Scalar strangeness content of the nucleon and baryon sigma terms

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arXiv:1404.4799

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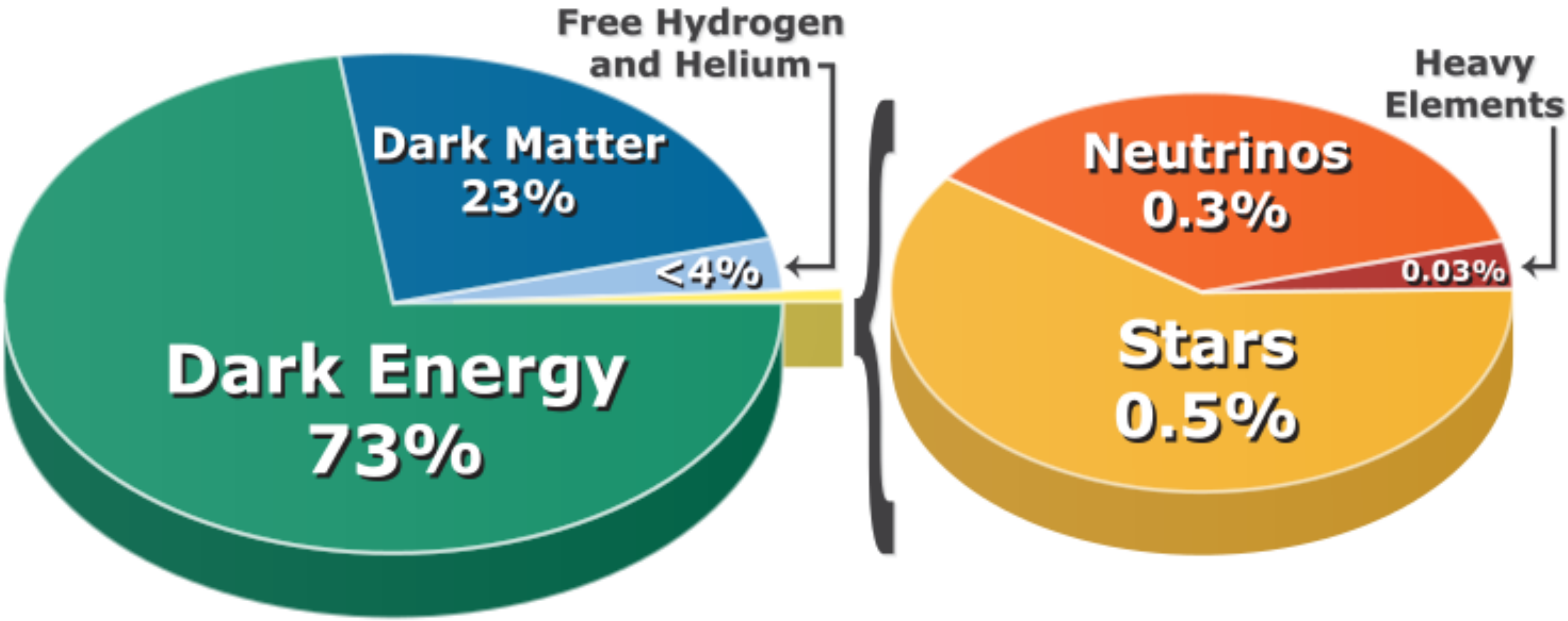
Motivation

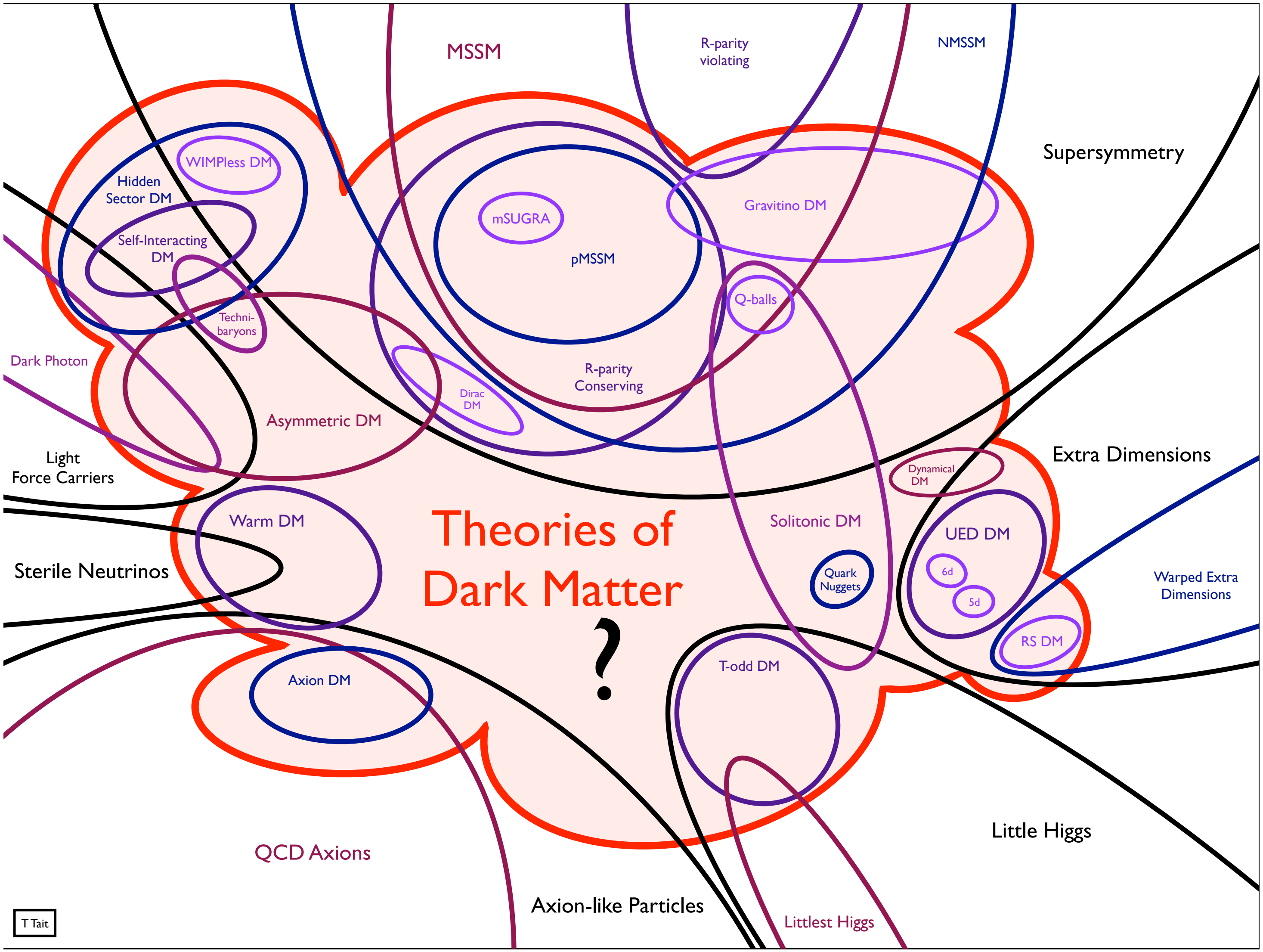
Energy-matter composition of the universe



What is dark matter?

- **What we know!**
  - Dark (electric neutral)
  - (Probably) Massive (cold/non-relativistic)
  - Still abound today (stable or with a lifetime of the age of the universe)
- **What we do not know!**
  - Mass, spin,…
  - Couplings: gravity, weak Interactions, Higgs, quarks/ gluons, leptons?
- **Questions can only be answered ultimately by experiments, but theories are needed to formulate the questions**

No lack of theories

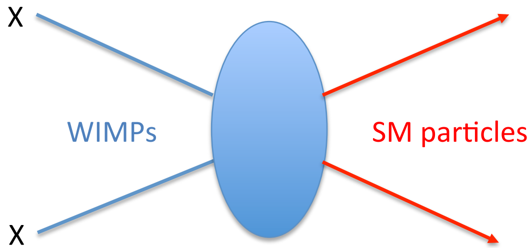


Tim M.P. Tait

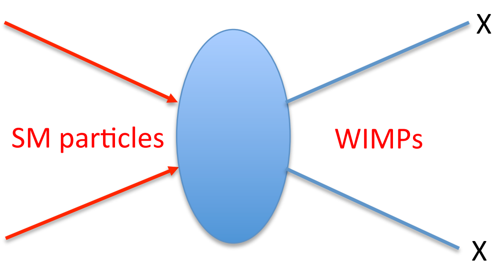
Some selected candidates

- **Axions, and axion-like particles**
- **Sterile Neutrinos**
- **Weakly Interacting Massive Particles (WIMPS)**
  - WIMPs naturally can account for the amount of dark matter we observe in the Universe
  - WIMPs automatically occur in many models of physics beyond the Standard Model

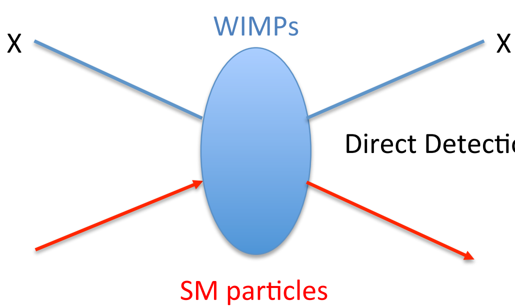
Particle searches for WIMPs



Indirect Detection

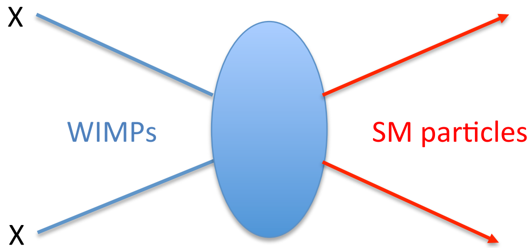


Collider Searches

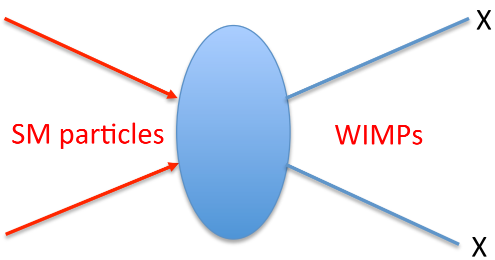


Direct Detection

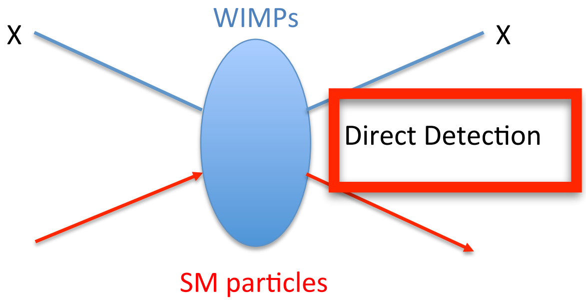
Particle searches for WIMPs



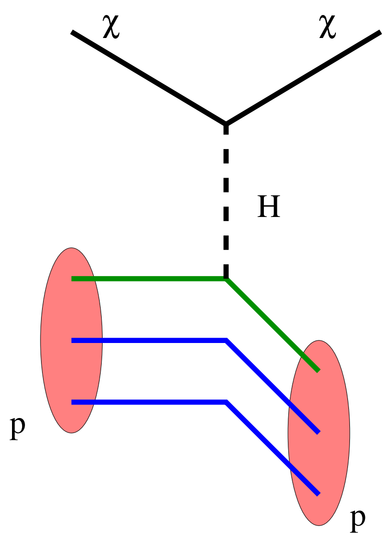
Indirect Detection



Collider Searches



Spin-independent neutrilino-nucleon scattering



$$\mathcal{L}_{int} = \lambda_N \bar{n} n \bar{\chi}\chi \rightarrow \mathcal{L}_{int} = \lambda_q \bar{q} q \bar{\chi}\chi$$

$$\lambda_N \longrightarrow \sum_{q=1}^{6} f_q^N \lambda_q$$

Spin indep. WIMP-N X-section

$$\sigma_{SI} = \frac{4M^2}{\pi}\left[Z f_P + (A-Z) f_N\right]$$

with

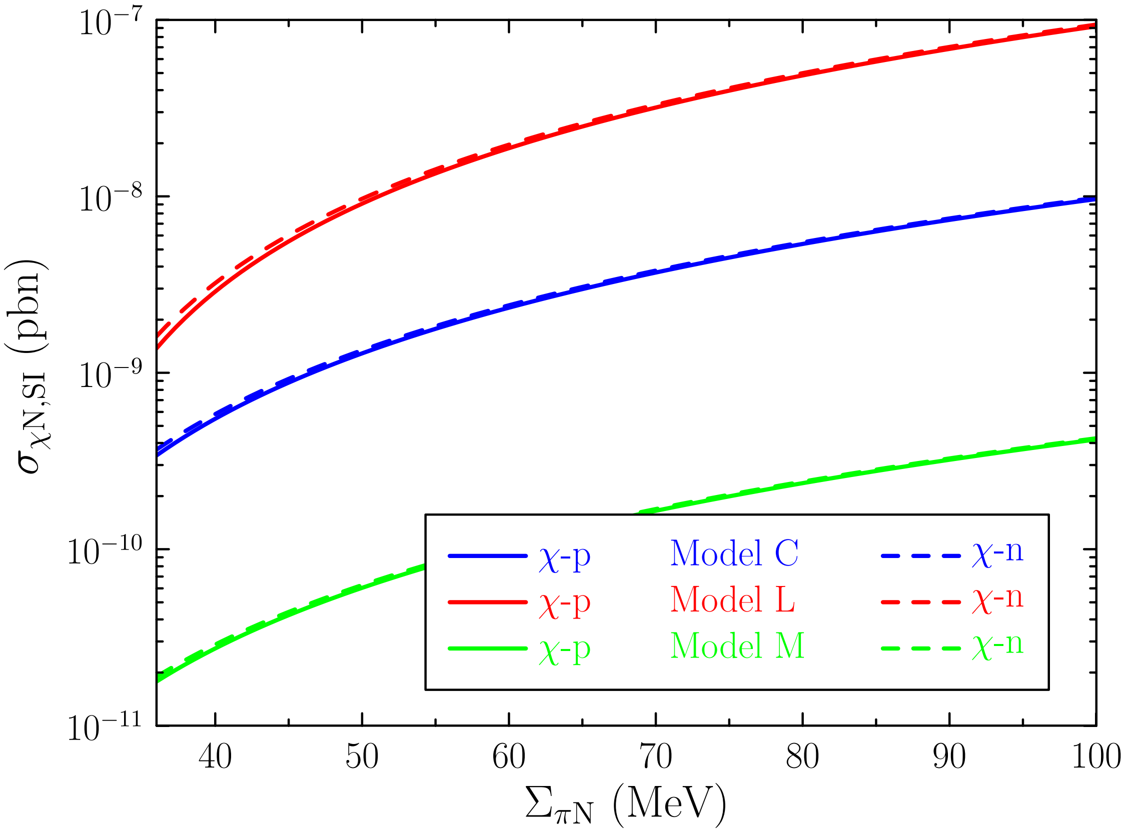
$$\frac{f_N}{M_N} = \sum_q f_q^N \frac{\lambda_q}{m_q}$$

pion- and strangeness sigma terms (SM physics)

$$\boxed{f_{ud}^N M_N = \sigma_{\pi N} = m_q \langle N | u\bar{u} + d\bar{d} | N \rangle}$$

$$\boxed{f_s^N M_N = \sigma_{sN}/2 = m_s \langle N | s\bar{u} | N \rangle}$$

Strong dependence on the strangeness sigma term



$$\sigma_{\chi p,\mathrm{SI}} \sim (\Sigma_{\pi N} - \sigma_0)^2.$$

$$\sigma_{sN} \propto \Sigma_{\pi N} - 36$$

Ellis, Olive, Savage, PRD77(2008)065026

Determination of the sigma terms

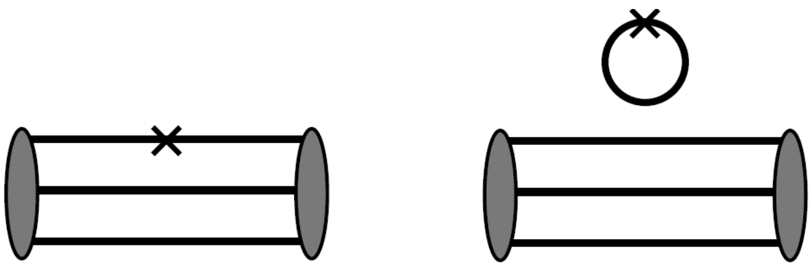
- Experimentally, the pion sigma term can be inferred from pion-nucleon scattering data at Cheng-Dashen point $(s = u = m_N^2, t = 2M_\pi^2)$

$$\sigma_{\pi N} = 45 \pm 8 MeV$$

- Because of lack of kaon-nucleon scattering data, the strangeness-sigma term cannot be obtained this way
  - Lattice QCD might be our hope to predict it from first principles

LQCD determination of sigma terms

- **Direct method**—calculates the 3-point connected and disconnect diagrams



  - JLQCD coll., PRD83,114506 (2011)
  - R. Babich *et al.*, PRD85,054510 (2012)
  - QCDSF coll., PRD85, 054502 (2012)
  - ETM coll., JHEP 1208,037(2012)
  - M. Engelhardt *et al.*, PRD86, 114510 (2012)
  - JLQCD coll., PRD87, 034509 (2013)

- **Spectrum method**-calculates the baryon masses, and relates the sigma terms to their quark mass dependence via the Feynman Hellman theorem

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l}$$

$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}.$$

  - JLQCD coll., PRD83,114506 (2011)
  - R. Babich *et al.*, PRD85,054510 (2012)
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Why ChPT?

