Chiral study of resonance dynamics in the π, K decay constants

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"2015强子物理与核物理前沿研讨会"

2015年1月，北京怀柔

Outline

- Importance and Status of F_π and F_K
- Setup in resonance chiral effective theory
- Chiral extrapolation of lattice QCD simulation
- Summary

Importance of F_π and F_K

Definition in QCD: the coupling of the axial current to π or K

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^+(p) = ip_\mu\sqrt{2}F_\pi\,, \qquad \langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^+(p) = ip_\mu\sqrt{2}F_K$$

Determination in Exp and the extraction of V_{us}:

$\pi^+ \to \ell^+\nu_\ell$ and $K^+ \to \ell^+\nu_\ell$

$$\Gamma(\pi \to \ell\nu_\ell) \propto F_\pi^2\,|V_{ud}|^2\,, \qquad \Gamma(K \to \ell\nu_\ell) \propto F_K^2\,|V_{us}|^2$$

$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{F_K^2}{F_\pi^2}\,\frac{|V_{us}|^2}{|V_{ud}|^2}$$

Goldberger-Treiman relation:

$$F_\pi\, g_{\pi N} = m_N\, g_A$$

Related quantities in other phenomenological models: wave functions at zero point

Status of F_π and F_K

Recent lattice simulations ($N_f = 2 + 1$):

RBC/UKQCD **[Aoki,** *et al.*, **PRD'11][Arthur,** *et al.*, **PRD'13]**
HPQCD/UKQCD **[Follana,** *et al.*, **PRL'08]**
MILC **[Bazavov,** *et al.*, **PoS'10]**
BMW **[Durr,** *et al.*, **PRD'10]**

Large uncertainties occur in the chiral extrapolation.

Higher order calculation in Chiral Perturbation Theory (χPT):

Full $O(p^6)$: **[Bijnens and Jemos, NPB'12]**
Partial $O(p^6)$: **[Ecker,** *et al.*, **EPJC'14]**

Suffering from the huge number of free couplings.

Our proposal (also bridging lattice data and chiral extrapolation):

[Guo, Sanz-Cillero, PRD'14]

Including the heavier d.o.f from QCD, i.e. resonances.

Setup in Resonance Chiral Theory (RχT)

Important aspect in QCD: approximate χ-symmetry

$$\mathcal{L}_{QCD} = \sum_{f=u,d,s} (i\bar{q}_L^f \gamma^\mu D_\mu q_L^f + i\bar{q}_R^f \gamma^\mu D_\mu q_R^f + \bar{q}_L^f m_q q_R^f + \bar{q}_R^f m_q q_L^f) + \cdots$$

Strong interaction in low energy QCD is determined by:
$SU_L(3) \otimes SU_R(3) \to SU_V(3)$: pseudo-Goldstone π, K, η_8

SU(3) χPT: [Weinberg,PhyA'79] [Gasser and Leutwyler, NPB'84'85]

$U_A(1) \equiv U_{L-R}$: violated at the quantum level, $\Rightarrow$ massive η_1.

$N_C \to \infty$: $M^2_{\eta_1} \sim \mathcal{O}(1/N_C)$, $\therefore$ η_1 becomes Goldstone.
[Witten, NPB'79]

$U(3)$ χPT: π, K, η and η'

Extension of χPT with meson resonances

Resonance Chiral Theory (RχT):

- Dynamic states in RχT: $\rho(770), a_1(1260), f_0, \pi', \cdots \quad + \pi, K, \eta$
- Respect the chiral symmetry in the very low energy.
- More QCD dynamics are being implemented: short-distance constraints, large N_C, etc.

