Institute of High Energy Physics
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Radio bursts from superconducting strings

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CYF, E. Sabancilar, T. Vachaspati, arXiv:1110.1631;
CYF, E. Sabancilar, D. Steer, T. Vachaspati, arXiv:1205.3170.

Motivation

- Cosmic strings are predicted by Grand Unified Theories and String Theories;
- Observation of cosmic strings can serve as a useful hint to understand fundamental theories of physics;
- Cosmic strings can be superconducting in a wide class of particle physics models and thus can produce electromagnetic effects;
- Radiation from superconducting strings can be source of astronomical observation.

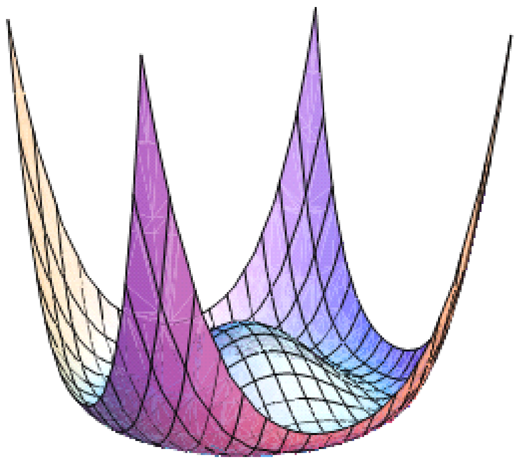
Outline

- Introduction
  - Abelian Higgs Model
  - Observable effects
- Theoretical description of superconducting strings
  - Setup
  - Electromagnetic bursts
- Event rate of Radio signals from superconducting strings
  - Theorist's variables
  - Observer's variables
- Numerical Results
- Summary

Abelian Higgs Model

Consider a Lagrangian:

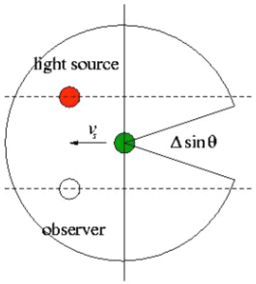
$$\mathcal{L} = D_\mu^* \phi^* D^\mu \phi - \frac{1}{4}\lambda(\phi^*\phi - \eta^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad <\phi> \sim \eta e^{i\theta}$$



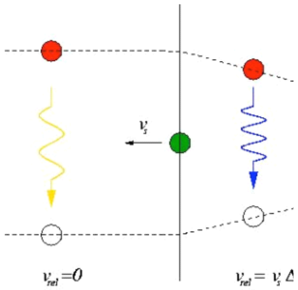
- The fundamental homotopy group of the vacuum manifold is nontrivial:
  $\pi_1(U(1)) = \mathbb{Z}$
- The model admits vortex solution
- Cosmic strings can form via Kibble mechanism

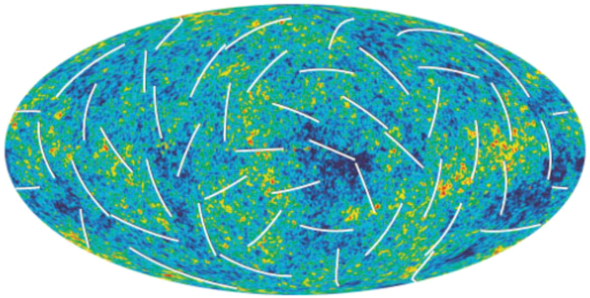
Observational Effects of Cosmic Strings

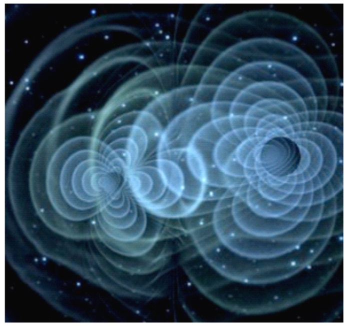
- Gravitational lensing: $G\mu \lesssim 10^{-7}$ (Christiansen et al. 2011)
- Scale invariant CMB fluctuations: $G\mu \lesssim 1.5 \times 10^{-7}$(Dvorkin et al. 2011)
- Pulsar timing measurements: $G\mu \lesssim 4 \times 10^{-9}$ (van Haasteren et al. 2011)



(a)


(b)





- Keiser-Stebbins effect might be seen by Planck (Keiser & Stebbins 1984)
- B-mode might be detectable by Planck (Pogosian & Wyman 2008)
- Gravitational Radiations (Vachaspati & Vilenkin 1985)
- Cosmic Rays:

  Neutrinos (MacGibbon et al. 1990)

