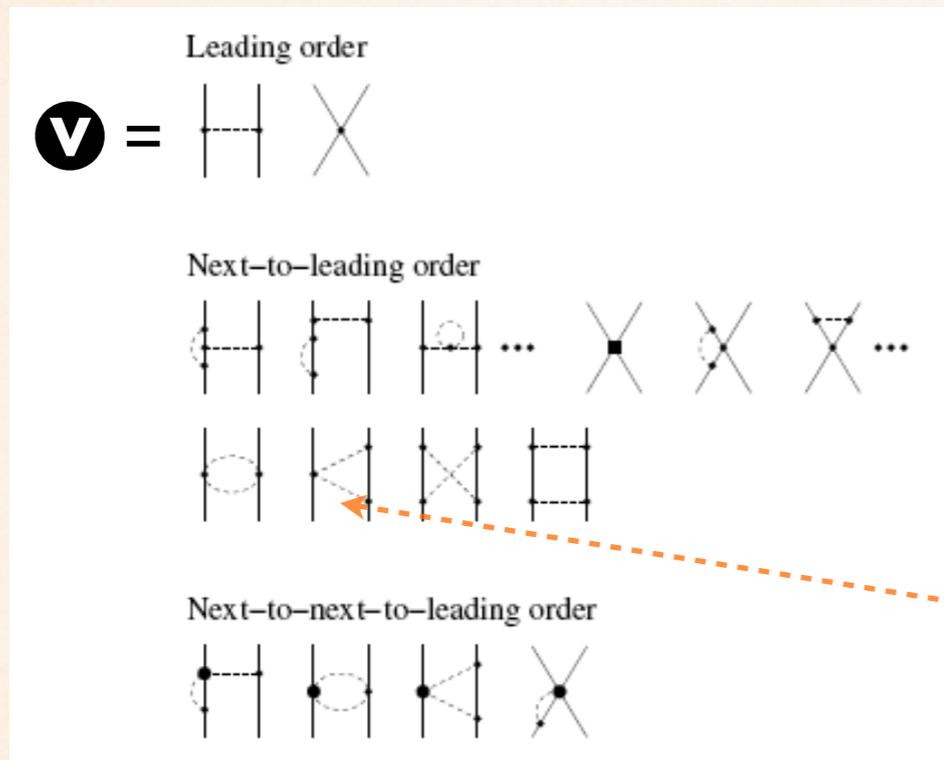
(3) generate a potential for low-energy nuclear physics

unconventional cutoff flows

contact coupling sensitive to cutoff physics is RG invariant



Renormalization of Sch. eqn.



Schrodinger
eqn.

Λ
momentum
cutoff

- ❖ Cutoff dependence of resummed amplitudes was not addressed in W counting
- Need to consider renormalization of the Sch. eqn.

Effective field theory

$$\text{EFT} = \text{Effective Lagrangian} + \text{Power Counting}$$

(Organization principle)

- ❖ Low-energy Dofs
- ❖ Symmetries

- ❖ *a priori* estimation of Feynman diagrams

- ❖ Normally need to be regularized to cutoff high momentum modes
- ❖ However, obs. independent of cutoffs — renormalization group invariance

Why EFT for nuclear physics

Controlled approximations of low-energy observables

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}} \right)^n \mathcal{F}_n \left(\frac{Q}{M_{lo}} \right)$$

Q: generic external momenta,

$$M_{hi} = \Lambda_{SB}, m_\rho, \dots \sim 1\text{GeV}$$

$$M_{lo} = m_\pi, f_\pi \sim 100\text{MeV}$$

- ❖ Respect all QCD symmetries
- ❖ A good strategy to fit a large number of parameters
- ❖ Can estimate theoretical errors