Big challenge in RχT: struggling to find a power counting system

A roadmap to build RχT: CCWZ formalism

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

$$\text{Resonances}: \qquad R \overset{G}{\Longrightarrow} h\,R\,h^\dagger, \qquad h \in H$$

$$\nabla_\mu R \overset{G}{\Longrightarrow} h\,(\nabla_\mu R)\,h^\dagger, \qquad \nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R]$$

$$\Gamma_\mu = \frac{1}{2}\left\{u^+[\partial_\mu - i(v_\mu + a_\mu)]u + u[\partial_\mu - i(v_\mu - a_\mu)]u^+\right\}$$

$$\text{pNGB and external sources}: \qquad X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu},$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger, \qquad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u\,,$$

$$f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u\,, \qquad h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu\,,$$

$$X \overset{G}{\Longrightarrow} h\,X\,h^\dagger$$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Relevant Chiral Lagrangian

Minimal RχT Lagrangian

$$\begin{aligned}
\mathcal{L}_{\rm kin}^{S} &= \frac{1}{2}\langle \nabla^{\mu} S \nabla_{\mu} S - \overline{M}_S^2 S^2 \rangle \, . \\
\mathcal{L}_{\rm kin}^{V} &= -\frac{1}{2}\langle \nabla^{\lambda} V_{\lambda\mu} \nabla_{\nu} V^{\nu\mu} - \frac{1}{2}\overline{M}_V^2 V_{\mu\nu} V^{\mu\nu} \rangle \, , \\
\mathcal{L}_{2S} &= c_d \langle S u_{\mu} u^{\mu} \rangle + c_m \langle S \chi_+ \rangle \, , \\
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{i G_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^{\mu}, u^{\nu}] \rangle
\end{aligned}$$

Mass splitting operators

$$\mathcal{L}_{RR}^{\rm split} = e_m^S \langle S S \chi_+ \rangle - \frac{1}{2} e_m^V \langle V_{\mu\nu} V^{\mu\nu} \chi_+ \rangle$$

Pertinent local operators

$$\begin{aligned}
\mathcal{L}_{\chi}^{\rm LO} &= \frac{\widetilde{F}^2}{4} \langle u_{\mu} u^{\mu} \rangle + \frac{\hat{F}^2}{4} \langle \chi_+ \rangle + \frac{F_0^2}{3} M_0^2 \ln^2 \det u \, , \\
\mathcal{L}_{\chi}^{\rm NLO} &= \widetilde{L}_4 \langle u_{\mu} u^{\mu} \rangle \langle \chi_+ \rangle + \widetilde{L}_5 \langle u_{\mu} u^{\mu} \chi_+ \rangle + \frac{\widetilde{L}_8}{2} \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle
\end{aligned}$$

Notice the difference between $L_i^{\chi\rm PT}$ and $\widetilde{L}_i$!

Scalar tadpole and field redefinition:

$$\mathcal{L}_{\rm kin}^{S} + \mathcal{L}_{S} = \frac{1}{2}\langle \nabla^{\mu} S \nabla_{\mu} S - M_S^2 S^2 \rangle + c_d \langle S u_{\mu} u^{\mu} \rangle + c_m \langle S \chi_+ \rangle$$

$$S \to \overline{S} + \frac{c_m}{M_S^2}\chi_+$$

This leads to

$$\begin{aligned} \mathcal{L}_{\rm kin}^{S} + \mathcal{L}_{S} &= \frac{1}{2}\langle \nabla^{\mu} \overline{S} \nabla_{\mu} \overline{S} - M_S^2 \overline{S}^2 \rangle + c_d \langle \overline{S} u_{\mu} u^{\mu} \rangle + \frac{c_m}{M_S^2}\langle \nabla_{\mu} \overline{S} \nabla^{\mu} \chi_+ \rangle \\ &+ \frac{c_d c_m}{M_S^2}\langle \chi_+ u_{\mu} u^{\mu} \rangle + \frac{c_m^2}{2M_S^2}\langle \chi_+ \chi_+ \rangle \\ &+ \frac{c_m^2}{2M_S^4}\langle \nabla_{\mu} \chi_+ \nabla^{\mu} \chi_+ \rangle \end{aligned}$$

[Guo and Sanz-Cillero, PRD'14]