- By themselves, both methods suffer from a number of drawbacks in addition to the inherent artifacts of LQCD simulations, e.g.,
  - Method 1 still too time consuming, noise/signal ratio, etc.
  - Method 2 requires calculations at quark masses both larger and smaller than their physical counterparts
- **ChPT can help not only in alleviating some of the drawbacks but also removing the LQCD artifacts**

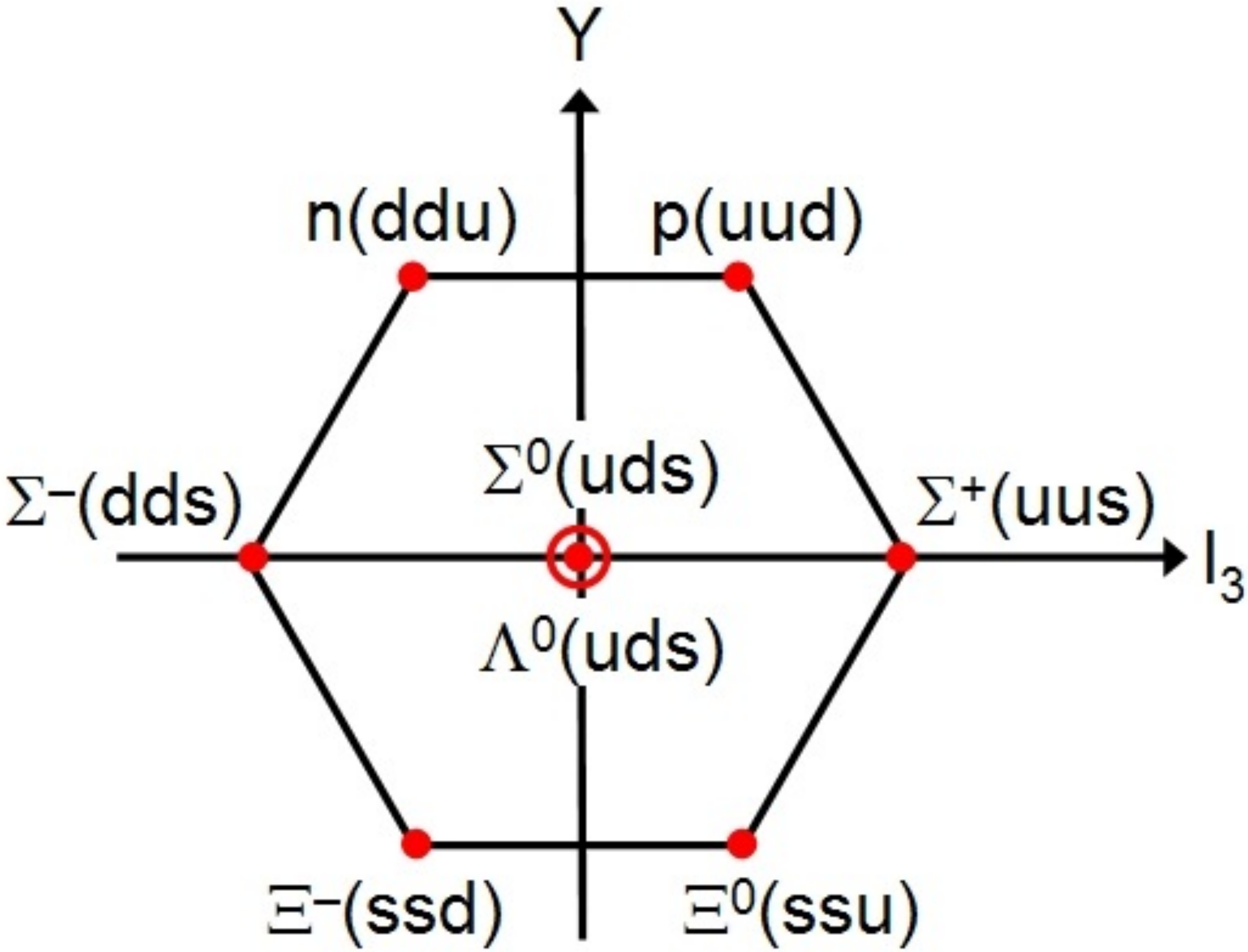
Our aim

- To apply the Feynman-Hellmann theorem to predict the baryon sigma terms using the covariant (EOMS) baryon chiral perturbation theory

$$\boxed{\begin{aligned}\sigma_{\pi B} &= m_l\langle B(p)|\bar{u}u+\bar{d}d|B(p)\rangle = m_l\frac{\partial M_B}{\partial m_l}\\ \sigma_{sB} &= m_s\langle B(p)|\bar{s}s|B(p)\rangle = m_s\frac{\partial M_B}{\partial m_s}.\end{aligned}}$$

- To fix the unknown low-energy constants of BChPT, we rely on the lQCD simulations of baryon masses

Quark-flavor structure of octet baryons



Naive quark model

Quark-flavor structure of the proton

- Naive quark model—minimal quark contents

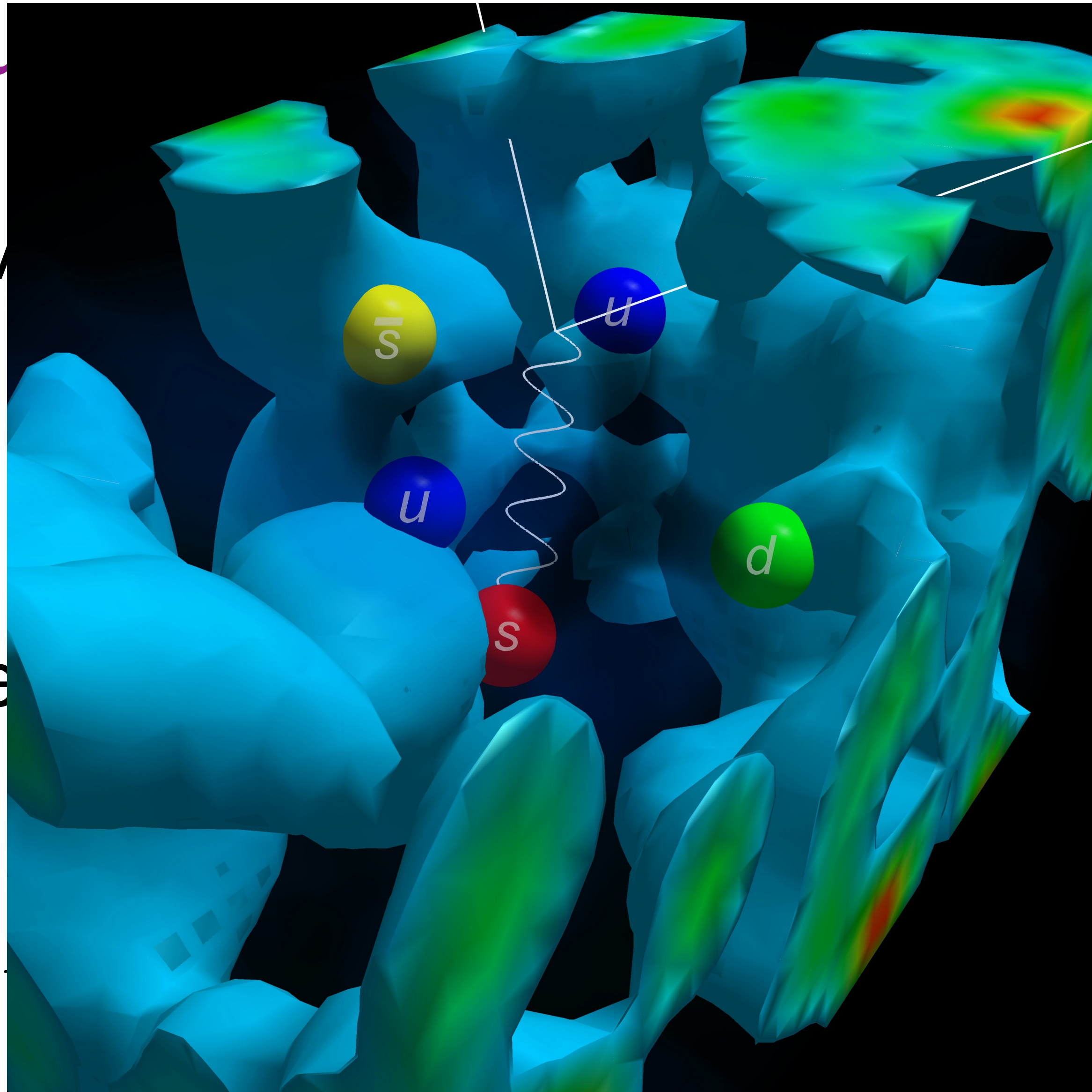
$$|p\rangle = |uud\rangle$$

- In reality,

$$|p\rangle = |uud\rangle(1 + |u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)$$

Qu oton

- Naiv contents



- In re

$$| \quad + |s\bar{s}\rangle)$$

Strangeness content of the proton

- to the spin
  - deep-inelastic lepton scattering
- to the electromagnetic form factors
  - parity-violating electron-proton scattering
- to the mass
  - scalar strangeness content, cannot be measured directly

$$\langle N | s\bar{s} | N \rangle$$

Strangeness content of the proton

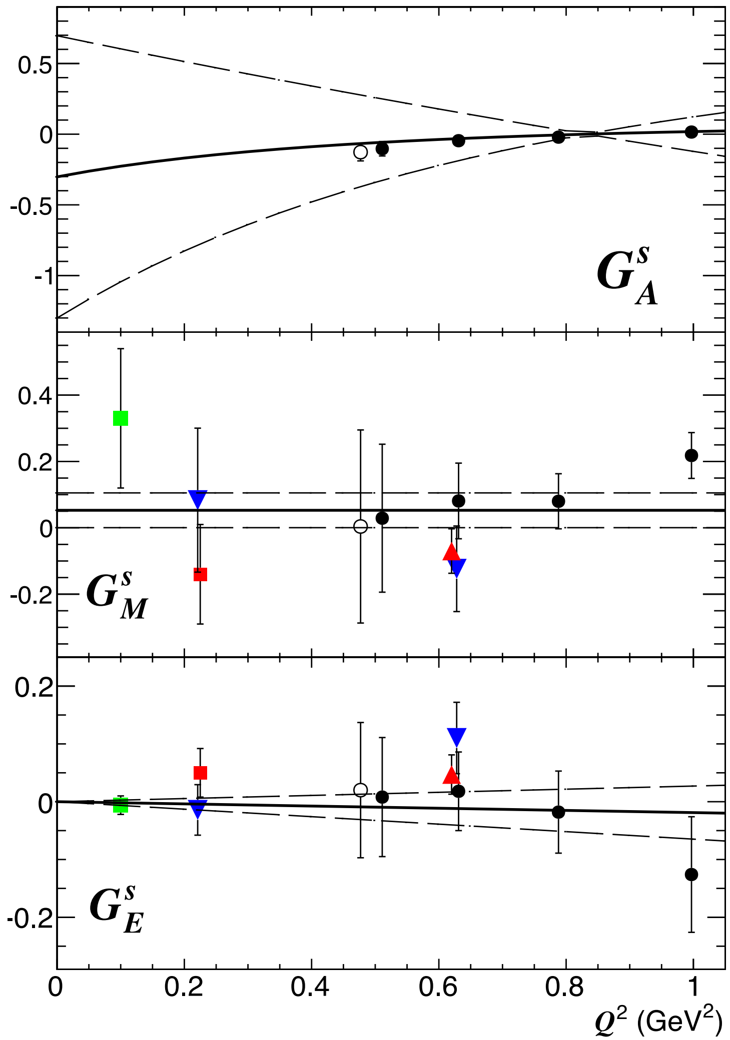
- to the spin
  - deep-inelastic lepton scattering
- to the electromagnetic form factors
  - parity-violating electron-proton scattering
- to the mass
  - scalar strangeness content, cannot be measured directly

$$\langle N | s\bar{s} | N \rangle$$

How to obtain the scalar strangeness content of the nucleon from the LQCD masses using Chiral Perturbation Theory