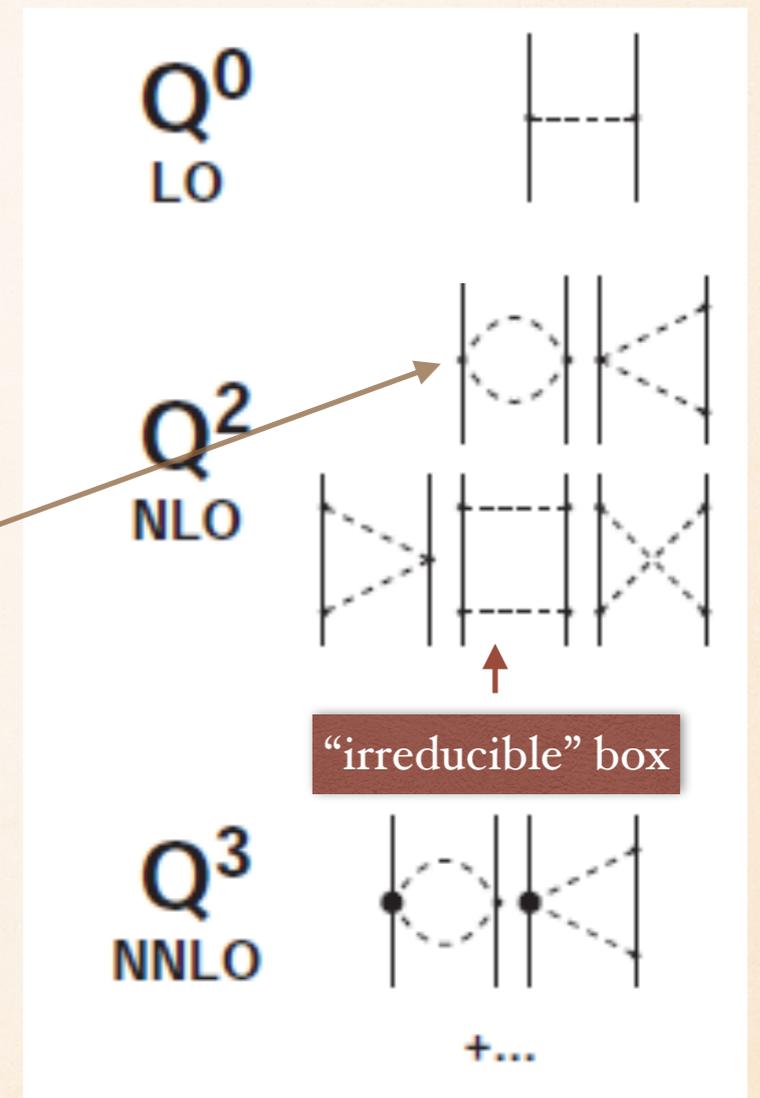
Power counting of pion-exchanges

Irreducible pion-exchanges diagrams:

- ❖ Represent long-range NN potentials
- ❖ Dictated by chiral symmetry

p.c. of non-analytic parts
pion line --- $1/Q^2$
nucleon line --- $1/Q$
loop integral --- Q^4

Weinberg (1990, 1991)



Power counting of pion-exchanges

1-pion exchange $V^{\text{OPEP}} = - \left(\frac{g_A}{2f_\pi} \right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{q^2 + M_\pi^2}$

2-pion exchange $V_{\text{NLO}}^{\text{TPEP}} = - \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{384\pi^2 f_\pi^4} L(q) \left\{ 4M_\pi^2 (5g_A^4 - 4g_A^2 - 1) \right.$

Epelbaum, et al. (1999)

$$+ q^2 (23g_A^4 - 10g_A^2 - 1) + \frac{48g_A^4 M_\pi^4}{4M_\pi^2 + q^2} \left. \right\} \longrightarrow \frac{V^{\text{OPE}}}{V^{\text{TPE}}} \sim \left(\frac{Q}{4\pi f_\pi} \right)^2$$