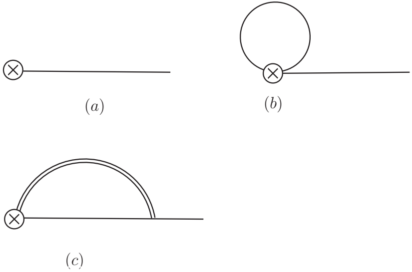


Figure: 1PI diagram for F_ϕ

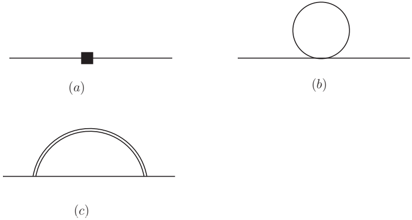


Figure: Self-energy for ϕ

Renormalization

$$F_\phi = F_0 \left(\frac{\widetilde{F}}{F_0} + \frac{4\widetilde{\widetilde{L}}_4\,(2m_K^2 + m_\pi^2)}{F_0^2} + \frac{4\widetilde{\widetilde{L}}_5\, m_\phi^2}{F_0^2} + \frac{F_{\phi,\,1\ell}^{\rm 1PI}}{F_0} + \frac{1}{2}\Sigma'_{\phi,\,1\ell} \right)$$

Define:

$$C_\chi = C_\chi^r(\mu) + \delta C_\chi(\mu)\,, \qquad C_\chi = \widetilde{F},\ \widetilde{\widetilde{L}}_4,\ \widetilde{\widetilde{L}}_5$$

$$\frac{dC_\chi^r(\mu)}{d\ln\mu^2} = -\frac{\Gamma^{C_\chi}}{32\pi^2}$$

We have

$$\Gamma^{\widetilde{\widetilde{L}}_4} = \frac{1}{8}\left[1 + \frac{4c_d c_m}{F_0^2} + \frac{2c_d^2}{F_0^2}(1 - 4e_m^S) - \frac{3G_V^2}{F_0^2}(1 - 4e_m^V)\right],$$

$$\Gamma^{\widetilde{\widetilde{L}}_5} = \frac{3}{8}\left[1 - \frac{4c_d c_m}{F_0^2} + \frac{2c_d^2}{F_0^2}(1 - 4e_m^S) - \frac{3G_V^2}{F_0^2}(1 - 4e_m^V)\right],$$

$$\frac{\delta\widetilde{F}}{F_0} = -\frac{1}{16\pi^2}\left(\frac{3c_d^2 M_S^2}{F_0^4} + \frac{2c_d^2 M_0^2}{3F_0^4} - \frac{9G_V^2 M_V^2}{2F_0^4}\right)\frac{1}{\hat{\epsilon}}, \quad (\overline{MS} - 1)\,.$$

Power breaking terms: to match with χPT

$$
\begin{aligned}
1-\frac{\widetilde{F}^r(\mu)}{F_0} &= \frac{1}{16\pi^2}\left[\frac{c_d^2 M_S^2}{F_0^4}\left(\frac{7}{6}-\frac{7}{3}\ln\frac{M_S^2}{\mu^2}\right)+\frac{c_d^2}{3F_0^4}\frac{M_S^4-M_0^4+2M_0^4\ln\frac{M_0^2}{\mu^2}-2M_S^4\ln\frac{M_S^2}{\mu^2}}{M_S^2-M_0^2}\right.\\
&\left.\quad+\frac{G_V^2 M_V^2}{F_0^4}\left(\frac{3}{4}+\frac{9}{2}\ln\frac{M_V^2}{\mu^2}\right)\right],\\
L_4^{\chi\mathrm{PT,r}}(\mu) &= \widetilde{\widetilde{L}}{}_4^r(\mu)+\frac{1}{16\pi^2 F_0^2}\left\{\frac{c_d^2}{144(M_S^2-M_0^2)^2}\left[(M_0^2-9M_S^2)(M_0^2-M_S^2)+8M_0^4\ln\frac{M_0^2}{\mu^2}\right.\right.\\
&\left.\quad-2(13M_0^4-18M_0^2M_S^2+9M_S^4)\ln\frac{M_S^2}{\mu^2}\right]\\
&\quad+\frac{1}{8}c_d c_m-\frac{1}{4}c_d c_m\ln\frac{M_S^2}{\mu^2}+\frac{1}{32}G_V^2+\frac{3}{16}G_V^2\ln\frac{M_V^2}{\mu^2}\\
&\left.\quad+c_d^2 e_m^S\left(\frac{1}{4}+\frac{1}{2}\ln\frac{M_S^2}{\mu^2}\right)-G_V^2 e_m^V\left(\frac{7}{8}+\frac{3}{4}\ln\frac{M_V^2}{\mu^2}\right)\right\},\\
L_5^{\chi\mathrm{PT,r}}(\mu) &= \widetilde{\widetilde{L}}{}_5^r(\mu)+\frac{1}{16\pi^2 F_0^2}\left\{\frac{c_d^2}{48(M_0^2-M_S^2)}\left[9(M_0^2-M_S^2)-16M_0^2\ln\frac{M_0^2}{\mu^2}\right.\right.\\
&\left.\quad-2(M_0^2-9M_S^2)\ln\frac{M_S^2}{\mu^2}\right]+\frac{c_d^2 e_m^S}{12(M_S^2-M_0^2)^2}\left[(M_0^2-9M_S^2)(M_0^2-M_S^2)+\cdots\right.
\end{aligned}
$$