Global fit of the strangeness vector and axial vector form factors of the nucleon

arXiv:1308.5694



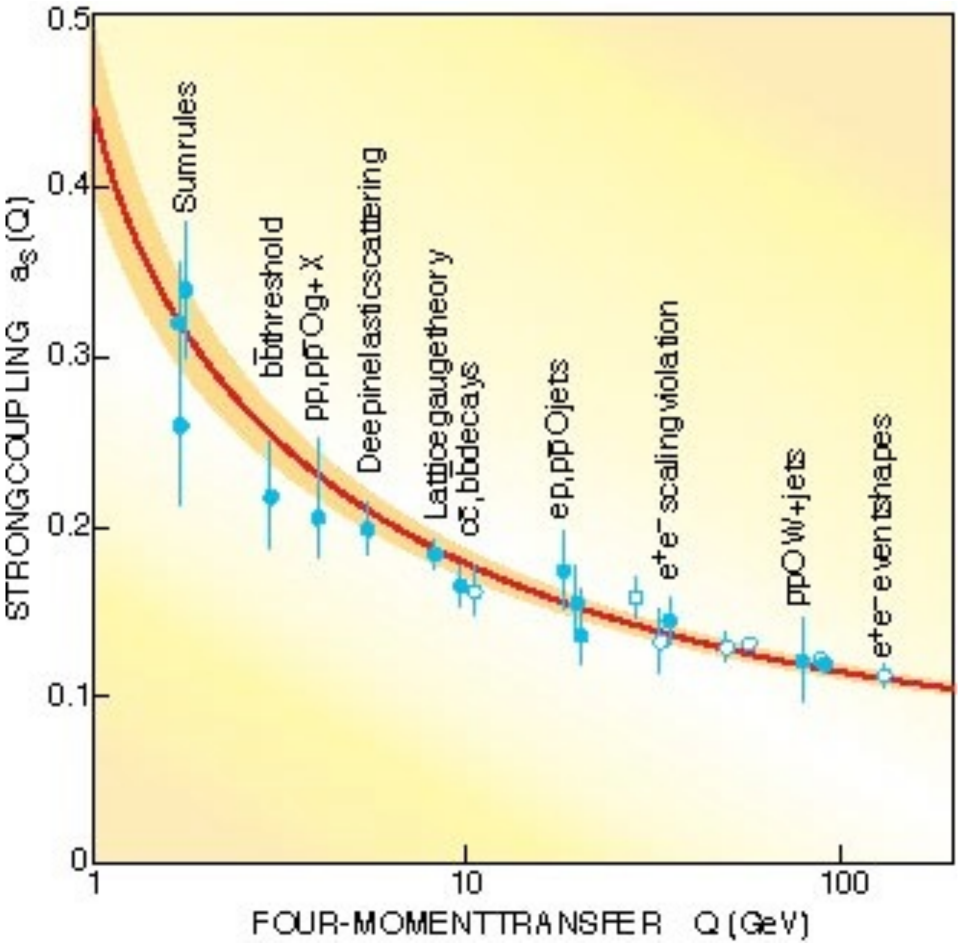
Parameter	Fit value
ρ_s	-0.071 ± 0.096
μ_s	0.053 ± 0.029
ΔS	-0.30 ± 0.42
Λ_A	1.1 ± 1.1
S_A	0.36 ± 0.50

- The electric and magnetic form factors are consistent with **zero, but not** the axial-vector form factor

A few words on LQCD and BChPT, and why BChPT is needed

QCD—non-perturbative at low energies

- Quantum ChromoDynamics—the theory of the strong interaction



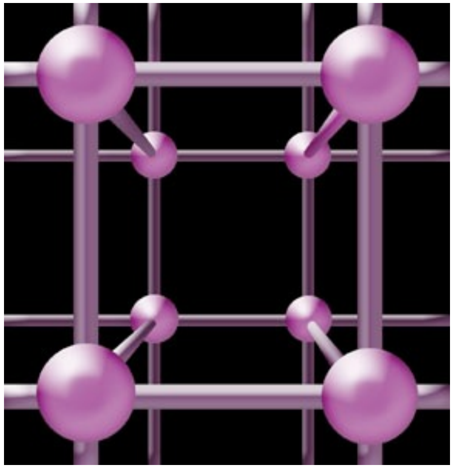
Asymptotic freedom—
Nobel prize in physics 2004

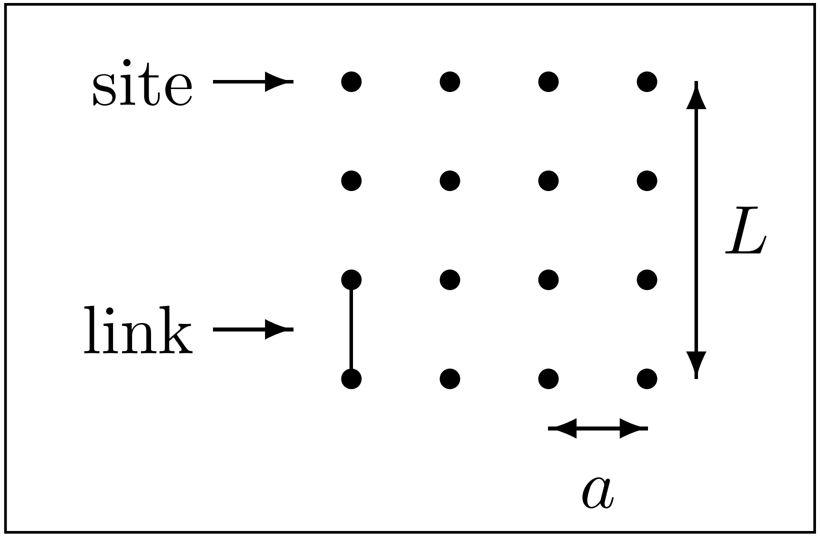
High energy: perturbative QCD successful

Low energy: non-perturbative problematic

- ✓ Phenomenological models
- ✓ Effective field theories
- ✓ LQCD

Brute Force: Lattice QCD





Basic idea： discretize space-time and solve non-perturbative strong interaction physics in a finite hypercube, utilizing monte carlo sampling techniques

Calculating path-integral in Euclidean space-time

- **Vacuum**

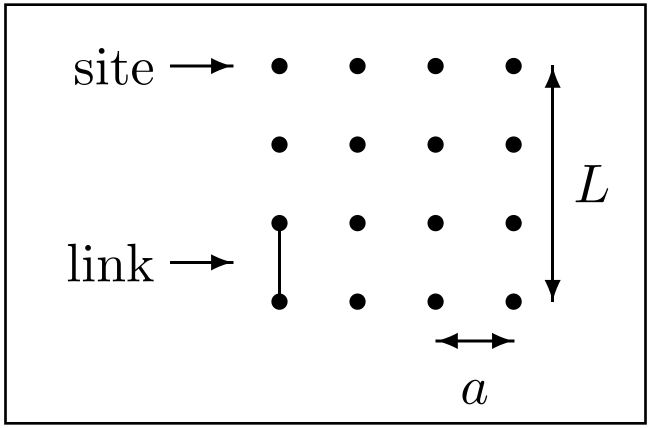
$$\boxed{Z = \int [DU] e^{-S_g(U) + \mathrm{Tr}\ln M[U]}}$$

- **Observable**

$$\boxed{\langle O \rangle = \int [DU] O(U) e^{-S_g(U) + \mathrm{Tr}\ln M[U]}}$$

Parameters and simulation costs

- light quark masses: m_u/m_d
- lattice spacing: a
- lattice volume: $V=L^4$



$$\text{cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_\pi^2 a}$$

- To reduce cost: employ larger than physical light quark masses, finite lattice spacing and volume.
- To obtain physical quantities, multiple extrapolations are needed

Multiple extrapolations

- **Chiral extrapolations:** **light quark masses to their physical values**

$$\boxed{m_q \to m_q(\mathrm{Phys.})}$$

- **Finite volume corrections:** **infinite space-time**

$$\boxed{L \to \infty}$$

- **Continuum extrapolations:** **zero lattice spacing**

$$\boxed{a \to 0}$$

Why Chiral Perturbation Theory needed?

- **All can be performed with the help of Chiral Perturbation Theory**
- **The low-energy effective field theory of QCD**
  - **provides a bridge to link LQCD simulations to the physical world**
  - **helps/guides to perform the aforementioned extrapolations**

Interplay between ChPT and LQCD Simulations

- As the low-energy EFT of QCD, ChPT provides a model-independent description of low-energy strong interaction phenomena by itself
- At higher orders, which are needed to achieve accuracy at the few percent level, there might be too many unknown low-energy constants (LECs), which can not easily be determined by experimental data alone
- LQCD simulations provide a solution to overcome the above difficulty

Chiral Perturbation Theory (ChPT) in essence

- **Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons**

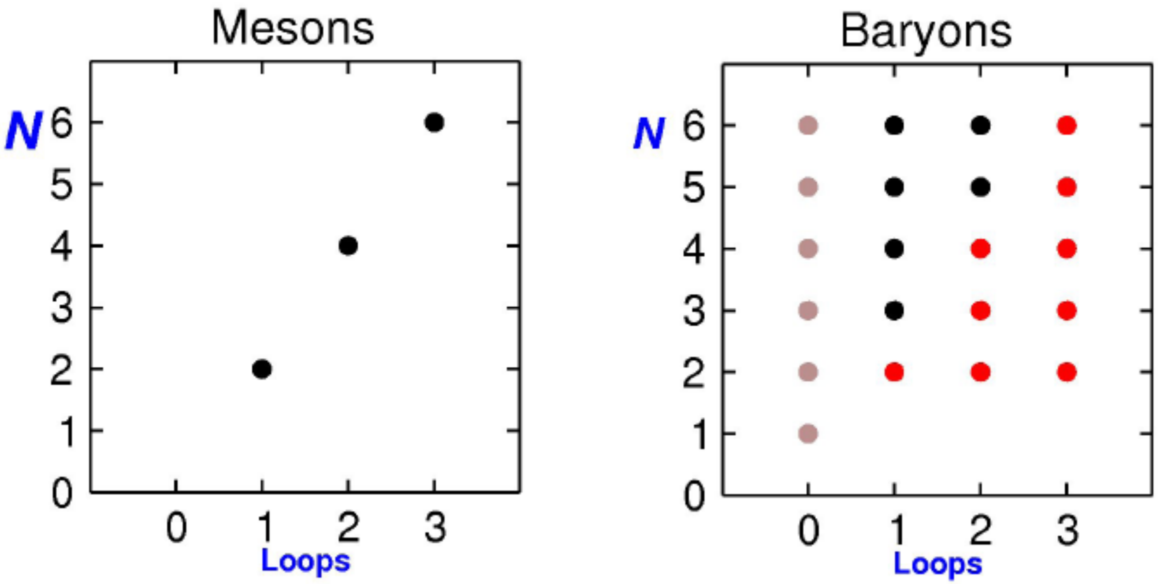
$$\boxed{\mathcal{L}_{\mathrm{QCD}}[q,\bar{q};G] \to \mathcal{L}_{\mathrm{ChPT}}[U,\partial U,\ldots,\mathcal{M},N]}$$

- U parameterizes the Nambu-Goldstone bosons
- ∂U vanishes at $E=\vec{p}=0$ (Nambu-Goldstone theorem)
- M parameterizes the explicit symmetry breaking
- N denotes interactions with matter fields
- Exact mapping via chiral Ward identities

- **ChPT exploits the symmetry of the QCD Lagrangian and its ground state; in practice, one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses. (J. Gasser, 2003)**

Power-counting-breaking (PCB) in the one-baryon sector

- **ChPT very successful in the study of Nanbu-Goldstone boson self-interactions. (at least in SU(2))**
- **In the one-baryon sector, things become problematic because of the nonzero (large) baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., a systematic power counting is lost!**



***red dots* denote possible PCB terms (pion-nucleon scattering)**

J. Gasser et al., NPB 307, 779(1988)

$$\boxed{\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k}$$

Power-counting-restoration methods

- **Heavy Baryon ChPT**: baryons are treated "semi-relativistically" by a simultaneous expansion in terms of external momenta and $1/M_N$ (*Jenkins et al., 1993*). It converges slowly for certain observables!

- **Relativistic baryon ChPT**: removing power counting breaking terms but retaining higher-order relativistic corrections, thus, keeping relativity.
  - Infrared baryon ChPT (*T. Becher and H. Leutwyler, 1999*)
  - Fully relativistic baryon ChPT–Extended On-Mass-Shell (EOMS) scheme (*J. Gegelia et al., 1999; T. Fuchs et al.,2003*)

- IR scheme separates the full integral into the Infrared and Regular parts:

$$H = \frac{1}{ab} = \int_0^1 dz \frac{1}{[(1-z)a + zb]^2} \equiv I + R = \int_0^\infty \ldots dz - \int_1^\infty \ldots dz$$

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- IR scheme separates the full integral into the Infrared and Regular parts:

$$H = \text{Infrared}$$

Extended-on-Mass-Shell (EOMS)

- "Drop" the PCB terms

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

$$\Big\Downarrow \; a = 0; b' = 0$$

$$\boxed{M_N = M_0 + b\ m_\pi^2 + cm_\pi^3 + \cdots \quad (\mathcal{O}(p^3))}$$

Extended-on-Mass-Shell (EOMS)

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- Equivalent to redefinition of the LECs

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

$$\Downarrow \; M_0^r = M_0(1 + aM_0^2); b^r = b^0 + b' M_0$$

$$\boxed{M_N = M_0^r + b^r m_\pi^2 + cm_\pi^3 + \cdots \quad (\mathcal{O}(p^3))}$$

Extended-on-Mass-Shell (EOMS)

- “Drop” the PCB terms

$$\boxed{\text{tree} = M_0 + bm_\pi^2} \quad + \quad \boxed{\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots}$$

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ChPT contains all possible terms allowed by symmetries, therefore whatever analytical terms come out from a loop amplitude, they must have a corresponding LEC

HB vs. Infrared vs. EOMS

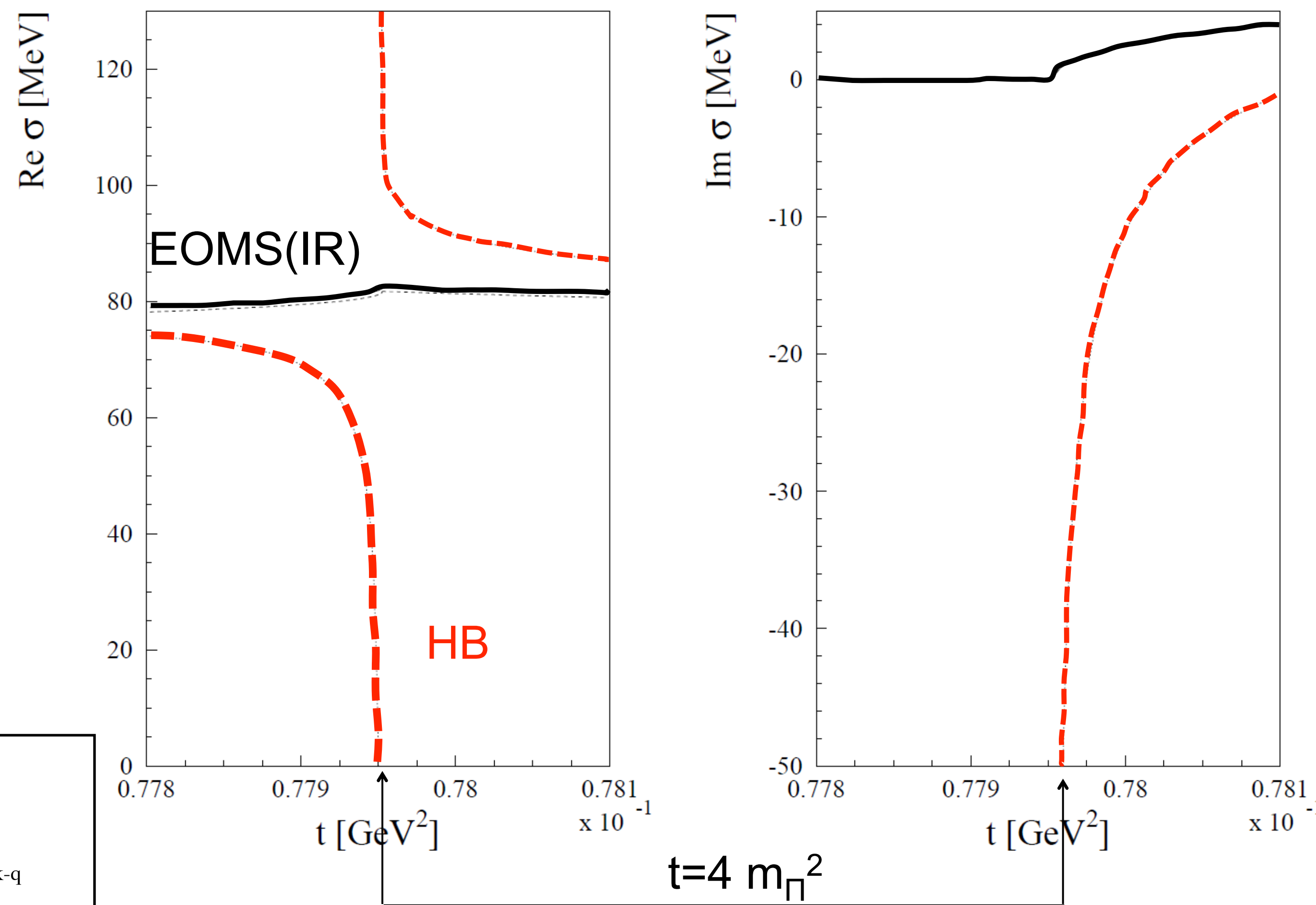
- **Heavy baryon (HB) ChPT**
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - strict PC and simple nonanalytical results
- **Infrared BChPT**
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor sector)
  - analytical terms the same as HBChPT
- **Extended-on-mass-shell (EOMS) BChPT**
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

