Heavy quark pair production at low transverse momentum

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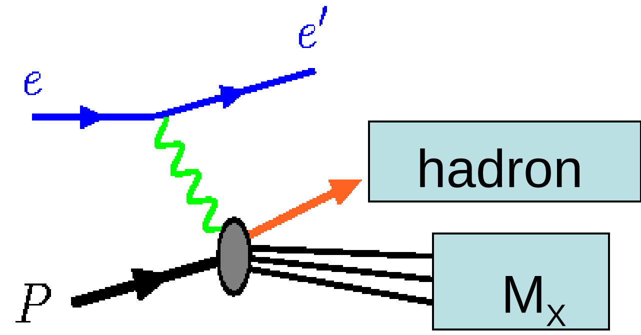
- Introduction to Transverse Momentum Dependent parton distribution functions (TMDs)

- Heavy quark pair production in DIS

- Heavy quark pair production in pp collision

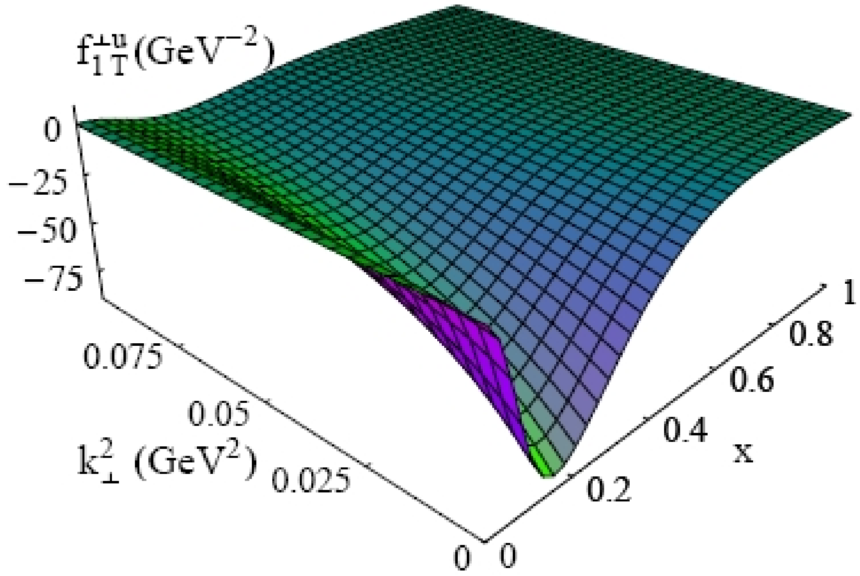
- Summary

1、 TMDs



Main focus of SIDIS and EIC studies:

- **parton spin in the nucleon (Refer to B.-Q. Ma)**
- **parton distributions (valence, sea, gluons)**
- **…**



- **What is the transverse imaging?**
- **What is the transverse momentum distribution?**

Difference between PDFs and TMDs

Parton distribution functions (PDFs) only account the longitudinal momentum information which is right in the large momentum limit or in the light-cone, however, TMDs include both the longitudinal and transverse momentum information.