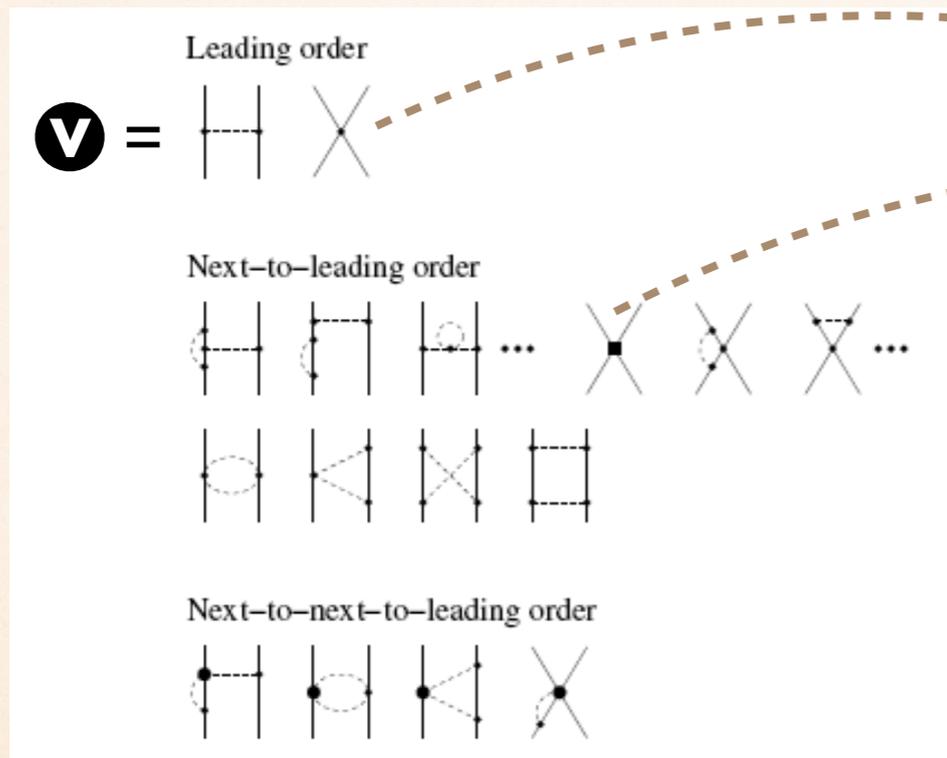
$$- \frac{3g_A^4}{64\pi^2 f_\pi^4} L(q) \{ \boldsymbol{\sigma}_1 \cdot \mathbf{q} \boldsymbol{\sigma}_2 \cdot \mathbf{q} - q^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \}$$

$$L(q) = \frac{1}{q} \sqrt{4M_\pi^2 + q^2} \ln \frac{\sqrt{4M_\pi^2 + q^2} + q}{2M_\pi}$$

- ❖ Weinberg p.c. is right about pion-exchanges
- ❖ UV div. will be accounted for when NN contact interactions are considered
- ❖ So, what about p.c. of NN contact interactions

Dr. W's prescription: NDA for contact interactions

Derivatives on short-range couplings always suppressed
by M_{hi} : naive dimensional analysis (NDA)



S-wave short-range pot.

$$\frac{g_A^2}{4f_\pi^2} \left[\tilde{C}_0 + \frac{\tilde{C}_2}{M_{\text{hi}}^2} (p^2 + p'^2) + \dots \right]$$

$C_{0,2}$: dimensionless $\sim O(1)$

What's wrong w/ NDA?