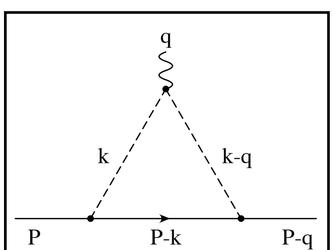
The nucleon scalar form factor at q^3

t

$$\boxed{\langle p(p',s')|\mathcal{H}_{\rm sb}(0)|p(p,s)\rangle = \bar{u}(p',s')u(p,s)\sigma(t), \quad t=(p'-p)^2}$$

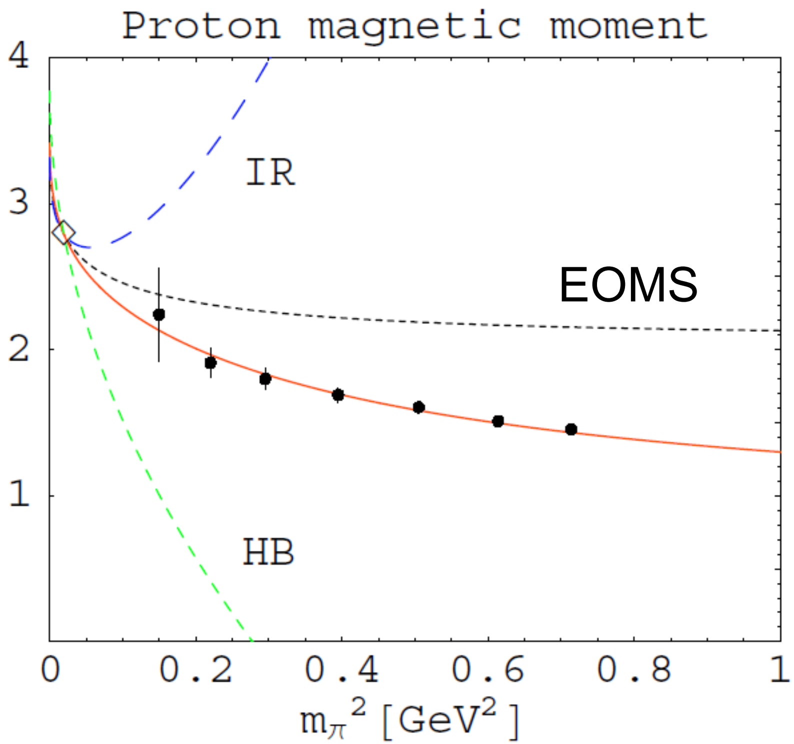
$$\mathcal{H}_{\rm sb} = \hat{m}(\bar{u}u + \bar{d}d)$$

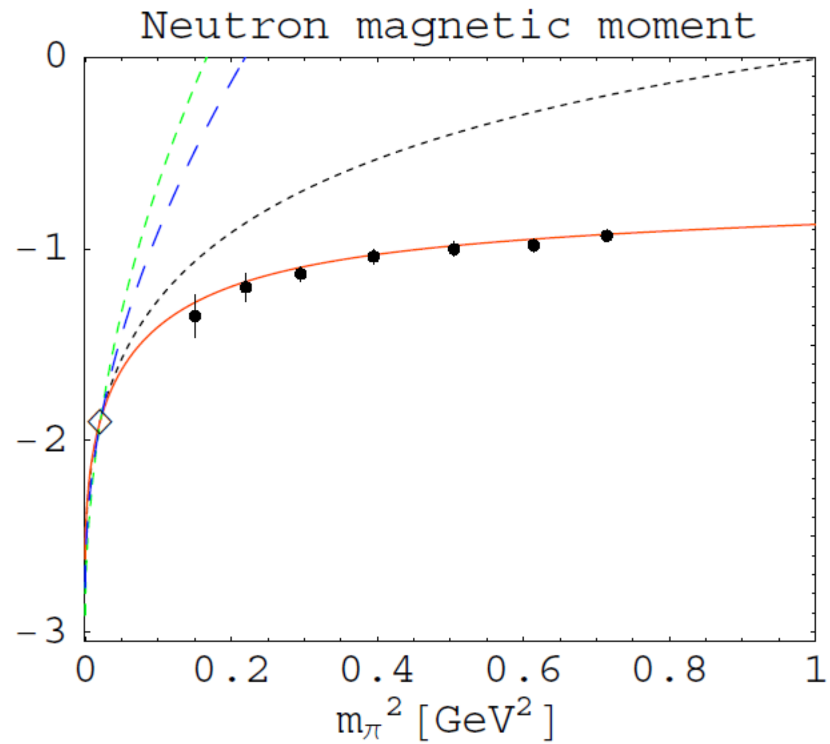




S. Scherer, Prog.Part.Nucl.Phys.64:1-60,2010

Proton and neutron magnetic moments: chiral extrapolation





V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.

Octet baryon magnetic moments at NLO BChPT

$$\chi^2 = \sum(\mu_{th} - \mu_{exp})^2$$

		p	n	Λ	Σ^-	Σ^0	Σ^+	Ξ^-	Ξ^0	$\Lambda\Sigma^0$	χ^2
LO	C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	**0.46**
NLO	HB	3.01	-2.62	-0.42	-1.35	0.42	2.18	-0.52	-0.70	1.68	**1.01**
	IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	**1.83**
	EOMS	2.58	-2.10	-0.66	-1.10	0.66	2.43	-0.95	-1.27	1.58	**0.18**
	Exp.	2.79	-1.91	-0.61	-1.16	—	2.46	-0.65	-1.25	1.61	—

- Contribution of the chiral series [LO(1+NLO/LO)]:

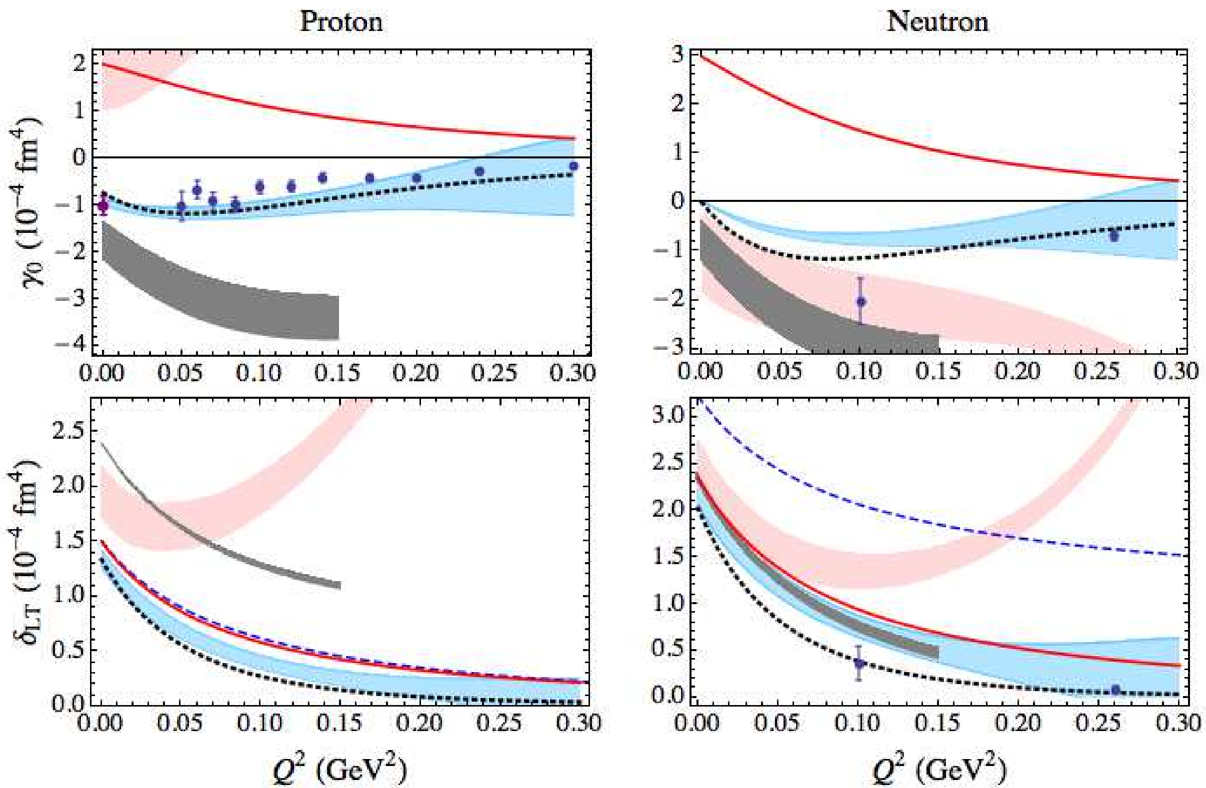
$$\mu_p = 3.47(1-0.257), \quad \mu_n = -2.55(1-0.175), \quad \mu_\Lambda = -1.27(1-0.482),$$

$$\mu_{\Sigma^-} = -0.93(1+0.187), \quad \mu_{\Sigma^+} = 3.47(1-0.300), \quad \mu_{\Sigma^0} = 1.27(1-0.482),$$

$$\mu_{\Xi^-} = -0.93(1+0.025), \quad \mu_{\Xi^0} = -2.55(1-0.501), \quad \mu_{\Lambda\Sigma^0} = 2.21(1-0.284).$$

LSG, J. Martin Camalich , L. Alvarez-Ruso, M.J. Vicente Vacas, Phys.Rev.Lett. 101:222002,2008

Generalized Spin Polarizabilities



Grey bands: EOMS + small scale

Blue dashed: O(p^4) HB

Red bands: IR calculation

Black dotted: MAID2007

Solid line: LO EOMS + delta

Blue bands: NLO EOMS + delta

Phys.Rev. C90 (2014) 055202

Problems reported in SU(3) HBChPT (1)

LHPC (A.Walker-Loud et al.), Phys.Rev.D79:054502, 2009.

TABLE XVII. Results from NLO bootstrap χ extrapolations of the octet baryon masses, using mixed action (MA) and $SU(3)$ heavy baryon χPT.

C=1.2(2), D=0.715(50), F=0.453(50)

FIT: NLO	Range	M_0 (GeV)	σ_M (GeV^{-1})	α_M (GeV^{-1})	β_M (GeV^{-1})	C	D	F	χ^2	d.o.f.
M_N, M_Λ,	007–020: MA	1.087(51)	−0.03(5)	−0.72(8)	−0.62(4)	0.15(9)	0.33(4)	0.14(3)	6.0	5
M_Σ, M_Ξ	007–020: $SU(3)$	1.014(32)	−0.07(4)	−0.77(10)	−0.56(5)	0.18(9)	0.30(6)	0.19(4)	5.5	5
	007–030: MA	1.149(57)	0.01(4)	−0.79(11)	−0.67(7)	0.12(9)	0.38(6)	0.16(3)	14.4	9
	007–030: $SU(3)$	1.091(66)	−0.04(3)	−0.99(28)	−0.73(19)	0.1(1)	0.44(14)	0.24(7)	11.9	9
	007–040: MA	1.147(52)	0.01(3)	−0.78(10)	−0.68(6)	0.13(9)	0.39(6)	0.16(3)	14.9	13
	007–040: $SU(3)$	1.090(61)	−0.04(3)	−0.99(26)	−0.73(18)	0.1(1)	0.45(13)	0.25(6)	12.5	13

TABLE XX. Results from NLO bootstrap χ extrapolations of the decuplet masses, using mixed action (MA) and $SU(3)$ heavy baryon χPT.

FIT: NLO	Range	$M_{T,0}$ (GeV)	$\bar{\sigma}_M$ (GeV^{-1})	γ_M (GeV^{-1})	C	H	χ^2	d.o.f.
M_Δ, M_{Σ^*},	007–020: MA	1.68(10)	−0.04(3)	1.2(3)	0.00(07)	1.2(2)	18.9	7
M_{Ξ^*}, M_{Ω^-}	007–020: $SU(3)$	1.52(05)	−0.20(4)	1.3(3)	0.00(15)	1.4(3)	20.3	7
	007–030: MA	1.64(08)	−0.05(2)	1.1(2)	0.00(07)	1.1(2)	21.0	11
	007–030: $SU(3)$	1.52(04)	−0.19(4)	1.3(3)	0.00(15)	1.4(3)	21.1	11
	007–040: MA	1.73(08)	−0.01(1)	1.2(2)	0.00(06)	1.2(2)	32.8	15
	007–040: $SU(3)$	1.57(04)	−0.18(4)	1.4(3)	0.00(14)	1.6(2)	34.8	15

mixed action heavy baryon chiral perturbation theory. Both the three-flavor and two-flavor functional forms describe our lattice results, although the low-energy constants from the next-to-leading order $SU(3)$ fits are inconsistent with their phenomenological values. Next-to-next-to-leading order $SU(2)$ continuum

Problems reported in SU(3) HBChPT (II)

PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502, 2009.

PACS-CS (S. Aoki et al.), Phys.Rev.D79:034503, 2009.

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D, F, and $\mathcal{C}$ at the phenomenological estimate.

	LO	NLO Case 1	NLO Case 2	Phenomenological
m_B	0.410(14)	0.391(39)	−0.15(9)	
α_M	−2.262(62)	−2.62(62)	−15.3(2.0)	
β_M	−1.740(58)	−2.6(1.5)	−21.3(3.0)	
σ_M	−0.53(12)	−0.71(34)	−9.6(1.4)	
D		$0.000(16) \times 10^{-8}$	0.80 fixed	0.80
F		$0.000(9) \times 10^{-8}$	0.47 fixed	0.47
$\mathcal{C}$		0.36(30)	1.5 fixed	1.5
χ^2/dof	1.10(63)	1.39(77)	153(82)	

We investigate the quark mass dependence of baryon masses in 2 + 1 flavor lattice QCD using SU(3) heavy baryon chiral perturbation theory up to one-loop order. The baryon mass data used for the analyses are obtained for the degenerate up-down quark mass of 3 to 24 MeV and two choices of the strange quark mass around the physical value. We find that the SU(3) chiral expansion fails to describe both the octet and the decuplet baryon data if phenomenological values are employed for the meson-baryon couplings. The SU(2) case is also examined for the nucleon. We observe that higher order terms are controlled only around the physical point. We also evaluate finite size effects using SU(3) heavy baryon chiral perturbation theory, finding small values of order 1% even at the physical point.