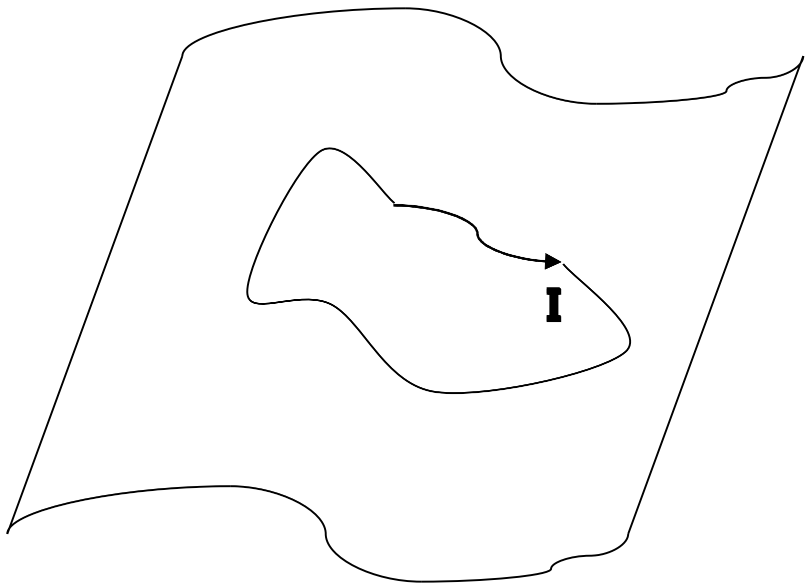
  Positrons (Brandenberger et al. 2009)
- 21 cm signatures: (Brandenberger et al. 2010)

Superconducting Strings

The effective action of a superconducting string

$$S = \int d\sigma d\tau \sqrt{-\gamma} \left\{ -\mu + \frac{1}{2}\gamma^{ab}\phi_a\phi_b - A_\mu X^\mu_{,a} J^a \right\} - \frac{1}{16\pi} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

Witten 1985
Vilenkin & Shellard 1994



String tension: μ
Metric of world-sheet: γ^{ab}
World-sheet field: ϕ
World-sheet current: J^a
Electromagnetic field: A_μ

One particular solution: $\phi' = 0 \,, \quad \dot{\phi} = \frac{\mathcal{I}}{e}$ $\rightarrow$ superconductivity!

Equations of motion

Varying the action w.r.t field variables:

$$\begin{aligned}\gamma^{ab}\partial_a\partial_b\phi &= -\frac{e}{2}\epsilon^{ab}F_{\mu\nu}X^{\mu}_{,a}X^{\nu}_{,b}\\ \mu\gamma^{ab}\partial_a\partial_b X^{\mu} &= -F^{\mu}_{\sigma}X^{\sigma}_{,a}J^{a} - (\Theta^{ab}X^{\mu}_{,a}), b\\ \partial_\sigma\partial^\sigma A^{\mu} &= 4\pi J^{\mu}\end{aligned}$$

where $\Theta_{ab} = \phi_{,a}\phi_{,b} - \frac{1}{2}\gamma_{ab}\phi_{,c}\phi^{,c}$ is the world-sheet stress energy tensor.

A superconducting string carries a current $\mathcal{I}$, its current density which is given by

$$J^{\mu}(t,\vec{x}) = \mathcal{I}\int d\sigma\ X^{\mu}_{,\sigma}\ \delta^{(3)}(\vec{x}-\vec{X}(t,\sigma))$$

For one Fourier mode of the current density, there is

$$J^{\mu}_{\omega}(\vec{k}) = \frac{2\mathcal{I}}{L}(I^{\mu}_{+}I^{0}_{-} + I^{0}_{+}I^{\mu}_{-}) \qquad I^{\mu}_{\pm}(\vec{k}) = \int_0^L d\sigma_{\pm}e^{ik\cdot X_{\pm}/2}X'^{\mu}_{\pm}$$

where $X^{\mu}_{\pm}$ are left-hand and right-hand string functions, respectively.

Saddle Point Analysis

A cosmic string allows saddle points or discontinuities existing in the derivative of X^{μ} along the spatial coordinate of world-sheet.

Let $\vec{k} = \omega \vec{n}$ where $\vec{n}$ is a unit vector. When there is a saddle point, it corresponds to $\vec{n} = \pm \vec{X}'_{\pm}$

Then expanding about this point yields

$$I_{\pm}^{\mu\,(\mathrm{saddle})} \approx \frac{L}{(\omega L)^{1/3}} \tilde{a}\, X'^{\mu}_{\pm} + i \frac{L^2}{(\omega L)^{2/3}} \tilde{b}\, X''^{\mu}_{\pm} + \ldots$$

with $\tilde{a} \simeq 1$ and $\tilde{b} \simeq 0.4$.