→ PDFs,

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle ,$$

$$\xi^\pm = \frac{1}{2}(\xi^0 \pm \xi^3) ,$$

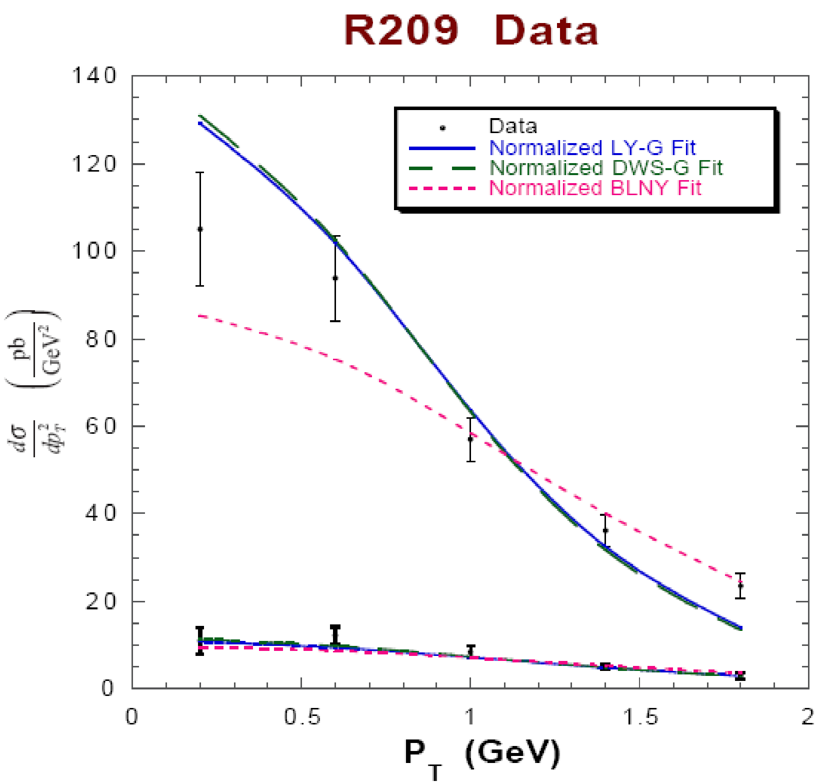
→ TMDS

$$f(x,k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp\cdot\vec{k}_\perp)} \times \langle P|\overline{\psi}(\xi^-,\xi_\perp) L^\dagger_{\xi_\perp}(\infty,\xi^-)\gamma^+ L_0(\infty,0)\psi(0)|P\rangle ,$$

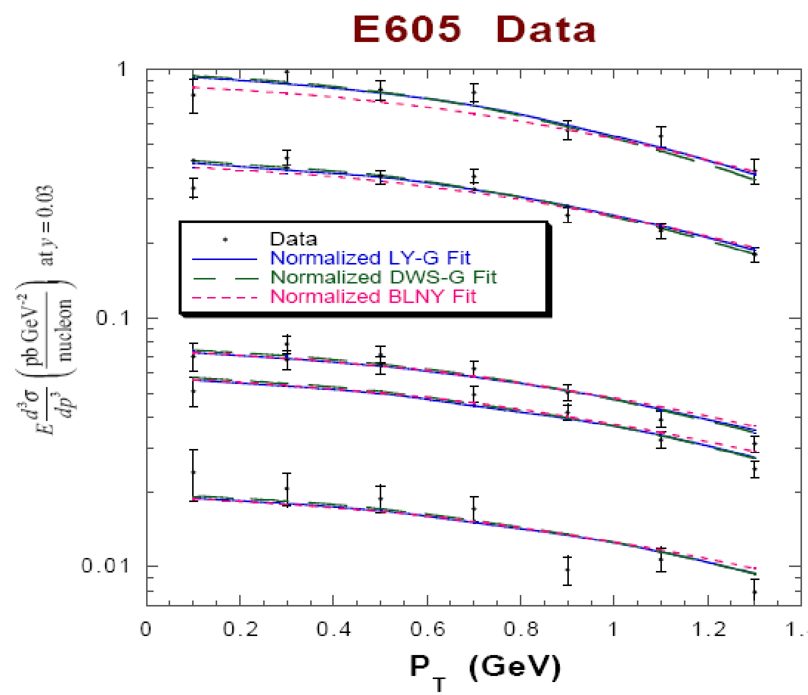
$$L_{\xi_\perp}(\infty,\xi^-) = P\exp\left(-ig\int_{\xi^-}^{\infty} A^+(\xi^-,\xi_\perp)d\xi^-\right) .$$

X.D. Ji, J.P. Ma, F. Yuan, PLB597,299(2004);
Collins, Metz.PRL93,252001(2004).

uccessful examples in TMD factorizatior



1: Comparison to the R209 data for the process $p+p \to \mu^+\mu^- + X$ process at $\sqrt{S} = 62\,\mathrm{GeV}$.



Comparison to the E605 data for the process $p + Cu \to \mu^+\mu^- + X$ at $\sqrt{S} = 38.8\,\mathrm{GeV}$.