Some successful applications of covariant BChPT (in the three-flavor sector)

- **Octet (decuplet) baryon magnetic moments:**

  Phys.Rev.Lett.101:222002,2008; Phys.Lett.B676:63-68,2009; Phys.Rev.D80:034027,2009

- **Octet and Decuplet baryon masses**

  Phys.Rev.D82:074504,2010; Phys.Rev.D84:074024,2011; JHEP12(2012)073; Phys.Rev.D D87:074001 (2013); Phys.Rev. D89:054034,2014 ; Eur.Phys.J. C74:2754,2014

- **Hyperon vector coupling $f_1(0)$**

  Phys.Rev.D79:094022,2009;arXiv:Phys.Rev. D89 (2014) 113007

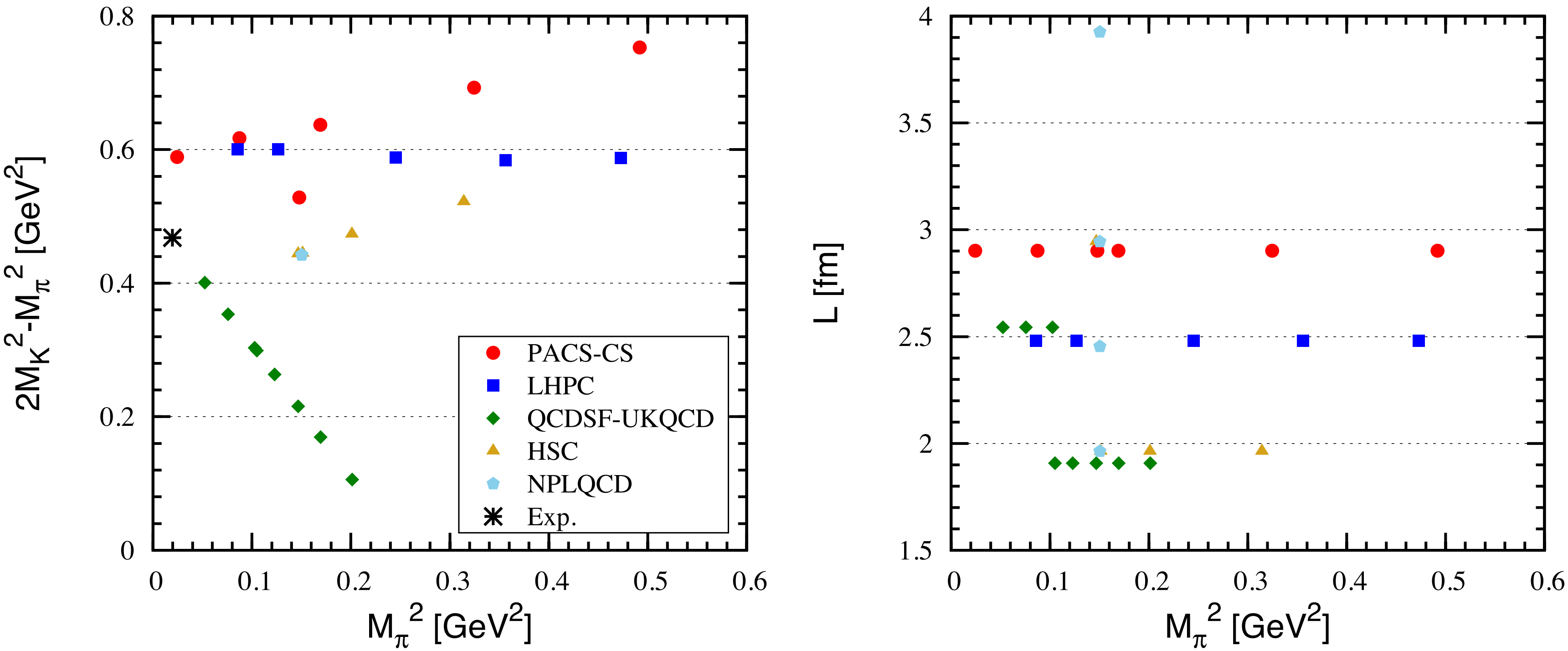
- **Octet baryon axial coupling**

  Phys.Rev.D78:014011,2008， Phys.Rev. D90 (2014) 054502

Two key factors for a reliable determination of the baryon sigma terms

- Lattice QCD simulations of baryon masses at various quark masses, volumes, and lattice spacings, and with various fermion/gauge actions
- A reliable formulation of ChPT, which not only can well describe the LQCD data, but also needs to satisfy all symmetry and analyticity constrains

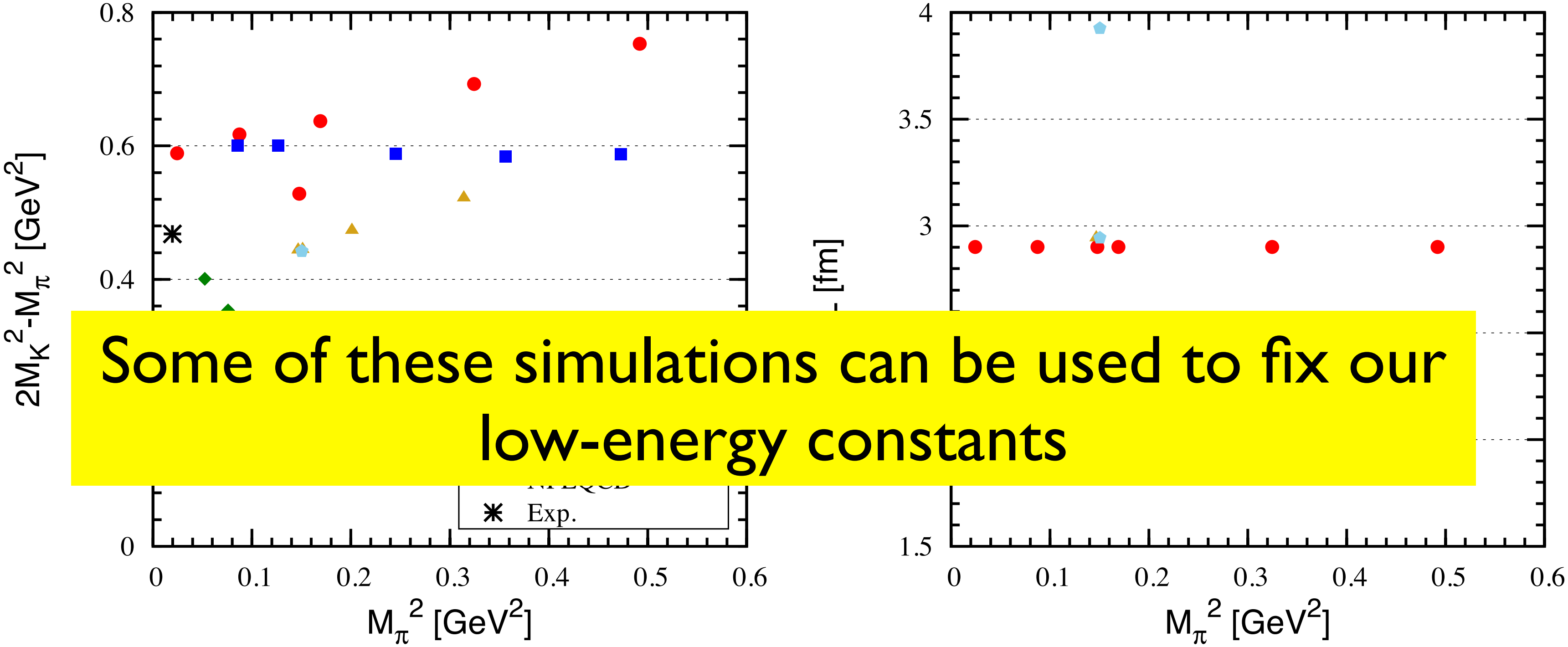
landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses



To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum: $a \to 0$
- Extrapolate to physical light quark masses: $m_q \to m_q(\text{Phys.})$
- Extrapolate to infinite space-time: $L \to \infty$

landscape of latest 2+1 f LQCD simulations of g.s. octet baryon masses



Some of these simulations can be used to fix our low-energy constants

To obtain g.s. baryon masses in the physical world

- Extrapolate to the continuum: $a \to 0$
- Extrapolate to physical light quark masses: $m_q \to m_q(\mathrm{Phys.})$
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Systematic Description of the LQCD data with the EOMS BChPT

- *NNLO EOMS BChPT study of the PACS-CS and LHPC data: Camalich, Geng, Vacas, PRD82(2010)074504*
- *Finite volume corrections: Geng, Ren, Camalich, Weise, PRD84(2011)074024;*
- *First systematic study of all publically available LQCD data: Ren, Geng, Camalich, Meng, Toki, JHEP12(2012)073;*
- *Effects of virtual decuplet baryons: Ren, Geng, Meng, Toki, PRD87(2013)074001*
- *Continuum extrapolations: Ren, Geng, Meng, Eur.Phys.J. C74:2754,2014*

Systematic Description of the LQCD data with the EOMS BChPT

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- *Finite volume corrections:* *Geng, Ren, Camalich, Weise, PRD84(2011)074024;*

The EOMS BChPT can be trusted to predict the baryon sigma terms

- *Effects of virtual decuplet baryons: Ren, Geng, Meng, Toki,* *PRD87(2013)074001*
- *Continuum extrapolations:* *Ren, Geng, Meng, Eur.Phys.J. C74:2754,2014*

Selection of LQCD data

- All n_f=2+1 LQCD simulations
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD, BWM
  - BWM—not publicly available
  - HSC and NPLQCD—Low statistics

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  - BWM—not publicly available
  - HSC and NPLQCD—Low statistics

PACS-CS, LHPC, QCDSF-UKQCD

An accurate determination of baryon sigma terms

- **Scale setting:** mass independent (given by the LQCD simulations or self-consistently determined) vs. mass dependent (r_0, r_1, X_π)
- **Isospin breaking effects:** better constrain the LQCD LECs
- **Theoretical uncertainties caused by truncating chiral expansions:** NNLO vs. N3LO; EOMS vs. FRR

Scale-setting effects on the determination of baryon sigma terms

arXiv:1301.3231
P.E. Shanahan*, A.W. Thomas and R.D. Young

- **Lattice-scale setting**
  - **PACS-CS data with mass independent scale-setting:**

$$\sigma_{sN} = 59 \pm 7 \ (\mathrm{MeV})$$

  - **PACS data with mass dependent (r_0) scale-setting:**

$$\sigma_{sN} = 21 \pm 6 \ (\mathrm{MeV})$$

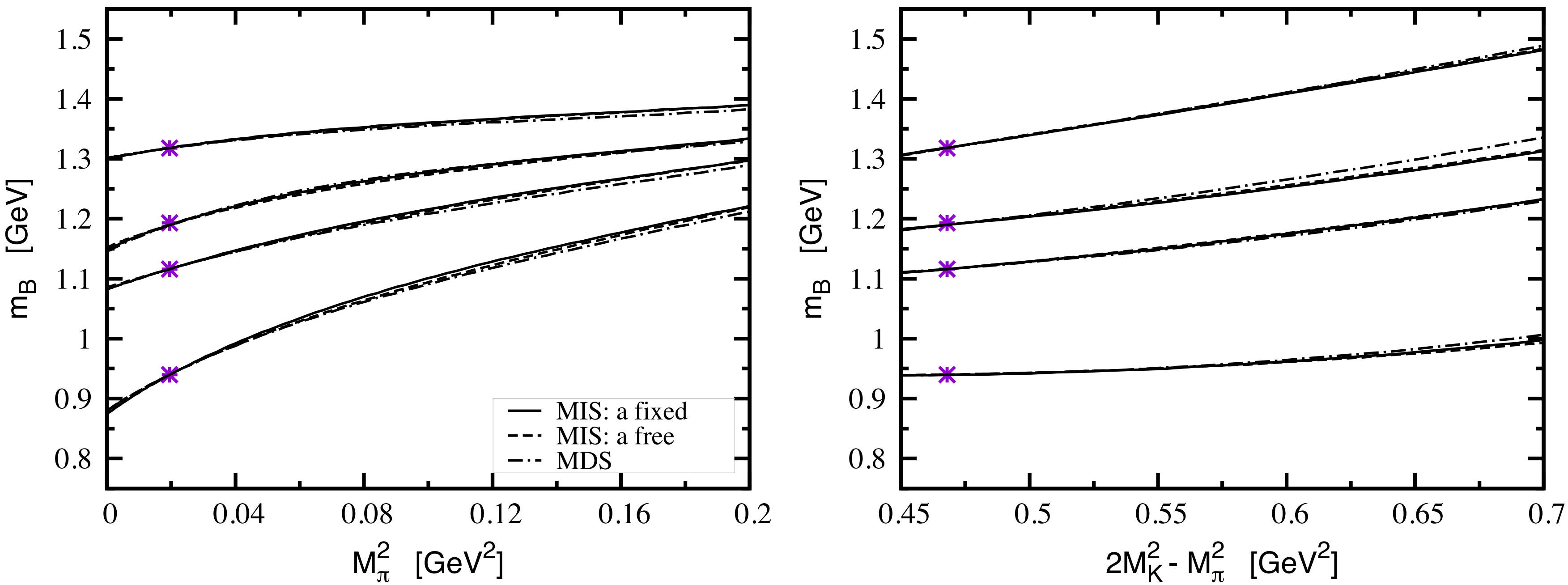
- **Whether other LQCD data will show the same trend?**

Three different fits at N^3LO

	MIS		MDS
	a fixed	a free	
m_0 [MeV]	884(11)	877(10)	887(10)
b_0 [GeV^{-1}]	$-0.998(2)$	$-0.967(6)$	$-0.911(10)$
b_D [GeV^{-1}]	0.179(5)	0.188(7)	0.039(15)
b_F [GeV^{-1}]	$-0.390(17)$	$-0.367(21)$	$-0.343(37)$
b_1 [GeV^{-1}]	0.351(9)	0.348(4)	$-0.070(23)$
b_2 [GeV^{-1}]	0.582(55)	0.486(11)	0.567(75)
b_3 [GeV^{-1}]	$-0.827(107)$	$-0.699(169)$	$-0.553(214)$
b_4 [GeV^{-1}]	$-0.732(27)$	$-0.966(8)$	$-1.30(4)$
b_5 [GeV^{-2}]	$-0.476(30)$	$-0.347(17)$	$-0.513(89)$
b_6 [GeV^{-2}]	0.165(158)	0.166(173)	$-0.0397(1574)$
b_7 [GeV^{-2}]	$-1.10(11)$	$-0.915(26)$	$-1.27(8)$
b_8 [GeV^{-2}]	$-1.84(4)$	$-1.13(7)$	0.192(30)
d_1 [GeV^{-3}]	0.0327(79)	0.0314(72)	0.0623(116)
d_2 [GeV^{-3}]	0.313(26)	0.269(42)	0.325(54)
d_3 [GeV^{-3}]	$-0.0346(87)$	$-0.0199(81)$	$-0.0879(136)$
d_4 [GeV^{-3}]	0.271(30)	0.230(24)	0.365(23)
d_5 [GeV^{-3}]	$-0.350(28)$	$-0.302(50)$	$-0.326(66)$
d_7 [GeV^{-3}]	$-0.435(10)$	$-0.352(8)$	$-0.322(7)$
d_8 [GeV^{-3}]	$-0.566(24)$	$-0.456(30)$	$-0.459(33)$
χ^2/d.o.f.	0.87	0.88	0.53