An overlooked infrared scale

Strength of OPE provides an infrared scale

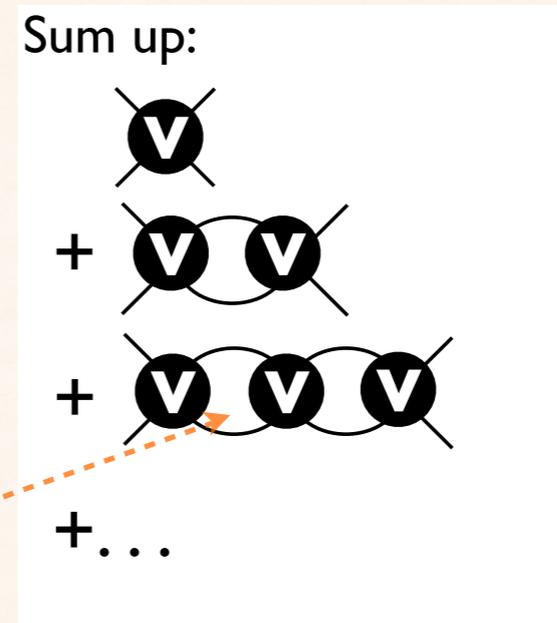
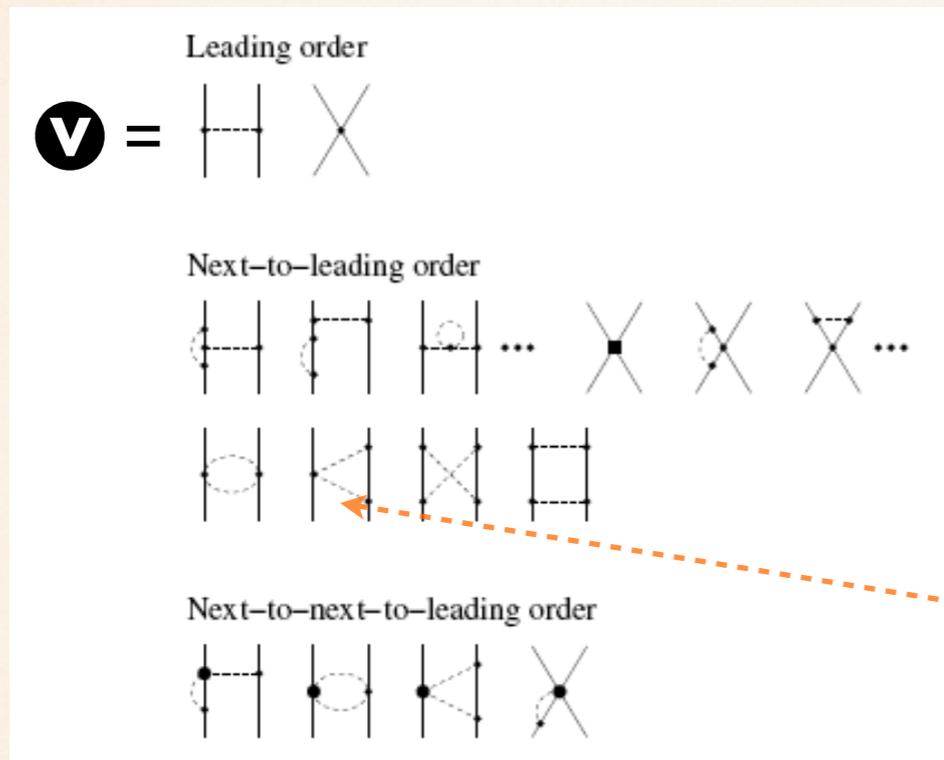
$$\text{OPE} \sim \frac{g_A^2}{f_\pi^2} \frac{Q^2}{Q^2 + m_\pi^2}$$

$$\text{Iterated OPE} \sim \left(\frac{g_A^2}{f_\pi^2} \frac{Q^2}{Q^2 + m_\pi^2} \right)^2 \frac{m_N Q}{4\pi} \sim \text{OPE} \frac{Q}{M_{NN}} \quad M_{NN} = 100 \sim 300 \text{MeV}$$

varies for different partial waves

- ❖ M_{NN} , though part of pion-exchange, can change the scaling of short-range interactions through renormalization
- ❖ Therefore, NDA is no longer powerful in counting contact interactions, because there are now two mass scales: M_{hi} and M_{NN}
- ❖ Renormalization is the reason to upset NDA

Renormalization of Sch. eqn.