Ladinsky, Brock, Nadolsky, Yuan,PRD67,073016(2003)

2、 Heavy quark pair production in DIS

R.L. Zhu, P. Sun, F. Yuan, PLB727,474(2013)

For the process $e^-(\ell) + H(P) \to e^-(\ell') + c(k_1) + \bar{c}(k_2) + X\,,$

→ The differential cross section

$$d\sigma = \frac{d^3\ell'}{2\ell'^0}\frac{e^4}{4\pi^2 SQ^4}L^{\mu\nu}(\ell,q)W_{\mu\nu}(q,P)\,,$$

$$L^{\mu\nu} = \frac{1}{2}\sum \bar{u}(\ell')\gamma^\mu u(\ell)\bar{u}(\ell)\gamma^\nu u(\ell')\,,$$

$$W_{\mu\nu} = \frac{1}{8\pi}\sum_S\sum_X \langle H(P,S)|J_\mu^{V\dagger}(0)|X\rangle\langle X|J_\nu^V(0)|H(P,S)\rangle(2\pi)^4\delta(P+q-p_X)\,,$$

→ In the small Pt (heavy quark pair) limit,

$$\frac{d\sigma}{dx_B dy dp_\perp^2 dM_{c\bar{c}}^2 d\cos\theta} = \int \frac{d^2b_\perp}{(2\pi)^2}e^{ip_\perp\cdot b_\perp}\frac{4\pi^2 S\alpha^2}{Q^4}\left(x_B y^2\sigma_1 W_1(x,b_\perp) + (1-y)\frac{\nu\sigma_2}{M_H}W_2(x,b_\perp)\right),$$

$$W_i = H_i(\tilde{Q},\mu)xg(x,b_\perp,\mu,\zeta)S(b_\perp,\mu)\;.$$

W function in b-space

→ into three parts $W_{c\bar{c}}(x, b_\perp) = H(\widetilde{Q}, \mu) x g(x, b_\perp, \widetilde{Q}, \mu) \bar{S}(b_\perp, \mu),$

Hard kernel **TMDs** **Soft factor**

→ A gauge invariant matrix element, i.e. soft factor is defined

→The expression can be turned into Pt-space using Fourier transformation

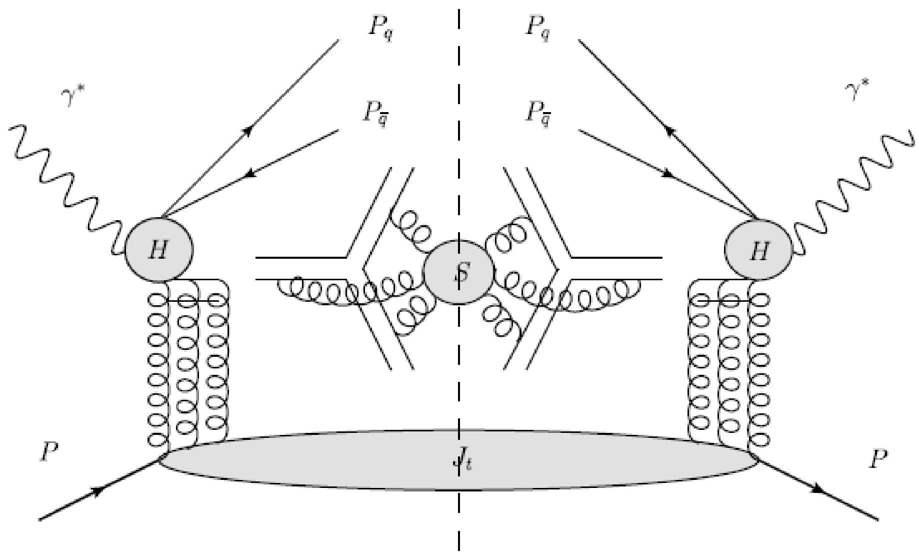


Fig. 1. TMD factorization for heavy quark pair production in DIS process.