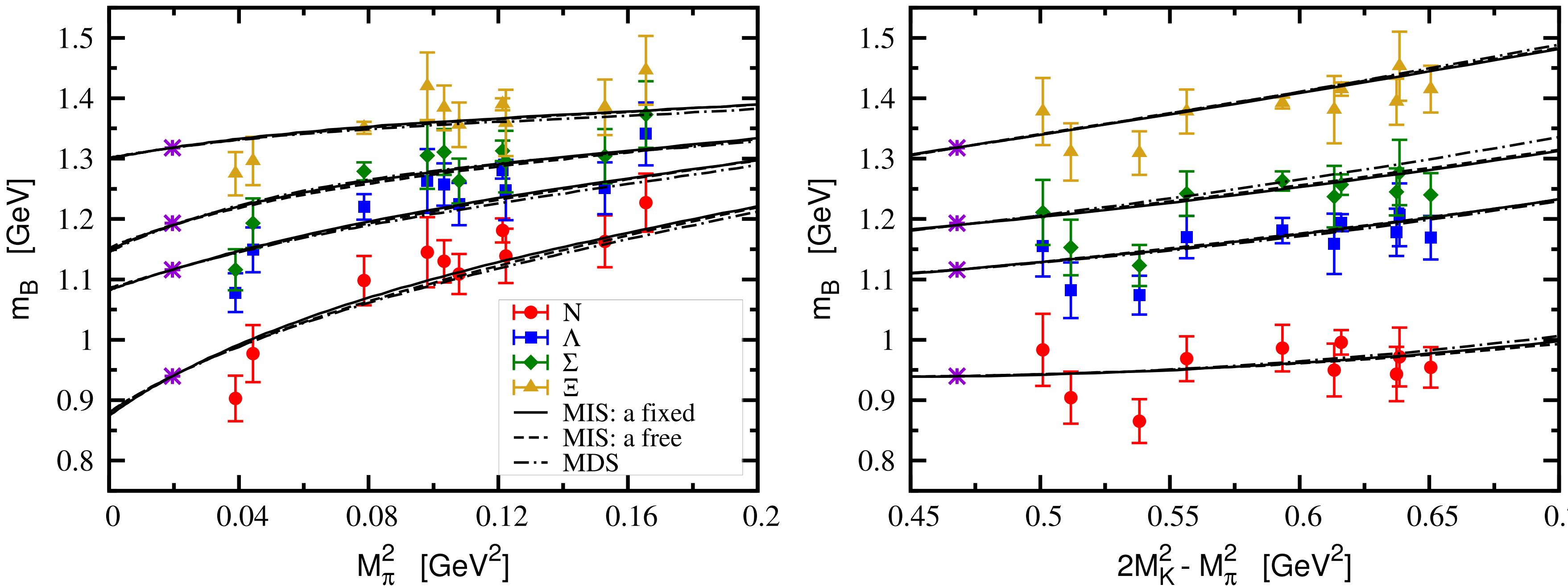
- **Mass independent**
  - Lattice spacing a fixed to the published value
  - Lattice spacing a determined self-consistently
- **Mass dependent**
  - r_0 for PACS-CS
  - r_1 for LHPC
  - X_π for QCDSF-UKQCD

Evolution of baryon masses with u/d and s quark masses



Only central values are shown!

Evolution of baryon masses with u/d and s quark masses in comparison with the BMW data



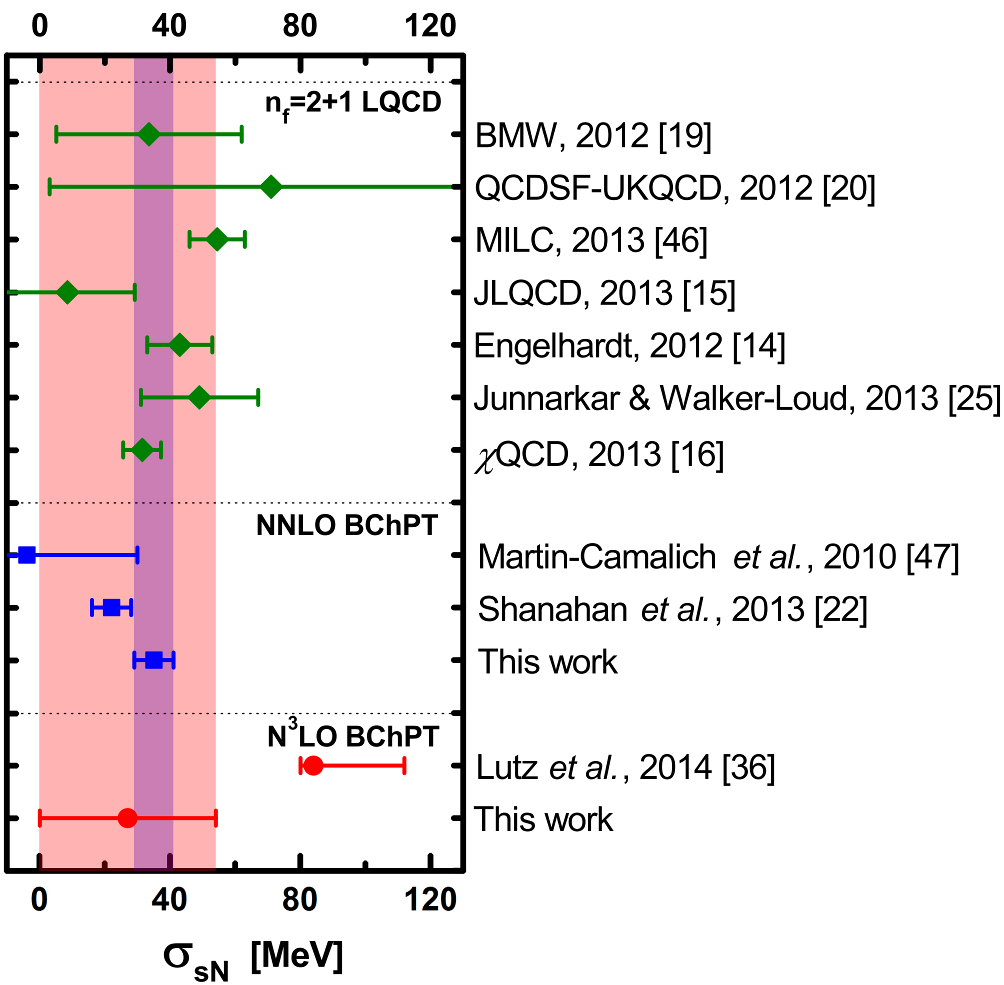
S. Durr et al., BMW collaboration, Phys.Rev. D85 (2012) 014509

Baryon sigma terms from N^3LO BChPT

	MIS		MDS
	a fixed	a free	
$\sigma_{\pi N}$	55(1)(4)	54(1)	51(2)
$\sigma_{\pi\Lambda}$	32(1)(2)	32(1)	30(2)
$\sigma_{\pi\Sigma}$	34(1)(3)	33(1)	37(2)
$\sigma_{\pi\Xi}$	16(1)(2)	18(2)	15(3)
σ_{sN}	27(27)(4)	23(19)	26(21)
$\sigma_{s\Lambda}$	185(24)(17)	192(15)	168(14)
$\sigma_{s\Sigma}$	210(26)(42)	216(16)	252(15)
$\sigma_{s\Xi}$	333(25)(13)	346(15)	340(13)

- All three scale-setting methods yield similar baryon sigma terms

Comparison with earlier studies



Nucleon Strangeness Sigma Term

- **Consistent with most recent LQCD studies and those of NNLO ChPT, e.g., that of Young and Shanahan**
- **Uncertainties at N^3LO substantially larger, because of the extra LECs**

Summary

Summary

- ✤ I have explained how the baryon sigma terms (particularly those of the nucleon) are related to dark matter direct searches and the understanding of the quark-flavor structure of the nucleon.

Summary

- ✤ I have explained how the baryon sigma terms (particularly those of the nucleon) are related to dark matter direct searches and the understanding of the quark-flavor structure of the nucleon.
- ✤ I have shown how a combination of lattice QCD simulations and baryon chiral perturbation theory allows us to make a reliable prediction of these terms.

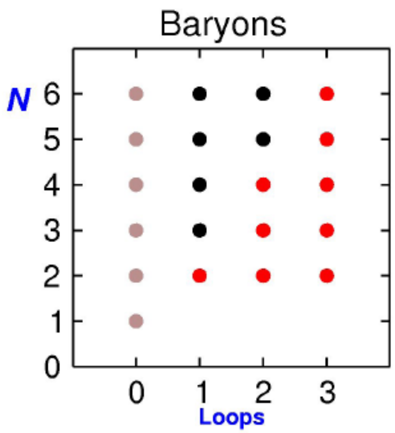
Thank you very much
for your attention!

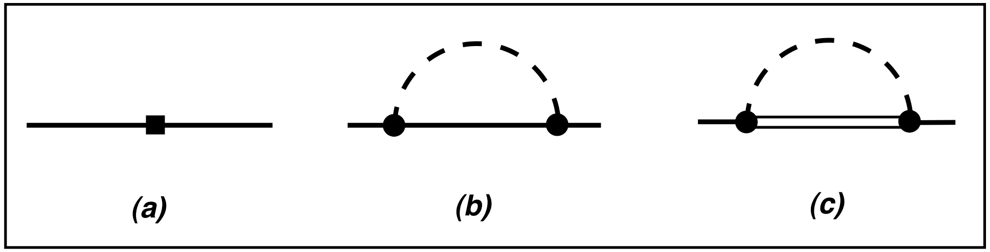
Covariant chiral perturbation theory in heavy-light systems

- **DK interactions and dynamically generated resonances**
  - Phys.Rev. D89 (2014) 054008
  - Phys.Rev. D89 (2014) 014026
  - Phys.Rev. D82 (2010) 054022
- **Decay constants of D/B/D*/B* mesons**
  - Phys.Lett. B713 (2012) 453-456
  - Phys.Lett. B696 (2011) 390-395

Chiral order = $4L - 2N_M - N_B + \sum_k kV_k.$

Nucleon mass up to $O(p^3)$

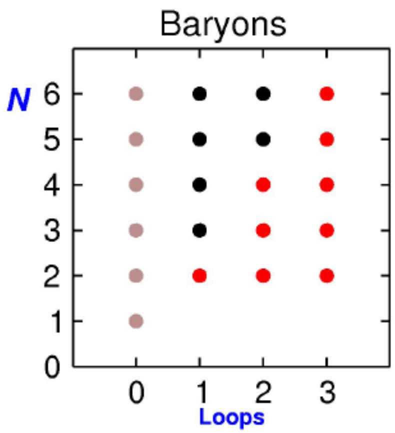


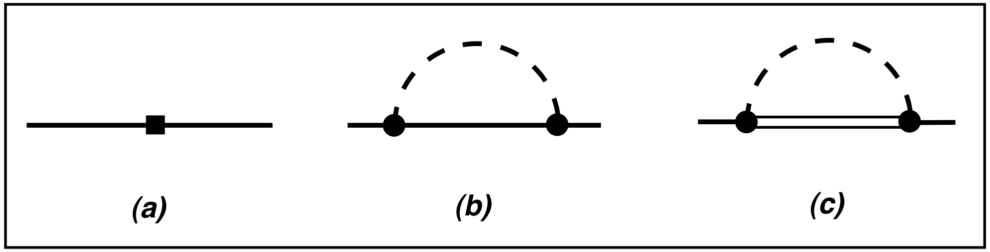


$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$

$\boxed{\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.}$

Nucleon mass up to $O(p^3)$





order of the loop $= 1 + 1 + 4 - 1 - 2 = 3$

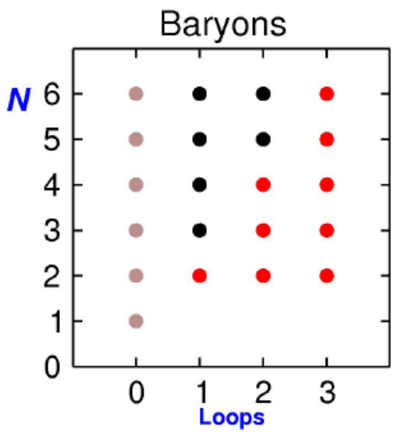
Naively (no PCB)

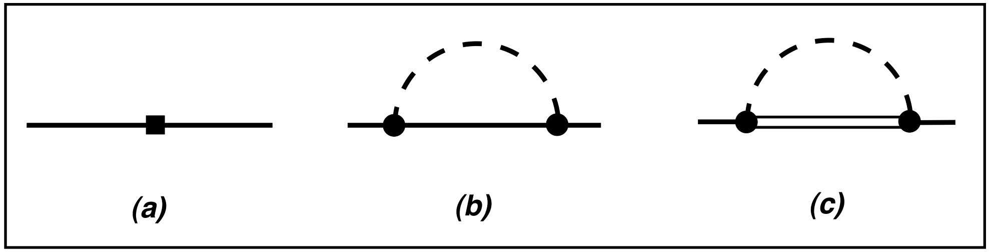
$$M_N = M_0 + bm_\pi^2 + \text{loop}$$

$$\text{loop}(= cm_\pi^3 + \cdots)$$

Chiral order = $4L - 2N_M - N_B + \sum_k kV_k.$

Nucleon mass up to $O(p^3)$





order of the loop $= 1 + 1 + 4 - 1 - 2 = 3$

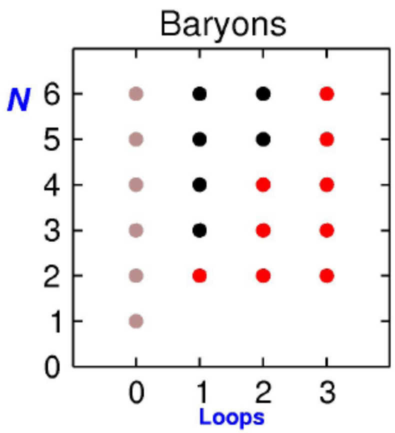
Naively (no PCB) $M_N = M_0 + bm_\pi^2 + \text{loop}$

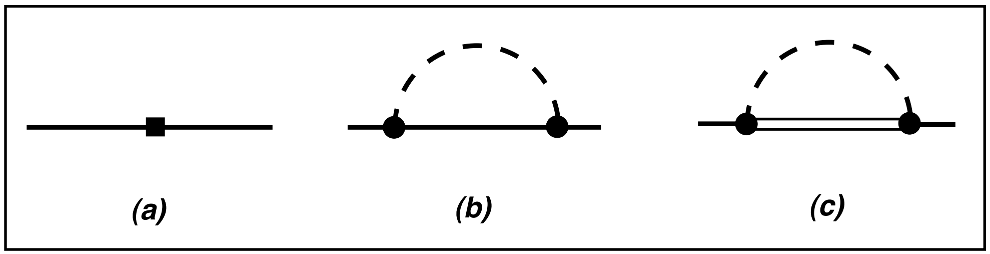
$$\text{loop}(= cm_\pi^3 + \cdots)$$

However $\boxed{\text{loop} = aM_0^3 + b' M_0 m_\pi^2 + cm_\pi^3 + \cdots}$

$$\boxed{\text{Chiral order} = 4L - 2N_M - N_B + \sum_k kV_k.}$$

Nucleon mass up to O(p^3)





$$\text{order of the loop} = 1 + 1 + 4 - 1 - 2 = 3$$

Naively (no PCB)

$$M_N = M_0 + b m_\pi^2 + \text{loop}$$

$$\text{loop}(= c m_\pi^3 + \cdots)$$

However

$$\boxed{\text{loop} = a M_0^3 + b' M_0 m_\pi^2 + c m_\pi^3 + \cdots}$$

No need to calculate, simply recall that *$M_0 \sim O(p^0)$*

Summary and Outlook

Summary and Outlook

✤ In addition, our (series of) studies showed

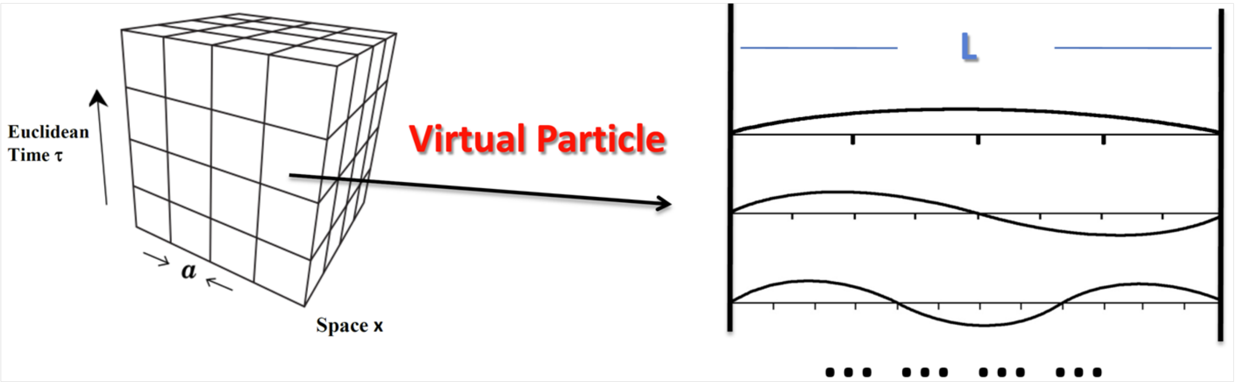
- The extended-on-mass-shell (EOMS) BChPT provides a reliable framework to study the properties of the ground-state octet baryons
- LQCD simulations can help determine the many unknown low-energy constants which otherwise cannot be fixed

✤ Many interesting observables remain unexplored within the EOMS framework

- Axial, Vector, and Electromagnetic form factors of the g.s. octet baryons
- Spin polarizabilities, TMDs and GPDs of the octet baryons
- Hyperon-nucleon (hyperon) forces
- ...