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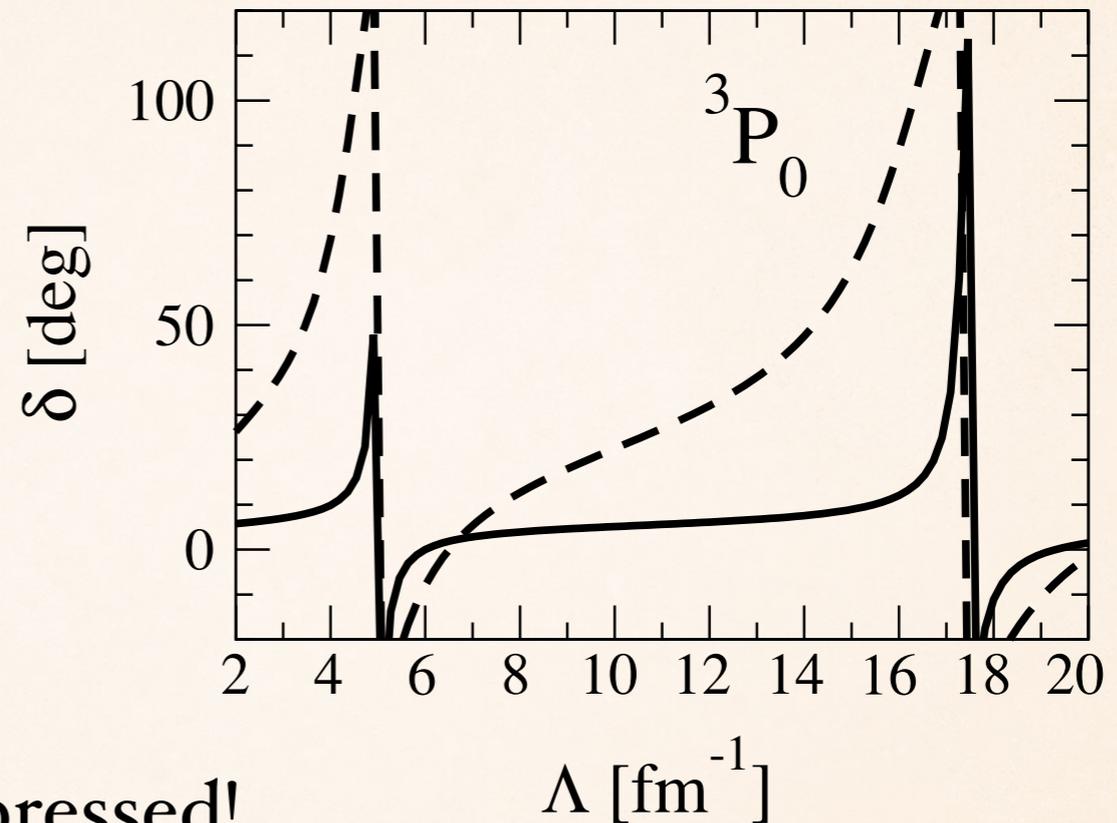
❖ However, cutoff dependence of resummed amplitudes was not addressed in W counting --- **Need to consider renormalization of the Sch. eqn.**

Cutoff dependence of W counting

Nogga, Timmerman & van Kolck (2005)

E.g., 3P_0

A **singular attractive** potential needs a counterterm — 4-nucleon operator acting in 3P_0 partial wave



➔ A derivative coupling not suppressed!

Solid: $T_{\text{lab}} = 10$ MeV, dashed: 50 MeV

Very large cutoffs were used to illustrate the cutoff dependence, but we don't insist on using them in practical calculations once power counting is established.

WPC for 3P0

LO	OPE	
$O(Q)$		
$O(Q^2)$	TPE_0	$C_2 p^2$
$O(Q^3)$	TPE_I	
$O(Q^4)$	$3PE$	$C_4 p^4$

Modified PC for $3P_0$

BwL, van Kolck (2008)
BwL, Yang (2011, 2012)
Pavon Valderrama (2011, 2012)

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Similar modifications to other attractive triplet channels ($3P_2$, maybe $3D_2$)