Note, the above imaginary component leads to the integrals to die off exponentially outside an angle

$$\theta_{\omega} \simeq (\omega L)^{-1/3}$$

Thus we obtain a beam-shape burst of radiation with a duration $\delta t_{\omega} \simeq \frac{L^{2/3}}{\omega^{1/3}}$.

EM bursts from a superconducting string

•When both I_+ and I_- have a saddle point, then this corresponds to a **cusp** since in this case the saddle point is null-like $|\dot{\vec{X}}| = 1$.

Using a little bit basic knowledge of electrodynamics, one can obtain the power emitted in photons per unit frequency, per unit solid angle from a cusp

$$\frac{\mathrm{d}^2 P_\gamma}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{\omega^2}{2\pi} \frac{L}{4\pi} |J^\mu_\omega|^2 \approx \mathcal{I}^2 L.$$

•Assuming a saddle point in one of the integrals and a discontinuity in the other, this case corresponds to a **kink**. Correspondingly,given the kink sharpness □□□, the spectrum of radiation is given by

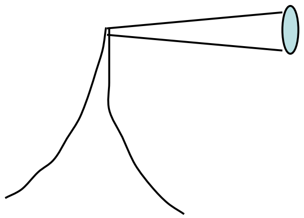
$$\frac{\mathrm{d}^2 P^{\mathrm{k}}_\gamma}{\mathrm{d}\omega \mathrm{d}\Omega} \approx \frac{\mathcal{I}^2 L \psi_+}{(\omega L)^{2/3}}$$

•Finally, a discontinuity in both two integrals corresponds to a **kink-kink collision**, of which the radiation spectrum is

$$\frac{\mathrm{d}^2 P^{\mathrm{kk}}_\gamma}{\mathrm{d}\omega \mathrm{d}\Omega} \approx \frac{\mathcal{I}^2 L \psi_+ \psi_-}{(\omega L)^2}$$

Features of EM bursts from different sources

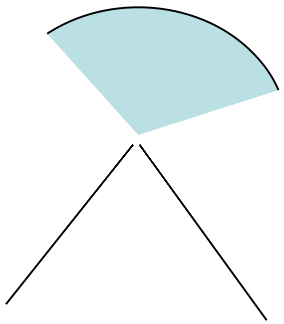
- **Cusps:**



Radiation from a cusp is emitted in a narrow solid angle

$$\Omega^c \simeq \theta_\omega^2 \simeq (\omega L)^{-\frac{2}{3}}$$

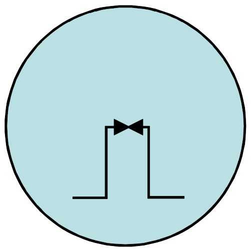
- **Kinks:**



Radiation from a kink is emitted in a "fan-shape" solid angle

$$\Omega^k \simeq 2\pi\theta_\omega \simeq 2\pi(\omega L)^{-\frac{1}{3}}$$

- **Kink-kink collisions:**



Radiation from a cusp is emitted in all directions.

Event rate of radio bursts from superconducting strings

•We have already analyzed radio bursts from a single string. What about radio bursts from a network of strings?

•The network of cosmic strings in our universe scales with the horizon.

•The distribution of string loops takes the form of

$$\mathrm{d}n(L,t) \simeq \frac{C_L}{t^2(L+\Gamma G\mu t)^2}\mathrm{d}L, \quad C_L \equiv 1+\sqrt{\frac{t_{\mathrm{eq}}}{L+\Gamma G\mu t}}.$$

Rocha 2007
Polchinski & Rocha 2007

•We shall study the radio transient events in the matter-dominated era, thus the redshift scales as

$$1+z=\left(\frac{t_0}{t}\right)^{2/3}$$

Event rate of radio bursts from superconducting strings

•Combining the above factors and assuming a loop containing N kinks in average, we can write down the event rate of radio bursts emitted in a spatial volume

$$d\dot{\mathcal{N}}(L,z) \simeq N^p \frac{(\theta_{\nu_o})^{\tilde{m}}}{L(1+z)} \, dn(L,z) \, dV(z)$$

where

$$\begin{aligned} p &= 0\,, \quad \tilde{m} = 2 \quad && \text{for cusp} \\ p &= 1\,, \quad \tilde{m} = 1 \quad && \text{for kink} \\ p &= 2\,, \quad \tilde{m} = 0 \quad && \text{for kink kink.} \end{aligned}$$

•Therefore, the event rate of radio bursts is determined by two variables:

loop length L and redshift z .