Quark and gluon TMDs

The nonperturbative long-distance matrix element we needed are

→ quark transverse momentum dependent distribution

$$xq(x,k_\perp,\mu,\zeta,\rho) = \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \times \frac{\left\langle P|\psi(\xi^-,\xi_\perp)\mathcal{L}_v^\dagger(\xi^-,\xi_\perp)\gamma^+\mathcal{L}_v(0,0_\perp)\psi(0)|P\right\rangle}{\langle 0|\mathcal{L}_{\bar{v}}^\dagger(b_\perp;\infty)\mathcal{L}_v^\dagger(\infty;b_\perp)\mathcal{L}_v(0;\infty)\mathcal{L}_{\bar{v}}(\infty;0)|0\rangle/N_c},$$

→ gluon transverse momentum dependent distribution

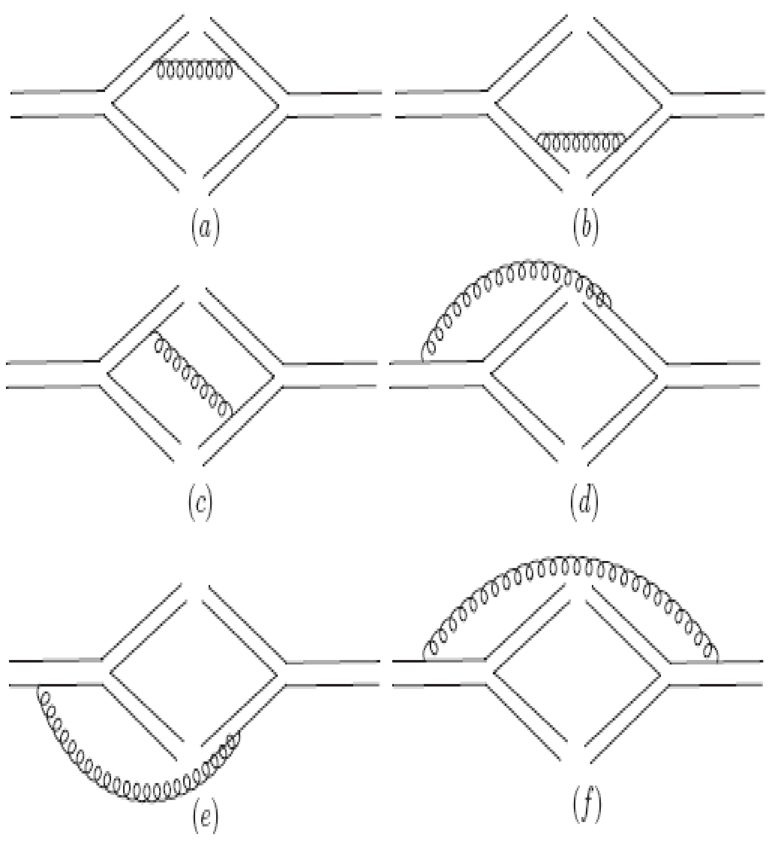
$$xg(x,k_\perp,\mu,\zeta,\rho) = \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \times \frac{\left\langle P|F_{a\ \mu}^{+}(\xi^-,\xi_\perp)\mathcal{L}_{vab}^\dagger(\xi^-,\xi_\perp)\mathcal{L}_{vbc}(0,0_\perp)F_c^{\mu+}(0)|P\right\rangle}{\langle 0|\mathcal{L}_{\bar{v}cb'}^\dagger(b_\perp;\infty)\mathcal{L}_{vb'a}^\dagger(\infty;b_\perp)\mathcal{L}_{vab}(0;\infty)\mathcal{L}_{\bar{v}bc}(\infty;0)|0\rangle/(N_c^2-1)}$$

X.D. Ji, J.-P. Ma, F. Yuan. PLB597,299(2004); PRD71,034005(2005).

Collins, Metz,PRL93,252001(2004).

Soft factor definition

$$\overline{S}(b_\perp,\mu,\rho) = \frac{\int_0^\pi \frac{d\phi}{\pi} \langle 0|\mathcal{L}^\dagger_{\bar{v}ca'}(b_\perp)\mathrm{Tr}\left[\mathcal{L}^\dagger_{n_c}(b_\perp)\mathrm{T}^{a'}\mathcal{L}^\dagger_{n_{\bar{c}}}(b_\perp)\mathcal{L}_{n_{\bar{c}}}(0)\mathrm{T}^a\mathcal{L}_{n_c}(0)\right]\mathcal{L}_{\bar{v}ac}(0)|0\rangle}{\mathrm{Tr}[\mathrm{T}^d\mathrm{T}^d]}.$$