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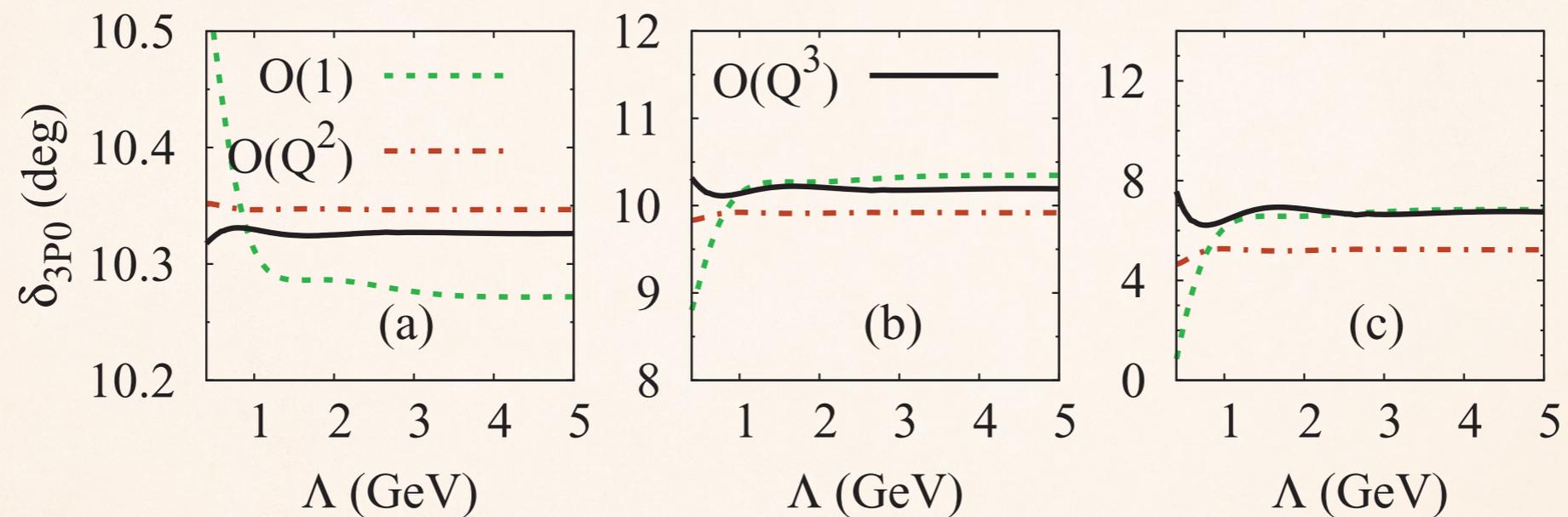
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Similar modifications to other attractive triplet channels ($3P_2$, maybe $3D_2$)

Numerics

3p0 phase shifts at a given energy vs. cutoffs



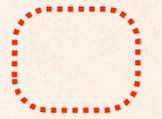
Plateaus at large cutoffs show RG invariance

Modified Game Plan

BwL & Yang (2012)
BwL (2013)

TABLE I. Power counting for pion exchanges and S - and P -wave counterterms up to $\mathcal{O}(Q^3)$. p (p') is the magnitude of the center-of-mass incoming (outgoing) momentum. The two-by-two matrices are for the coupled channels.

$\mathcal{O}(1)$	OPE, C_{1S_0} , $\begin{pmatrix} C_{3S_1} & 0 \\ 0 & 0 \end{pmatrix}$, $C_{3P_0} p' p$, $\begin{pmatrix} C_{3P_2} p' p & 0 \\ 0 & 0 \end{pmatrix}$
$\mathcal{O}(Q)$	$D_{1S_0} (p'^2 + p^2)$
$\mathcal{O}(Q^2)$	TPE0, $E_{1S_0} p'^2 p^2$, $\begin{pmatrix} D_{3S_1} (p'^2 + p^2) & E_{SD} p^2 \\ E_{SD} p'^2 & 0 \end{pmatrix}$, $D_{3P_0} p' p (p'^2 + p^2)$, $p' p \begin{pmatrix} D_{3P_2} (p'^2 + p^2) & E_{PF} p^2 \\ E_{PF} p'^2 & 0 \end{pmatrix}$, $C_{1P_1} p' p$, $C_{3P_1} p' p$
$\mathcal{O}(Q^3)$	TPE1, $F_{1S_0} p'^2 p^2 (p'^2 + p^2)$



couplings
enhanced relative
to WPC

Summary and outlook

- ❖ RG invariance demands modifying Weinberg's scheme: Some of the NN contact operators need promotions
- ❖ Need to produce a new potential
- ❖ Need to look at few-nucleon sector and electroweak reactions:
Can we solve the “Ay puzzle” with the new potential?