•How can we relate this event rate to radio observations?

Observable variables

For observers, the relevant quantities are not loop length and redshift, but the observed energy flux S and the burst duration Δ .

- The observed energy flux depends on the power spectrum of radiation as was discussed before. Explicitly,

$$S \approx \frac{L^2\mathcal{I}^2}{r(z)^2\Delta}\psi^p(\nu_o L(1+z))^{-q}$$

$$\begin{array}{lll} p=0\,, & q=0 & \text{for cusp} \\ p=1\,, & q=2/3 & \text{for kink} \\ p=2\,, & q=2 & \text{for kink kink} \end{array}$$

- Regarding the burst duration,

$$\Delta^2_{\rm radio}(z) \;=\; \Delta t_s^2(z)+\nu_o^{-2}$$

  - Intrinsic beam duration: $\frac{1}{\nu_o}$
  - time delay due to scattering with the turbulent intergalactic medium

$$\Delta t_s(z) \simeq \delta t_1 \left(\frac{1+z}{1+z_1}\right)^{1-\beta}\left(\frac{\nu_o}{\nu_1}\right)^{-\beta}$$

Lee & Jokipii 1976
Kulkarni, et al. 2007

Event rate in observable variables

Now we are able to translate the variables from **(L, z) to (S, △)** through a Jacobian transformation.

- Event rate for Kinks and Cusps

$$d\dot{\mathcal{N}}(S,\Delta) \simeq \tilde{A}\frac{N^p}{S}[L(x,S)]^m f_m(x,S)\mathrm{d}S\mathrm{d}\Delta,$$

where

$$\tilde{A} = \frac{A t_0 \nu_o^m}{(2-q)(\beta-1)},$$

and

$$f_m(x,S) = \frac{x}{x^2-1}C_L(z) \times \frac{(1+z)^{m+1/2}[\sqrt{1+z}-1]^2}{[(1+z)^{3/2}L(x,S)+\Gamma G\mu t_0]^2},$$

- Kink-kink collisions

$$d\dot{\mathcal{N}}(S) \simeq \frac{A N^2 t_{\rm eq}^{1/2} S_0}{(\Gamma G\mu)^{5/2} t_0^{3/2}(\beta-1)} \frac{x}{(x^2-1)|\mathrm{d}P/\mathrm{d}x|} \times (1+z)^{5/4}[\sqrt{1+z}-1]^2\frac{\mathrm{d}S}{S^2},$$

$$1+z = d\,(x^2-1)^{-1/(2\beta-2)}$$

where $d = 82\,\nu_1/\nu_0$ and

$$x \equiv \Delta\,\nu_o.$$

Numerical Computation

The event rate of radio burst from superconducting strings involves the following five parameters:

$$(\mathcal{I}, G\mu, \nu_o, \Delta, S)$$

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String parameters:
namely,
$\mathcal{I} = 10^5\ \mathrm{GeV}\ , \quad G\mu = 10^{-10}$

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String parameters:
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$\mathcal{I} = 10^5\ \mathrm{GeV}\ ,\quad G\mu = 10^{-10}$

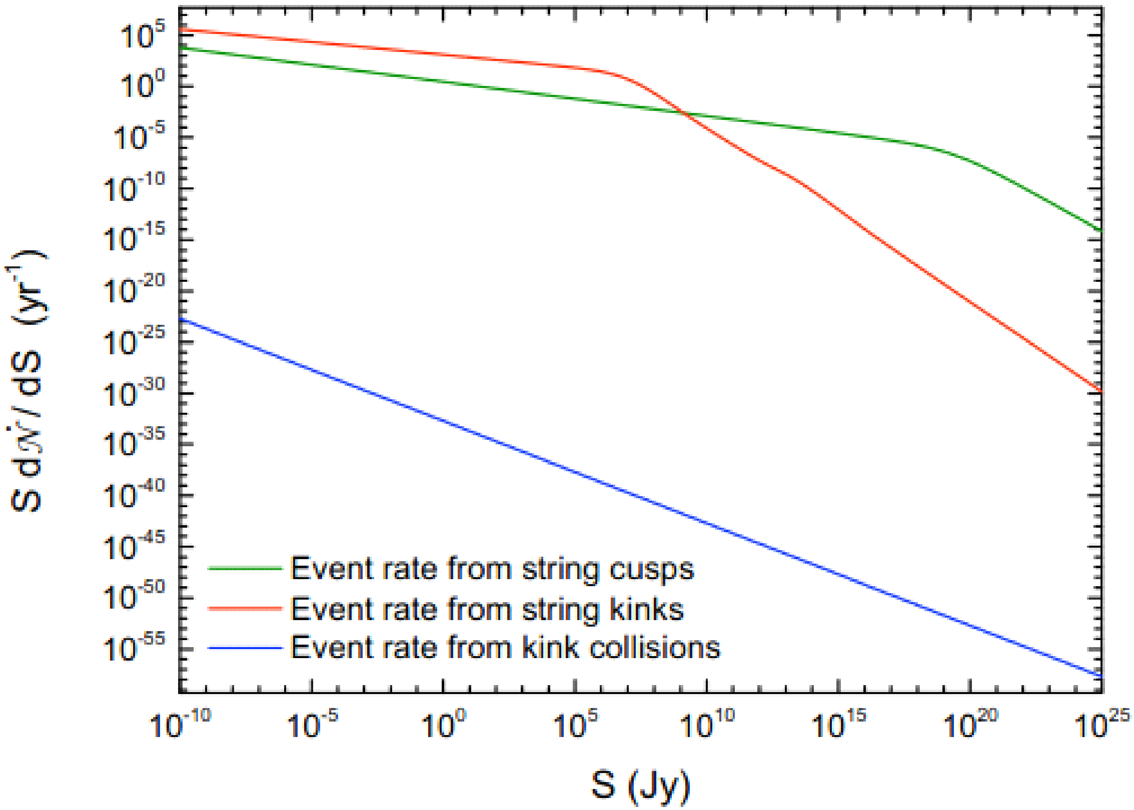
Experimental parameters:
namely **Parkes** survey can probe