Gauge invariant,
can be calculated in
Perturbative Theory

Results for soft factor,hard kernel

$$\bar{S}^{(1)}_{\rm JMY}(b_\perp,\mu,\rho)=\frac{\alpha_s}{2\pi}\left\{C_A\ln\frac{c_0^2}{b_\perp^2\mu^2}\left(B_{final}+\ln\rho^2+\ln\frac{\tilde{Q}^2}{\zeta^2}-1\right)+C_{final}\right\},$$

$$B_{final}=\frac{1}{2N_c^2}\frac{1+\beta^2}{\beta}\ln\frac{1-\beta}{1+\beta}-2\frac{C_F}{N_c}+\ln\frac{t_1u_1}{\tilde{Q}^2m_c^2},$$

$$C_{final}=\frac{1}{2N_c}f_{c\bar{c}}+2C_F\ln\frac{1-\beta^2\cos^2\theta}{1-\beta^2}+C_A\mathrm{Li}_2\left(-\frac{\beta^2\sin^2\theta}{1-\beta^2}\right).$$

$$f_{c\bar{c}}=\frac{(1+\beta^2)}{\beta}f^a_{c\bar{c}}-\frac{4(1+\beta^2)(1-\beta\cos\theta)}{1-\beta^2}f^b_{c\bar{c}},$$

$$f^a_{c\bar{c}}=\left(\ln\frac{b_1}{b_4}\right)^2-\left(\ln\frac{b_3}{b_2}\right)^2+2\ln\frac{b_3b_4}{b_1b_2}\ln\frac{b_1\cot^2(\frac{\theta}{2})}{b_2}+2\left(\mathrm{Li}_2\left(\frac{b_2}{b_4}\right)+\mathrm{Li}_2\left(\frac{b_4}{b_1}\right)-\mathrm{Li}_2\left(\frac{b_2}{b_3}\right)-\mathrm{Li}_2\left(\frac{b_3}{b_1}\right)\right),$$

$$H^{(1)}_{\rm JMY}(\mu,\rho)=\frac{\alpha_sC_A}{\pi}\left\{\left(\beta_0-\frac{B_{final}}{2}+\frac{\ln\rho}{2}-\frac{3}{4}\right)\ln\frac{\tilde{Q}^2}{\mu^2}+\frac{\ln^2\rho}{4}-\frac{3}{4}\ln\rho+\frac{\pi^2}{6}+\frac{7}{4}+B_f^{V,g\gamma}\right\},$$

Resummation

CSS Eq.

$$\frac{\partial}{\partial \ln \zeta} g(x, b_\perp, \mu, \zeta, \rho) = \left(K(b_\perp, \mu) + G(\zeta, \mu)\right) g(x, b_\perp, \mu, \zeta, \rho) \ ,$$

$$K(b_\perp, \mu) = -\frac{\alpha_s C_A}{\pi} \ln \frac{\mu^2 b_\perp^2}{4} e^{2\gamma_E} \ , \quad G(\zeta, \mu) = -\frac{\alpha_s C_A}{\pi} \left(\ln \frac{\zeta^2}{\mu^2} - \frac{3}{2} \right) \ ,$$

RG Eq.

$$\frac{d}{d \ln \mu} S(b_\perp, \mu, \rho) = \gamma_S(\mu, \rho) \ S(b_\perp, \mu, \rho) \ ,$$

$$\gamma_S(\mu, \rho) \ = \ -\frac{\alpha_s(\mu) C_A}{\pi} (B_{final} + \ln \rho - 1) \ .$$

Resummation fomurla

$$\begin{aligned} W_{h\bar{h}}(x, b_\perp, \overline{Q}^2) \ &= \ g(x, b_\perp, \overline{Q}_0, \overline{Q}_0) S(b_\perp, \overline{Q}_0) H(\overline{Q}, \overline{Q}) \\ &\times \ \exp\left[-\int_{\overline{Q}_0}^{\overline{Q}} \frac{d\mu}{\mu} \left(\ln \frac{\overline{Q}}{\mu} \gamma_K(\mu) - \gamma_S(\mu, 1) + \frac{\alpha_s C_A}{\pi} (1 - 2\beta_0 - \ln \frac{\overline{Q}_0^2 b_\perp^2}{4} e^{2\gamma_E}) \right) \right] \ . \end{aligned}$$