Finite volume corrections

- **Physical origin: existence of boundary conditions**



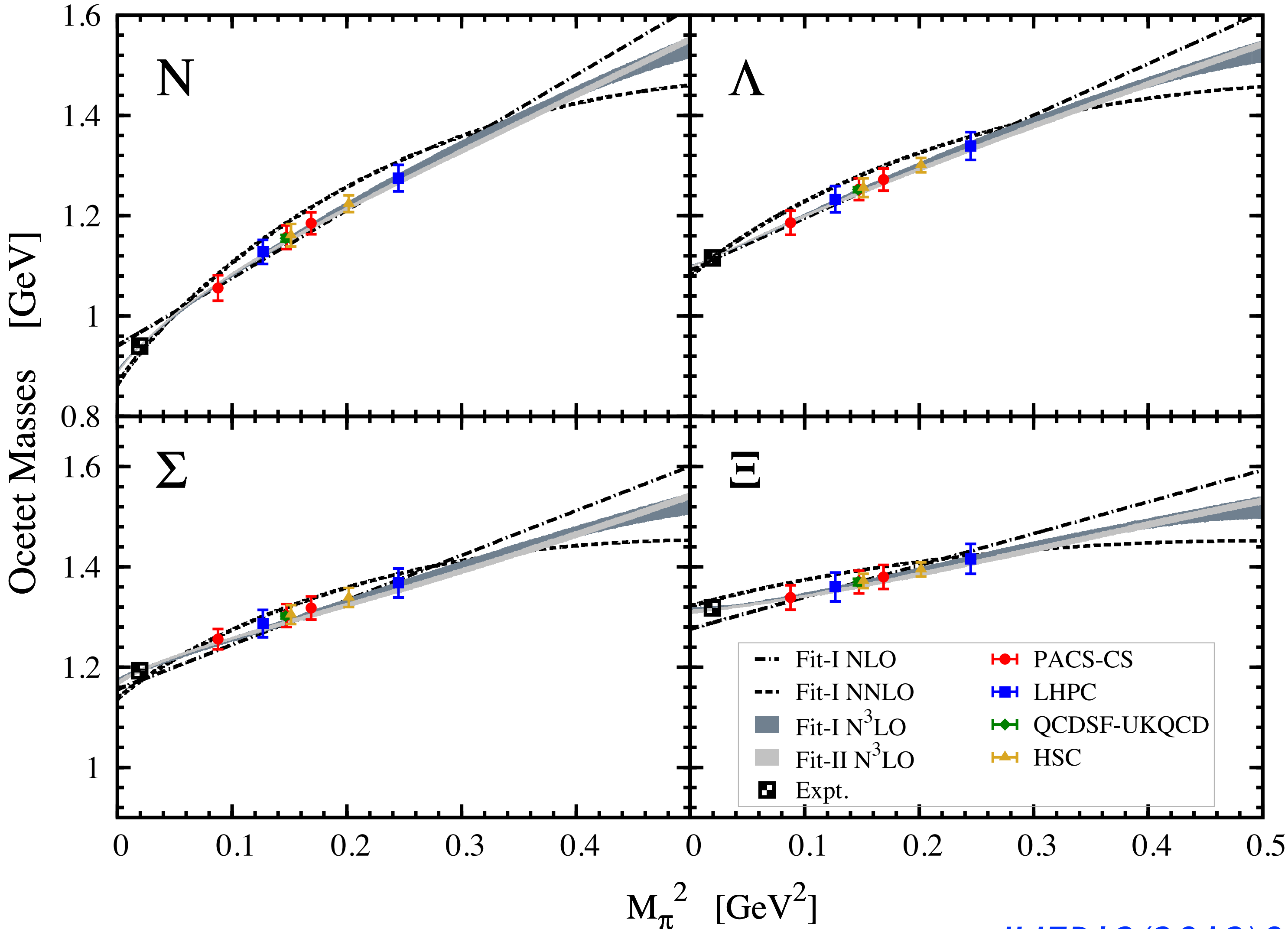
- **Momenta of virtual particles are discretized**

$$k_i = 2\pi \frac{n_i}{L},\ (i = 0, 1, 2, 3)$$

$$\boxed{\int_{-\infty}^{\infty} dk \Rightarrow \sum_{n=-\infty}^{\infty} \left(\frac{2\pi}{L}\right) \cdot n.}$$

Geng, Ren, Martin-Camalich, Weise, PRD84(2011)074024

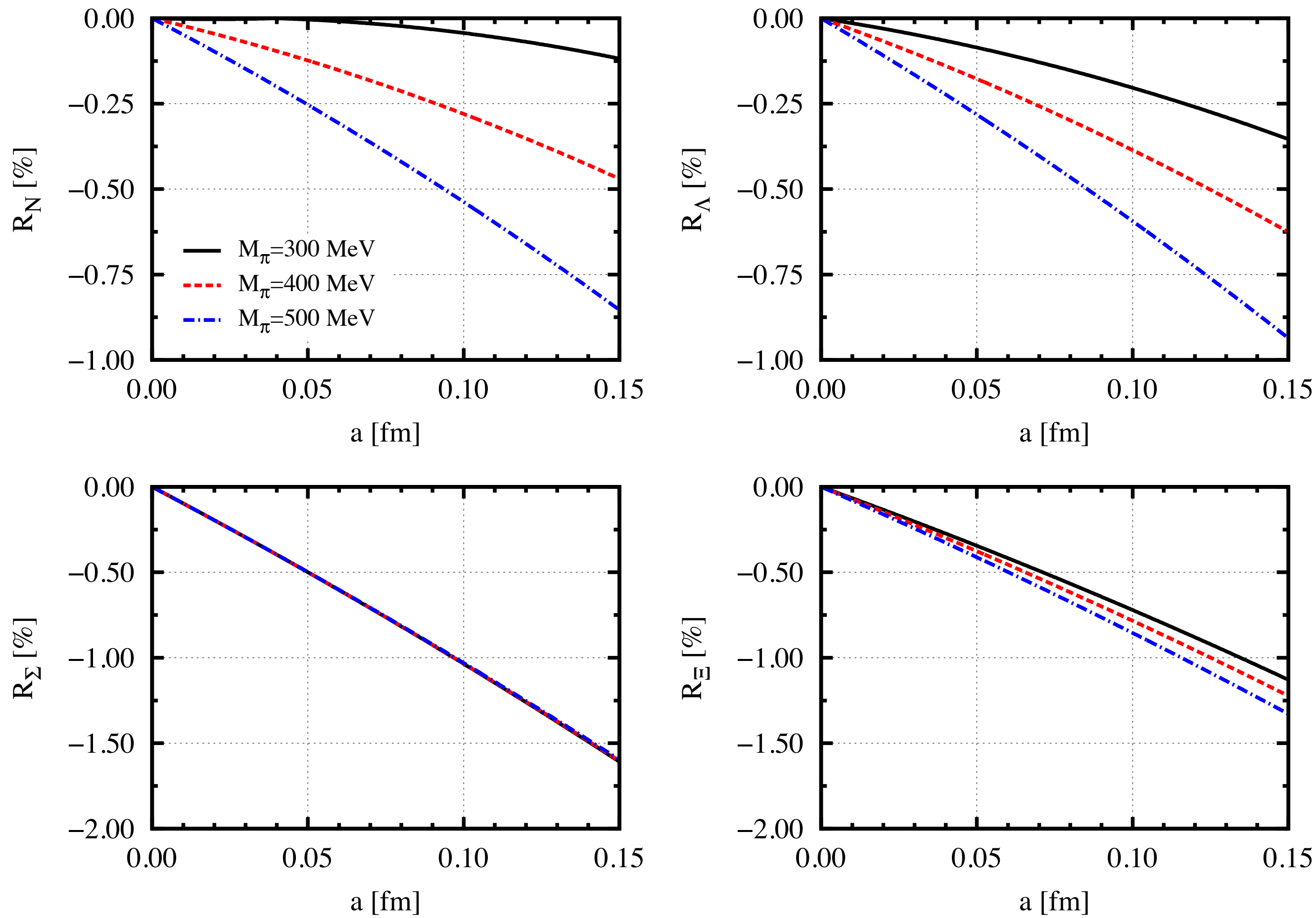
Chiral extrapolations upto N3LO in BChPT



JHEP12(2012)073;

Ren, Geng, Meng, EPJC74 (2014) 2754

Lattice spacing evolutions



$$R_B = m_B^{(a)}/m_B$$

- **For LQCD simulations with m_π<500 MeV and a<0.15 fm, discretization effects are about 1 to 2 percent**

NNLO fits

TABLE I. Values of the LECs obtained from the best fits to the LQCD simulations and the experimental octet baryon masses and the corresponding χ^2/d.o.f.. The underlined numbers denote the values at which they are fixed.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
m_0 [MeV]	$757(7)$	$808(1)$	$829(7)$	$805(9)$
b_0 [GeV^{-1}]	$-0.907(6)$	$-0.710(2)$	$-0.820(7)$	$-0.922(20)$
b_D [GeV^{-1}]	$0.0582(22)$	$0.0570(22)$	$0.101(2)$	$0.116(3)$
b_F [GeV^{-1}]	$-0.508(2)$	$-0.411(11)$	$-0.464(2)$	$-0.510(8)$
f_0 [GeV]	$\underline{0.0871}$	$0.105(3)$	$\underline{0.0871}$	$\underline{0.0871}$
Λ or μ [GeV]	$\underline{1.0}$	$\underline{1.0}$	$\underline{1.0}$	1.24(5)
χ^2/d.o.f.	3.0	1.6	2.4	1.8

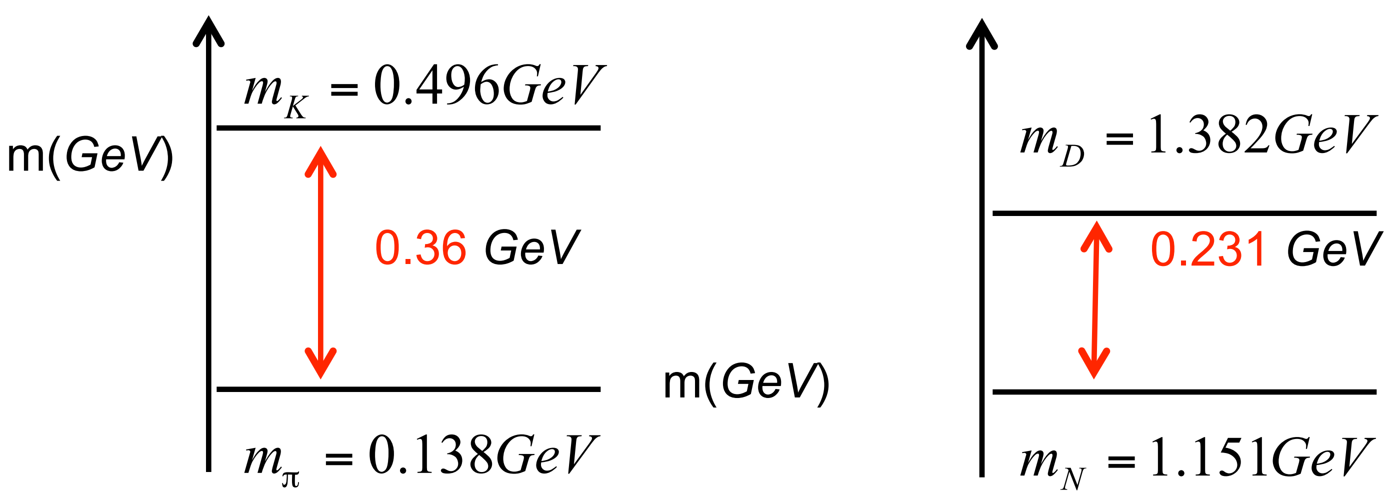
NNLO sigma terms

TABLE II. Sigma terms of the octet baryons at the physical point, predicted by the NNLO BChPT with the LECs of Table I.