$$\nu_o \in (1.230, 1.518)\ \mathrm{GHz}\ ,$$
$$\Delta \in (10^{-3}, 1)\ \mathrm{s}\ ,$$
$$S \in (10^{-5}, 10^{5})\ \mathrm{Jy}\ ,$$

Lorimer, et al. 2007

Numerical Computation

With the above parameters, we consider a fixed observed frequency $\nu_o = 1.23\text{GHz}$, and then numerically integrate out the observed duration Δ.



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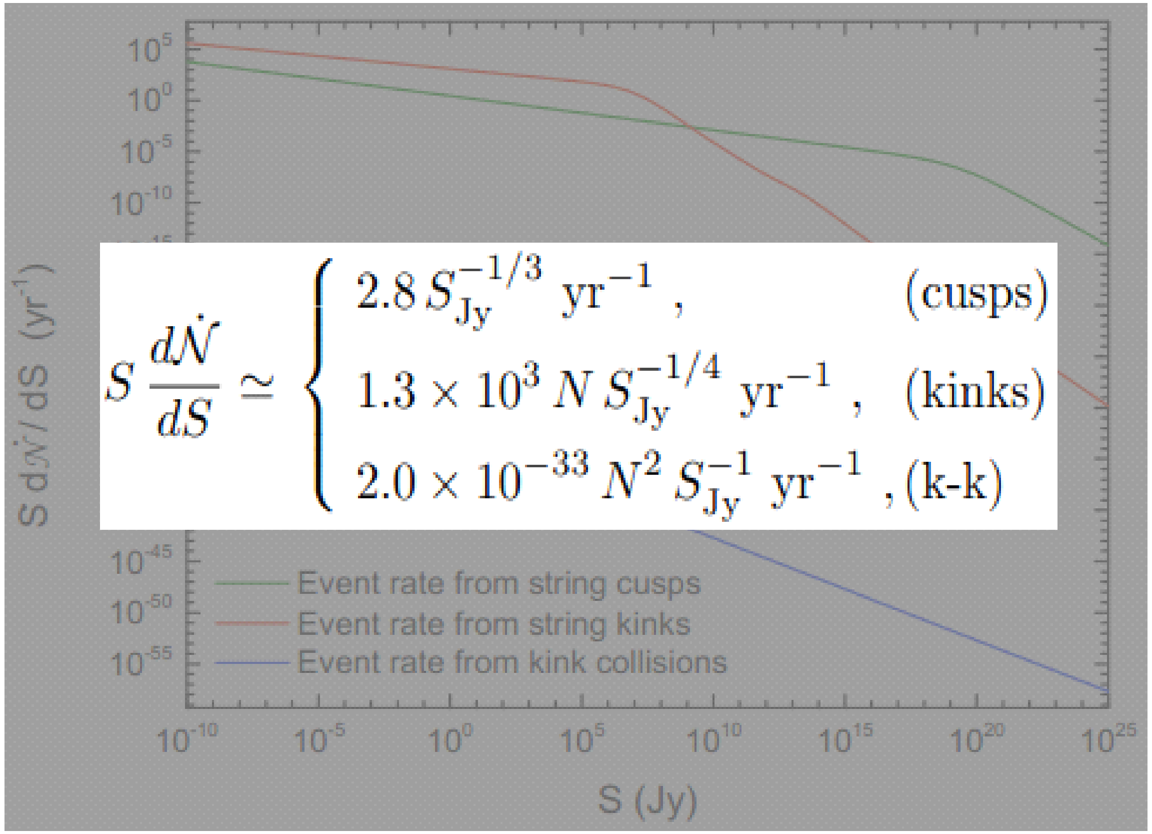
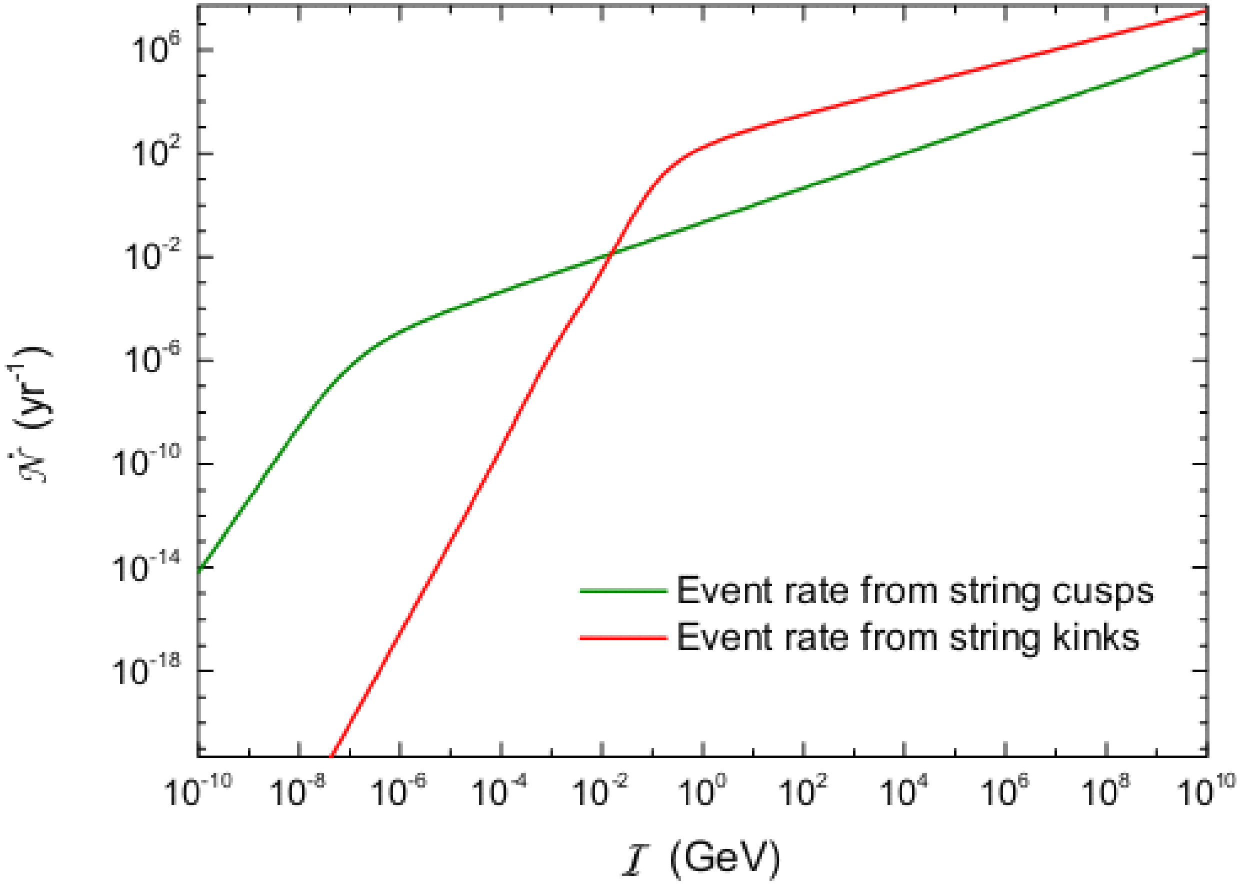


$$S\frac{d\dot{N}}{dS} \simeq \begin{cases} 2.8\, S_{\rm Jy}^{-1/3}\ {\rm yr}^{-1}\ , & {\rm (cusps)} \\ 1.3\times 10^{3}\, N\, S_{\rm Jy}^{-1/4}\ {\rm yr}^{-1}\ , & {\rm (kinks)} \\ 2.0\times 10^{-33}\, N^{2}\, S_{\rm Jy}^{-1}\ {\rm yr}^{-1}\ , & {\rm (k\text{-}k)} \end{cases}$$

Numerical Computation

Event rate of radio bursts from cusps and kinks on superconducting string loops at fixed observed frequency, $\nu_o = 1.23\text{GHz}$, as functions of the current.