3、 Heavy quark pair production in pp collision

C.-F. Qiao, P. Sun, F. Yuan, R.-L. Zhu, to be prepared

- Factorization valid in heavy quark pair production in DIS
- It is natural to applying to pp collision
- Quark’s and gluon’s tomography, both are involved
- The soft factor may be different
- Proof to all orders of strong coupling constant is lacking, factorization breaking?
- …

Differential cross section

For a process $H_1(P_1) + H_2(P_2) \to c(k_1) + \bar{c}(k_2) + X$.

→ In the low Pt region, we can write as follows

$$\frac{d^3\sigma(M_{c\bar{c}}^2, q_\perp, y)}{dM_{c\bar{c}}^2 dy d\cos\theta d^2q_\perp} = \sum_{ab} \frac{\pi\beta\alpha_s^2}{8M_{c\bar{c}}^2 S d_{ab}} \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} W_{ab\to c\bar{c}}(x_1, x_2, b_\perp) + Y_{ab\to c\bar{c}} \right] ,$$

$$W_{ij}(x_i, b_\perp, M_{c\bar{c}}^2) = f_i(x_1, b_\perp, \zeta_1, \mu, \rho) f_j(x_2, b_\perp, \zeta_2, \mu, \rho) \times Tr[\mathrm{H}_{ij}(M_{c\bar{c}}^2, \mu, \rho)\mathrm{S}_{ij}(b_\perp, \mu, \rho)] ,$$

where Y term represents the higher-order in the expansion of Pt

→ when analyzing the collinear gluon contribution, we return into

$$\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}_\perp} W_{ab\to c\bar{c}}^{collinear}(x_1, x_2, b_\perp) =$$

$$\frac{\alpha_s C_{ab}\sigma_{ab}^{(0)}}{2\pi} \frac{1}{q_\perp^2} \int_{\xi_1}^1 \int_{\xi_2}^1 dx_1' dx_2' f_1(x_1) f_2(x_2) \left[\{x_1'\mathcal{P}(x_1')\delta(x_2'-1) + (x_1' \leftrightarrow x_2')\} \right]$$

Quark and gluon TMDs

The nonperturbative long-distance matrix element we needed are

→ quark transverse momentum dependent distribution

$$xq(x,k_\perp,\mu,\zeta,\rho) = \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \times \frac{\left\langle P|\psi(\xi^-,\xi_\perp)\mathcal{L}_v^\dagger(\xi^-,\xi_\perp)\gamma^+\mathcal{L}_v(0,0_\perp)\psi(0)|P\right\rangle}{\langle 0|\mathcal{L}_{\bar{v}}^\dagger(b_\perp;\infty)\mathcal{L}_v^\dagger(\infty;b_\perp)\mathcal{L}_v(0;\infty)\mathcal{L}_{\bar{v}}(\infty;0)|0\rangle/N_c},$$