Our theoretical predictions for F_ϕ will depend only on

$$\begin{array}{ll} \text{Tree - level contributions :} & \widetilde{F}^r \{\to F_0\}, \\ & \widetilde{\widetilde{L}}_4^r \{\to L_4^{\chi PT,r}\}, \\ & \widetilde{\widetilde{L}}_5^r \{\to L_5^{\chi PT,r}\}, \\ \text{One - loop contributions :} & e_m^V\,, \quad e_m^S\,, \quad c_m\,, \\ & c_d\,, \quad G_V\,, \\ & M_V\,, \quad M_S\,, \quad M_0\,. \end{array}$$

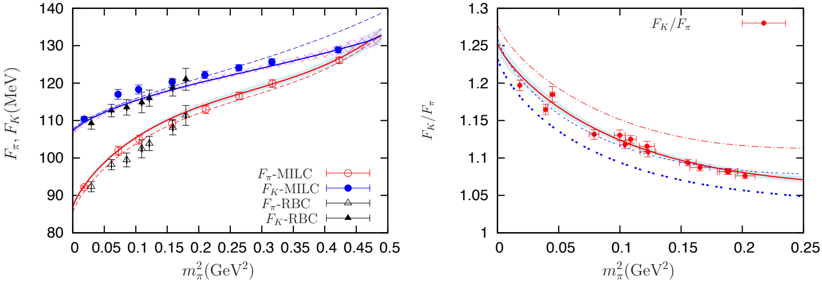
$c_d = F_0^2/4c_m$ **[Jamin,Oller,Pich, NPB'01]**

$G_V = F_0^2/4c_m$ **[Guo, Sanz-Cillero, Zheng, JHEP'07]**

M_V , M_S **[Guo, Sanz-Cillero, PRD'09]**

M_0 **[Guo, Oller, PRD'11, PLB'12, PRD'12]**

Chiral Extrapolation of the Lattice QCD simulations



$$F_0 = 80.0 \pm 1.0 \text{ MeV}\,, \qquad c_m = 54.5 \pm 3.3 \text{ MeV}\,,$$
$$L_4^{\chi PT} = (-0.11 \pm 0.06) \times 10^{-3}\,, \qquad L_5^{\chi PT} = (0.59 \pm 0.08) \times 10^{-3}\,,$$
$$e_m^V = -0.236 \pm 0.005\,, \qquad e_m^S = -0.204 \pm 0.024\,,$$

χ^2/d.o.f= 90.8/(52 − 6). **[Guo and Sanz-Cillero, PRD'14]**

High energy constraints implemented

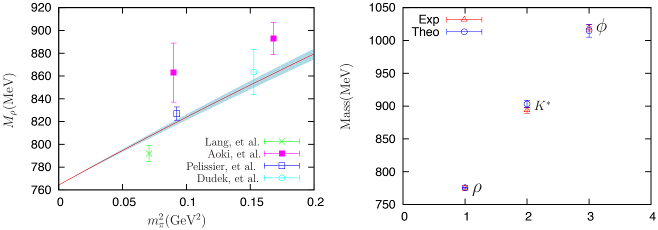


Figure: The $\rho(770)$ masses with varying m_π^2.

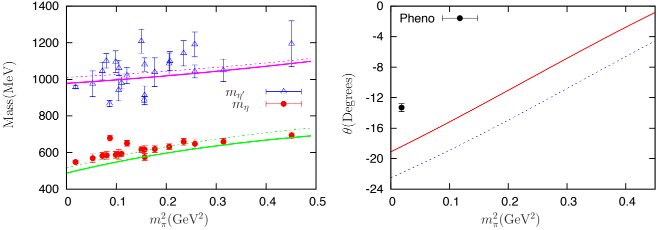


Figure: The m_η, $m_{\eta'}$ and the leading order mixing angle with varying m_π^2.