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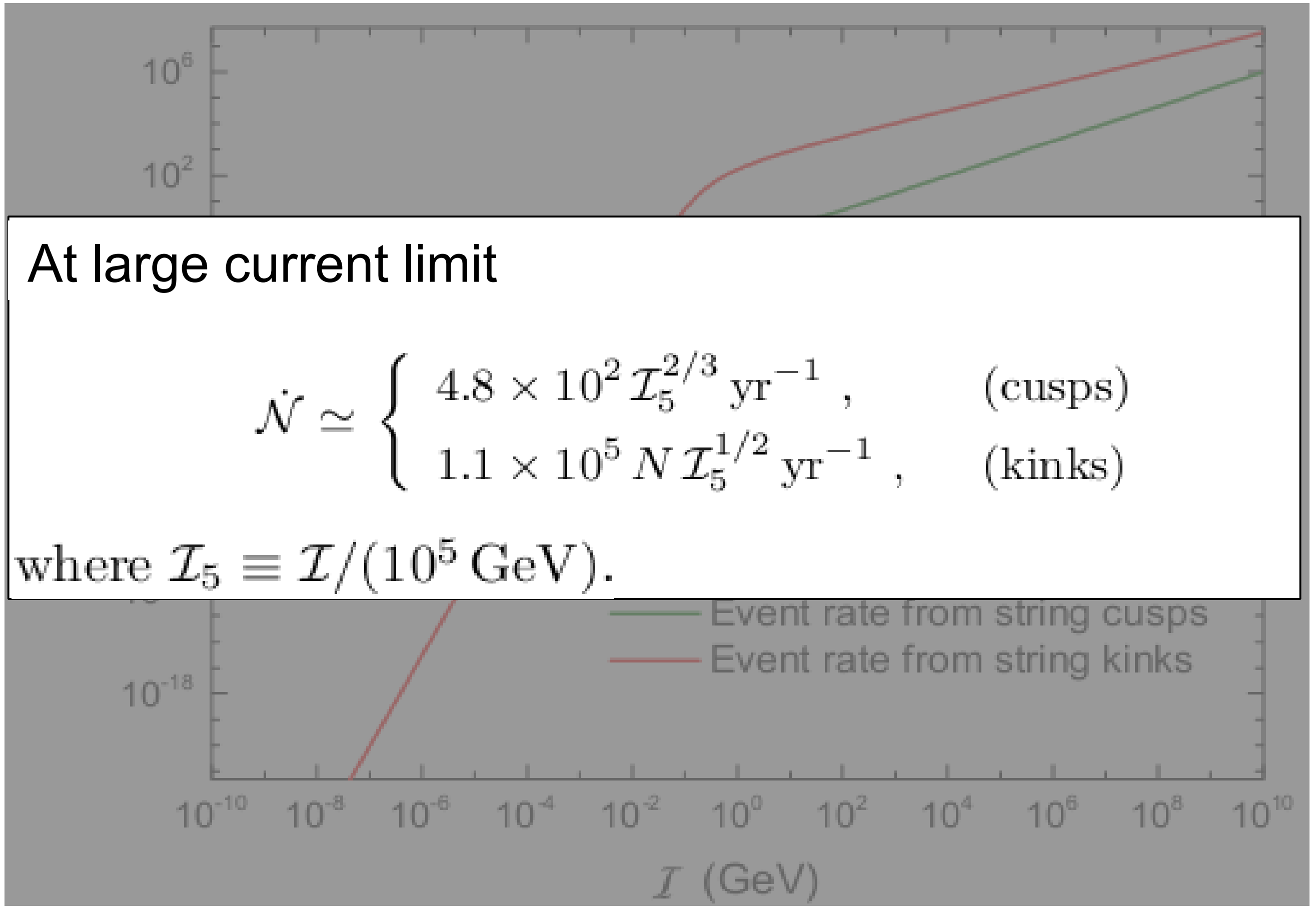