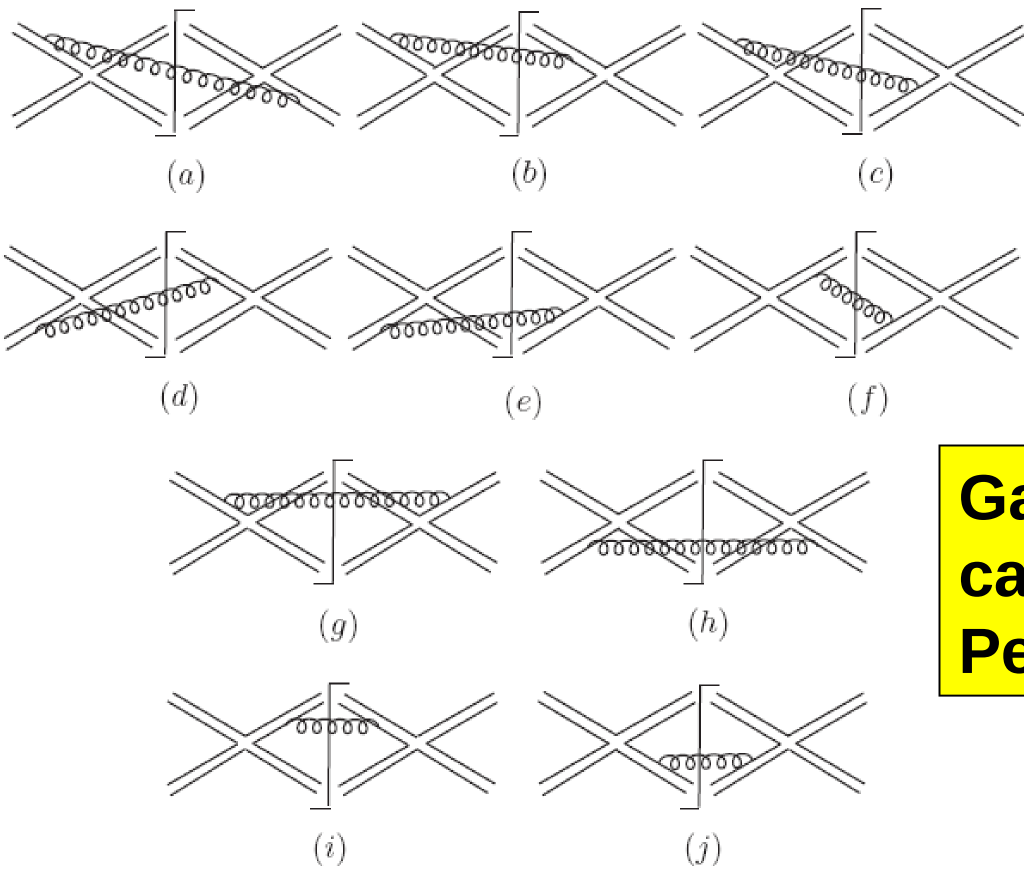
→ gluon transverse momentum dependent distribution

$$xg(x,k_\perp,\mu,\zeta,\rho) = \int \frac{d\xi^- d^2\xi_\perp}{P^+(2\pi)^3} e^{-ixP^+\xi^- + i\vec{k}_\perp\cdot\vec{\xi}_\perp} \times \frac{\left\langle P|F_{a\ \mu}^{+}(\xi^-,\xi_\perp)\mathcal{L}_{vab}^\dagger(\xi^-,\xi_\perp)\mathcal{L}_{vbc}(0,0_\perp)F_c^{\mu+}(0)|P\right\rangle}{\langle 0|\mathcal{L}_{\bar{v}cb'}^\dagger(b_\perp;\infty)\mathcal{L}_{vb'a}^\dagger(\infty;b_\perp)\mathcal{L}_{vab}(0;\infty)\mathcal{L}_{\bar{v}bc}(\infty;0)|0\rangle/(N_c^2-1)}$$

X.D. Ji, J.-P. Ma, F. Yuan. PLB597,299(2004);
PRD71,034005(2005).
Collins, Metz,PRL93,252001(2004).

Soft factor definition

$$S_{IJ} = \int_0^{\pi} \frac{d\phi}{\pi} \frac{C^{bb'}_{Iii'} C^{aa'}_{Jll'}}{d_R} \langle 0 | \mathcal{L}^{\dagger}_{vcb'} \mathcal{L}_{vbc} \mathcal{L}^{\dagger}_{\bar{v}ca'} \mathcal{L}_{\bar{v}ac} \mathcal{L}^{\dagger}_{tji} \mathcal{L}_{ti'k} \mathcal{L}^{\dagger}_{\bar{t}kl} \mathcal{L}_{\bar{t}l'j} | 0 \rangle,$$



Gauge invariant, can be calculated in Perturbative Theory

Results for soft factor

It can be decomposed into two parts $\mathrm{S}^{(1)}_{\bar{i}i,IJ} = \sum_{j,k} W^{jk}_{IJ} I_{jk}\,,$