	EOMS		FRR	
	Fit-I	Fit-II	Fit-III	Fit-IV
$\sigma_{\pi N}$ [MeV]	56(0)	47(1)	47(0)	53(1)
$\sigma_{\pi\Lambda}$ [MeV]	35(1)	30(1)	31(1)	34(1)
$\sigma_{\pi\Sigma}$ [MeV]	32(0)	27(1)	25(0)	27(1)
$\sigma_{\pi\Xi}$ [MeV]	13(1)	12(1)	13(1)	13(1)
σ_{sN} [MeV]	35(6)	27(7)	21(6)	20(7)
$\sigma_{s\Lambda}$ [MeV]	147(7)	152(7)	162(7)	153(7)
$\sigma_{s\Sigma}$ [MeV]	218(7)	222(7)	226(7)	214(7)
$\sigma_{s\Xi}$ [MeV]	295(7)	313(8)	332(7)	312(8)

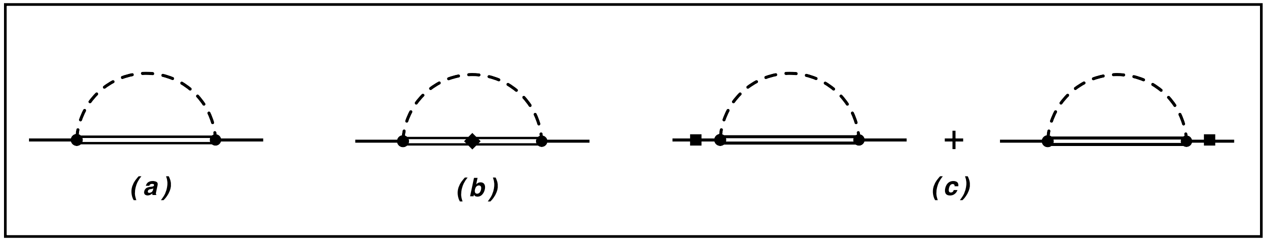
Effects of dynamical decuplet baryons

- **ChPT relies on the assumption that all high-energy degrees of freedom can be integrated out--not necessarily true for SU(3) BChPT**



Feynman diagrams/Lagrangians-no new unknown LECs

- **Feynman diagrams**



- **Lagrangians**

  - **Octet-Decuplet-Pseudoscalr coupling**

$$\mathcal{L}^{(1)}_{\phi BT} = \frac{i\mathcal{C}}{m_D F_\phi}\varepsilon^{abc}(\partial_\alpha \bar{T}^{ade}_\mu)\gamma^{\alpha\mu\nu}B^e_c\partial_\nu\phi^d_b + \text{H.c.},$$

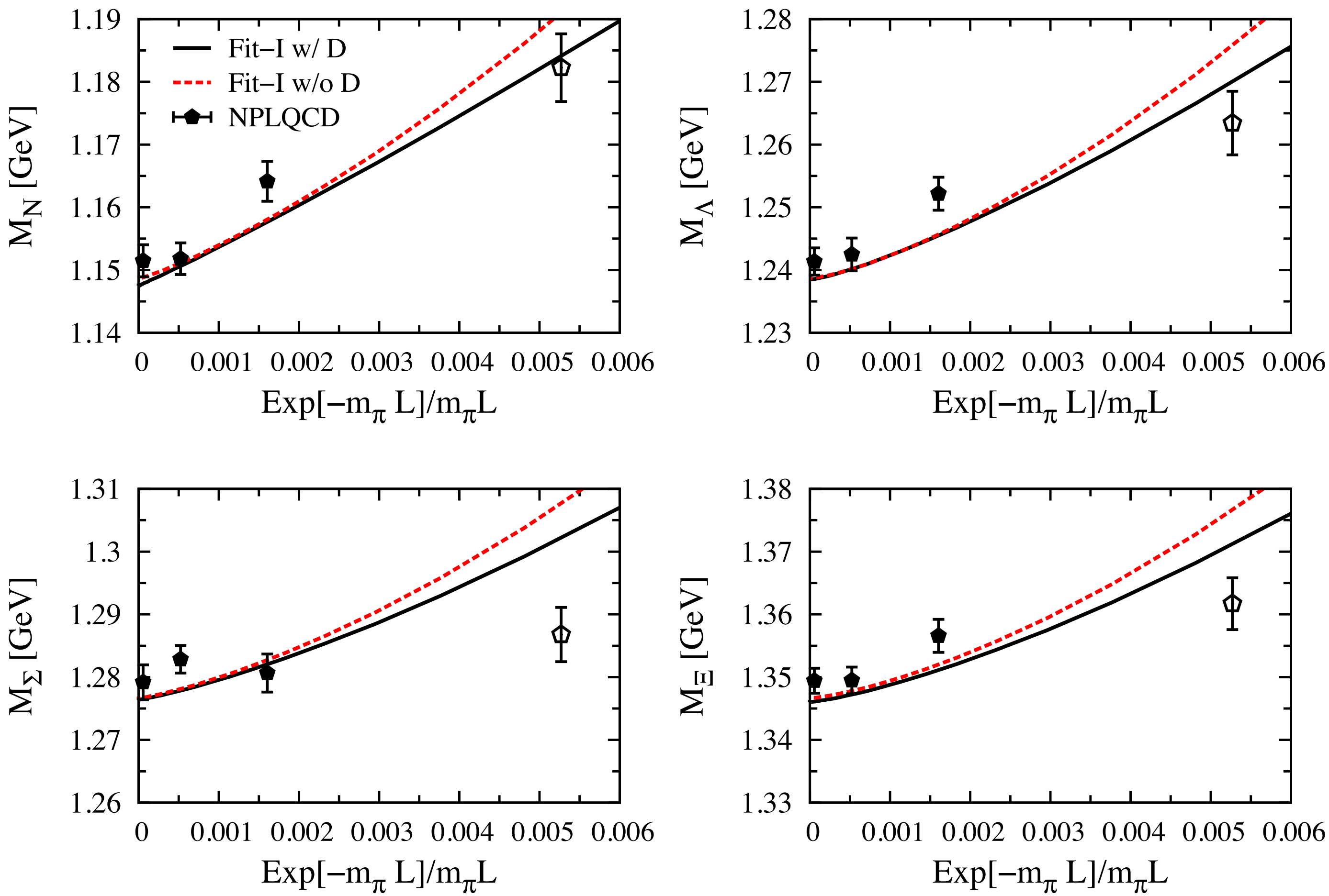
fixed from decay of a decuplet into an octet baryon and a pseudoscalar

  - **mass corrections**

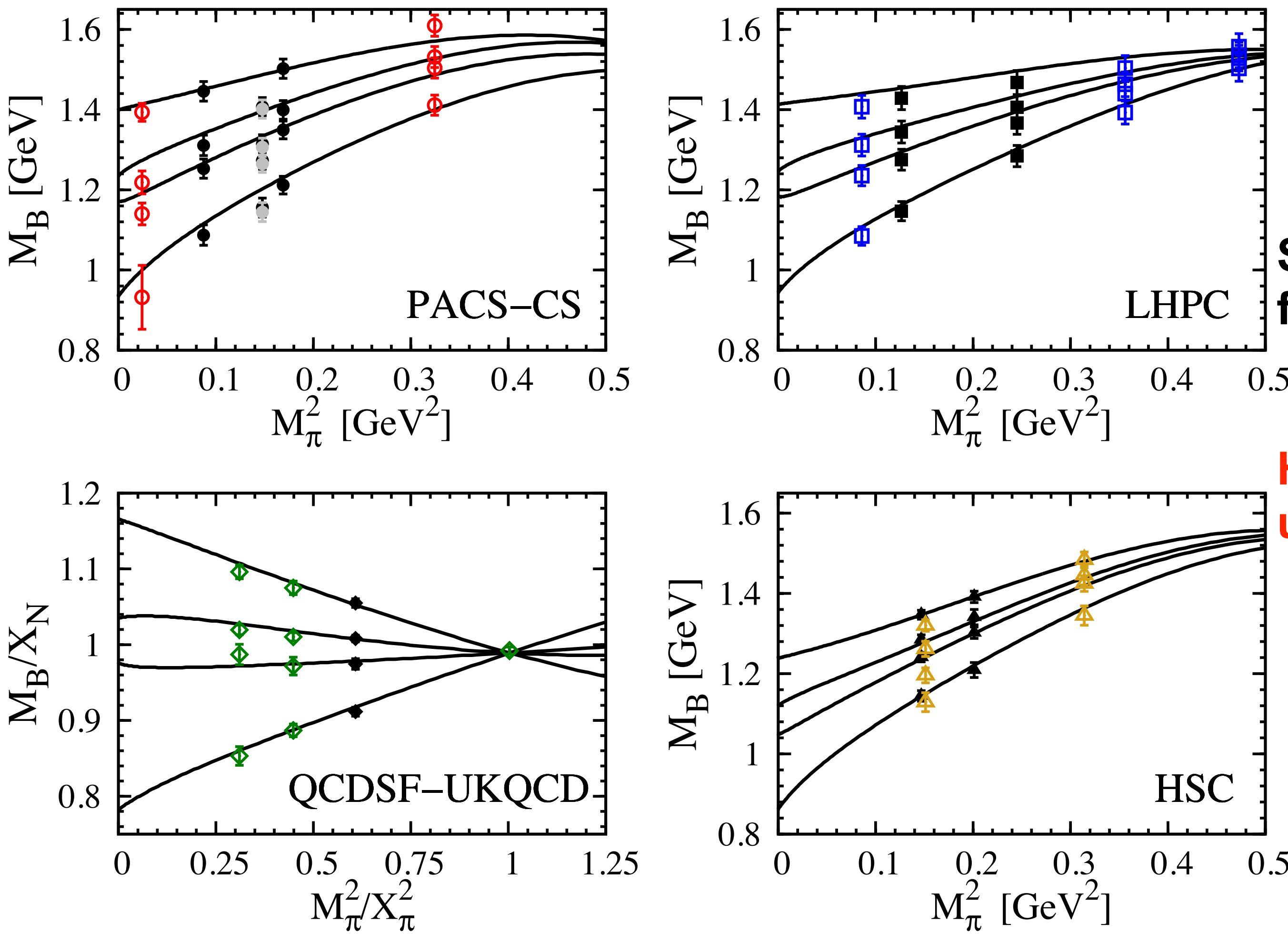
$$\mathcal{L}^{(2)}_T = \frac{t_0}{2}\bar{T}^{abc}_\mu g^{\mu\nu}T^{abc}_\nu\langle\chi_+\rangle + \frac{t_D}{2}\bar{T}^{abc}_\mu g^{\mu\nu}(\chi_+, T_\nu)^{abc},$$

fixed from the experimental decuplet masses

Slightly better description of the volume dependence of the NPLQCD data



Unfitted data can also reasonably well described



Solid black: fitted

Hollow colored: unfitted

Baryon Pion and Strangeness Sigma terms

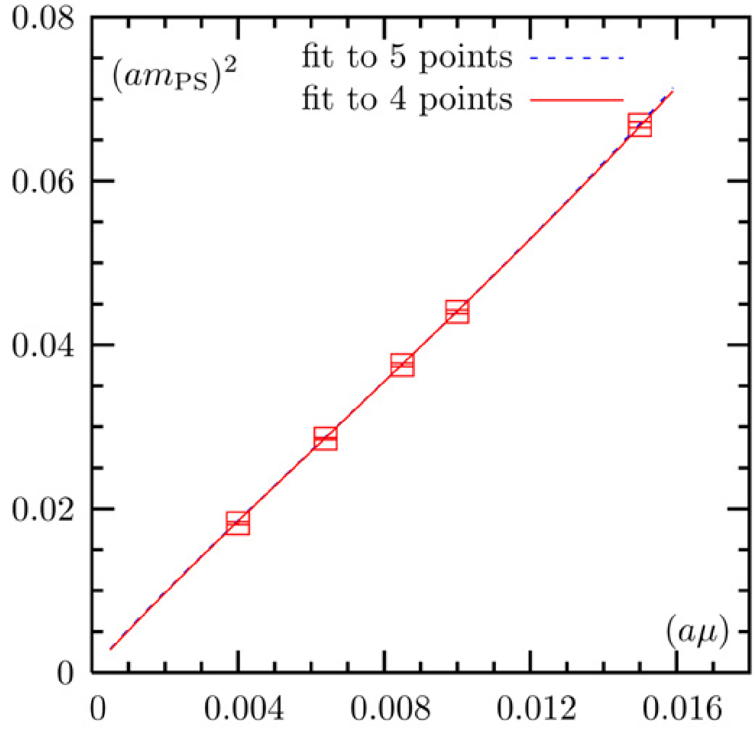
- **Feynman-Hellmann theorem states**

$$\boxed{\begin{aligned} \sigma_{\pi B} &= m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle = m_l \frac{\partial M_B}{\partial m_l} \\ \sigma_{sB} &= m_s \langle B(p)|\bar{s}s|B(p)\rangle = m_s \frac{\partial M_B}{\partial m_s}. \end{aligned}}$$

- **Using** **leading-order** **ChPT meson masses**

$$\boxed{\begin{aligned} \sigma\pi B &= \frac{m_\pi^2}{2}\left(\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi} + \frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{1}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) m_B \\ \sigma_s &= \left(m_K^2 - \frac{m_\pi^2}{2}\right)\left(\frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{2}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) m_B \end{aligned}}$$

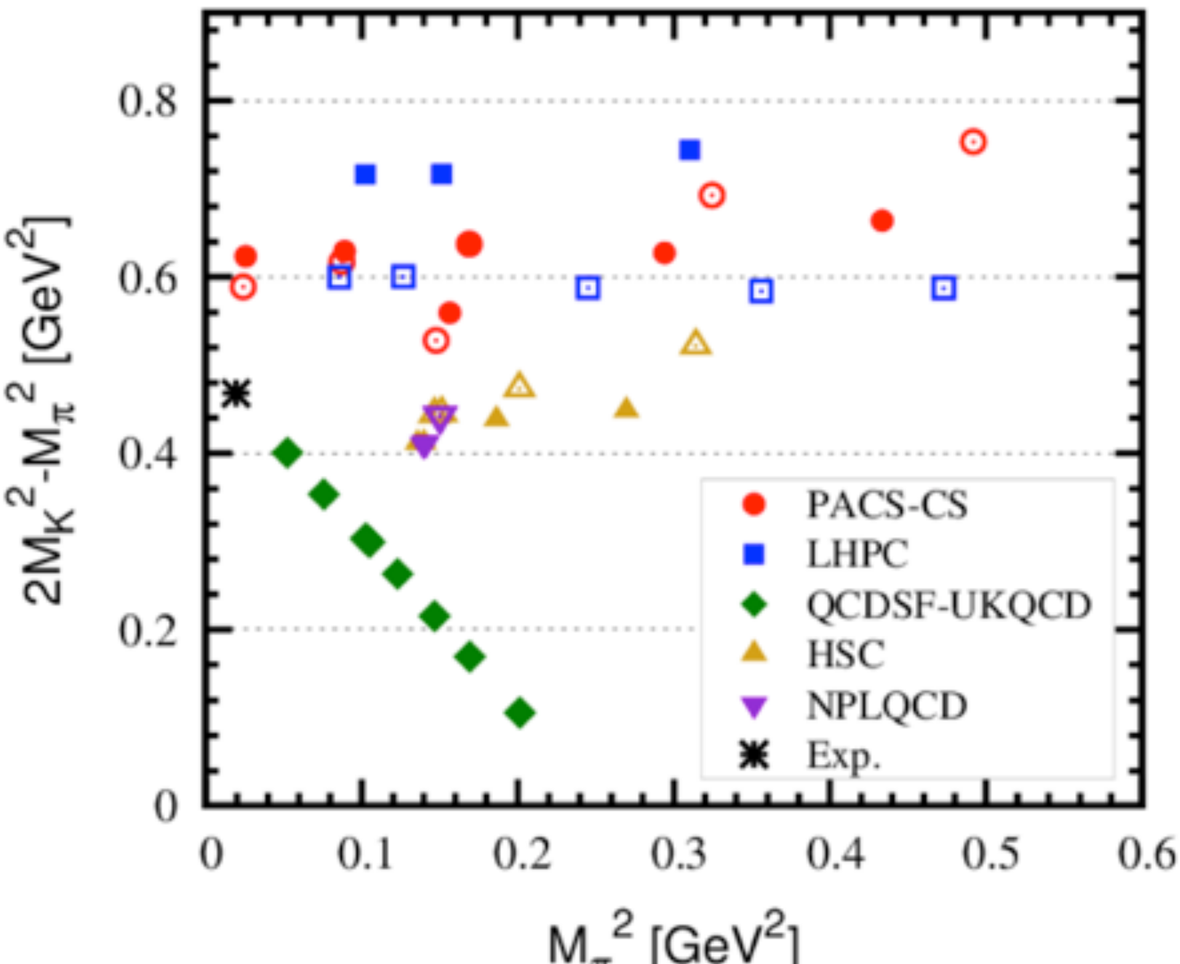
Pion mass vs. light quark mass



ETM collaboration, hep-lat/0701012

$$m_\pi^2 \propto m_q$$

Scale-setting effects on the octet baryon masses



- **Full symbols:**
  scale dependent
- **Hollow symbols:**
  scale independent