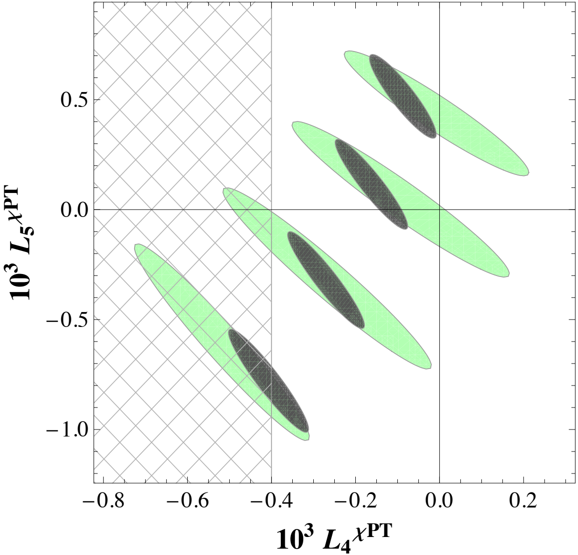


Figure: 68% CL regions for $L_4^{\chi PT}(\mu)$ and $L_5^{\chi PT}(\mu)$.

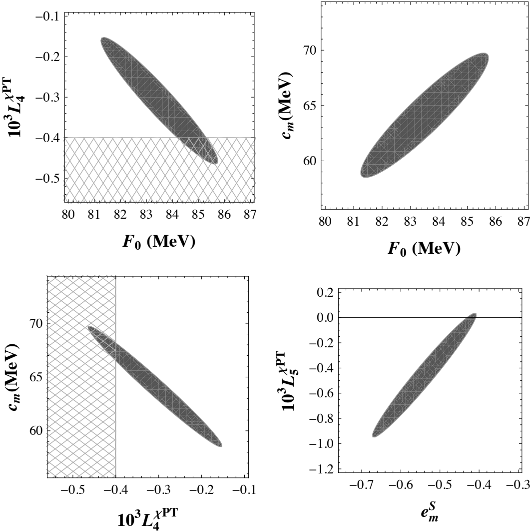


Figure: 68% CL regions for sets of two couplings.

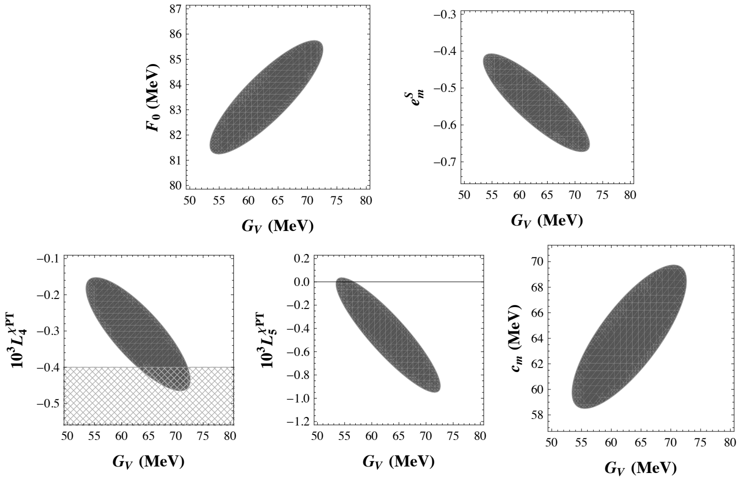


Figure: 68% CL regions for G_V ($\rho\pi\pi$ coupling strength) and other parameter.

Summary

(1) We provide an alternative and economical way to study F_π, F_K beyond $\mathcal{O}(p^4)$ χPT, rather than in the conventional $\mathcal{O}(p^6)$ χPT framework.

(2) This is a pioneer work to perform the one-loop calculation with resonances in the case of non-vanishing quark masses in RχT.

(3) Strong correlations are observed for the determination of χPT LECs and resonance couplings.

(4) Rather stable values for $F \simeq 80$ in the chiral limit are obtained for the 3 flavor case.