→ color matrix elements $W^{j\bar{k}}_{IJ}$

→ master integrals

$$I_{jk} \;=\; -\frac{g^2\mu^{2\epsilon}}{(2\pi)^{3-2\epsilon}}\int_0^\pi \frac{(\sin\phi)^{-2\epsilon}}{a_1}d\phi\int d^{\,4-2\epsilon}k\; e^{ik_t\cdot\, b}\frac{n_j\cdot n_k}{k\cdot n_j\;k\cdot n_k}\delta(k^2)\theta(k_0)\,,$$

One loop results for soft factor

$$\begin{aligned}S^{(1)}(b_\perp,\mu) \;=\; \frac{\alpha_s}{2\pi}\Bigg\{&\ln\frac{b_\perp^2\mu^2}{b_0^2}\left(2W_{33}-W_{34}\frac{1+\beta^2}{\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)+4W_{13}\ln\left(\frac{-t_1}{M_{c\bar{c}}\,m}\right)\right.\\&\left.+4W_{23}\ln\left(\frac{-u_1}{M_{c\bar{c}}\,m}\right)\right)+2W_{34}f_{c\bar{c}}+2W_{33}\ln\frac{t_1u_1}{M_{c\bar{c}}^2m^2}\\&+2(W_{13}+W_{23})\mathrm{Li}_2(1-\frac{t_1u_1}{M_{c\bar{c}}^2m^2})\Bigg\}\,.\end{aligned}$$

In agreement with SCET results: H.X. Zhu, et. al.PRL110,082001(2013)

Resummation formula

General formalism for heavy quark pair hadroproduction

$$\frac{d\sigma}{dy dq_\perp^2 dM_{c\bar{c}}^2 d\cos\theta} = \frac{\pi^2\beta\alpha_s^2}{8M_{c\bar{c}}^2 S d_{i\bar{i}}}\frac{1}{(2\pi)^2}\int d^2b_\perp e^{iq_\perp\cdot b_\perp} W_{kl}(b_\perp, M_{c\bar{c}}, x_1, x_2) \, ,$$

W function:

$$W_{kl}\left(x_i, b_\perp, \frac{C_1^2}{C_2^2 b_\perp^2}\right) = f_k(x_1, C_1^2/(C_2^2/b_\perp^2)) f_l(x_2, C_1^2/(C_2^2/b_\perp^2)) e^{-\mathcal{S}_{Sud}(M_{c\bar{c}}^2, b_\perp, C_1/C_2)}$$
$$\times Tr\left[\mathrm{H}(M_{c\bar{c}}^2, M_{c\bar{c}}^2)\mathrm{Exp}\{-\int_{C_1^2/b_\perp^2}^{M_{c\bar{c}}^2}\frac{d\mu}{\mu}\gamma_{i\bar{i}}^{s\dagger}\}\mathrm{S}(b, \frac{C_1^2}{C_2^2 b_\perp^2})\mathrm{Exp}\{-\int_{C_1^2/b_\perp^2}^{M_{c\bar{c}}^2}\frac{d\mu}{\mu}\gamma_{i\bar{i}}^{s}\}\right] ,$$

Sudakov factor:

$$\mathcal{S}_{Sud} = \int_{C_1^2/b_\perp^2}^{C_2^2 M_{c\bar{c}}^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{C_2^2 M_{c\bar{c}}^2}{\mu^2}\right) A(C_1, \mu) + B(C_1, C_2, \mu)\right] \, .$$

$$A_{qq}^{(1)} = 2C_F, \quad B_{qq}^{(1)} = 2C_F\left[\ln\left(\frac{C_1^2}{4C_2^2}e^{2\gamma_E}\right) - \frac{3}{2}\right] ,$$
$$A_{gg}^{(1)} = 2C_A, \quad B_{gg}^{(1)} = 2C_A\left[\ln\left(\frac{C_1^2}{4C_2^2}e^{2\gamma_E}\right) - \beta_0\right] .$$

Summary

- **TMD factorization solves the low Pt problem and provides more information of partons in the nucleon**

- **We study heavy quark pair production with low Pt in DIS and pp collision**

- **Soft factors are defined in field language, and can be calculated in Perturbative Theory.**

- **Large Logarithm terms are resumed using the CSS and renormalization equation.**

Thank You !