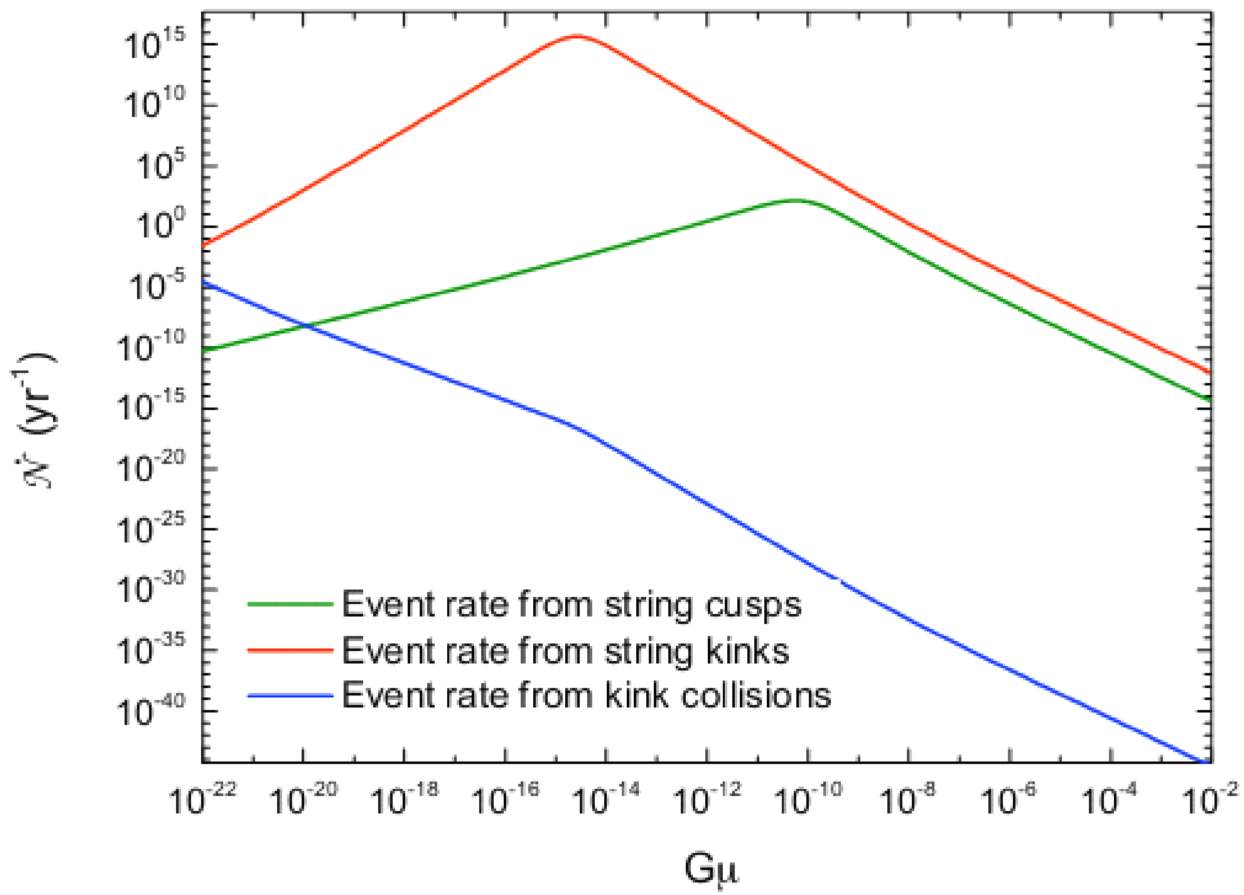
At large current limit

$$\dot{\mathcal{N}} \simeq \begin{cases} 4.8 \times 10^2\, \mathcal{I}_5^{2/3}\, \text{yr}^{-1}\,, & \text{(cusps)} \\ 1.1 \times 10^5\, N\, \mathcal{I}_5^{1/2}\, \text{yr}^{-1}\,, & \text{(kinks)} \end{cases}$$

where $\mathcal{I}_5 \equiv \mathcal{I}/(10^5\,\text{GeV})$.

Numerical Computation

Event rate of radio bursts from superconducting string loops with fixed observed frequency, $\nu_o = 1.23\text{GHz}$, as functions of $G\mu$.



Summary

- Superconducting strings can emit radio bursts through cusps, kinks and kink-kink collisions;
- The energy of radiation emitted from a superconducting string is mainly caused by cusps;
- The radio signals from cusps, kinks, and kink-kink collisions are of different shapes, among which the solid angle of radiation from cusps is smallest;
- In the observable parameter space, the main contribution of event rate of radio bursts is from string kinks.

Summary

- If we consider radio bursts emitted by kinks with observable frequency *1.23 GHz* and flux greater than *300 mJy*, the event rate is about *0.75 per hour*, which is a factor of 30 larger than the upper bound given by the Parkes survey (0.025 per hour).

- This result implies that current radio burst experiments might already rule out some parameter space of the string current $\mathcal{I}$ and the string tension $G\mu$.

Thank You !